The Effect of Astigmatism Aberration on Point Spread Function for Optical System Using Different Apertures

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Abstract:

In this research , Point Spread Function (PSF) of an optical system has been studied, with a number of different apertures (circular, square, rectangle (horizontal)). In case for diffraction limited system and with a different values of astigmatism aberration (W_{22}=0.25λ،0.5 λ,0.75 λ and1 λ) and with different rotation angles (ψ = 0,45,90). The results showed that if the angle (ψ =0,45) is used, the rectangle (horizontal) aperture is the best in the distribution of intensity of the image plane and resolution power for optical system. While, The circular aperture was the best in decreasing the secondary peaks. When using the angle (ψ=90), and for all apertures, we observe the increasing of width of the function curve and increasing the secondary peaks**,** especially at ($W_{22=}$ 0.75 and 1). which effects negatively on the clarity image

Keywords: point spread function, astigmatism aberration, apertures (circular, square and rectangle (horizontal)).

1. Introduction

Diffraction plays an important role in the optical system, where the phenomenon of connecting of light waves around corners and their spreading into the geometric shadow of an object is called diffraction [1]. A defect or distortion in an image is called an aberration. An aberration can be caused by a defect in a lens or mirror, but even with an ideal optical surface some degree of aberration is inevitable [2].

Many studies on astigmatism aberration and point spread function have been previously adopted, where in 2005, S.Y. Hassan, calculated a Zernike polynomials and point spread function of different wave front aberrations For Multi-Square Aperture [3]. In 2010, Majid Badieirostami et.al, compared the theoretical localization precision for an unbiased estimator of the DH-PSF to that for 3D localization by astigmatic and biplane imaging using fisher information analysis including pixelation and varying levels of background.[4]. While 2014, Zhaolou Cao and Keyi Wang, appeared that the intensity pattern is robust to both astigmatism and coma, while astigmatism affects the rotating angle of the PSF, and coma affects the lateral position of the PSF [5]. In 2016, Laura Oudjedi et.al, developed a 3D super-resolution microscopy method that enables deep imaging in cells [6]. In this research, we will study the one of the types of aberrations , which is an astigmatism aberration on point spread function for optical system using different apertures. The astigmatism is similar to the coma aberration in that it is made up of object point outside the axis. The difference between them is that the image formed in the coma is spread in a perpendicular level to the axis of the lens while the image formed in the astigmatism is along the axis of the lens .

There are two levels for the new axis, known as sagittal (horizontal) and meridional (vertical) levels. Every forms its own image point and between these points the circle is the least distorted. The difference in image resulting from the sagittal and meridional levels on the new axis is called the astigmatic [7].

2. The Equation of Point Spread Function (PSF)

The point spread function is the irradiance in the image of a point source in an optical system.[8] This function may be obtained by the Fourier transform of the pupil function, where the complex amplitude for point located at the coordinates (u ,v) in the image plane was given by [9].

$$F(u,v)=\frac{1}{A}\int_{y}\int_{x} f(x,y)\, e^{2\pi i(ux+vy)}dxdy \quad (1)$$

Where $F(u,v)$ represent complex amplitude in point (u ,v) , while (A)represent the exit pupil area, and $f(x,y)$ represents the pupil function, can be expressed by [10]

$$f(x,y)=\tau(x,y)e^{ikw(x,y)} \quad (2)$$

$\tau(x,y)$: represents the real amplitude distribution in exit pupil and it is called "pupil transparency" often equals one unit if the illumination is uniform.

$e^{ikw(x,y)}$ is the wave front of aberration function. $w(x,y)$ is the aberration factor.

Then the point spread function is then given by the complex square of the amplitude in the image [11]

$$G(u,v)=|F(u,v)|^2 \quad (3)$$

$$PSF=G(u,v)$$
$$=n.f\left|\int_{y}\int_{x} f(x,y)\, e^{2\pi i(ux+vy)}dxdy\right|^2 \quad (4)$$

Where $(n.f)$: a normalization constant.

Assume that $z=2\pi u$ and $2\pi v=m$ the equation (4) become:

$$PSF=G(z,m)$$
$$=n.f\left|\int_{y}\int_{x} f(x,y)e^{i(zx+my)}\,dxdy\right|^2 \quad (5)$$

Because of the symmetry, we will take only one axis, so equation (5) becomes as follows:

$$PSF=G(z)\ =n.f\left|\int_y\int_x f(x,y)e^{i(zx)}\,dxdy\right|^2 \ (6)$$

The boundaries of integration for the point spread function are given through the optical system aperture, and in this research a number of different apertures were used.

3. The Equations Astigmatism Aberration

From wave front of aberration function $e^{ikw(\mathrm{x,y})}$:

W(x ,y) is the aberration factor and k is wave number ($k = 2\pi/\lambda$)

