State Reconstruction in Sobolev Space

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Abstract:

This paper is devoted on the reconstruction of the state for a class of distributed parameter systems in Sobolev space. The crossing method from internal reconstruction to boundary will be explained and discussed such that many results will be shown in the evolution of the state to the Sobolev space .

Keywords: Γ_E- state reconstruction, Sobolev space, crossing method, and Γ_E-observability.

1. Introduction

Many works on distributed parameter systems (DPSs) have been devoted to the state reconstruction problem [1-2]. It has often been studied independently of any geometric considerations, and most of the works were focused on the observation and state reconstruction in a certain observation space. The notion of sensors and actuators introduced in the 1980s by El Jai and Pritchard allows for a better description of measurements and actions see [3]. In addition, the study of observability and controllability can be considered with respect to the structure, number and location of sensors and actuators [4-5].

For a distributed parameter system evolving on a spatial domain $\Omega \subset R^n$, the notion of regional observability concerns the reconstruction of the initial state on a sub region of Ω. Characterization results and approaches for the reconstruction of regional state are given in [6-7]. Similar results were developed for the state on boundary region of the boundary of Ω. This led to the so called regional boundary observability and concerns the possibility to reconstruct the state on a boundary subregion Γ without the knowledge of the system state [8-9].

The purpose of this paper, is to crossing the state reconstruction from the space $L^2(\omega)$ into $H^1(\omega)$ and then into the space $H^1(\Gamma)$ that is to estimate the current state of consider system exponentially on boundary sub region Γ of the boundary $\partial\Omega$, in Sobolev space of order one $H^1(\Gamma)$. The results are considered in a particular case of parabolic systems.

The principal reason behind introducing this concept is that it provides a means to deal with some physical problems concern the determination of laminar flux conditions, developed in steady state by vertical uniformly heated plate (see figure 1).

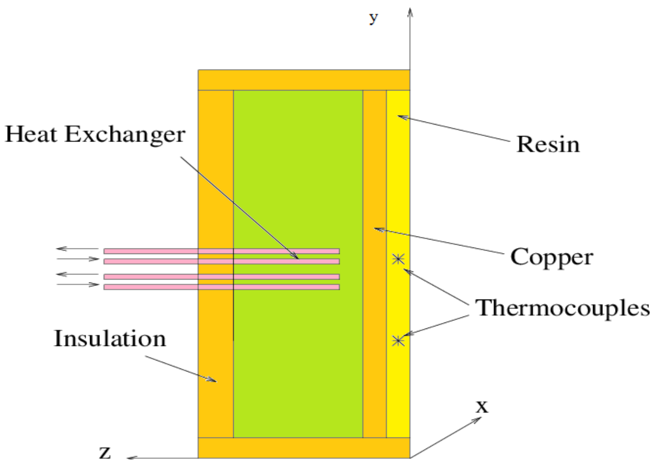

Figure1. Profile of active plate

This reconstruction can be used to find the unknown boundary convective condition on the front face of the active plate, as in [10]. The rebuilding of the state is focused on the knowledge of the dynamical system and the measurements given by internal pointwise sensors (that means by the thermocouples in case of active plate).

This paper is organized as follows: section one is concerned on preliminaries and the problem formulation. In the next section, the characterization notion of exponential observation on a critical bounded region is given by using of optimal sensors and the exponential reconstruction of the state on Γ will be explained and analyzed .

2. Problem statement

Let Ω be a regular, bounded and open set of $\mathbb{R}^{\boldsymbol{n}}$, with boundary $\boldsymbol{\partial\Omega}$. Let Γ be a nonempty given sub-region of $\boldsymbol{\partial\Omega}$, with positive measurement.

We denote $\boldsymbol{Q} = \Omega \times\,]0,\infty[$ and $\boldsymbol{\Sigma} = \boldsymbol{\partial\Omega} \times\,]0,\infty[$. We consider the system described by the following parabolic partial deferential equation:

$$\begin{cases} \frac{\partial z}{\partial t}(\mu,t) & = Az(\mu,t) + Bu(t) & Q \\ z(\mu,0) & = z_0(\mu) & \bar{\Omega} \\ \frac{\partial z}{\partial v}(\eta,t) & = 0 & \Sigma \end{cases} \qquad (1)$$

with the output function

$$y(.,t) = Cz(.,t) \qquad (2)$$

where $\bar{\Omega}$ holds for the closure of Ω and $z_0(\mu)$ is supposed to be unknown in $Z = H^1(\bar{\Omega})$. The system (1) is defined with a Neumann condition, $\partial z/\partial v_A$ holds for outward normal derivative.

