A novel approach on neutrosophic crisp sets

Received :27/ 12 / 2017 **Accepted : 7 / 6 /2018**

Saied A. Johnny
AL-Qadisiyah Education - Iraq
Saied.jhony@iraqima.org

Hashmiya Ibrahim Nasser
AL-Qadisiyah Education - Iraq
hashimea.ibrahem@iraqima.org

ABSTRACT

The focus of this paper is to introduce a new type of neutrosophic crisp sets as the neutrosophic crisp soft sets and which is the generalization of an ordered triple in the definition of Salama set's [9]. After given the fundamental definitions of generalized neutrosophic crisp set operations, we obtain several properties and we discussed some theorems in this concept. Finally , the concept to the neutrosophic crisp soft lattices.

Keyword: $\mathcal{nc}$-set, $\mathcal{ncs}$-set, $\mathcal{ncs}$-lattices.

1. Introduction:

Soft set theory was firstly introduced by molodtsov in 1999 , [1] , as a general mathematical for dealing tool for dealing with problems that contain uncertainty . The algebraic structure of soft set theory also has been studied in more detail [2] , [3] , [4] and [5]. Smarandache defined the notion of neutrosophic sets , which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set . Neutrosophic crisp sets have been investigated by Salama et al. [9] , [10] , [11] and [13]. In this paper is to introduce and study some new neutrosophic crisp soft notions via neutrosophic crisp soft lattices.

2. Terminologies:

We recollect some relevant basic preliminaries and in particular , the work of Molodtsov D. in [1] , Faruk K. [8] and Salama et al. [9] , [11] and [13].

Definition (2.1) [1]:

Let X be the initial universe set and A be a set of parameters. Let $P(X)$ denote the power set of X. Consider a non-empty set A, $A \subseteq X$. A pair (F, A) is called a soft set over X (for short s-set), where F is a mapping given by: $F: A \to P(X)$.

Remark (2.2) [2]:

$(\boldsymbol{i})$ The s-set can be represented by F_A.

$(\boldsymbol{ii})$ Every set is s-set.

$(\boldsymbol{iii})$ If $A = |1|$, a s-set can be considered a crisp set.

$(\boldsymbol{iv})$ $\mathcal{S}(X)$ is the collection of all s-sets over X.

Definition (2.3) [9]:

Let X be a non-empty fixed set. A neutrosophic crisp set (for short $\mathcal{nc}$-set) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$, where A_1, A_2 and A_3 are subsets of X satisfying:

$A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_2 \cap A_3 = \emptyset$.

Remark (2.4) [11]:

Every crisp set in X is obviously a (NC-set) having the form $\langle A_1, A_2, A_3 \rangle$.

Definition (2.5) [13]:

The object having the form $\langle A_1, A_2, A_3 \rangle$ is called:

$(\boldsymbol{i})$ $\mathcal{nc}$-set with type I if it satisfies $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and $A_2 \cap A_3 = \emptyset$. (NC-set type I).

$(\boldsymbol{ii})$ $\mathcal{nc}$-set with Type II if it satisfies: $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$, $A_2 \cap A_3 = \emptyset$ and $A_1 \cup A_2 \cup A_3 = X$ (NC-set type II).

$(\boldsymbol{iii})$ $\mathcal{nc}$-set with type I if it satisfies: $A_1 \cap A_2 \cap A_3 = \emptyset$, $A_1 \cup A_2 \cup A_3 = X$ ($\mathcal{nc}$-set type III).

Definition (2.6) [8]:

Let $\mathcal{L} \subseteq \mathcal{S}(X)$, $\curlyvee$ and $\curlywedge$ be two binary operations on $\mathcal{L}$. If $\mathcal{L}$ is equipped with two commutative and associative binary operations $\curlyvee$ and $\curlywedge$, which are connected by the absorption law, then algebraic structure $(\mathcal{L}, \curlyvee, \curlywedge)$ is called soft lattice.

3. Main result

Definition (3.1):

Let X be a universe and A be a set of parameters that are describe the elements of a set X. A neutrosophic crisp soft set (for short $\mathcal{ncs}$-set) over X (denoted by $\widehat{\mathcal{N}}$) is a set defined by:

$\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$, where F_A, G_A and H_A are disjoint s-sets over X.

Example (3.2):

Let $X = \{x_1, x_2, x_3\}$ be a universe set and $A = \{e_1, e_2, e_3\}$ a set of parameters $F: A \to P(X)$ be a mapping such that:

$F(e_1) = \{x_1, x_3\}$; $F(e_2) = \{x_1, x_2\}$ & $F(e_3) = \emptyset$.