In this research, we will use the astigmatism aberration to study its effect on point spread function
equation astigmatism aberration is [12,13]

$$W = W_{222}\sigma^2 r^2 \cos^2\Phi \qquad (7)$$

In Cartesian coordinate (x , y):

x= r sin Φ, y= r cos Φ

$$W(x,y) = W_{222}\,(y_1)(y_1) \qquad (8)$$

When the Cartesian coordinates are rotated by an angle ψ , the new coordinates become:

$$x_1 = x\cos\psi - y\sin\psi$$

$$y_1 = x\sin\psi + y\cos\psi \qquad (9)$$

Where ψ is rotation angle of Cartesian coordinates, The aberration function becomes:

$$W = W_{222}\,(x\sin\psi + y\cos\psi)(x\sin\psi + y\cos\psi)$$

$$= W_{222}\,(x^2\sin^2\psi + y^2\cos^2\psi + 2xy\sin\psi)$$

$$= W_{222}\,(x^2\sin^2\psi + y^2\cos^2\psi + xy\sin 2\psi) \ (10)$$

4. Point Spread Function for Circular Aperture

The use of circular aperture in optical systems gave it a preference in terms of comparison with other apertures. The boundaries of equation (6) are defined by the circular pupil aperture with the normalizing area (π) and radius equals (1). Thus, equation (6) will be for an optical system with a astigmatism aberration for circular aperture, as follows:

$$PSF = n.f$$

$$\left|\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{i\left[\begin{array}{c}2\pi W_{222}\,(x^2\sin^2\psi+y^2\cos^2\psi+xy\sin 2\psi)\\+(zx)\end{array}\right]}\,dxdy\right|^2 \quad (11)$$

5. Point Spread Function for Square Aperture

The boundaries of the square aperture is defined by the square pupil aperture with the normalizing area (π) and the length of its side $\sqrt{\pi}$. Thus, equation (6) will be for an optical system with a astigmatism aberration for square aperture, as follows:

$$PSF = n.f$$

$$\left|\int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}}\int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} e^{i[2\pi W_{222}\,(x^2\sin^2\psi+y^2\cos^2\psi+xy\sin 2\psi)+(zx)]}\,dxdy\right|^2 \quad (12)$$

6. Point Spread Function for rectangle (horizontal) Aperture

The boundaries of the rectangle (horizontal) aperture is defined by the rectangle pupil aperture with the normalizing area (π) , the length of horizontal side $\left(\frac{\pi}{2}\right)$ and the length of vertical side (2). Thus, equation (6) will be for an optical system with an astigmatism aberration for rectangle (horizontal) aperture, as follows:

$$PSF = n.f$$

$$\left|\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_{-1}^{1} e^{i[2\pi W_{222}\,(x^2\sin^2\psi+y^2\cos^2\psi+xy\sin 2\psi)+(zx)]}\,dxdy\right|^2 \quad (13)$$

7. Results and Discussion

In this research, The equations (6,11,12 and13) derived in the last section for PSF were programmed in A MATLAB code (version7), the aperture is used in different shapes (circular, square, rectangle (horizontal)) in state found diffraction limited system and with a different values of astigmatism aberration W_{22}(0.25 λ، 0.5 λ ،0.75λ and 1 λ), to get the results, as follows:

7.1. PSF for Different Apertures (Circular, Square And Rectangle (horizontal)) for a Perfect Optical System

The results were shown in fig.(1), we observed that the central intensity (Strehl ratio) of the point spread function of the different apertures (Circular, square and rectangle (horizontal)) starts with (1) in the case of the diffraction limited system, because the of normalization function. Moreover, it is showed the width of the function curve was lessed in rectangle (horizontal) aperture, while the circular aperture was better decreasing the secondary peaks.

7.2. PSF for Different Apertures (Circular, Square and Rectangle (horizontal)) for Astigmatism Aberration

a- When (ψ=0)

Different values of Astigmatism aberration ($W_{22=}$ 0.25,0.5,0.75 and 1) for the rotation angle (ψ=0) were taken to calculate values of a point spread function of a different apertures, and they were shown in fig (2, 3 and 4). The value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.857, 0.544, 0.281 and 0.174) respectively, with the circular aperture, while in square aperture, the value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.872, 0.570, 0.270 and 0.106) respectively. But, with the rectangle (horizontal) aperture, the value central intensity changed from 1 for free of aberration to (0.919, 0.711, 0.454 and 0.237) respectively, it is noticed the effect of astigmatism aberration less in rectangle (horizontal) aperture, and the rectangle (horizontal) aperture leads to increase the image intensity and to reduce curve width, more than apertures (circular and square), The circular aperture was a preferable in decreasing the secondary peaks.

b - When (ψ=45)

The fig. (5,6 and7) described the effect of astigmatism aberration ($W_{22=}$ 0.25,0.5,0.75 and 1) for the rotation angle (ψ=45) on a Point Spread Function(PSF) of a different apertures, and the value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.857, 0.544, 0.281 and 0.174) respectively, with the circular aperture, while in square aperture, the value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.796, 0.464, 0.309 and 0.252) respectively. But, with the rectangle (horizontal) aperture, the value central intensity changed from 1 for free of aberration to (0.790, 0.453, 0.301, and 0.245) respectively, in the circular aperture, the central intensity values are only similar to the state (ψ= 0), but the other values are different. The square and rectangle (horizontal) apertures were a preferable in increasing the central intensity of the point spread function, but the circular aperture is better decreasing the secondary peaks.

c - When (ψ=90)

Fig (8, 9 and10) represents the intensity distribution curves for point spread function with different values astigmatism aberration (0.25,0.5,0.75 and 1) when a different apertures for angle (ψ=90), and the value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.857, 0.544, 0.281 and 0.174) respectively, with the circular aperture, while in square aperture, the value central intensity (Strehl ratio) was decreased from 1 for free of aberration to (0.872, 0.570, 0.270 and 0.106) respectively, But, with the rectangle (horizontal) Aperture, the value central intensity changed from 1 for free of aberration to (0.800, 0.395, 0.124 and 0.089) respectively. In the circular and square apertures, the central intensity values are only similar to the state (ψ= 0), but the other values are different and the curve width of a Point Spread Function increases, Leading to resolution power was very low, especially at ($W_{22=}$ 0.75 and 1). In the square and rectangle (horizontal) apertures increases the secondary peaks**,**
especially at ($W_{22=}$ 0.5, 0.75 and 1). Which effects negatively on the clarity image .