Suppose that Z,U and $\mathcal{O}$ be separable spaces where Z is the space of the state U is the control space and $\mathcal{O}$ is the observability space. we consider $U = L^2(0,\infty, \mathbb{R}^p)$ and $\mathcal{O} = L^2(0,\infty, \mathbb{R}^q)$ where p and q hold for the number of controls and sensors [13].

A is a second order linear differential operator which generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on Z and is self-adjoint with compact resolvent.

The translator $B \in L(\mathbb{R}^p, Z)$ and $C \in L(H^1(\bar{\Omega}),, \mathbb{R}^q)$ depend on the structure of actuators and sensors [13, 15]. Under the given assumptions, the system (3.1) has a unique solution [12] given by:

$$z(t) = S_A(t)z_0 + \int_0^t S_A(t - \tau)Bu(\tau)d\tau \qquad (3)$$

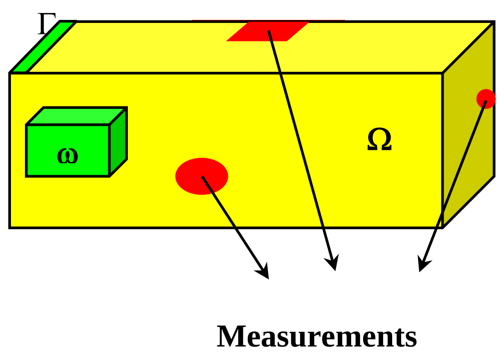

Figure 7. Domain Ω, regions ω, Γ and locations of measurements.

The measurements can be obtained by the use of zone, pointwise or lines sensors which may be located in Ω (or $\partial\Omega$), (see figure 7) [5, 9,11].

- Let us recall that a sensor by any couple (D, f) where D denote closed subsets of $\overline{\Omega}$, which is spatial support of sensor and $f \in L^2(D)$ defines the spatial distribution of measurement on D.

- According to the choice of the parameters D and f, we have various types of sensors. A sensor may be a zone type when $D \subset \Omega$. The output function (3.2) can be written in the form:

$$y(t) = Cz(.,t) = \int_D z(\mu,t) f(\mu) d\mu \qquad (4)$$

- A sensor may also be a pointwise when $D = \{b\}$ and $f = \delta(. - b)$ where δ is the Dirac mass concentrated in b. Then, the output function (3.2) may be given by the form:

$$y(t) = Cz(.,t) = \int_\Omega z(\mu,t)\delta_b(\mu - b) d\mu \qquad (5)$$

- In the case of boundary zone sen sor, we consider $D = \Gamma$ with the $\Gamma \subset \partial\Omega$ and $f \in L^2(\Gamma)$. The output function (3.2) can then be written in the form:

$$y(t) = C\,z(.,t) = \int_\Gamma z(\eta,t) f(\eta) d\eta \qquad (6)$$

The operator C is unbounded and some precautions must be taken in [5, 9,11].

- The formulation of translator K is :

$$K : H^1(\Omega) \to L^2(0,\infty, R^q)$$
$$z \quad \to CS_A(t)\, z$$

and in the case of internal zone sensors is, linear and bounded with an adjoint

$$K^* \quad : L^2(0,\infty,\mathbb{R}^q) \to H^1(\Omega)$$

$$y^* \to \int_0^t S_A^*(\tau) C^* y^*(.,t) d\tau$$

- The zero order trace translator

$$\gamma_0 : H^1(\Omega) \to H^{1/2}(\partial\Omega)$$

is linear, surjective and continuous with adjoint denoted by γ_0^*.

- let a critical region Γ of the boundary and

$$\chi_\Gamma : H^{1/2}(\partial\Omega) \to H^{1/2}(\Gamma)$$
$$z \quad \to \chi_\Gamma z = z$$

such that $z_{|\Gamma}$ is the constraint of the state z to Γ. We denote by χ_Γ^* the adjoint of χ_Γ .

$$\chi_\omega : H^1(\Omega) \to H^1(\omega)$$
$$z \quad \to \chi_\omega z = z_{|\omega}$$

- Let χ_ω be the function defined by

Such that $z_{|\omega}$ is the constraint of the state z to ω.

