Weakly ALC-Spaces

(WALC-Spaces)

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Abstract:

The aim of this paper is the study certain operation and properties of weakly $ALC-space$ $(WALC-space)$ as spaces in which every almost Lindelöf subset is closed, we continue the investigation of more relationships. .

KEYWORDS: $LC-space, ALC-space, WALC-space$,Lindelöf, almost Lindelöf, locally Lindelöf spaces, locally $LC-spaces$.

1. Introduction:

Atopological space whose Lindelof subsets are closed is called an $LC-space$ by Mukherji and Sarkar $[16]$ and by Gauld ,Mrsevic ,Reilly and Vamanamurthy $[9]$. $LC-spaces$ are also known as $L-$closed spaces($[10],[11],[13],[14]$).They generalize $KC-spaces$ (= compact subsets are closed) $[22]$ and Hausdorff $P-spaces$ ($=F_\sigma-$sets are closed) $[15]$.Every $LC-space$ is a $cid-space$ (=countable subsets are closed and discrete) $[7]$ and so T_1 and anticompact (=compact subsets are finite).Not that $cid-spaces$ have been called weak $LC-spaces$ by Mukherji and Sarkar $[16]$.

In 2002, Sarsak $[20]$introduced the notion of $ALC-spaces$ as spaces in which every subset of X which is almost Lindelöf in X is closed.

In 2008, Hdeib and Sarsak $[12]$introduced the notion of weakly $ALC-spaces$ as spaces in which every almost Lindelöf subset is closed. Weakly $ALC-spaces$ are placed between $ALC-spaces$ and $LC-spaces$. Several properties, mapping properties of such spaces are studied extensively, it is also shown that in a regular space X if every point has a weakly ALC neighborhood, then X is weakly ALC.

In this paper we give the basic definitions and known theorems about weakly $ALC-space$ $(WALC-space)$ and we continue the investigation of more relationships.

2. Almost Lindelöf Spaces

Definition2.1 [5]:

A topological space $(X,T)$is called Lindelöf if every open cover of X has a countable subcover .

Definition2.2: A topological space (X,T) is almost Lindelöf if for every open cover β of X there exists a countable subfamily $\psi\subset\beta$ such that $X=\bigcup_{V\in\psi}\overline{V}$.It follows immediately from the definition that every Lindelöf space is almost Lindelöf $[6]$, $[21]$. It was pointed out in$[20]$ that if A is an almost Lindelöf subspace of a space X , then A is almost Lindelöf in X but not conversely.

Definition2.3 [23]:

A subspace Y of a space is almost Lindelöf in X if for every open cover β of X there exists a countable subset ψ of β such that $Y=\bigcup_{V\in\psi}\overline{V}$.

From the above definitions it is clear that if Y is Lindelöf in X , then Y is almost Lindelöf in X .

Definition2.4 [5]:

A topological space $(X,T)$is 2^{nd} countable (C_{11})if and only if X has a countable basis.

Definition2.5 [5]: A topological space $(X,T)$is a completely regular space (sometimes $T_{3\frac{1}{2}}$) if given any $x \in X$ and closed subset F *of* X, $x \notin F$, there is a continuous function $f : X \to [0,1]$such that $f(x)=0$ *and* $f(y)=1$.for all $y \in F$.

A topological space $(X,T)$is a Tychonoff space if X is T_1 and completely regular.

Definition2.6 [5]:

A topological space (X,T) is locally compact if each point has a relatively compact neighborhood.

Theorem2.7 [17]: A regular almost Lindelöf space is Lindelöf.

Theorem2.8 [23]:

Let X be a regular space and Y a subspace of X. If Y is almost Lindelöf in X, then Y is Lindelöf in X.

Theorem 2.9[23]:

A continuous image of an almost Lindelöf space is almost Lindelöf.

Theorem2.10 [17]:

If X is almost Lindelöf, then any clopen subset of X is almost Lindelöf.

Theorem2.11 [12]:

(i) The countable union of subspaces of a space X each of which almost Lindelöf in X is almost Lindelöf in X.

(ii) If $f : X \to Y$ is a continuous function and A is almost Lindelöf in X, then $f(A)$ is almost Lindelöf in Y.