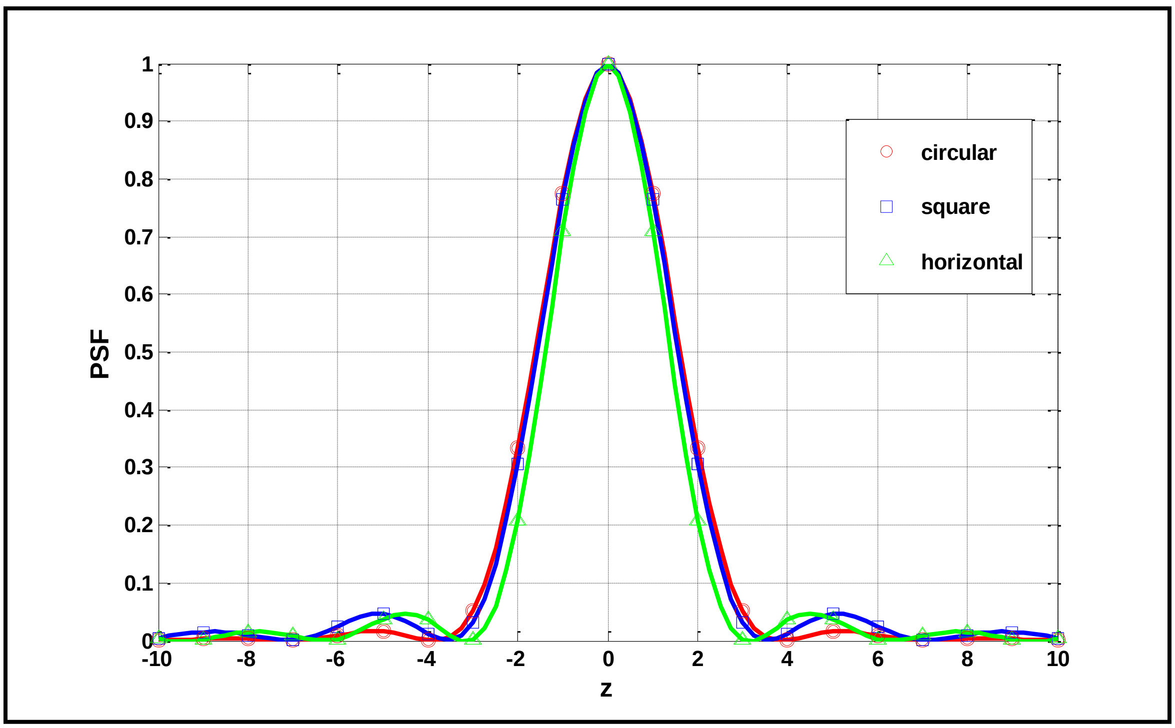

Figure 1: PSF for different apertures (circular, square and rectangle (horizontal)) of perfect optical system.

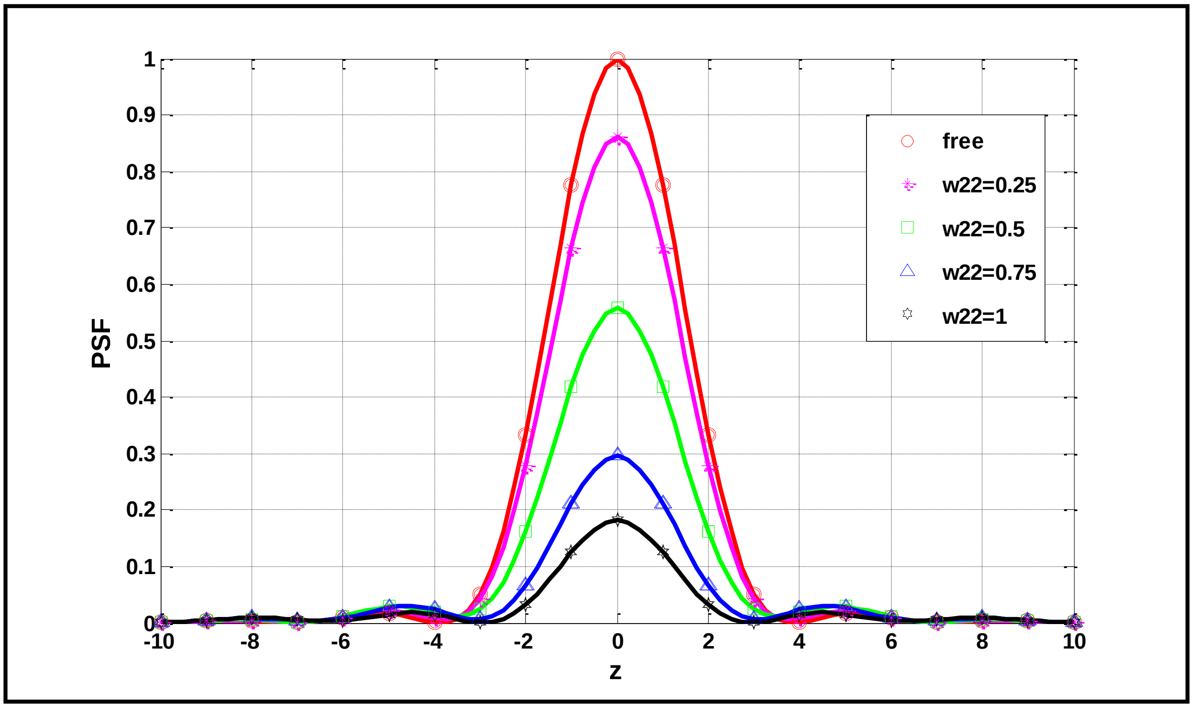

Figure 2: PSF for circular aperture of optical system, different values of astigmatism aberration when (ψ =0).

