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Centralizing Higher Left Centralizers On Prime Rings

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Abstract :
In this paper we study the commutativity of prime rings satisfying certain identities involving higher left centralizer on it .

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Key words: prime rings , higher left centralizer

1.Introduction :

Throughout this paper R is denote to an associative ring and it is center will denoted by $Z(R)$ which equal to the set of all elements $x \in R$ such that $xy = yx$ for all $y \in R$.

Now for any $x, y \in R$, the symbols $[x, y]$ and $\langle x, y\rangle$ will denoted to $xy - yx$ and $xy + yx$ respectively which are called commutator(Lie product) and anti- commutator (Jordan product) respectively .[1],[2] . A ring R is called commutative if $[x, y] = 0$ for all $x, y \in R$.

The above commutator and anti- commutator

satisfies the following[1] ,[2] :

1) $[xy, z] = [x, z]y + x[y, z]$
2) $[x, yz] = y[x, z] + [x, y]z$
3) $\langle x, yz\rangle = \langle x, y\rangle z - y[x, z]$
$= y\langle x, y\rangle + [x, y]z$
3) $\langle xy, z\rangle = x\langle y, z\rangle - [x, z]y$
$= \langle x, z\rangle y + x[y, z]$

A ring R is called prime if $xRy = \{0\}$ implies that $x = 0$ or $y = 0$ and it is called semi-prime if $xRx = \{0\}$ implies that $x = 0$[3] .

An additive mapping $F: R \to R$ is called centralizing on asubset S of ring R if $[F(x), x] \in Z(R)$ and it is called commuting if $[F(x), x] = 0$ for all $x \in S$[4], [5].

An additive mapping $T: R \to R$ is called left(right) centralizer on a ring R if $T(xy) = T(x)y$ ($T(xy) = xT(y)$) holds for all $x, y \in R$[6] .

Many authors covers the concept of left centralizer and study the relation between the commutativity of ring and left centralizers .

K.K.Dey and A.C. Paul in [7] study the commutativety of Γ- ring in which satisfying certain identities involving left centralizers .

In this paper , we obtain the commutativity of a ring satisfying certain identities involving higher left centralizers on ring R , this work motivated from the work of K.K.Dey and A.C.Paul [7] .

We generalized the definition of higher k- left centralizer on a Γ- ring [8] into a higher left centralizer on a ring R by taking k as the identity automorphism as the following

Definition 1.1 :let R be a ring and let $T = (T_i)_{i\in N}$ be a family of left centralizers on R . then $T_n: R \to R$ is called higher left centralizer on R if

$$T_n(xy) = \sum_{i=1}^{n} T_i(x)y$$

holds for all $x, y \in R$.

2. Commutativity of prime gamma rings : : in this section we study the commutativity of the ring R by using higher left centralizer on it .

Theorem 2.1 : let R be a prime ring and I be anon –zero ideal of R. suppose that R admits a family of non – zero higher left centeralizers T = $(T_i)_{i\in n}$ such that $\sum_{i=1}^{n} T_i(x) \neq x$ for all $x \in I$ and $i \in N$. if $T_n([x,y]) - [x,y] = 0$ for all x,y $\in I$ then R is commutative .

Proof : Given that T = $(T_i)_{i\in N}$ afamily of left centralizens of R such that.

$$T_n([x,y]) - [x,y] = 0 \quad \ldots\ldots\ (1)$$

for all x,y $\in I$.