It is Clear that:

$F_A = \{\{x_1, x_3\}, \{x_1, x_2\}, \emptyset\}$,

$G_A = F_A^c$ and $H_A = \widetilde{\emptyset}$ are disjoint s-sets. Then we'll give a $\mathcal{ncs}$-set is $\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$.

Remark (3.3):

$(\boldsymbol{i})$Every $\mathcal{ncs}$-set formed by three disjoint s-sets.

$(\boldsymbol{ii})$ Every $\mathcal{nc}$-set is a $\mathcal{ncs}$-set.

The difference between $\mathcal{nc}$-set and $\mathcal{ncs}$-set arise from this fact in remark (2.2.ii).

$(\boldsymbol{iii})$ When $A = |1|$ a $\mathcal{ncs}$-set is a $\mathcal{nc}$-set.

Definition (3.4):

Let $\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$ & $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle$ be two $\mathcal{ncs}$-sets over a universe set X. Then:

$(\boldsymbol{i})$ $\widehat{\mathcal{N}}_1$ is called a $\mathcal{ncs}$-subset of $\widehat{\mathcal{N}}_2$ ($\widehat{\mathcal{N}}_1 \subseteq \widehat{\mathcal{N}}_2$), if $F_A \,\widetilde{\subseteq}\, F_B$, $G_A \,\widetilde{\subseteq}\, G_B$ and $H_A \,\widetilde{\supseteq}\, H_B$.

$(\boldsymbol{ii})$ $\widehat{\mathcal{N}}_1$ & $\widehat{\mathcal{N}}_2$ are called an equal ($\widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_2$), if $\widehat{\mathcal{N}}_1 \subseteq \widehat{\mathcal{N}}_2$ and $\widehat{\mathcal{N}}_2 \subseteq \widehat{\mathcal{N}}_1$.

$(\boldsymbol{iii})$The complement of $\widehat{\mathcal{N}}$ is denoted by $\widehat{\mathcal{N}}_1^{\,c}$ and may be defined as $\widehat{\mathcal{N}}_1^{\,c} = \langle F_A^c \,, G_A^c \,, H_A^c \rangle$.

$(\boldsymbol{iv})$ The union of $\widehat{\mathcal{N}}_1$ and $\widehat{\mathcal{N}}_2$ denoted by $\widehat{\mathcal{N}}_1 \cup \widehat{\mathcal{N}}_2$ may be defined as:

$\widehat{\mathcal{N}}_1 \cup \widehat{\mathcal{N}}_2 = \langle F_A \,, G_A \,, H_A \rangle \cup \langle F_B \,, G_B \,, H_B \rangle$

$= \langle\, F_A \widetilde{\cup} F_B \,, G_A \widetilde{\cup} G_B \,, H_A \widetilde{\cap} H_B \,\rangle$

or $\widehat{\mathcal{N}}_1 \cup \widehat{\mathcal{N}}_2 =$

$\langle\, F_A \widetilde{\cup} F_B, G_A \widetilde{\cap} G_B, H_A \widetilde{\cap} H_B \,\rangle$.

$(\boldsymbol{v})$ The intersection of $\widehat{\mathcal{N}}_1$ and $\widehat{\mathcal{N}}_2$, is denoted by $\widehat{\mathcal{N}}_1 \cap \widehat{\mathcal{N}}_2$ may be defined as:

$\widehat{\mathcal{N}}_1 \cap \widehat{\mathcal{N}}_2 = \langle F_A \,, G_A \,, H_A \rangle \cap \langle F_B \,, G_B \,, H_B \rangle$

$= \langle\, F_A \widetilde{\cap} F_B \,, G_A \widetilde{\cap} G_B \,, H_A \widetilde{\cup} H_B \,\rangle$.

or $\widehat{\mathcal{N}}_1 \cap \widehat{\mathcal{N}}_2 =$

$\langle\, F_A \widetilde{\cap} F_B \,, G_A \widetilde{\cup} G_B \,, H_A \widetilde{\cup} H_B \,\rangle$

$(\boldsymbol{vi})$ The difference of $\widehat{\mathcal{N}}_1$ and $\widehat{\mathcal{N}}_2$ is denoted by $\widehat{\mathcal{N}}_1 \backslash \widehat{\mathcal{N}}_2$ and defined as $\widehat{\mathcal{N}}_1 \backslash \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_1 \cap \widehat{\mathcal{N}}_2^{\;c}$.