Corollary2.12 [12]:

(i) The countable union of subspaces of a space X each of which almost Lindelöf is almost Lindelöf.

(ii) If $f : X \to Y$ is a continuous function and A is an almost Lindelöf subset of X ,then $f(A)$ is almost Lindelöf subset of Y.

Theorem2.13 [20]:

A regular closed subset of an almost Lindelöf space X is almost Lindelöf.

Corollary2.14: Every regular almost Lindelöf space is Normal.

Corollary2.15:

For a Hausdorff locally compact space X the following are equivalent:

(a) X is an almost Lindelöf space.

(b) X is a Lindelöf space.

3. Weakly ALC-Spaces (WALC-Spaces)

Definition3.1[20]:

A topological space(X,T) is called $ALC-space$ if every subset of X which is almost Lindelöf in X is closed.

Definition3.2[12]: A topological space (X,T) is called weakly $ALC-space$ $(WALC-space)$ if every almost Lindelöf subset of X is closed .

Clearly, every $ALC-space$ is an $WALC-space$ and every $WALC-space$ is an $LC-space$.

Remark3.3:

It is clear that every $LC-space$ is $cid-space$.

Definition3.4[8]: A topological space (X,T) is called a Locally $LC-space$ if each point of X has a neighborhood which is an $LC-subspace.$

Clearly every $LC-space$ is locally $LC-space$ .In general the converse needs not be true$[4]$, however every regular locally $LC-space$ is $LC-space$..

Definition3.5 [4]: A topological space(X,T) is a $R_1-space$ if x and y have disjoint neighborhoods whenever $cl\{x\}\neq cl\{y\}$ .Clearly a space is Hausdorff if and only if its T_1 and R_1 .

Definition3.6 [11]:

A topological space$(X,T)$is called hereditarily Lindelöf if every subspace of X is Lindelöf.

Definition 3.7[1]:

A topological space (X,T) is called a $Q-set$ space if each subset of X is an $F_\sigma-closed$ sets.

Definition3.8 [1]: A topological space (X,T) is said to be anti – Lindelöf if each Lindelöf subset of X is countable.

Definition3.9[2]: A topological space (X,T) is called locally Lindelöf (resp. weakly locally Lindelöf) if each point of X has a closed Lindelöf (resp. Lindelöf) neighborhood. It follows immediately from the definition that every locally Lindelöf space is a weakly locally Lindelöf.

Note that a weakly locally Lindelöf space need not be a locally Lindelöf space.

Definition3.10 [2] :

A topological space $(X,T)$is called an $L_3-space$ if every Lindelöf subset L is an $F_\sigma-closed$.

Definition3.11 [8]: A topological space (X,T) is called an $LC-space$ if each point of X has a closed neighborhood that is an $LC-subspace.$

Theorem 3.12[2]:

If (X,T) is an $LC-space$, then (X,T) is a $L_3-space$.

Theorem3.13 [2]:

Every hereditarily Lindelöf $L_3-space$ is a $Q-set$ space.

Corollary3.14: Every hereditarily Lindelöf $LC-space$ is a $Q-set$ space.

Proof. Let X be $LC-space$,then X is an $L_3-space$ by Theorem3.12, since X is a hereditarily Lindelöf ,then X is a $Q-set$ space by Theorem3.13.

Theorem3.15[18]: Every locally compact $KC-space$ X is a Hausdorff.

Corollary3.16: Every locally compact $LC-space$ is a Hausdorff.

Theorem3.17[5]:

(i) Every locally compact Hausdorff space is T_3 .

(ii) Every locally compact Hausdorff space is a Tychonoff.

Theorem3.18:

For anti – Lindelöf space X the following are equivalent:

(a) X is an $LC-space$.

(b) X is a $cid-space$.

Proof. (a) $\Longrightarrow$(b): This is obvious by Remark 3.3.

(b) $\Longrightarrow$(a): Let L be a Lindelöf subset of X ,then L is countable (since X is anti – Lindelöf), so L is a closed set (since X is $cid-space$),

hence X is an $LC-space$.