2.1 Definition and characterization

In this subsection well crossing the notions related to state reconstruction from the space $L^2(\omega)$ into the space $H^1(\omega)$ and well do the necessary changes.

Def. 2.2:[1] Consider the system (1)-(2) together with the dynamical system:

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = F_\omega w(\mu,t) + G_\omega u(t) + H_\omega y(t) & Q \\ w(\mu,0) = w_0(\mu) & \overline{\Omega} \\ \frac{\partial w}{\partial v}(\eta,t) = 0 & \Sigma \end{cases} \quad 7$$

where F_ω generates a strongly continuous semi-group $(S_{F_\omega}(t))t \geq 0$ which is stable on Hilbert space W, $G_\omega \in L(\mathbb{R}^p, W)$ and $H_\omega \in L(W, \mathbb{R}^q)$. The system (7) defines an ω_E-estimator for $\chi_\omega Tz(\mu,t)$if:

1- $\lim_{t\to\infty} \|w(.,t) - \chi_\omega Tz(\mu,t)\|_{H^1(\omega)} = 0$

2- $\chi_\omega T$ Maps D (A) into D (F_ω) such that $w(\mu,t)$ the solution of the dynamic system (7).

Def. 2.3:[13] The dynamical system (7) observes the current regional state of the system (1)-(2) if the following conditions hold:

- there exist $M_\omega \in L(\mathbb{R}^q, H^1(\omega))$ and $N_\omega \in L(H^1(\omega))$ such that:

$$M_\omega C + N_\omega \chi_\omega T = I_\omega,$$

- $\chi_\omega TA + F_\omega \chi_\omega T = H_\omega C$ and $G_\omega = \chi_\omega TB$,
- The dynamical system (7) defines an ω_E-estimator for the state of the system (1).

Rmark 2.4: If Z = W and $\chi_\omega T = I_\omega$ in definition 2.3, then we have

$F_\omega = (A - H_\omega C)$ and $G_\omega = B$.

Thus, In this case, the dynamical system (7) becomes:

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = Aw(\mu,t) + Bu(t) \\ \quad + H_\omega(Cw(\mu,t) - y(.,t)) & Q \\ w(\mu,0) = w_0(\mu) & \overline{\Omega} \\ \frac{\partial w}{\partial v}(\eta,t) = 0 & \Sigma \end{cases} \quad (8)$$

Now, from the previous definitions and characterizations, we can obtain the following results

Lemma 2.5:

(1) The system (1)-(2) is ω_E-observable if there exists an ω_E-estimator (8) which construct the regional state of this system exponentially.

(2) The suite of sensors $(D_i, f_i)_{1\leq i\leq q}$ is said to be regional exponential strategic (or ω_E- strategic) if the consider system is ω_E-observable.

2.6 Regional boundary exponential detectability

The main reason for introducing Γ-detectability is, the possibility to construct an Γ-estimator for the current state of the original system. This concept is extended by Al-Saphory and El Jai *et al.* as in ref.s [2-5, 7-10, 12] to the asymptotic case. For this objective we develop some definitions devoted this concept to the boundary regional exponential case. In this subsection well crossing the notions related to state reconstruction from the space $H^1(\omega)$ into the space $H^{1/2}(\partial\Omega)$ and then into $H^{1/2}(\Gamma)$ with do the necessary changes.

Def. 2.7[14]: The operator of semi-group $(S_A(t))t \geq 0$ is exponential stable in $H^{1/2}(\partial\Omega)$ if, for some positive $F_{\partial\Omega}$ and $\sigma_{\partial\Omega}$, then

$$\|\gamma_0 S_A(.)\|_{L(H^{1/2}(\partial\Omega),\ Z))} \leq F_{\partial\Omega}\, e^{-\sigma_{\partial\Omega} t}, t \geq 0$$

Remark 2.8: If $(S_A(t))_{t\geq 0}$ is stable semi-group on $H^{1/2}(\partial\Omega)$, then for all $z_0 \in H^{1/2}(\partial\Omega)$, the solution of the associated autonomous to system (1) satisfies

$$\lim_{t\to\infty}\|\gamma_0 z(.,t)\|_{H^{1/2}(\partial\Omega)} = \lim_{t\to\infty}\|\gamma_0 S_A(t) z_0(.)\|_{H^{1/2}(\partial\Omega)} = 0 \qquad (9)$$

Def. 2.9 [14]: The system (1) is boundary exponentially stable on $H^{1/2}(\partial\Omega)$ (or $\partial\Omega_E$ - stable) if the operator A generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ which is $\partial\Omega_E$ -stable.