Remark (3.5):

Let $\{\widehat{\mathcal{N}}_j : j \in J\}$ be an arbitrary family of $\mathcal{ncs}$-sets over X. Then:

$(\boldsymbol{i})$ $\cup_j \widehat{\mathcal{N}}_j = \langle \widetilde{\cup} F_{jA}, \widetilde{\cup} G_{jA}, \widetilde{\cap} H_{jA} \rangle$ or

$\cup_j \widehat{\mathcal{N}}_j = \langle \widetilde{\cup} F_{jA}, \widetilde{\cap} G_{jA}, \widetilde{\cap} H_{jA} \rangle$.

$(\boldsymbol{ii})$ $\cap_j \widehat{\mathcal{N}}_j = \langle \widetilde{\cap} A_{j1} \,, \widetilde{\cap} A_{j2} \,, \widetilde{\cup} A_{j3} \rangle$ or

$\cap_j \widehat{\mathcal{N}}_j = \langle \widetilde{\cap} A_{j1} \,, \widetilde{\cup} A_{j2} \,, \widetilde{\cup} A_{j3} \rangle$.

Definition (3.6):

May be define $\widehat{\Phi}$ and $\widehat{\mathcal{X}}$ as follows:

$(\boldsymbol{i})$ $\widehat{\Phi} = \langle \widetilde{\emptyset} \,, \widetilde{\emptyset} \,, \widetilde{X} \rangle$ or $\langle \widetilde{\emptyset} \,, \widetilde{X} \,, \widetilde{\emptyset} \rangle$ or $\langle \widetilde{X} \,, \widetilde{X} \,, \widetilde{\emptyset} \rangle$

(NC null s-set).

$(\boldsymbol{ii})$ $\widehat{\mathcal{X}} = \langle \widetilde{X} \,, \widetilde{X} \,, \widetilde{\emptyset} \rangle$ or $\langle \widetilde{X} \,, \widetilde{\emptyset} \,, \widetilde{X} \rangle$ or $\langle \widetilde{\emptyset} \,, \widetilde{X} \,, \widetilde{X} \rangle$

(NC absolute s-set).

Definition (3.7):

A NCS-set over X, is called a NCS-point over X, denoted by $\hat{\mathcal{p}} = \langle P_e^x, P_e^y, P_e^z \rangle$, $P_e^x \neq P_e^y \neq P_e^z$, $P_e^x, P_e^y, P_e^z \,\widetilde{\in}\, \mathcal{S}_{\mathcal{p}}(X)$, ($\mathcal{S}_{\mathcal{p}}(X)$ is the collection of all soft points over a universe X)

Definition (3.8):

Let $\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$ be a $\mathcal{ncs}$-sets over X. Then $\widehat{\mathcal{N}}$ is called a singleton $\mathcal{ncs}$-set over X, if for each $F_A \,, G_A$ & H_A are singleton s-sets over X .

Remark (3.9):

Every $\mathcal{ncs}$-point is a singleton $\mathcal{ncs}$-set. The converse is not true as the following example shows.

Example (3.10):

$(\boldsymbol{i})$ Given $X = \{x, y, z\}$ is a universe set and $A = \{e_1, e_2, e_3\}$ be a set of parameters. Then:

$\widehat{\mathcal{N}} = \langle P_{e_1}^{x}, P_{e_2}^{y}, P_{e_3}^{z} \rangle$ is a $\mathcal{ncs}$-point.

$(\boldsymbol{ii})$ Let $X = \{\alpha, \beta, \gamma, \delta\}$ be a universe set and $A = \{\ell_1, \ell_2, \ell_3, \ell_4\}$ be a set of parameters. Then $\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$ is a singleton $\mathcal{ncs}$-set but it is not $\mathcal{ncs}$-point such that:

$F_A = \{(\ell_1, \{\alpha\}), (\ell_2, \{\beta\}), \emptyset, \emptyset\}$

$G_A = \{\emptyset, \emptyset, (\ell_3, \{\gamma\}), \emptyset\}$.

$H_A = \{\emptyset, \emptyset, \emptyset, (\ell_4, \{\delta\})\}$.

Remark (3.11):

$(\boldsymbol{i})$ The collection of all $\mathcal{ncs}$-sets over a universe X is denoted by $\widehat{\mathcal{N}}(X)$.

$(\boldsymbol{ii})$ The collection of all $\mathcal{ncs}$-points over X is denoted by $\widehat{\mathcal{N}}_{\wp}(X)$.

Theorem (3.12):

Let $\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$ & $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle$ be two $\mathcal{ncs}$-sets over a universe X. Then:

$(\boldsymbol{i})$ $\widehat{\mathcal{N}}_1 \subseteq \widehat{\mathcal{N}}_2$ iff $\hat{\wp} \in \widehat{\mathcal{N}}_2$ for all $\hat{\wp} \in \widehat{\mathcal{N}}_1$.