Corollary3.19:

Every $R_1 LC-space$ is a Hausdorff.

Theorem3.20 [3]: Every locally $LC-space$ is T_1 .

Theorem3.21[19]:

For a locally compact $R_1-space$ X the following are equivalent:

(a) X is an $LC-space$.

(b) X is a locally $LC-space$.

Theorem3.22:

If (X,T) a regular space has an open cover by locally $LC-subspaces$, then X is an $LC-space$.

Proof. Let $X=\bigcup_{i\in I} G_i$ be an open cover of X where each G_i is alocally $LC-space$,and let $x\in X$ .Choose $j\in I$ such that $x\in G_j$ .If U_J is an open and closed neighborhood(since X is a regular) of x in G_j such that U_J is an $LC-space$ of G_j ,then U_J is also open and closed in (X,T). By Definition3.11, (X,T) is an $LC-space$.

Theorem3.23:

If (X,T) a regular space has an open cover by $LC-subspaces$, then X is an $LC-space$.

Proof. Let $X=\bigcup_{i\in I}G_i$ be an open cover of X where each G_i is $LC-space$,and let $x\in X$ .Choose $j\in I$ such that $x\in G_j$ .If U_J is a closed neighborhood(since X is a regular)of x in G_j such that U_J is an $LC-space$ of G_j ,then U_J is also closed in (X,T). By Definition3.11, (X,T) is an $LC-space$.

Theorem3.24[12]:

For a hereditarily almost Lindelöf space X , the following are equivalent:

(a) X is an $ALC-space$.

(b) X is an $WALC-space$.

(c) X is a countable discrete space.

Corollary 3.25[12]:

For a hereditarily Lindelöf space X , the following are equivalent:

(a) X is an $ALC-space$.

(b) X is an $WALC-space$.

(c) X is an $LC-space$.

(d) X is a countable discrete space.

Corollary3.26:

For a 2^{nd} countable (C_{11})space X the following are equivalent:

(a) X is an $ALC-space$.

(b) X is an $WALC-space$.

(c) X is an $LC-space$.

(d) X is a countable discrete space.

Proof. This is obvious by Corollary 3.25.

Corollary3.27: For a countable space X the following are equivalent:

(a) X is an $ALC-space$.

(b) X is an $WALC-space$.

(c) X is an $LC-space$.

(d) X is a countable discrete space.

Proof. This is obvious by Corollary 3.25.

Theorem3.28: For a regular space X the following are equivalent:

(a) X is an $LC-space$.

(b) X is an $WALC-space$.

Proof. (a) $\Longrightarrow$(b): If L is an almost Lindelöf subset in X ,which is a regular space , then L is a

Lindelöf subset in X by Theorem2.8, but X is an $LC-space$, so L is a closed set, hence

X is an $WALC-space$.

(b) $\Longrightarrow$(a) This is obvious by Definition 3.2.

Corollary3.29:

(i) Every $WALC-space$ is a $KC-space$.

(ii) Every $WALC-space$ is T_1.

(iii) Every $WALC-space$ is cid.

(iv) Every $WALC-space$ is a locally $LC-space$.

Corollary3.30:

(i) Every locally compact $WALC-space$ is a Hausdorff.

(ii) Every $R_1\,WALC-space$ is a Hausdorff.

(iii) Every locally compact $WALC-space$ is T_3.

(iv) Every locally compact $WALC-space$ is a Tychonoff $(T_{3\frac{1}{2}})$ space .

Proof. (i) This is obvious by Definition 3.2 and Corollary 3.16.

(ii) This is obvious by Definition 3.2 and Theorem 3.19 .

(iii) This is obvious by Definition 3.2 and Theorem 3.17 (i).

(iv) This is obvious by Definition 3.2 and Theorem 3.17 (ii).

Theorem3.31:

For a locally compact $WALC-space$ X the following are equivalent:

(a) X is an almost Lindelöf.

(b) X is a Lindelöf.

Theorem3.32:

For a regular anti – Lindelöf space X the following are equivalent:

(c) X is an $LC-space$.

(d) X is a $cid-space$.

(e) X is an $WALC-space$.

Proof. (a) $\Rightarrow$(b): This is obvious by Remark 3.3.