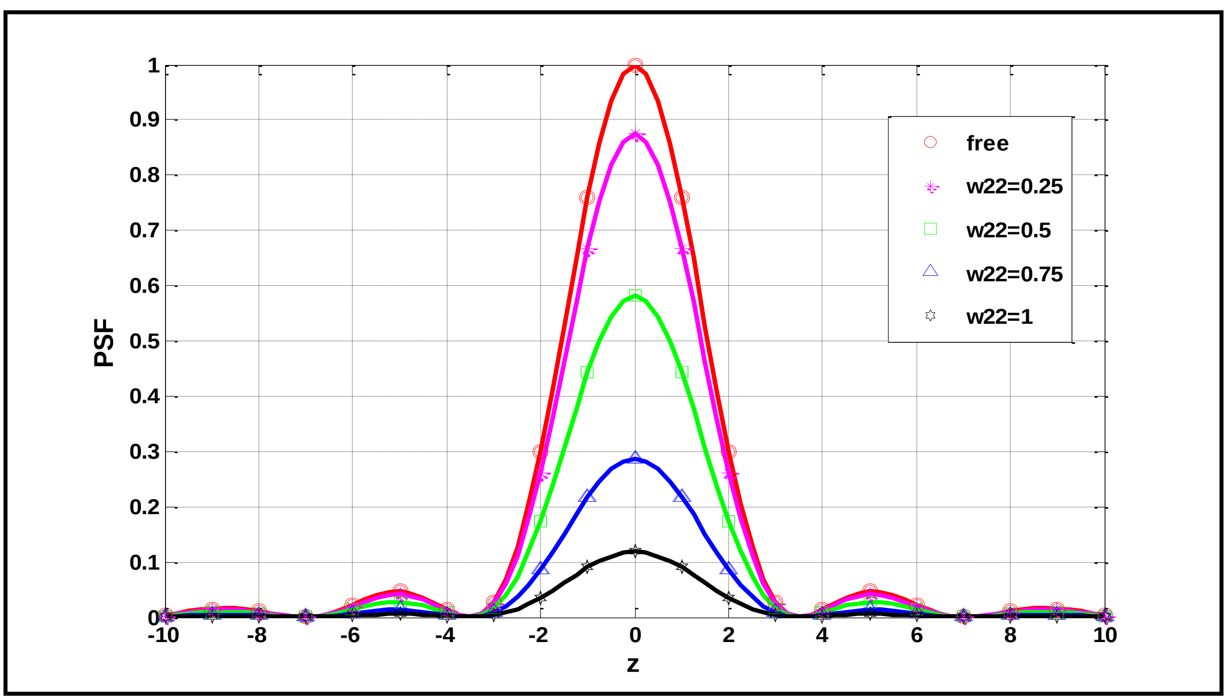

Figure 3: PSF for square aperture of optical system, different values of astigmatism aberration when (ψ =0).