$(\boldsymbol{ii})$ $\widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_2$ iff $\widehat{\mathcal{N}}_1 \subseteq \widehat{\mathcal{N}}_2$ and $\widehat{\mathcal{N}}_2 \subseteq \widehat{\mathcal{N}}_1$.

Proof: Obvious.

Theorem (3.13):

Let $\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$ be a $\mathcal{ncs}$-sets over X. Then:

$(\boldsymbol{i})$ $\widehat{\mathcal{N}} = \bigcup_{\hat{\wp} \in \widehat{\mathcal{N}}} \{\hat{\wp}\}$.

$(\boldsymbol{ii})$ $\widehat{\mathcal{N}} = \bigcup_{\hat{\wp} \in \widehat{\mathcal{N}}} \{\hat{\wp}\} =$
$\bigcup \langle \{P_e^x : P_e^x \tilde{\in} F_A\}, \{P_e^y : P_e^y \tilde{\in} G_A\}, \{P_e^z : P_e^z \tilde{\in} H_A\} \rangle$

.Proof: Clear.

Theorem (3.14):

Let $\{\widehat{\mathcal{N}}_j : j \in J\}$ be an arbitrary family of $\mathcal{ncs}$-sets over X. Then:

$(\boldsymbol{i})$ $\hat{\wp} \in \bigcup_j \widehat{\mathcal{N}}_j$ iff $\hat{\wp} \in \widehat{\mathcal{N}}_j$ for some $j \in J$.

$(\boldsymbol{ii})$ $\hat{\wp} \in \bigcap_j \widehat{\mathcal{N}}_j$ iff $\hat{\wp} \in \widehat{\mathcal{N}}_j$ for each $j \in J$.

Theorem (3.15):

Let $f: X \to Y$ be a mapping. Then:

$(\boldsymbol{i})$ If $\widehat{\mathcal{N}} = \langle F_A, G_A, H_A \rangle$ be a $\mathcal{ncs}$-sets over X, then $f(\widehat{\mathcal{N}}) = \langle \tilde{f}(F_A), \tilde{f}(G_A), \tilde{f}(H_A) \rangle$ is a $\mathcal{ncs}$-sets over Y.

$(\boldsymbol{ii})$ The image of $\mathcal{ncs}$-point $\hat{\wp} = \langle P_e^x, P_e^y, P_e^z \rangle$ over X under a mapping f denoted by $f(\hat{\wp})$ and defined as $f(\hat{\wp}) = \langle \tilde{f}(P_e^x), \tilde{f}(P_e^y), \tilde{f}(P_e^z) \rangle$.

Definition (3.16):

Let $\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$ & $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle$ be two NCS-sets over X. Then:

$(\boldsymbol{i})$A Cartesian product of $\widehat{\mathcal{N}}_1$ & $\widehat{\mathcal{N}}_2$ is defined as:

$\widehat{\mathcal{N}}_1 \widehat{\times} \widehat{\mathcal{N}}_2$
$= \langle F_A , G_A , H_A \rangle \widehat{\times} \langle F_B , G_B , H_B \rangle$
$=$
$\langle F_A \widetilde{\times} F_B, G_A \widetilde{\times} G_B , H_A \widetilde{\times} H_B \rangle$.

$(\boldsymbol{ii})$ A relation from $\widehat{\mathcal{N}}_1$ to $\widehat{\mathcal{N}}_2$ is a soft subset of $\widehat{\mathcal{N}}_1 \widehat{\times} \widehat{\mathcal{N}}_2$.

Definition (3.17):

Let $\mathcal{R}$ be a relation from $\widehat{\mathcal{N}}_1 = \langle F_A , G_A , H_A \rangle$ to $\widehat{\mathcal{N}}_2 = \langle F_B , G_B , H_B \rangle$. Then:

$(\boldsymbol{i})$ The domain of $\mathcal{R}$ is defined as the NCS-set $\widehat{\mathcal{N}}_* = \langle F_C , G_C , H_C \rangle$,
$C = \{a \in A : W(a, b) \in R\}$ such that:
$W(a, b) = \{ F(a) \times F(b) :$
for some $b \in B$ } .

$(\boldsymbol{ii})$ The range of R is defined as the NCS-set $\widehat{\mathcal{N}}^* = \langle F_D , G_D , H_D \rangle$, $D = \{b \in B : H(a, b) \in R\}$ such that:
$H(a, b) = \{ F(a) \times F(b) :$
for some $a \in A$ }.