(b) $\Rightarrow$ (a): This is obvious by Theorem 3.18.

(b) $\Rightarrow$(c): Let X be a $cid-space$,since X is anti – Lindelöf space, then X is an $LC-space$ by Theorem 3.18, since X is a regular space, then X is an $WALC-space$.

(c)$\Rightarrow$(b):This is obvious by Definition3.2and Remark 3.3.

Corollary3.33: For a countable space X the following are equivalent:

(a) X is a $cid-space$.

(b) X is an $WALC-space$.

Proof. (a) $\Rightarrow$(b): If $L \subseteq X$ is almost Lindelöf , but X is countable, then L is countable in X, which is $cid-space$ so L is closed and discrete , then X is an $WALC-space$.

(b) $\Rightarrow$ (a): This is obvious by Definition3.2and Remark 3.3.

Corollary3.34: For a countable space X the following are equivalent:

(a) X is an $ALC-space$.

(b) X is an $WALC-space$.

(c) X is a $cid-space$.

(d) X is an $LC-space$.

(e) X is a countable discrete space.

Proof. This is obvious by Corollary 3.27 and Corollary 3.33.

Theorem3.35:

Let (X,T) be a topological space and $Y \subseteq X$, $Y = \bigcup_{i=1}^{n} Y_i$, where Y_i , $i=1,2,...,n$ are clopen $WALC-subspace$s in X, then Y is an $WALC-subspace$.

Proof. Let L be an almost Lindelöf subset of Y, then $L \cap Y_i$, $i=1,2,...,n$ are clopen in L,which is almost Lindelöf so $L \cap Y_i$, $i=1,2,...,n$ are almost Lindelöf subset of Y_i , $i=1,2,...,n$. Since $L \cap Y_i$ is subset of Y_i , $i=1,2,...,n$ which is $WALC-subspace$, then $L \cap Y_i$ is a closed in Y_i , $i=1,2,...,n$. Since Y_i , $i=1,2,...,n$ is closed in X , then $L \cap Y_i$, $i=1,2,...,n$ is closed in X . But $L = \bigcup_{i=1}^{n}(L \cap Y_i)$, so L is closed in X and also in Y , hence Y is $WALC-subspace$.

Corollary3.36: Every regular locally $LC-space$ is an $WALC-space$.

Proof. Let X be a locally $LC-space$, since X is a regular, then X is an $LC-space$ by Definition3.4. Since X is a regular, then X is an $WALC-space$ by Theorem 3.28.

The following example shows that we cannot replace 'regular' by 'Hausdorff'.

Example3.37: [7,8]There exists a Hausdorff, locally $LC-space$ (X,T), which is not an $LC-space$. Let Z be a set of cardinality N_1 with a distinguished point z_0. The topology on Z is definited as follows: each $z \neq z_0$ is isolated while the basic neighborhoods of z_0 are the Co-countable

subsets of Z containing z_0. Note that Z is a Lindelöf $LC-space$.The space (X,T) will be constructed from copies of Z .

For each $n \in \omega$, let X_n be a copy of Z, where x_n denotes the non-isolated point of X_n. Let $X^* = \sum_{n\in\omega} X_n$ denote the topological sum of the spaces X_n and let $X = X^* \cup \{p\}$ with $p \in X^*$. A topology T on X can be defined if, in addition, we specify the basic open neighborhoods of p. They are the union of $\{p\}$ and co-countable subset of $\cup\{X_n \setminus \{x_n\} : n \geq k\}$ for some $k \in \omega$. (X,T) is a Hausdorff space than fails to be an $LC-space$ [7] and also an $WALC-space$. However, as shown in [8], (X,T) is a locally $LC-space$.

<u>Corollary3.38:</u> For a regular X the following are equivalent:

(a) X is an $WALC-space$.

(b) X is an $LC-space$.

(c) X is a locally $LC-space$.

Proof.

This is obvious by Definition3.2, Definition3.4 and Theorem 3.28.

<u>Theorem3.39:</u> If every $LC-subspace$ of every almost Lindelöf subset of a topological space (X,T) is Lindelöf, then X is a locally $LC-space$ if and only if X is an $WALC-space$.