Then $T_n(xy - yx) - (xy - yx) = 0$

So that

$$(\sum_{i=1}^{n} T_i(x)y - \sum_{i=1}^{n} T_i(y)x) - (xy - yx) = 0$$

Which leads to

$$(\sum_{i=i}^{n} T_i(x) - x)y - \sum_{i=1}^{n} T_i(y) - y)x = 0 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Replace x by xr in (2) we get

$$(\sum_{i=i}^{n} T_i(xr) - xr)y - \sum_{i=1}^{n} T_i(y) - y)xr = 0$$

Hence

$$(\sum_{i=1}^{n} T_i(x) - x)ry - (\sum_{i=1}^{n} T_i(y) - y)xr = 0 \quad \ldots\ldots\ldots\ldots(3)$$

For all $x, y \in I, r \in R$

Using (2) in (3) to simplify , we obtain

$$(\sum_{i=1}^{n} T_i(x) - x)[r,y] = 0 \quad \ldots\ldots\ldots\ldots\ldots(4)$$

For all $x, y \in I, r \in R$.

Again replacing r by rs in (4)

$$(\sum_{i=1}^{n} T_i(x) - x)r[s,y] = 0$$

For all $x, y \in I$ and $r, s \in R$

$$(\sum_{i=1}^{n} T_i(x) - x)R[s,y] = 0$$

for all $x, y \in I$,$s \in R$

by primness' of R and since $\sum_{i=i}^{n}(T_i(x) - x) \neq 0$

hence $[s,y] = 0$ for all $y \in I$, $s \in R$

there fore $I \subset Z(R)$ and hence R is commutative . ■

Corollary 2.2 : In theorem 2.1 , if the family T of higher left centralizers is zero then R is commutative

proof : suppose that $T_n([x,y]) - [x,y] = 0$ for any $x, y \in I$

if $T_n = 0$ then $[x, y] = 0$ for all $x, y \in I$

There fore I is commutative hence R is commutative . ■

Theorem 2.3 : let R be a prime ring and I be a non – zero ideal of R suppose that R admits a family T of non – zero higher left centralizer

T = $(T_i)_{i \in N}$ such that $\sum_{i=1}^{n} T_i(x) \neq -x$ for all $x \in I$ and $i \in N$, for ther if $T_n([x, y]) + [x, y] = 0$ for all $x, y \in I$, then R is commutative .

Proof : Given that T = $(T_i)_{i \in N}$ is a family of higher left centralizers of R such that .

$T_n([x, y]) + [x, y] = 0$ for all $x, y \in I$ ....(1)

Then

$$T_n(xy - yx) + (xy - yx) = 0$$

So that

$$\left(\sum_{i=1}^{n} T_i(x)y - \sum_{i=1}^{n} T_i(y)x\right) + (xy - yx) = 0$$

Which leads to

$(\sum_{i=1}^{n} T_i(x) + x)\, y - (\sum_{i=1}^{n} T_i(y) + y)x = 0$ ....(.2)

In (2) replace x by xr to get

$$(\sum_{i=1}^{n} T_i(xr) + xr)\, y - (\sum_{i=1}^{n} T_i(y) + y)xr = 0$$

Hence

$(\sum_{i=1}^{n} T_i(x) + x)ry - (\sum_{i=1}^{n} T_i(y) + y)xr = 0$ ......(.3)

For all $x, y \in I, r \in R$

Using (2) in (3) to simplify , we obtain

$(\sum_{i=1}^{n} T_i(x) + x)\ \beta\, [r, y]_\alpha = 0$ ....(4)

For all $x, y \in I, r \in R$

Replace r by rs in (4)

$$(\sum_{i=1}^{n} T_i(x) + x)\ r\, [s, y] = 0$$

For all $x, y \in I, r, s \in R$

in other words

$(\sum_{i=1}^{n} T_i(x) + x)\ R[s, y]$ =o

For all $x, y \in I, s \in R$

By primness of R and since $\sum_{i=i}^{n}(T_i(x) + x) \neq 0$

we get $[s, y]$ = 0 for all $y \in I, s \in R$

Therefore $I \subset Z(R)$ and hence R is commutative . ■

Theorem 2.4 : - let R be a prime ring and I be anon – zero ideal of R . suppose that R adimits afamily of non – zero higher left centralizers . $T = (T_i)_{i \in n}$ such that $\sum_{i=1}^{n} T_i(x) \neq x$ for all $x \in I$ and $i \in N$, further if

$$T_n(\langle x, y \rangle) = \langle x, y \rangle$$

For all $x, y \in I$, then R is commutative .