Definition (3.18):

Let $\mathcal{R}$ be a relation on a $\mathcal{ncs}$-set $\widehat{\mathcal{N}}$ over a universe X. Then:

$(\boldsymbol{i})$ $\mathcal{R}$ is reflexive:
if $(\hat{p}, \hat{p}) \in \mathcal{R}$, for all $\hat{p} \in \widehat{\mathcal{N}}$.

$(\boldsymbol{ii})$ $\mathcal{R}$ is symmetric:
if $(\hat{p}_1, \hat{p}_2) \in \mathcal{R}$, then $(\hat{p}_2, \hat{p}_1) \in \mathcal{R}$ for all $\hat{p}_1, \hat{p}_2 \in \widehat{\mathcal{N}}$.

$(\boldsymbol{iii})$ R is transitive:
if $(\hat{p}_1, \hat{p}_2), (\hat{p}_2, \hat{p}_3) \in \mathcal{R}$, then $(\hat{p}_1, \hat{p}_3) \in \mathcal{R}$, for all $\hat{p}_1, \hat{p}_2, \hat{p}_3 \in \widehat{\mathcal{N}}$.

$(\boldsymbol{iv})$ $\mathcal{R}$ is anti-symmetric:
if $(\hat{p}_1, \hat{p}_2), (\hat{p}_2, \hat{p}_1) \in \mathcal{R}$, then $\hat{p}_1 = \hat{p}_2$ for all $\hat{p}_1, \hat{p}_2 \in \widehat{\mathcal{N}}$.

4. $\mathcal{ncs}$-Lattice:

Definition (4.1):

Let $\mathfrak{N} \subseteq \widehat{\mathcal{N}}(X)$, $\curlyvee$ and $\curlywedge$ be two binary operations on $\mathfrak{N}$. If the set $\mathcal{N}$ is equipped with two commutative and associative binary operations $\curlyvee$ and $\curlywedge$, which are connected by the absorption law .

Then algebraic structure $(\mathfrak{N}, \curlyvee, \curlywedge)$ is called $\mathcal{ncs\ell}$.

Theorem (4.2):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $\mathcal{ncs\ell}$ & $\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$
, $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle \in \mathfrak{N}$. Then $\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_1$ iff $\widehat{\mathcal{N}}_1 \curlyvee \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_2$.

Proof:

$\widehat{\mathcal{N}}_1 \curlyvee \widehat{\mathcal{N}}_2 = (\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2) \curlyvee \widehat{\mathcal{N}}_2$
$= \widehat{\mathcal{N}}_2 \curlyvee (\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2) = \widehat{\mathcal{N}}_2 \curlyvee (\widehat{\mathcal{N}}_2 \curlywedge \widehat{\mathcal{N}}_1)$
$= (\widehat{\mathcal{N}}_2 \curlywedge \widehat{\mathcal{N}}_2) \curlywedge (\widehat{\mathcal{N}}_2 \curlywedge \widehat{\mathcal{N}}_1) = \widehat{\mathcal{N}}_2 \curlywedge \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_2$.

Conversely,

$\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_1 \curlywedge (\widehat{\mathcal{N}}_1 \curlyvee \widehat{\mathcal{N}}_2)$
$= \widehat{\mathcal{N}}_1 \curlywedge (\widehat{\mathcal{N}}_2 \curlyvee \widehat{\mathcal{N}}_1)$
$= (\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2) \curlyvee (\widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_1) = \widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_1$.

Example (4.3):

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a universe set and $A = \{e_1, e_2, e_3\}$ be a set of parameters with $\mathfrak{N} = \{\widehat{\mathcal{N}}_1, \widehat{\mathcal{N}}_2, \widehat{\mathcal{N}}_3, \widehat{\mathcal{N}}_4, \widehat{\mathcal{N}}_5\} \subseteq \widehat{\mathcal{N}}(X)$. such that:

$\widehat{\mathcal{N}}_1 = \langle \{\{x_1, x_2, x_3\}\}, \{\{x_4, x_5\}\}, \widetilde{\emptyset} \rangle$.

$\widehat{\mathcal{N}}_2 = \langle \widetilde{\emptyset}, \widetilde{\emptyset}, \widetilde{\emptyset} \rangle$.

$\widehat{\mathcal{N}}_3 = \langle \{x_2\}, \widetilde{\emptyset}, \widetilde{\emptyset} \rangle$.

$\widehat{\mathcal{N}}_4 = \langle \{x_1, x_3\}, \widetilde{\emptyset}, \widetilde{\emptyset} \rangle$.

$\widehat{\mathcal{N}}_5 = \langle \{x_1, x_2, x_3\}, \widetilde{\emptyset}, \widetilde{\emptyset} \rangle$.