Proof. Assume that X is a locally $LC-space$. Let $A \subseteq X$ almost Lindelöf and let $x \notin X$. Since X is an locally $LC-space$, there exists $U \in T$ such that $x \in U$ and $(U, T/U)$ is an $LC-space$. Since every subspace of an $LC-space$ is an $LC-space$, $U \cap A$ is an $LC-space$. By assumption, $U \cap A$ is Lindelöf and hence closed in $(U, T/U)$. Thus, U \ A is open in (X,T) , contains x and is disjoint from A. This shows that A is closed and consequently X is an $WALC-space$.

<u>Corollary3.40:</u>

Every weakly locally Lindelöf $WALC-space$ is a locally Lindelöf.

Proof. This is obvious by Definition 3.2.

<u>Corollary3.41:</u> For a $WALC-space$ X the following are equivalent:

(a) X is locally Lindelof .

(b) X is a weakly locally Lindelof .

Proof. (a) $\Rightarrow$ (b): This is obvious by Definition 3.9.

(b) $\Rightarrow$ (a): This is obvious by Corollary 3.40.

<u>Corollary3.42:</u>

If (X,T) a regular space has an open cover by locally $LC-subspaces$,then is an $WALC-space$.

Proof. This is obvious by Theorem 3.22 and Theorem 3.28.

Corollary3.43:

If (X,T) a regular space has an open cover by $LC-subspaces$, then is an $WALC-space$.

Proof. This is obvious by Theorem 3.23 and Theorem 3.28.

Theorem3.44:

For a hereditarily compact space X the following are equivalent:

(a) X is an $LC-space$.

(b) X is an $WALC-space$.

Proof. (a)$\Rightarrow$(b): Let L be an almost Lindelöf subset of X, then L is a compact (since X is a hereditarily compact) , so L is a Lindelöf , then L is a closed set (since X is an $LC-space$) , hence X is an $WALC-space$.

(b) $\Rightarrow$(a): This is obvious by Definition3.2.

Corollary3.45:

(i) Every $WALC-space$ having a dense almost Lindelöf Subset is almost Lindelöf.

(ii) Every $WALC-space$ having a dense Lindelöf Subset almost Lindelöf.

Theorem3.46:

For a locally compact $R_1-space$ X the following are equivalent:

(c) X is an $LC-space$.

(d) X is a locally $LC-space$.

(e) X is an $WALC-space$.

Proof. (a) $\Rightarrow$(b): This is obvious by Definition3.9.

(b) $\Rightarrow$ (a): This is obvious by Theorem 3.21.

(b)$\Rightarrow$ (c): Let X be a locally $LC-space$,then X is a $T_1-space$ by Theorem3.20.

Since X is a $R_1-space$, then X is a Hausdorff by Definition3.5. Since X is a locally compact, so X is a regular by Theorem 3.17 , hence X is an $LC-space$ and is an $WALC-space$ by Definition 3.4 and Theorem 3.28.

(c) $\Rightarrow$(b): This is obvious by Definition3.2 and Definition3.9.

Corollary3.47: For a regular space X the following are equivalent:

(a) X is a Lindelöf $LC-space$.

(b) X is an almost Lindelöf $WALC-space$.

Proof.

This is obvious byDefinition2.2, Theorem 2.7, Definition 3.2 and Theorem 3.28.

Corollary3.48:

Every hereditarily Lindelof $WALC-space$ is a $Q-set$ space.

Proof. This is obvious by Definition 3.2 and Theorem 3.14.

Theorem3.49: If $f:X\longrightarrow Y$ is a continuous injective function from a space X into an $WALC-space\, Y$ then X is an $WALC-space$.

Proof. Let L be any almost Lindelöf subset of a space X ,then $f(L)$ is an almost Lindelöf in Y (Acontinuous image of an almost Lindelöf is almost Lindelöf),since Y is an $WALC-space$, then $f(L)$ is a closed subset of Y ,therefore $f^{-1}(f(L))=L$ is a closed subset of X (because f is a continuous injective function) ,thus X is an $WALC-space$.