Proof : - Given that

$T_n(\langle x, y \rangle) - \langle x, y \rangle = 0$ ....(1)

For all $x, y \in I$

This implies that

$(\sum_{i=1}^{n} T_i(x) - x)\, y + (\sum_{i=1}^{n} T_i(y) - y)x = 0$ ....(.2)

Replace x by xr in (2) we obtain .

$(\sum_{i=1}^{n} T_i(xr) - xr)y + (\sum_{i=1}^{n} T_i(y) - y)xr = 0$

Hence

$(\sum_{i=1}^{n} T_i(x) - x)ry - (\sum_{i=1}^{n} T_i(y) - y)xr = 0$ ……….(3)

For all $x, y \in I, r \in R$

Using (2) in (3) we get

$(\sum_{i=1}^{n} T_i(x) - x)ry + (\sum_{i=1}^{n} T_i(x) - x)yr = 0$ ……………(4)

For all $x, y \in I, r \in R$

That is

$(\sum_{i=1}^{n} T_i(x) - x)[r, y] = 0$ …………(5)

For all $x, y \in I, r \in R$

Replace r by rs in (5) we get .

$(\sum_{i=1}^{n} T_i(x) - x)r[s, y] = 0$ ……..(6)

For all $x, y \in I, r, s \in R$

i.e : $(\sum_{i=1}^{n} T_i(x) - x)R[s, y] = 0$

By primness of R and since $\sum_{i=1}^{n}(T_i(x) - x) \neq 0$

Then $[s, y] = 0$ for all $y \in I$

Hence $I \subset Z(R)$ there for R is commutative ■

Theorem 2.5 : - let R be a prime ring and I be anon – zero ideal of R . suppose that R adimits afamily of non – zero higher left centralizers . $T = (T_i)_{i \in N}$ such that $\sum_{i=1}^{n} T_i(x) \neq -x$ for all $x \in I$ and $i \in N$, further if

$$T_n(<x, y>) + \langle x, y \rangle = 0$$

For all $x, y \in I$, then R is commutative

Proof : - Given that $T = (T_i)_{i \in N}$ be a family of non – zero higher left centralizers of R such that

$T_n(<x, y>) + <x, y> = 0$ ………..(1)

For all $x, y \in I$.

Then

$(\sum_{i=1}^{n} T_i(xy + yx)) + (xy + yx) = 0$

H hence

$\sum_{i=1}^{n} T_i(x)y + \sum_{i=1}^{n} T_i(y)x + (xy + yx) = 0$

$(\sum_{i=1}^{n} T_i(x) + x)y + (\sum_{i=1}^{n} T_i(y) + y)x) = 0$ … ……….(.2)

In the above relation eplace x by xr we obtain .

$(\sum_{i=1}^{n} T_i(xr) + xr)y + (\sum_{i=1}^{n} T_i(y) + y)xr = 0$

So we get

$(\sum_{i=1}^{n} T_i(x) + x)ry + (\sum_{i=1}^{n} T_i(y) + y)xr = 0$ ……….(3)

For all $x, y \in I, r \in R$

Substitute (2) in (3) to get

$(\sum_{i=1}^{n} T_i(x) + x)[r, y] = 0$ …………(4)

For all $x, y \in I, r \in R$

Now again replace r by rs in (4) we have

$(\sum_{i=1}^{n} T_i(x) + x)r[s, y] = 0$ ……..(5)

For all $x, y \in I$ and $r, s \in R$

i.e : $(\sum_{i=1}^{n} T_i(x) + x)R[s, y] = 0$

By primness of R and since $\sum_{i=1}^{n}(T_i(x)+x)\neq 0$

We have $[s,y]=0$ for all $y\in I, s\in R$.

