On Almost Countably Compact of Bitopological Spaces

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Abstract

In this paper we introduce the notion of almost countably compact spaces in bitopological spaces , give new definitions , and study some of the important properties related to this concept , the relationship between almost countably compact spaces and countably compact spaces.

Key words, compactness, p- compactness, countably p- compact of bitopological space, almost countably p- compact of bitopological space, p- regular space

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introduction

endowed with two topological structures given on it and denoted by (X , τ_1 , τ_2), generally not at all related to one another, the concepts being pairwise Hausdorff , pairwise regularity , pairwise normality , and Separation axioms in bitopological spaces

Introduced by Kelly also, many papers

Devoted to bitopological spaces have been published at the present time, the overwhelming majority of them contain generalizations of various concepts and assertions of the theory of topological spaces to bitopological spaces in the sense of Kelly, for more details see [5]. Fletcher Hoyle and Patty provided an early definition of bitopological compactness , see [6] , then many authors studied the topological properties such as connectedness , compactness , and others topological properties , for more details see [7] . Singal , defined pairwise countably compact [8] , in this paper we study and introduce the notions , countably compact and almost countably compact in bitopological spaces depending on Fletcher definition of compactness and investigate the relationship between almost countably compact and countably compact of bitopological spaces , study some of the important properties

Recall that a topological space X is said to be compact if every open covering μ of X contains a finite subcollection that also covers X. A space X is said to be countably compact if every countable open covering of X contains a finite subcollection that covers X. And for a T_1 space X, countable compactness is equivalent to limit point compactness, see Munkres p178-181, [1]. By the definition of the countably compact space it is clear that every compact space is countably compact .As a generalization of countable compactness, Bonanzinga , Matveev and Pareek , [2] defined a topological space (X, τ) to be almost countably compact , if for every countable open cover μ of X , there exists a finite subcollection O of μ such that $\cup\{\bar{o} : o \in O\} = X$ .Clearly, every countably compact topological space is almost countably compact, but the converse does not hold (see Examples 2.3 and 2.4), [3]. In 1963 the study of bitopological spaces started with the paper of Kelly [4] , Indeed , in this fundamental paper the concept of bitopological space itself is clearly formulated as a non empty set X

1-2. Preliminaries

topology i . The cardinality of a set A is denoted by |A|. We mean by ω , ω_1 , c , p- , the first infinite cardinal , the first uncountable cardinal , the cardinality of the set of all real numbers , and pairwise , respectively . For a bitopological space (X,τ_1, τ_2) and a subset $A \subset X$, an induced topology τ_i (A) is defined as $\tau_i(A) = A \cap \tau_i$, $i \in \{1,2\}$

Throughout this paper , all spaces (X , τ) , (X , τ_1 , τ_2) (or simply *X*) are always meant as topological spaces and bitopological spaces, respectively unless explicitly stated. By *i*-open set, we shall means the open set with respect to topology τ_i in X, open cover of X, means that the cover of *X* by *i*-open sets in *X;* By *i*-int(A) and *i*-cl(A) we shall mean the interior and the closure of a subset A of X with respect to

2- Countably p-compact and almost countably p-compact

open if $\mu \subset \tau_1 \cup \tau_2$ and μ contains at least one non- empty member of τ_1 and at least one- non empty member of τ_2 . [6]

space (X , τ_1 , τ_2) is defined to be pairwise compact [6] .

pairwise compact which is differing from Singal and Singal definitions [8].

each countable pairwise open cover μ of (X , τ_1 , τ_2) has a finite subcover .

Definition 2-1. A cover μ of a bitopological space (X, τ_1, τ_2) is defined to be pairwise

Definition 2-2. If each pairwise open cover of (X , τ_1 , τ_2) has a finite subcover , then the

With the sense of Reilly of definition 'Pairwise Lindelof bitopological spaces', [9], we introduce the definition of countably

Definition 2-3. A bitopological space (X, τ_1 , τ_2) is said to be countably pairwise compact if

$\in O \cap \tau_i$, for $i \in \{1,2\}$ = X (i- ō the closure of a subset i- o) . Directly from the definition it can be concluded the following proposition

subcover O of μ such that X = ∪{i-o} ⊂ ∪ {i- ō } for i = {1,2 } and so ∪ {i- ō }also covers X , that is X = ∪ {i- ō } but ∪ {i- ō } ⊂ X , then ∪ {i- ō : i- o $\in O \cap \tau_i$, for $i \in \{1,2\}$} = X .

exists a finite subcollection O of μ such that D ⊆ ∪ {i- ō : i- o ∈ O } , since D is relatively countably pairwise compact in X .But D is i-dense in X, that is *i*-cl (D) = X for i = 1,2 so X ⊆ ∪ {i- ō : i- o ∈ O } , then X = ∪ {i- ō : i- o ∈ O } which completes the proof .

not be countably pairwise compact, see also example 2-4 [3].
compact, since R is a discrete closed in X, and X is almost countably pairwise compact, since ω is i-dense in X and every infinite subset of ω has a limit point in X (Recall from [10] that a subspace Y of a space X is relatively countably compact in X if every infinite subset of Y has a limit point in X), which completes the proof.