Remark 2.10: If the system (1) is $\partial\Omega_E$ - stable that is the solution of autonomous system associated with (1)-(2), converges exponentially to zero when t tends to ∞.

Def. 2.11: The system (1)-(2) is boundary exponentially detectable on $H^{1/2}(\partial\Omega)$ (or $\partial\Omega_E$ - detectable) if there exist an operator $H_{\partial\Omega}: \mathcal{O} \to H^{1/2}(\partial\Omega)$, such that the operator $(A - H_{\partial\Omega}C)$ generates a strongly semi-group and continuous $(S_{H_{\partial\Omega}}(t))_{t\geq 0}$ which is $\partial\Omega_E$ - stability.

Lemma 2.12: If a system is $\partial\Omega_E$ - detectable, then it is possible to construct an exponential $\partial\Omega$ -estimator ($\partial\Omega_E$ -estimator) for the original system.

Now, we consider the following system

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = Aw(\mu,t) + Bu(t) \\ \quad +H_{\partial\Omega}(Cw(\mu,t) - y(.,t))\ \mathcal{Q} \\ w(\mu,0) = w_0(\mu) & \overline{\Omega} \\ \frac{\partial w}{\partial \nu}(\eta,t) = 0 & \Sigma \end{cases} \qquad (10)$$

then $w(\mu,t)$ estimates exponentially the state $z(\mu,t)$ because the error $e(\mu,t)$ satisfies

$$\frac{\partial e}{\partial t}(\mu,t) = (A - H_{\partial\Omega}C)\, e(\mu,t)$$

with

$$e(\mu,t) = z(\mu,t) - w(\mu,t).$$

Then, if the system is $\partial\Omega$ -detectable, it is possible to choose $H_{\partial\Omega}$ which realizes

$$\lim_{t\to\infty} \|e(.,t)\|_{H^{1/2}(\partial\Omega)} = 0 .$$

Def. 2.13[1]: The semi-group $(S_A(t))t \geq 0$ is regionally boundary exponentially stable in $H^{1/2}(\Gamma)$ (or Γ_E-stable) if, for some positive constants F_Γ and σ_Γ, then

$$\|\chi_\Gamma \gamma_0 S_A(.)\|_{L(H^{1/2}(\Gamma),\, Z)} \leq F_\Gamma\, e^{-\sigma_\Gamma t}, t \geq 0$$

In this work, we only need the relation (9) to be true on a given sub-region Γ of the region $\partial\Omega$ in the following result.

Remark 2.14: If the semi-group$(S_A(t))_{t\geq 0}$ is Γ_E-stable on $H^{1/2}(\Gamma)$, then for all $z_0 \in H^{1/2}(\Gamma)$, the solution associated with the autonomous system of (1) satisfies

$$\mathrm{Lim}_{t\to\infty} \|\chi_\Gamma \gamma_0 z(.,t)\|_{H^{1/2}(\Gamma)} =$$

$$\lim_{t\to\infty} \|\chi_\Gamma \gamma_0 S_A(t) z_0(.)\|_{H^{1/2}(\Gamma)} = 0$$

(11)

Def. 2.15[1]: The system (1) is said to be regionally boundary exponentially stable on Γ (or Γ_E-stable), if the operator A generates a semi-group which is exponentially stable on the space $H^{1/2}(\Gamma)$.

Def. 2.16: The system (3.1)-(3.2) is said to be regionally boundary exponentially detectable on Γ (or Γ_E-detectable) if there exists an operator

$H_\Gamma : \mathbb{R}^q \to H^{1/2}(\Gamma)$ such that $(A - H_\Gamma C)$,

generates a strongly continuous semi-group$\left(S_{H_\Gamma}(t)\right)_{t\geq 0}$ which is Γ_E-stable

However, one can deduce the following results :

Corollary 2.17[14]: If the system (1) together with output function (2) is exactly Γ-observable, then it is Γ_E-detectable.