Theorem3.50:

Every continuous function f from Lindelöf space X into $WALC-space\, Y$ is a closed function.

Proof. Let F be a closed subset of a space X which is a Lindelöf then F is a Lindelöf in X ,so $f(F)$

is a Lindelöf in Y (continuous image of a Lindelöf is Lindelöf),then $f(F)$ is an almost Lindelöf

subset in a space Y ,hence $f(F)$ is a closed subset in a space Y (since Y is an $WALC-space$),

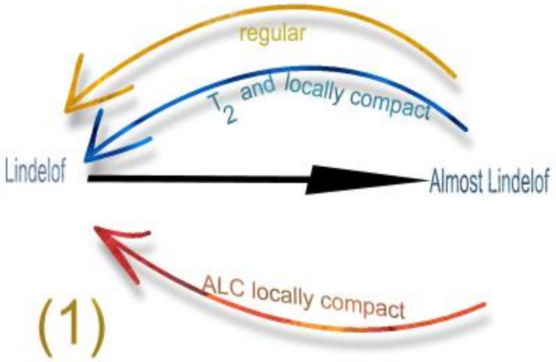
therefore f is a closed function.

Theorem3.51:

If $f:X\longrightarrow Y$ is a closed and open bijective function from an $WALC-space$e X into an space Y then Y is an $WALC-space$.

Proof. Let L be any almost Lindelöf subset of a space Y ,then $f^{-1}(L)$ is an almost Lindelöf in X (A continuous image of an almost Lindelöf is almost Lindelöf),since X is an $WALC-space$, then $f^{-1}(L)$ is a closed subset of X ,therefore $f(f^{-1}(L))=L$ is a closed subset of Y (because f is a closed surjective function) ,thus Y is an $WALC-space$.

We have the following diagrams:



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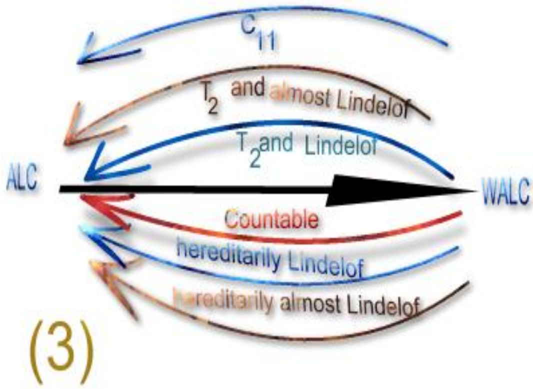

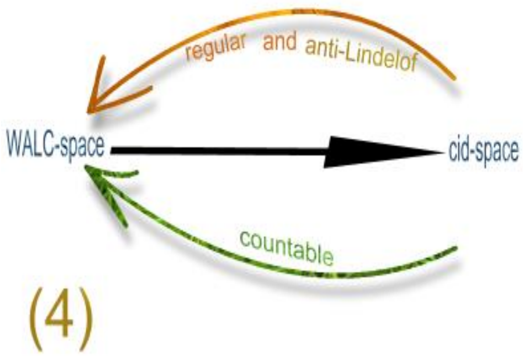

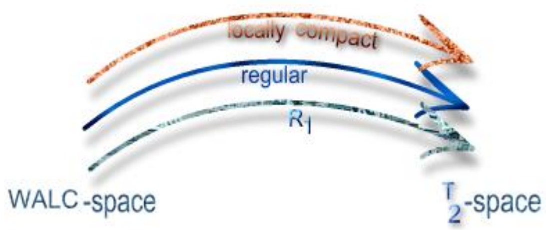

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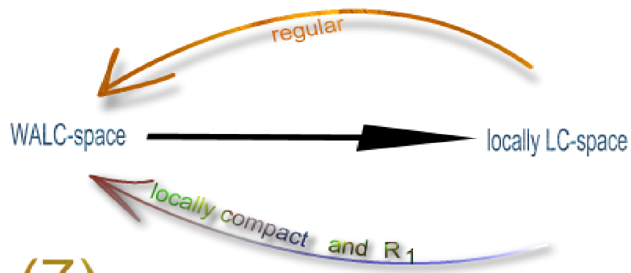

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