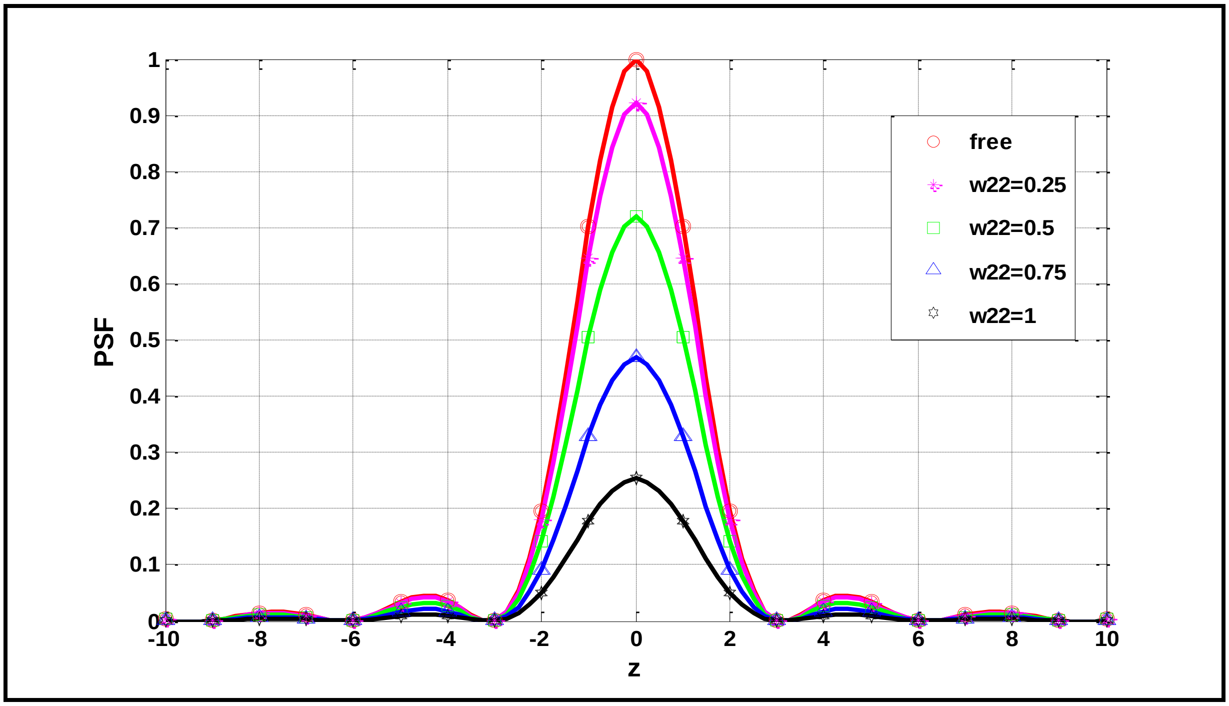

Figure 4: PSF for rectangle (horizontal) aperture of optical system, different values of astigmatism aberration when (ψ =0).

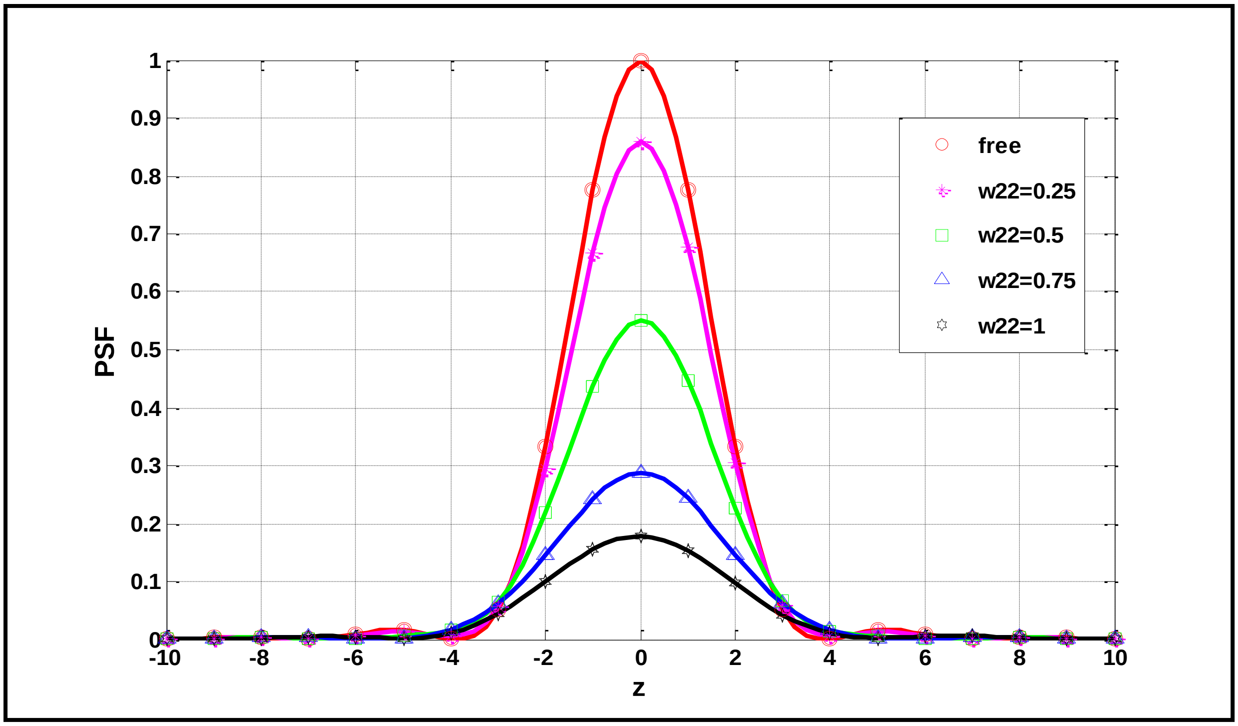

Figure 5: PSF for circular aperture of optical system, different values of astigmatism aberration when (ψ = 45).