τ_2) is said to be pairwise regular if τ_1-is regular with respect to τ_2 and τ_2 is regular with respect to τ_1, [4], Kelly .With the manner of Kelly the following definition is equivalents

of X containing x, there exists an j-open set U such that x ∈ U ⊆ i-cl (U) ⊆ V , X is said to be pairwise regular (or briefly, p-regular) if it is both i-regular and j-regular or (i , j) regular . See also, [5].

$\tau_1 \neq \varphi$, $\mu \cap \tau_2 \neq \varphi$) has a countable subcover. Reilly, 1973, [9].

space is compact, so we can conclude the following proposition ,
A be any pairwise open cover of X. For each x ∈ X, there exists an $A_x \in A$
such that $x \in A_x$ and there exists an open set B_X of x such that $x \in B_X \subseteq$ i - cl $B_X \subseteq A_x$, let B = { B_X : x∈ X } then B is an open cover of

Definition 2-4. A bitopological space (X , τ_1, τ_2) is said to be almost countably pairwise compact if for every countable pairwise open cover μ of X, there exists a finite subcollection O of μ such that ∪ {i- ō : i- o

Proposition 2-5. Every countably pairwise compact bitopological space (X, τ_1, τ_2) is almost countably pairwise compact.

Proof: Let (X , τ_1 , τ_2) be countably pairwise compact space , so for any countable pairwise open cover μ of X , there exist a finite

Definition 2-6. A subset A of a bitopological space (X , τ_1 , τ_2) is i- dense in X if *i*-cl (A) = X for i = 1,2 , [5] .

Proposition 2-7. Let D be an i-dense subset of (X , τ_1 , τ_2). If D is relatively countably pairwise compact in X, then X is almost countably pairwise compact.

Proof: Let μ be any countable pairwise open cover of X and let D be an i-dense subspace , relatively pairwise compact in X , then there

In the following example we explain that the almost countably pairwise compact space need

Example 2-8. The Isbell-Mr´owka space it is almost countably pairwise compact, but it is not countably pairwise compact.

Proof: Let X = ω ∪ R be the Isbell-Mr´owka space ($\tau_1 = \tau_2$) , where R is a maximal almost disjoint family of infinite subsets of ω such that |R| = c. Then X is not countably pairwise

Definition 2-9.In a space (X , τ_1 , τ_2), τ_1 is said to be regular with respect to τ_2 if for each x ∈ X and a τ_1-closed set *F* such that *x* ∉ F there exist a τ_1-open set *U* and a τ_2-open set *V* such that x ∈ *U, F* ⊆ V and U ∩ V = φ. (X, τ_1,

Definition 2-10. A bitopological space (X , τ_1 , τ_2) is said to be i-regular if for each point x ∈ X and for each i-open set V of X containing x, there exists an i-open set U such that x ∈ U ⊆ j-cl (U) ⊆ V . and is said to be j-regular if for each point x ∈ X and for each j-open set V

Definition 2-11. A space (X , τ_1 , τ_2) is said to be pairwise Lindelof (p-Lindelof) if each pairwise open cover μ , (i.e. $\mu \subseteq \tau_1 \cup \tau_2$, μ ∩

In a topological space we know that for every Lindelof and countably compact topological

Proposition 2-12. Every p- regular, almost countably pairwise compact space and p-Lindelof space is pairwise compact.

Proof: Let X be a p- regular almost countably pairwise compact space, p- Lindelof space and

1,2,…,m} ,since X is almost countably pairwise compact .clearly , { B_{Xnk} : k = 1,2,…,m} is a finite subcover of A which completes the proof .

homeomorphism) if the induced functions f: (X , τ_i) → (Y, Υ_i) are continuous (open , closed , homeomorphism) , [5]

Since X is almost countably pairwise compact, there exists a finite subset { n_i : i = 1 ,2 ,3, …, m } such that ∪{ cl ($f^{-1}(U_i)$) : i = 1 , 2 ,…, m } = X . Hence Y = f (X) = f (∪{ cl ($f^{-1}(U_i)$) : i = 1 , 2 ,…, m }) = ∪ {f cl ($f^{-1}(U_i)$) : i = 1 , 2 ,…, m } = ∪{(cl f ($f^{-1}(U_i)$) : i = 1 , 2 ,…, m } = ∪ { cl (U_i) : i = 1 , 2 ,…, m } , This shows that Y is almost countably compact .