Then $(\mathfrak{N}, \cup, \cap)$ is $NCSL$. Tables of the operations are as follows, respectively;

$\cup$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$
$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_5$

And

$\cap$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_1$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$
$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$
$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$
$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_4$
$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_5$	$\widehat{\mathcal{N}}_2$	$\widehat{\mathcal{N}}_3$	$\widehat{\mathcal{N}}_4$	$\widehat{\mathcal{N}}_5$

Theorem (4.4):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $ncs\ell$ such that:

$\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$, $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle \in \mathcal{N}$. Then a relation $\preccurlyeq$ that is defined by:

$\widehat{\mathcal{N}}_1 \preccurlyeq \widehat{\mathcal{N}}_2 \Leftrightarrow \widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_1$ or $\widehat{\mathcal{N}}_1 \curlyvee \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_2$. is an ordering on $\mathfrak{N}$.

Proof:

i. $\preccurlyeq$ is reflexive. $\widehat{\mathcal{N}}_1 \preccurlyeq \widehat{\mathcal{N}}_1 \Leftrightarrow \widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_1$.

ii. $\preccurlyeq$ is antisymmetric. Let $\widehat{\mathcal{N}}_1 \preccurlyeq \widehat{\mathcal{N}}_2$ & $\widehat{\mathcal{N}}_2 \preccurlyeq \widehat{\mathcal{N}}_1$.

Then $\widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_1 \curlywedge \widehat{\mathcal{N}}_2 = \widehat{\mathcal{N}}_2 \curlywedge \widehat{\mathcal{N}}_1 = \widehat{\mathcal{N}}_2$.

iii. $\preccurlyeq$ is transitive. Let $\widehat{N}_1 \preccurlyeq \widehat{N}_2$ & $\widehat{N}_2 \preccurlyeq \widehat{N}_3$. Then

$$\widehat{N}_1 \curlywedge \widehat{N}_3 = \left(\widehat{N}_1 \curlywedge \widehat{N}_2\right) \curlywedge \widehat{N}_3$$
$$= \widehat{N}_1 \curlywedge (\widehat{N}_2 \curlywedge \widehat{N}_3) = \widehat{N}_1 \curlywedge \widehat{N}_2 = \widehat{N}_1.$$

This implies that $\widehat{N}_1 \preccurlyeq \widehat{N}_3$.

Theorem (4.5):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $NCSL$ such that:

$\widehat{\mathcal{N}}_1 = \langle F_A, G_A, H_A \rangle$ and $\widehat{\mathcal{N}}_2 = \langle F_B, G_B, H_B \rangle \in \mathcal{N}$. Then:

$(\boldsymbol{i})$ $\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2} \preccurlyeq \widehat{\mathcal{N}_1}$ & $\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2} \preccurlyeq \widehat{\mathcal{N}_2}$.

$(\boldsymbol{ii})$ $\widehat{\mathcal{N}_1} \preccurlyeq \widehat{\mathcal{N}_1} \curlyvee \widehat{\mathcal{N}_2}$ & $\widehat{\mathcal{N}_2} \preccurlyeq \widehat{\mathcal{N}_1} \curlyvee \widehat{\mathcal{N}_2}$.

Proof:

By definition (4.1) , we have:

$(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) \curlyvee \widehat{\mathcal{N}_1} = \widehat{\mathcal{N}_1} \curlyvee (\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) = \widehat{\mathcal{N}_1}$. From theorem (4.4), we get $(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) \curlyvee \widehat{\mathcal{N}_1} \preccurlyeq \widehat{\mathcal{N}_1}$. It can be show that $(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) \preccurlyeq \widehat{\mathcal{N}_2}$.

The proof $(\boldsymbol{ii})$ can made similarity.

Theorem (4.6):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $NCSL$ such that: $\widehat{\mathcal{N}_1} = \langle F_A, G_A, H_A \rangle$, $\widehat{\mathcal{N}_2} = \langle F_B, G_B, H_B \rangle$,

$\widehat{\mathcal{N}_3} = \langle F_C, G_C, H_C \rangle$ & $\widehat{\mathcal{N}_4} = \langle F_D, G_D, H_D \rangle \in \mathcal{N}$. If $\widehat{\mathcal{N}_1} \preccurlyeq \widehat{\mathcal{N}_2}$ & $\widehat{\mathcal{N}_3} \preccurlyeq \widehat{\mathcal{N}_4}$, then $\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_3} \preccurlyeq \widehat{\mathcal{N}_2} \curlywedge \widehat{\mathcal{N}_4}$.