Hence $I\subset Z(R)$ there for R is commutative ■

Corollary 2 . 6 : In theorem 2.4 and 2.5 if a higher left centralizers T_n is zero . then R is commutative .

Proof : For any $x,y\in I$, we have

$T_n(<x,y>=<x,y>$

if $T_n=0$ then $<x,y>=0$ for all $x,y\in I$

replace x by xz and using the fact

$yx=-xy$ we conclude that

$x[z,y]=\{0\}$ for all $x,y,z\in I$

In other words we have

$IR[z,y]=0$ for all $y,z\in I$.

Since R is prime and $I\neq\{0\}$

So that $[z,y]=0$ for all $y,z\in I$

then I is commutative and hence R is commutative . ■

Theorem 2.7 :- let R be a prime ring and I be anon zero ideal of R . suppose that R admits a family of non –zero higher left centralizers $T=(T_i)_{i\in N}$ such that $\sum_{i=1}^{n}T_i(x)\neq x$ for all $x\in I$ and for all $i\in N$, further if $T_n(xy)\mp(xy)=0$ for all $x,y\in I$, then R is commutative .

proof :- for any $x,y\in I$ we have

$T_n(xy)=(xy)$

this implies that

$T_n([x,y])-([x,y])=0$

and hance by theorem 2.1 we have R is commutative

on the other hand if R is satisfy the condition $T_n(xy)+(xy)=0$ for all $x,y\in I$.

then for any $x,y\in I$

we have $T_n(xy+yx)=-(xy+yx)$

So that $T_n(\langle x,y\rangle)+(\langle x,y\rangle)=0$ for all x,$y\in I$.

Then by theorem 2.5 we have R is commutative ■

Corollary 2.8 : -let R be a prime ring and I be anon zero ideal of R . suppose that R admits a family of non –zero higher left centralizers $T=(T_i)_{i\in N}$ such that $\sum_{i=1}^{n}T_i(x)\neq\mp x$ for all $x\in I$ and for all $i\in N$, further if $T_n(xy)\mp(yx)=0$ for all $x,y\in I$ then R is commutative .

proof : For any $x,y\in I$ we have $T_n(xy)\mp(yx)=0$

now if $T_n(xy)=(yx)$ this implies that $T_n([x,y])-([y,x])=T_n([x,y])+([x,y])=0$

then by theorem 2.5 we have R is commutative

Now when $T_n(xy)+(yx)=0$ then $T_n([x,y])+([y,x])=0$

this implies that $T_n([x,y])+([x,y])=0$ and hance by theorem 2.1 we have R is commutative ■

3.The main results: in this section we introduce the main results of this paper

Theorem 3.1: :let R be a prime ring and I be anon zero ideal of R . suppose that R admits a family of non –zero higher left centralizers $T=(T_i)_{i\in N}$ such that $\sum_{i=1}^{n}T_i(x)\neq x$ for all $x\in I$

and for all $i \in N$, then the following conditions are equivalent :

(i) $T_n([x,y]\ ) - ([x,y]\ ) = 0$ for all $x,y \in I$

(ii) $T_n([x,y]\ ) + ([x,y]\ ) = 0$ for all $x,y \in I$

(iii) for all $x,y \in I$, either $T_n([x,y]\ ) - ([x,y]\ ) = 0$ or $T_n([x,y]\ ) + ([x,y]\ ) = 0$

(iv) R is commutative

Proof :

(i)→(iv) suppose that $T_n([x,y]\ ) - ([x,y]\ ) = 0$

Then by theorem 2.1 we have R is commutative

(iv)→(i) suppose that R is commutative then $[x,y]\ = 0$

and hence $T_n([x,y]\ ) - ([x,y]\ ) = 0$

(ii) →(iv) suppose that

$T_n([x,y]\ ) + ([x,y]\ ) = 0$

for all $x,y \in I$

Then by theorem 2.3 we have R is commutative

(iv)→(ii) suppose that R is commutative then $[x,y]\ = 0$ for all $x,y \in I$

And hence - $[x,y]\ = 0$ for all $x,y \in I$

Which implies that $T_n([x,y]\ ) - ([x,y]\ ) = 0$ for all $x,y \in I$

(iii) →(iv) suppose that for all $x,y \in I$ either $T_n([x,y]\ ) - ([x,y]\ ) = 0$ or