This result leads to : $\exists \gamma > 0$ such that

$$\|\chi_\Gamma \gamma_0 S_A z_0(.)\|_{H^{1/2}(\Gamma)} \leq \gamma \|C S_A z_0(.)\|_{L^2(0,\infty,O)}, \forall z_0 \in H^{1/2}(\Gamma)$$

Thus, the notion of Γ_E-detectability is a weaker property than the exact Γ-observability as in (ref.s [1-6]).

3.4 Regional boundary exponential reconstruction

In this section, we explore an approach which allows to characterize a regional exponential estimator of $z(\mu,t)$ on Γ region, based on the internal exponential ω-estimator as in [2, 7].

3.1 Definition and characterization

We consider the system (3.1) and the output function (3.2) specified by:

$$\begin{cases} \frac{\partial z}{\partial t}(\mu,t) = Az(\mu,t)+Bu(t) & Q \\ z(\mu,0) = z_0(\mu) & \bar{\Omega} \\ \frac{\partial z}{\partial v}(\eta,t) = 0 & \Sigma \\ y(.,t) = Cz(.,t) & Q \end{cases} \quad (12)$$

and the system

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = L_\Gamma w(\mu,t)+G_\Gamma u(t) \\ \quad +H_\Gamma y(t) & Q \\ w(\mu,0) = w_0(\mu) & \bar{\Omega} \\ \frac{\partial w}{\partial v}(\eta,t) = 0 & \Sigma \end{cases} \quad (13)$$

where $\boldsymbol{L}_\Gamma$ generates a strongly continuous semi-group $(S_{L_\Gamma}(t))_{t\geq 0}$ which is Γ_E-stable on the state space W, $G_\Gamma \in L(\mathbb{R}^p, W)$ and $H_\Gamma \in L(\mathbb{R}^q, W)$. The system (13) defines an Γ_E-estimater for $T_\Gamma z(\mu,t)$ if

1. $\lim_{t\to\infty} \|w(.,t) - T_\Gamma z(.,t)\|_{H^{1/2}(\Gamma).} = 0$
2. T_Γ maps $D(A)$ into $D(L_\Gamma)$ where $T_\Gamma = \chi_\Gamma \gamma_0 T$ and $w(.,t)$ is the solution of system (13).

This subsection is focused on some concepts which are related to the notion of regional boundary exponential observation

Def. 3.2: The dynamical system (13) observes the current regional boundary state of the system (3.12) if the following conditions hold:

- There exists operators $M_\Gamma \in L\left(\mathcal{O}, H^{1/2}(\Gamma)\right)$ and $N_\Gamma \in L\left(H^{1/2}(\Gamma)\right)$ such that: $M_\Gamma C + N_\Gamma T_\Gamma = I_\Gamma$
- $T_\Gamma A + L_\Gamma T_\Gamma = H_\Gamma C$ and $G_\Gamma = T_\Gamma B$
- The system (3.13) defines an Γ_E-estimator for the system (12).

Corollary 3.3: If $Z = W$ and $T_\Gamma = I_\Gamma$ then, in the above case, we have

$$L_\Gamma = A - H_\Gamma C, \text{and } G_\Gamma = B$$

and then, the dynamic system (3.13) become:

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = Aw(\mu,t)+Bu(t) + \\ H_\Gamma(Cw(\mu,t) - y(\mu,t)) & Q \\ w(\mu,0) = 0 & \bar{\Omega} \\ \frac{\partial w}{\partial v}(\eta,t) = 0 & \Sigma \end{cases} \quad (14)$$

From the previous result we can extend the following definitions

Def. 3.4: The system (12) is Γ_E-observable if there exists an Γ_E- estimator (.13) which observe the regional boundary state of this system exponentially.

Def. 3.5: The suite of sensors $(D_i, f_i)_{1\leq i\leq q}$ is said to be regional exponential strategic (or Γ_E- strategic) if the considered system (12) is Γ_E-observable.

3.6 Sensor and Γ_E - reconstruction method

This subsection is related to the reconstruction problem which consists of estimate the current state of the consider system exponentially in a given boundary sub region Γ of $\partial\Omega$.