Proof:

From hypothesis and theorem (4.4), we have:

$$\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2} = \widehat{\mathcal{N}_1} \;\&\; \widehat{\mathcal{N}_3} \curlywedge \widehat{\mathcal{N}_4} = \widehat{\mathcal{N}_3}$$

$(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_3}) \curlywedge (\widehat{\mathcal{N}_2} \curlywedge \widehat{\mathcal{N}_4})$

$= [(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_3}) \curlywedge \widehat{\mathcal{N}_2}] \curlywedge \widehat{\mathcal{N}_4}$

$= [\widehat{\mathcal{N}_1} \curlywedge (\widehat{\mathcal{N}_3} \curlywedge \widehat{\mathcal{N}_2})] \curlywedge \widehat{\mathcal{N}_4}$

$= [\widehat{\mathcal{N}_1} \curlywedge (\widehat{\mathcal{N}_2} \curlywedge \widehat{\mathcal{N}_3})] \curlywedge \widehat{\mathcal{N}_4}$

$= [(\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) \curlywedge \widehat{\mathcal{N}_3}] \curlywedge \widehat{\mathcal{N}_4}$

$= (\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}) \curlywedge (\widehat{\mathcal{N}_3} \curlywedge \widehat{\mathcal{N}_4})$

$= \widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_3}$.

Then from theorem (4.4) , $\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_3} \preccurlyeq \widehat{\mathcal{N}_2} \curlywedge \widehat{\mathcal{N}_4}$.

Theorem (4.7):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $NCSL$ such that:

$\widehat{\mathcal{N}_1} = \langle F_A, G_A, H_A \rangle$, $\widehat{\mathcal{N}_2} = \langle F_B, G_B, H_B \rangle$,

$\widehat{\mathcal{N}_3} = \langle F_C, G_C, H_C \rangle$ & $\widehat{\mathcal{N}_4} = \langle F_D, G_D, H_D \rangle \in \mathcal{N}$. If $\widehat{\mathcal{N}_2} \preccurlyeq \widehat{\mathcal{N}_1}$ & $\widehat{\mathcal{N}_4} \preccurlyeq \widehat{\mathcal{N}_3}$, then $\widehat{\mathcal{N}_2} \curlyvee \widehat{\mathcal{N}_4} \preccurlyeq \widehat{\mathcal{N}_1} \curlyvee \widehat{\mathcal{N}_3}$.

Proof:

Proof is made similarity to theorem (4.6).

Example (4.8):

From example (4.3) **,** Since $\widehat{N}_2 \subseteq \widehat{N}_3$& $\widehat{N}_4 \subseteq \widehat{N}_5$, we have: $\widehat{N}_2 \cap \widehat{N}_3 \subseteq \widehat{N}_4 \cap \widehat{N}_5$.

Remark (4.9):

Let $(\mathfrak{N}, \curlyvee, \curlywedge)$ be a $NCSL$ such that:

$\widehat{\mathcal{N}_1} = \langle F_A, G_A, H_A \rangle$, $\widehat{\mathcal{N}_2} = \langle F_B, G_B, H_B \rangle \in \mathfrak{N}$. Then:

$\widehat{\mathcal{N}_1} \curlywedge \widehat{\mathcal{N}_2}$ & $\widehat{\mathcal{N}_1} \curlyvee \widehat{\mathcal{N}_2}$ are the least upper and the greatest lower bound of $\widehat{\mathcal{N}_1}$ and $\widehat{\mathcal{N}_1}$ respectively.

Theorem (4.10):

Let $\mathfrak{N} \subseteq \widehat{\mathcal{N}}(X)$. Then $(\mathcal{N}, \curlyvee, \curlywedge, \preccurlyeq)$ is a $NCSL$**.**

Proof:

For all $\widehat{N}_1 = \langle F_A, G_A, H_A \rangle$, $\widehat{N}_2 = \langle F_B, G_B, H_B \rangle$

and $\widehat{N}_3 = \langle F_C, G_C, H_C \rangle \in \mathcal{N}$.