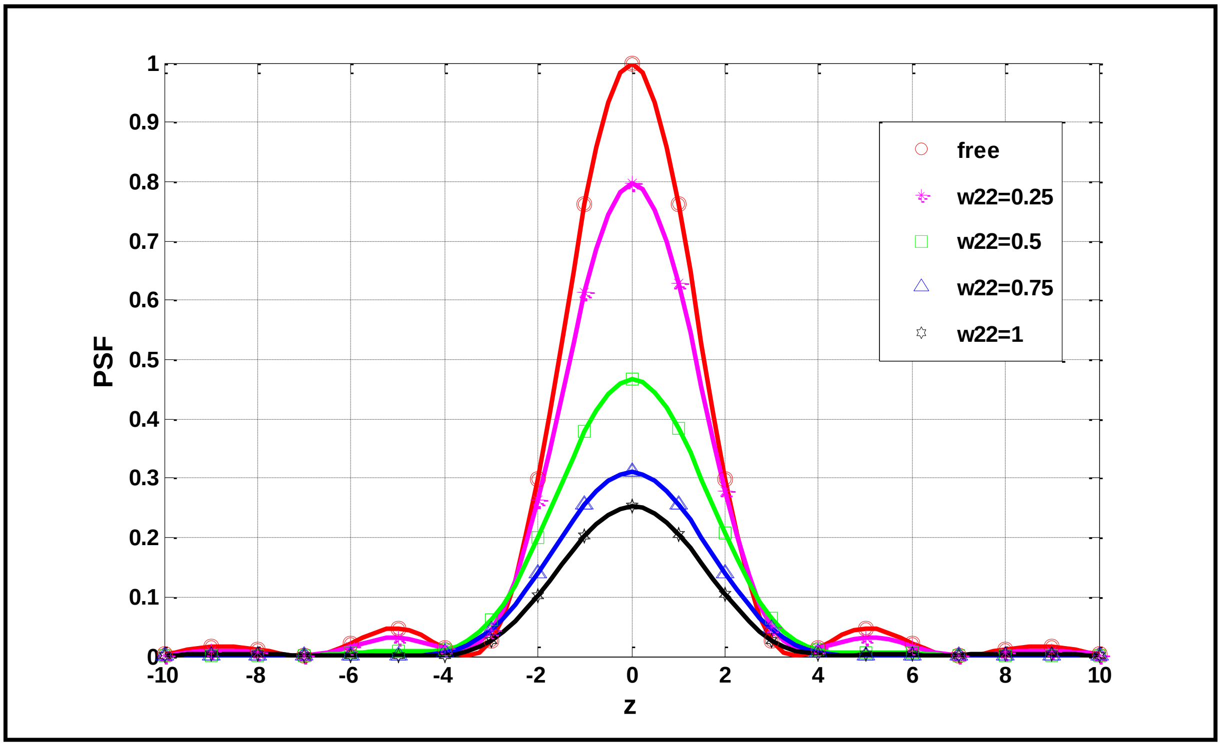

Figure 6: PSF for square aperture of optical system, different values of astigmatism aberration when (ψ = 45).

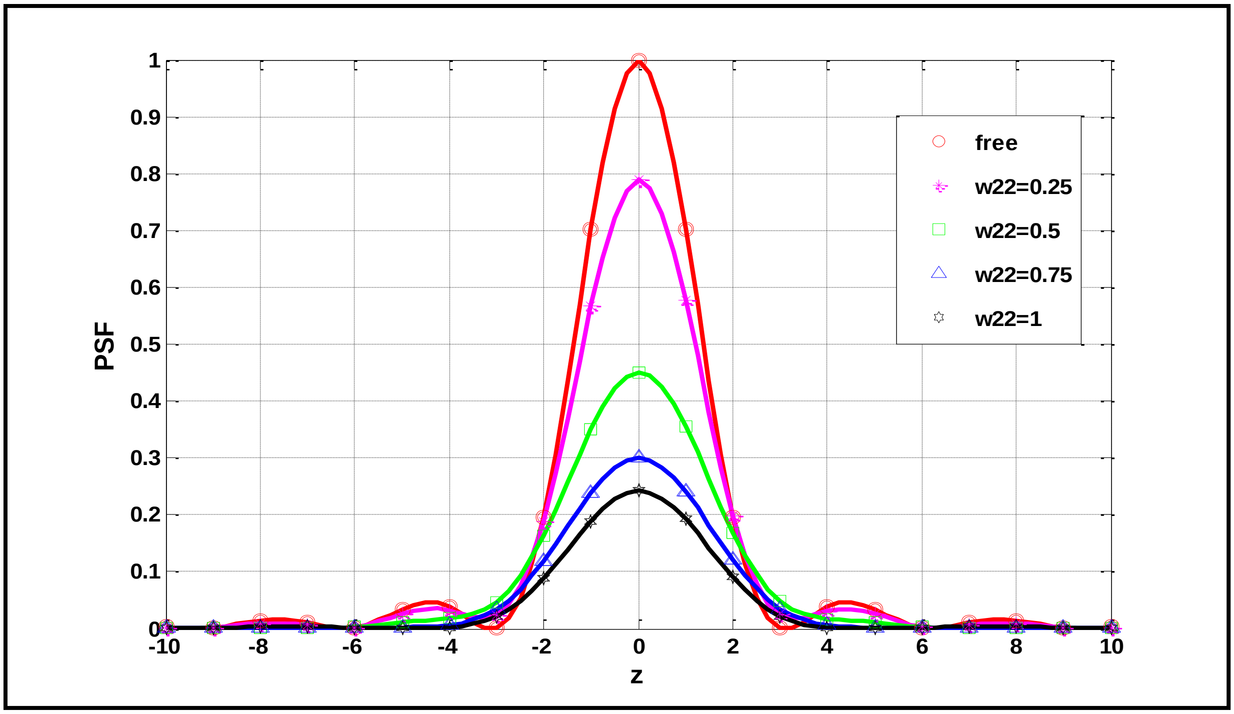

Figure 7: PSF for rectangle (horizontal) aperture of optical system, different values of astigmatism aberration when (ψ = 45).