$T_n([x,y]\ ) + ([x,y]\ ) = 0$

Then by theorem2.1 or theorem 2.3 we have R is commutative

(iv)→(iii) suppose that R is commutative

For each fixed $y \in I$ we set

$I_1 = \{x \in I | T_n([x,y]\ ) - ([x,y]\ ) = 0\}$

$I_2 = \{x \in I | T_n([x,y]\ ) + ([x,y]\ ) = 0\}$

Then I_1 and I_2 are additive subgroups of I such that $I = I_1 \cup I_2$.

But a group cannot be the set theoretic union of two proper subgroups , hance we have either

$I_1 = I$ or $I_2 = I$.

Further , using a similar argument , we obtain

$I = \{y \in I | I_1 = I\}$ or $I = \{y \in I | I_2 = I\}$

Thus we obtain that either $T_n([x,y]\ ) - ([x,y]\ ) = 0$ for all $x,y \in I$

or $T_n([x,y]\ ) + ([x,y]\ ) = 0$ for all $x,y \in I$

Hence R is commutative in both cases by theorem 2.1 (respectively theorem 2.3) ■

Theorem 3.2 : :let R be a prime ring and I be anon zero ideal of R . suppose that R admits a family of non –zero higher left centralizers $T = (T_i)_{i\in N}$ such that $\sum_{i=1}^{n} T_i(x) \neq x$ for all $x \in I$ and for all $i \in N$, further if $T_n(xy) - (xy) \in Z(M)$ for all $x,y \in I$ then R is commutative .

Proof : for any $x,y \in I$ we have

$$T_n(xy) - (xy) \in Z(R) \quad \text{………………(1)}$$

This can be written as $\sum_{i=1}^{n} T_i(x)y - xy \in Z(R)$ for all $x,y \in I$ ……….(2)

That is $[(\sum_{i=1}^{n} T_i(x) - x)y\, , r] = 0$ for all $x,y \in I, r \in R$ ……….(3)

Which implies that

$(\sum_{i=1}^{n} T_i(x) - x)[y,r] + [\sum_{i=1}^{n} T_i(x) - x\, ,r]\; y = 0$ …………..(4)

for all $x, y \in I, r \in R$

in (4) replace x by xz , we have

$(\sum_{i=1}^{n} T_i(x) - x)z[y,r] + [(\sum_{i=1}^{n} T_i(x) - x)z\, ,r]\; y = 0$ ………….(5)

for all $x, y, z \in I, r \in R$

from (3) we get that (5) becomes

$(\sum_{i=1}^{n} T_i(x) - x)z[y,r] = 0$ for all $x, y, z \in I, r \in R$.

This yields that

$(\sum_{i=1}^{n} T_i(x) - x)RI[y,r] = \{0\}$ for all $x, y \in I, r \in R$

By primness of R implies that

$I[y,r] = \{0\}$ or $\sum_{i=1}^{n} T_i(x) - x = 0$

and since $I \neq \{0\}$ and $\sum_{i=1}^{n} T_i(x) \neq x$ for all $x \in I$

we get that I is central and hence R is commutative ■

Theorem 3.3: let R be a prime ring and I be anon zero ideal of . suppose that R admits a family of non –zero higher left centralizers $T = (T_i)_{i \in N}$ such that $\sum_{i=1}^{n} T_i(x) \neq -x$ for all $x \in I$ and for all $i \in N$, further if $T_n(xy) - (xy) \in Z(M)$ for all $x, y \in I$, then R is commutative .

proof : suppose that $T = (T_i)_{i \in N}$ be a family of non –zero higher left centralizers satisfying the

property $T_n(xy) - (xy) \in Z(R)$ for all $x, y \in I$

then the non- zero higher left centralizers $(-T)$ satisfies the condition

$(-T_n)(xy) - (xy) \in Z(R)$ for all $x, y \in I$

Hance by theorem 3.2 we have R is commutative . ■

Remark 3.4: in theorem 3.2 if the higher left centralizer is zero , then R is commutative .