- ***Crossing method from internal to boundary case***

The regional boundary exponential observability in Γ (or Γ_E -observability) may be seen as internal regional exponential observability in $\omega_r \subset \Omega$ (or ω_{r_E} -observability) in this case, we consider the following extension operator (see [10]):

$$\Re : H^{1/2}(\partial\Omega) \longrightarrow H^1(\Omega)$$

which is defined by:

$$\gamma_0 \Re h(\mu,t) = h(\mu,t),\ \forall h \in H^{1/2}(\partial\Omega) \quad (15)$$

- Let $r > 0$ is an arbitrary and let

$$E = \bigcup_{z\in\Gamma} B(z,r) \text{ and } \omega_r = E \cap \Omega \quad (16)$$

where $B(z,r)$ is the ball of radius r centered in $z(\mu,t)$ and Γ is a part of $\overline{\omega}_r$ (see figure 8).

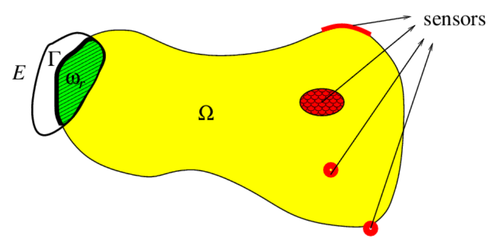

Figure 8: The domain Ω, sub-domain ω_r and the region Γ.

Now, the method of crossing from internal ω_{r_E} -detectability into Γ_E -detectability [9] will be given in the following proposition.

Proposition 3.7:

The system (12) is Γ_E -detectable If it is $\overline{\omega}_{r_E}$-detectable .

Proof: See [14]

During this subsection, we present method which allows the observation of the current state $z(.,t)$ on Γ based on internal regional Γ_E -observability. Next, the following proposition shows that $\overline{\omega}_{r_E}$ -observability leads to Γ_E -observability.

Proposition 3.8: If the system (12) is $\overline{\omega}_{r_E}$-observability, then it is Γ_E -observability.

- ***The Sufficient condition for Γ_E - rebuilding***

In the following, we provide an approach which estimates the current state $z(\mu,t)$ of the original system (12) exponentially.

Theorem 3.9:

Suppose that the sequence of sensors $(b_i, \delta_{b_i})_{1\le i\le q}$ is Γ-strategic and the spectrum of A contain J eigenvalues with non-negative real parts. Then (3.12) is Γ_E- observable by the dynamical system

$$\begin{cases} \frac{\partial w}{\partial t}(\mu,t) = Aw(\mu,t) + Bu(t) - \\ H_\Gamma(Cw(\mu,t) - y(\mu,t)) & Q \\ w(\mu,0) = w_0(\mu) & \bar{\Omega} \\ \frac{\partial w}{\partial \nu}(\mu,t) = 0 & \Sigma \end{cases} \quad (18)$$

Proof: see [1]

Def. 3.10:

The suite of sensors $(b_i, \delta_{bi})_{1\le i\le q}$ is said to be Γ_E-strategic if the corresponding system is Γ_E- observable.

Remark 3.11:

The important case in this subsection is that a state which is not exponential observability in the usual sense may be exponential observability on Γ, this is illustrated by the following counter example.

▪ ***Example* 3.12:**

Consider the case of two dimensional distributed parameter diffusion system defined in $\Omega =]0,1[\times]0,1[$ and described by the parabolic equation

$$\begin{cases} \frac{\partial z}{\partial t}(\mu_1,\mu_2,t) = \frac{\partial^2 z}{\partial \mu_1^2}(\mu_1,\mu_2,t) + \\ \frac{\partial^2 z}{\partial \mu_2^2}(\mu_1,\mu_2,t) + z(\mu_1,\mu_2,t) & Q \\ z(\mu_1,\mu_2,0) = z_0(\mu_1,\mu_2) & \bar{\Omega} \\ \frac{\partial z}{\partial \nu}(\mu_1,\mu_2,t) = 0 & \Sigma \end{cases} \quad (21)$$

the above system represents the heat-conduction problem. The measurement is given via boundary zone sensor (Γ_0, f) defined by $\Gamma_0 = \{0\} \times]0,1[$ and $f(\eta_1,\eta_2) = \cos\pi\eta_2$ (as in figure 9). Thus the augmented output function may be written in the form

$$y(t,.) = \int_{\Gamma_0} z(\eta_1,\eta_2,t) f(\eta_1,\eta_2)\, d\eta_1\, d\eta_2 \quad (22)$$