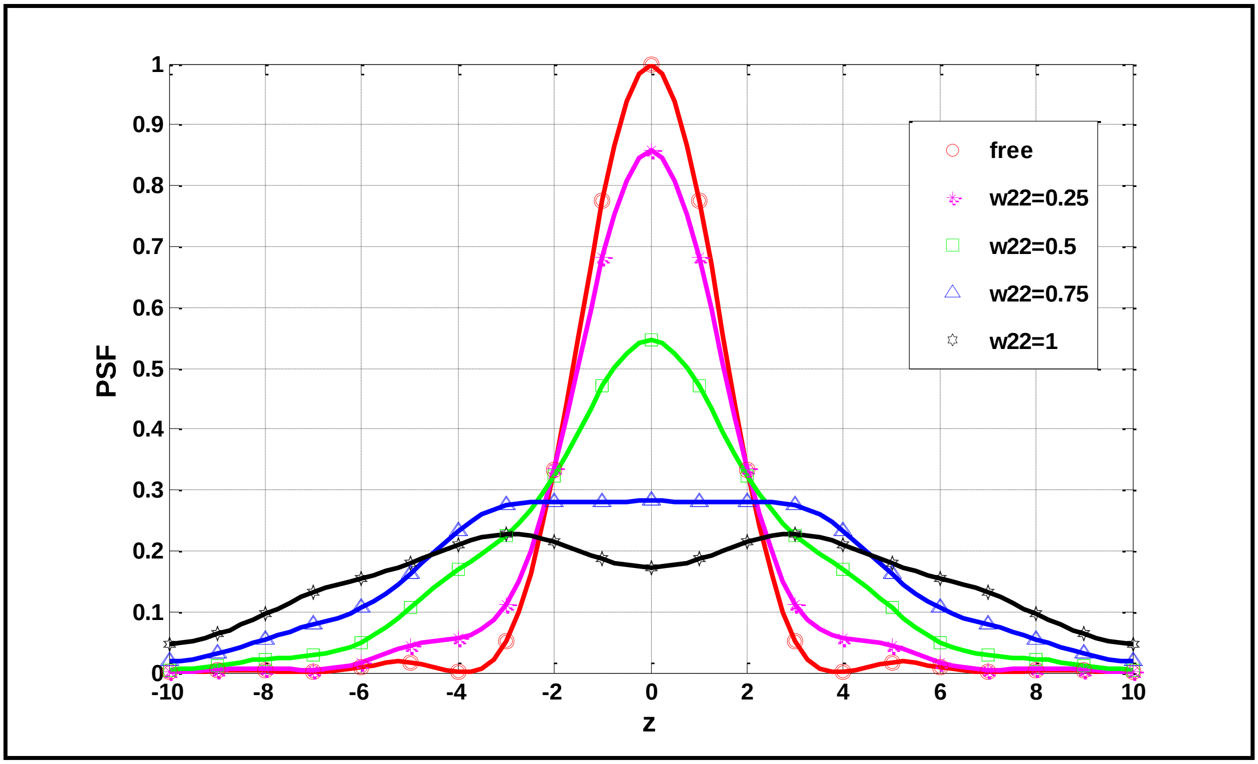

Figure 8: PSF for circular aperture of optical system, different values of astigmatism aberration when (ψ =90).

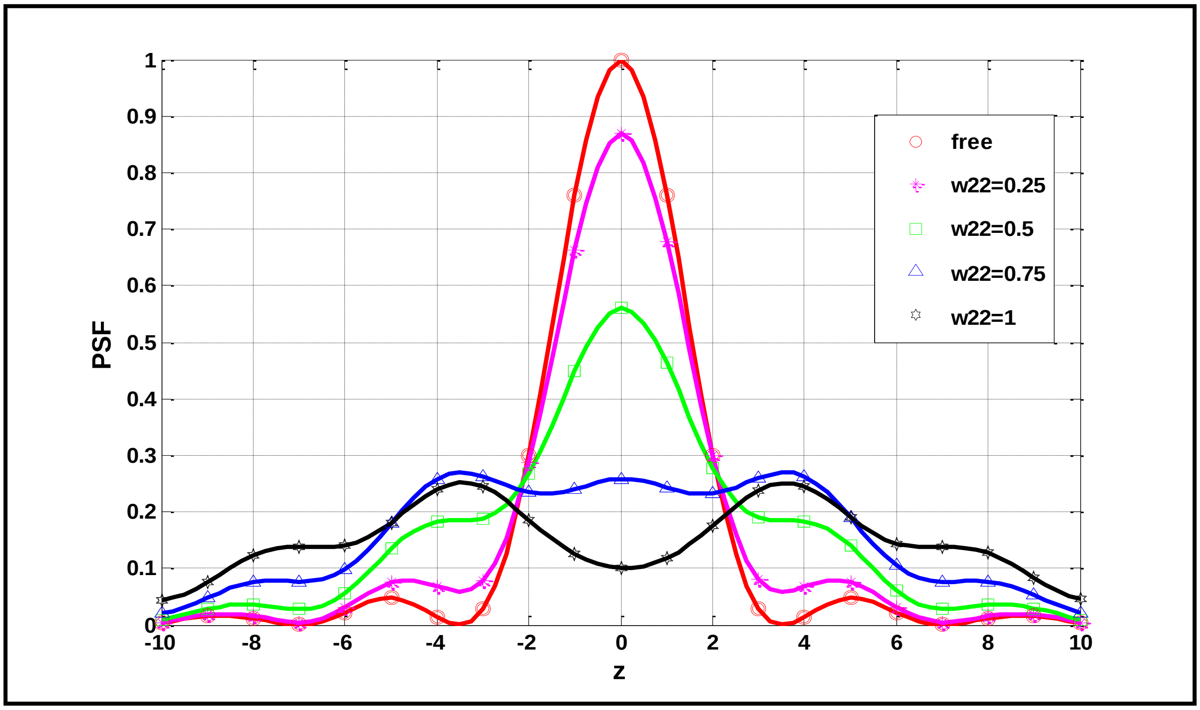

Figure 9: PSF for square aperture of optical system, different values of astigmatism aberration when (ψ =90).