From remark (4.9) ,

$\widehat{N}_1 \curlywedge \widehat{N}_2 \preccurlyeq \widehat{N}_1$ and $\widehat{N}_1 \curlywedge \widehat{N}_2 \preccurlyeq \widehat{N}_2$.
Then from
theorem (4.6), we have $\widehat{N}_1 \curlywedge \widehat{N}_2 \preccurlyeq \widehat{N}_2 \curlywedge \widehat{N}_1$.
Similarly , $\widehat{N}_2 \curlywedge \widehat{N}_1 \preccurlyeq \widehat{N}_1 \curlywedge \widehat{N}_2$.
Implies that:
$\widehat{N}_1 \curlywedge \widehat{N}_2 = \widehat{N}_2 \curlywedge \widehat{N}_1$. By the same way, the proof of $\widehat{N}_1 \curlyvee \widehat{N}_2 = \widehat{N}_2 \curlyvee \widehat{N}_1$. can be made.
Now, from theorem (3.5) , we have:
$(\widehat{N}_1 \curlywedge \widehat{N}_2) \curlywedge \widehat{N}_3 \preccurlyeq \widehat{N}_2$ and $(\widehat{N}_1 \curlywedge \widehat{N}_2) \curlywedge \widehat{N}_3 \preccurlyeq \widehat{N}_3$. Also , from theorem (3.6) , implies that:
$(\widehat{N}_1 \curlywedge \widehat{N}_2) \curlywedge \widehat{N}_3 \preccurlyeq \widehat{N}_1 \curlywedge \widehat{N}_2 \preccurlyeq \widehat{N}_1$.
Similarly , $\widehat{N}_1 \curlywedge (\widehat{N}_2 \curlywedge \widehat{N}_3) \preccurlyeq (\widehat{N}_1 \curlywedge \widehat{N}_2) \curlywedge \widehat{N}_3$. Then $\widehat{N}_1 \curlywedge (\widehat{N}_2 \curlywedge \widehat{N}_3) = (\widehat{N}_1 \curlywedge \widehat{N}_2) \curlywedge \widehat{N}_3$.
By the same way , the proof of $\widehat{N}_1 \curlyvee (\widehat{N}_2 \curlyvee \widehat{N}_3) = (\widehat{N}_1 \curlyvee \widehat{N}_2) \curlyvee \widehat{N}_3$ can be made.
Finally , can be made:
$\widehat{N}_1 \curlywedge (\widehat{N}_1 \curlyvee \widehat{N}_2) = \widehat{N}_1$ and $\widehat{N}_1 \curlyvee (\widehat{N}_1 \curlywedge \widehat{N}_2) = \widehat{N}_1$.

References:

[1] Molodtsov D. , (1999) , " **Soft set theory - first results** " , Comput. Math. Appl. Vol. 37 , pp. 19-31.

[2] Maji K. , Biswas R. & Roy R. , (2003) , " **Soft Set Theory** " , Comput. Math. Appl. Vol. 45 , pp. 555-562.

[3] Aktas H. and Cagman N. , (2007) , " **Soft sets and soft groups** " , Inf. Sci. , Vol. 177 , pp. 2726-2735.

[4] Babitha K. and Sunil J. , (2010) , " **Soft set relations and functions** " , Comput. Math. Appl. Vol. 60 No. 7 , pp. 1840-1849.

[5] Aslam M. and Qurashi S. , (2012) , " **Some contributions to soft groups** " , Ann. of fuzzy Math. and Inf. , Vol. 4 , pp. 177-195.

[6] Feng F. , Irfan M. and Muhammed , (2013) , " **Soft relations applied to semigroup**" , Filomat , Vol. 27 , No. 7 , pp. 1183-1196.

[7] Bekir T. & Gozde Y. , (2014) , " **New structure on partially ordered soft sets and soft scott topology** " , Ann. fuzzy math. inf. , Vol. 7 , No. 1, pp. 89-97.

[8] Faruk Karaaslan , Cagman N. and Serdar N. , (2012) , " **Soft Lattices** " , Journal of new results in science , Vol. 1. pp. 5-17.

[9] Salama A. , (2013) , " **Neutrosophic Crisp Point & Neutrosophic Crisp Ideals** " , Neutrosophic Sets and System , Vol. 1 , No. 1 , pp. 50 – 54.

[10] Salama A. Said B. and Samarndache F. , (2014) , "**Neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals** " , I. J. Inf. Eng. And Elect. Business , Vol. 3 , pp. 1-8 .

[11] Salama A. , Smarandache F. and Valeri K. , (2014) , " **Neutrosophic Crisp Sets And Neutrosophic Crisp Topological Spaces** " , Neutrosophic Sets and Systems , Vol. 2, pp. 25-30.

[12] Salama A. and Hewayda E. , (2014) , " **Neutrosophic crisp set and relations**" , Neutrosophic sets & Systems , Vol. 6 , pp. 12-16.

[13] Salama A. , Said B. and Samarandache F. , (2014), " **Some types of neutrosophic crisp set and relations**" , Int. J. Inf. Engineering and Electronic Business , Vol. 9 , pp. 1-9.

[14] Mandal D. , (2014) , " **Neutrosophic soft semi ring** " , Ann. of fuzzy math. Inf. Vol. 1 pp. 1-7.