The operator $\left(\frac{\partial^2}{\partial \mu_1^2} + \frac{\partial^2}{\partial \mu_2^2} + 1\right)$ generates a strongly continuous semi-group on the state space $H^1(\Omega)$ given by the form:

$$S_A(t)z = \sum_{n,m=0}^{\infty} e^{\lambda_{nm}} t < z, \varphi_{nm} >_{H^1(\Omega)} \varphi_{nm}$$

such that:

$\lambda_{nm} = -(n^2+m^2)\pi^2$, $\varphi_{nm}(\mu_1,\mu_2) = 2a_{nm}\cos(n\pi\mu_1)\cos(m\pi\mu_2)$ and $2a_{nm} = (1-\lambda_{nm})^{-1/2}$.

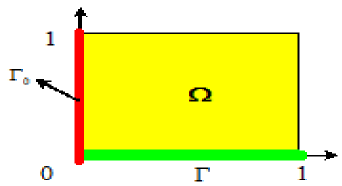

Figure 9: Domain Ω, region Γ and location Γ_0 of boundary zone sensor.

Consider now, the dynamical system

$$\begin{cases} \frac{\partial w}{\partial t}(\mu_1,\mu_2,t) = \frac{\partial^2 w}{\partial \mu_1^2}(\mu_1,\mu_2,t) + \frac{\partial^2 w}{\partial \mu_2^2}(\mu_1,\mu_2,t) + w(\mu_1,\mu_2,t) \\ \quad +HC(w(\mu_1,\mu_2,t) - z(\mu_1,\mu_2,t) & Q \\ w(\mu_1,\mu_2,0) = w_0(\mu_1,\mu_2) & \bar{\Omega} \\ \frac{\partial w}{\partial v}(\mu_1,\mu_2,t) = 0 & \Sigma \end{cases} \quad (23)$$

where $H \in L(\mathcal{O}, W)$, W is the state space of the above system. If the state z_0 is defined in Ω by $z_0(\mu_1,\mu_2) = \cos(\pi\mu_1)\cos(2\pi\mu_2)$, then the system (21)- (22) is not weakly observable in Ω, i.e. the sensor (Γ_0, f) is not strategic and therefore the system (21)- (22) is not exponentially detectable in Ω. Thus, the dynamical system (23) is not observe exponentially the system (21)- (22) (see [24-26]). Here, we consider the region $\Gamma =]0,1[\times\{0\} \subset \partial\Omega$ and the dynamical system

$$\begin{cases} \frac{\partial w}{\partial t}(\mu_1,\mu_2,t) = \frac{\partial^2 w}{\partial \mu_1^2}(\mu_1,\mu_2,t) + \frac{\partial^2 w}{\partial \mu_2^2}(\mu_1,\mu_2,t) + w(\mu_1,\mu_2,t) \\ \quad +H_\Gamma C(w(\mu_1,\mu_2,t) - z(\mu_1,\mu_2,t) & Q \quad (24) \\ w(\mu_1,\mu_2,0) = w_0(\mu_1,\mu_2) & \bar{\Omega} \\ \frac{\partial w}{\partial v}(\mu_1,\mu_2,t) = 0 & \Sigma \end{cases}$$

where $H_\Gamma \in L(\mathcal{O}, H^{1/2}(\Gamma))$. In this case, the system (21)-(22) is weakly observable in Γ and the sensor (Γ_0, f) is Γ-strategic [9]. Thus, the system (21)-(22) is Γ-detectable [12]. Finally the dynamical system (3.24) is observing exponentially the system (21)-(22) i.e., this system is Γ_E- observable [9].

Remark 3.13: The results in counter example can be extend to the case of internal (zone or pointwise sensor) and boundary pointwise.

Conclusion

We developed in this paper, an estimator which can be reconstruct the current state of the original system exponentially in a critical boundary region Γ of the boundary of the whole domain Ω in Sobolev space of order one, and the concepts which are related to the exponential regional boundary reconstruction are given in connection with the strategic sensors. Many questions still opened. This is the case of, for example, the problem of reconstruction the state of non-linear systems exponentially on a considered sub region.

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