each i-open subset u of Y and i = {1 , 2 } ,[5]

compact ,for each y∈Y .Then f is called a perfect map,[9].

that f^{-1}(y) is compact ,for each y∈Υ_i , i = 1, 2 Then f is called an i-perfect map . for every y ∈Y, there exists V_n ∈ $\mathcal{V}$ such that f^{-1}(y) ⊆ V_n . Since f^{-1}(y) is compact, then $w_n = \Upsilon_i - f(X_i - V_n)$ is an open neighborhood of y. Since Y is almost countably pairwise compact , then there exists a finite subfamily { w_{ni} : i = 1,2,…,m } of $\mathcal{W}$ such that Y= ∪ (cl $w_{ni\,,\, i\le m}$). Since f is almost open , then X = f^{-1}(Y) = f^{-1} ∪ cl $w_{ni\,,\, i\le m}$ ⊆ ∪ f^{-1}cl $w_{ni\,,\, i\le m}$ ⊆ ∪ cl f^{-1} $w_{ni\,,\, i\le m}$ ⊆ ∪ cl $V_{ni\,,\, i\le m}$ and since every element of $\mathcal{V}$ is the union of a finite subfamily of μ. This shows that X is almost countably pairwise compact, which completes the proof.

X hence B has a countable subcover, saying { B_{Xn} : n∈ ω }, since X is lindelof Thus { B_{Xn} : n∈ ω } has a finite subset { B_{Xnk} : k = 1,2,…,m} such that X = ∪{ i-cl B_{Xnk} : k =

Definition 2-13. Let (X, τ_1, τ_2) and (Y, Υ_1, Υ_2) be bitopological spaces. Then a function f: (X , τ_1 , τ_2) → (Y, Υ_1 ,Υ_2) is said to be i-continuous (i-open, i-closed , i-

Proposition 2-14. A continuous image of an almost countably pairwise compact space is almost countably pairwise compact.

Proof: Suppose that X is an almost countably pairwise compact space and let f: (X , τ_1 , τ_2) → (Y, Υ_1 ,Υ_2) an i-continuous Function . Let U = {U_n: n ∈ ω } be a countable pairwise open cover of Y. Then V = { $f^{-1}(U_n)$: U_n ∈ U } is a countable pairwise open cover of X.

Definition 2-15. A map f from a space (X , τ_1 , τ_2) to a space (Y, Υ_1 ,Υ_2) is called almost i-open function if i-f^{-1}(cl(u)) ⊆ cl(i-f^{-1}(u)) for

Definition 2-16. Let f : X → Y be a closed continuous surjective map such that f^{-1}(y) is

With the same sense we give the following definition,

Definition 2-17. Let f: (X , τ_1 , τ_2) → (Y, Υ_1 ,Υ_2) be an i-continuous surjective map such

Proposition 2-18. Let X, Y is two bitopological spaces, Y be an almost countably pairwise compact space and f : X → Y be an almost i-open and i-perfect mapping. Then X is almost countably pairwise compact.

Proof: Let μ be a countable pairwise open cover of X and let $\mathcal{V}$ = {V: there exists a finite subfamily Ƒ of μ such that V = ∪ Ƒ} Then $\mathcal{V}$ is countable, since μ is countable. Hence we can enumerate $\mathcal{V}$ as {V_n: n ∈ ω }. For each n ∈ ω ,let $w_n = \Upsilon_i - f(X_i - V_n)$, then w_n is an i-open subset of Υ_i for each i = 1,2 since f is i-closed . Let $\mathcal{W}$ = { w_n : n ∈ ω } , then $\mathcal{W}$ is a countable pairwise open cover of Y .In fact ,

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حول التراص العدي الغالب للفضاءات ثنائية التوبولوجيا

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العراق

الخلاصة

في هذه الورقة ادخلنا مفهوم التراص العدي الغالب على الفضاءات ثنائية التوبولوجيا واعطينا تعاريف جديدة ثم درسنا بعض الخواص المهمة المرتبطة بهذا المفهوم ،وايضا العلاقة بين الفضاءات المتراصة العدية ومفهوم الفضاءات المتراصة العدية غالبا

الكلمات المفتاحية – التراص ، التراص p ، التراص p العدي للفضاء ثنائي التوبولوجي ، التراص– p العدي الغالب للفضاءات ثنائية التوبولوجيا ، الفضاء– p المنتظم

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