Theorem 3.5 : let R be a prime ring and I be anon zero ideal of R . suppose that R admits a family of non –zero higher left centralizers $T = (T_i)_{i \in N}$ such that $\sum_{i=1}^{n} T_i(x) \neq x$ for all $x \in I$ and for all $i \in N$, further if $T_n(xy) - (yx) \in Z(R)$ $for\ all\ x, y \in I$ then R is commutative .

Proof : we are given that a higher left centralizer of R such that

$$T_n(xy) - (yx) \in Z(R)$$

for all $x, y \in I$

this implies that

$[T_n(xy) - (yx)\, ,\, r] = 0$ ………………….(1)

holds for all $x, y \in I, r \in R$

which implies that

$[\sum_{i=1}^{n} T_i(x)y - yx\, ,\, r] = 0$ …………………..(2)

for all $x, y \in I, r \in R$

replacing y by yx in the above relation and use it hence

$[\sum_{i=1}^{n} T_i(x)yx - yx^2\, ,\, r] = 0$ ……………………(3)

for all $x, y \in I, r \in R$

we find that

$$\left(\sum_{i=1}^{n} T_i(x)y - yx\right)[x,r] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

for all $x, y \in I, r \in R$

again replace r by rs in (4) to get

$$\left(\sum_{i=1}^{n} T_i(x)y - yx\right) r[x,s] + \left(\sum_{i=1}^{n} T_i(x)y - yx\right)[x,r]\, s = 0 \qquad \ldots\ldots\ldots\ldots(5)$$

for all $x, y \in I, r, s \in R$

From (4) the relation (5) becomes

$$\left(\sum_{i=1}^{n} T_i(x)y - yx\right) r[x,s] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

for all $x, y \in I, r, s \in R$

i.e.

$$\left(\sum_{i=1}^{n} T_i(x)y - yx\right) R\, [x,s] = 0$$

for all $x, y \in I, s \in R$

the primness of R implies that either $[x,s] = 0$ or $\sum_{i=1}^{n} T_i(x)y - yx = 0$

for all $x, y \in I, s \in R$

now put $I_1 = \{x \in I \mid [x,s] = 0 \; for\ all\ s \in R\}$

$I_2 = \{x \in I \mid \sum_{i=1}^{n} T_i(x)y - yx = 0 \; for\ all\ x, y \in I\}$

Then clearly that I_1 and I_2 are additive subgroups of R . moreover by the discussion given I is the set- theoretic union of I_1 and I_2 but can not be the set- theoretic of two proper subgroups .

Hence $I_1 = I$ or $I_2 = I$.

If $I_1 = I$, then $[x,s] = 0$ for all $x \in I, s \in R$ and hence R is commutative .

On the other hand if $I_2 = I$ then $\sum_{i=1}^{n} T_i(x)y = yx$ for all for all $x, y \in I$.

That is $\sum_{i=1}^{n} T_i(x)y - yx = 0$ for all for all $x, y \in I$

This implies that $T_n([x,y]) - ([x,y]) = 0$ for all for all $x, y \in I$.

Hence apply theorem 2.1 yields the required result . ■

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التمركزات العليا اليسرى على الحلقات الأولية

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الملخص :

في هذا البحث ندرس ابدالية الحلقات الاولية التي تحقق شروط معينة تتضمن تمركزات يسرى من الدرجات العليا معرفة على تلك الحلقات الاولية.

الكلمات المفتاحية الحلقات الأولية , تمركزات يسرى