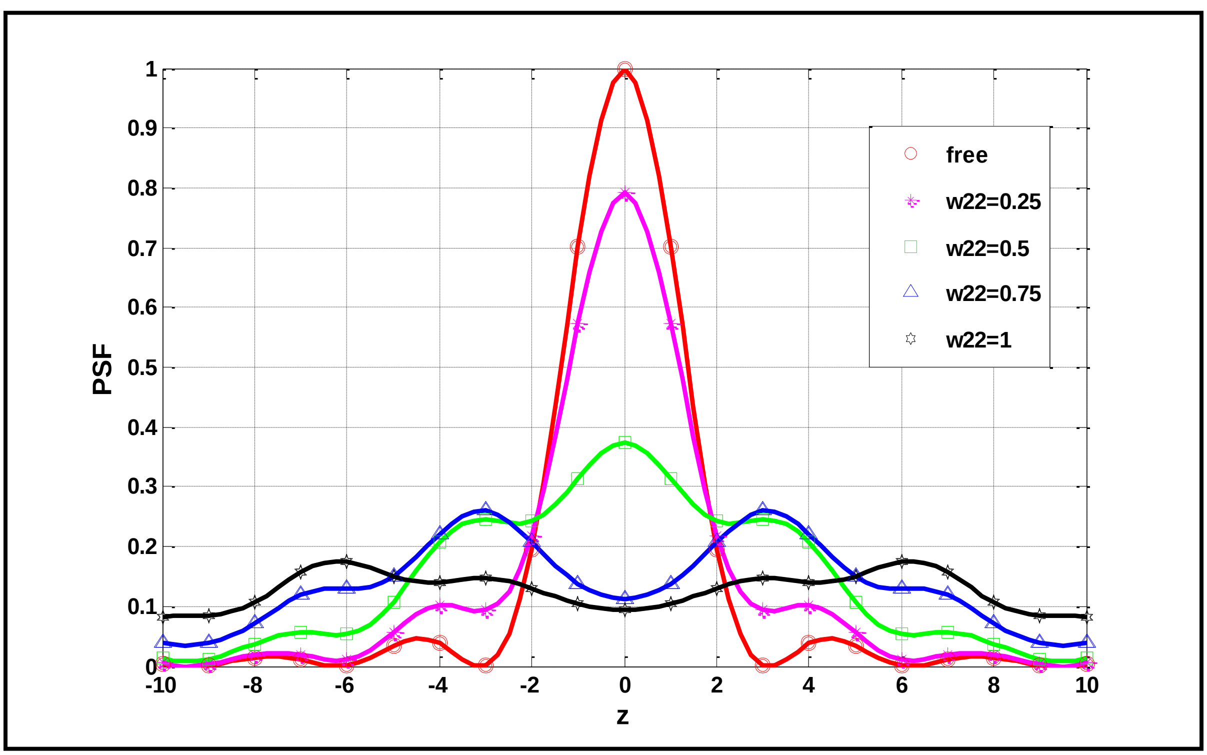

Figure 10: PSF for rectangle (horizontal) aperture of optical system, different values of astigmatism aberration when (ψ =90).

References

[1] D. Meschede, (2007), *Optics, Light and Lasers: The Practical Approach to Modern Aspects of Photonics and Laser Physics*, 2nd Edition. Wiley-VCH, Germany & Co. KGaA, Weinheim.

[2] B. Crowell, (2003), *Optics*, 2nd Edition. Fullerton, California: the Light and Matter series of free introductory physics textbooks, Benjamin Crowell.

[3] S. Y. Hassan, (2005), “Resolving power of multi- square aperture optical system,” Almustansiriya University.

[4] M. Badieirostami, M. D. Lew, M. A. Thompson, W. E. Moerner, M. Badieirostami, M. D. Lew, M. A. Thompson, and W. E. Moerner,(2010), “Three-dimensionallocalization precision of the double-helix point spread function versus astigmatism and biplane Three-dimensional localization precision of the double-helix point spread,” vol. 161103, no.

[5] Z. Cao and K. Wang, (2014), “Effects of astigmatism and coma on rotating point spread function,” vol. 53, no. 31, pp. 7325–7330.

[6] L. Oudjedi, J. Fiche, S. Abrahamsson, L. Mazenq, A. Lecestre, P. Calmon, A. Cerf, and M. Nöllmann, (2016), “Astigmatic multifocus microscopy enables deep 3D super-resolved imaging,” vol. 7, no. 6, pp. 914–915.

[7] R. C. Hilborn and T. D. Rossing,(2003), *Optics: Learning by Computing, with Examples Using MathCad*. Springer-Verlag New York, Inc. USA.

[8] M. A. X. Born, (1999), *Principles of optics*, 7thEdition. Cambridge University Press, UK.

[9] D. Malacara,(2004),*Handbook of Optical Design*, 2nd Edition. New York, Basel: Marcel Dekker, Inc. , USA.

[10] K.K. Sharma, (2006), *Optics Principles and Applications*. Elsevier Inc. USA.

[11] J. W. Goodman,(1996), *Introduction to Fourier Optics*, 2nd ed. McGraw-Hill Series in Electrical and Computer Engineering.

[12] F. L. Pedrotti, L. M. Pedrotti, and L. S. Pedrotti,(2006), *Introduction of Optics*, 3rd Edition. Prentice -Hall, Inc., USA.

[13] J. C. W. and K. Creath,(1992), “Basic Wavefront Aberration Theory for Optical Metrology,” vol. Xl, Applied Optics and Optical Engineering.

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