

ON FUZZY ANS – IDEAL OF B – ALGEBRA

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Received : 2/6/2019

Accepted :23/9/2019

ABSTRACT:

The purpose of this investigation is to characterize the ideal of fuzzy ANS – ideal of B – algebra and the relationship between it and the fuzzy α – translation , fuzzy ρ – multiplication, fuzzy extension and fuzzy magnified. Then we attach it by some theorems .

KEYWORDS: : B – algebra, fuzzy ANS – ideal of B – algebra, fuzzy α – translation, fuzzy ρ – multiplication , fuzzy extension and fuzzy magnified.

1. INTRODUCTION

After the starter of fuzzy subsets by L. A. zadeh (Zadeh, et al.,1965) several researchers explored on the generalization of the notion of fuzzy subset . K. Iseki and Imai announced two classes of algebras BCK – algebra , BCI – algebra (Iseki, et al.,1978; Iseki, et al.,1980) .It is known that the the class of BCK-algebras is a subclass of the class of BCI-algebras. H.S.kim and Neggers (Neggers, et al.,1999) presented a new ideal, named a B – algebra which is a broad view of BCK – algebra during. 1998. C. Yamini and S. kailasavalli introduced the B – ideals and fuzzy B – ideals and investigate how to deal with cartesian

product of B-ideals and strongest fuzzy relation in (Yamini, et al.,2014) .

Dr. A. Prasanna, M. Premkumar and Dr. S. Ismail Mohideen presented the ideal of a fuzzy translation and fuzzy multiplication on B – algedra (Prasanna, et al.,2018) .

In this papers we characterize the fuzzy ANS – ideal of B – algebra and we attach it with the notions of the fuzzy α – translation , fuzzy ρ – multiplication, fuzzy extension and fuzzy

magnified by use some definitions, examples and theorems.

2- Preliminaries:

Definition (2.1):(Neggers, et al.,1999)

A **B – algebra** is a non – blank set Λ with a constant 0 and a binary operation $*$ fulfilling the subsequent axioms :

1. $r * r = 0$,
2. $r * 0 = r$,
3. $(r * p) * q = r * (q * (0 * p))$

For all r, p, q in Λ .

Definition (2.2):(Yamini, et al.,2014)

A non – blank subset η of a B – algebra $(\Lambda; *, 0)$ is named a **B – Ideal of Λ** if it holds for $r, p, q \in \Lambda$:

1. $0 \in \eta$,
2. $(r * p) \in \eta, (q * r) \in \eta$, imply $(p * q) \in \eta$.

Definition (2.3):(Yamini, et al.,2014)

Let $(\Lambda; *, 0)$ be a B – algebra, a fuzzy subset η in Λ is named a **fuzzy B – ideal of Λ** if it holds the ensuing requirements $\forall r, p, q \in \Lambda$.

1. $\eta(0) \geq \eta(r)$,
2. $\eta(p * q) \geq \min\{\eta(r * p), \eta(q * r)\}$

Definition (2.4):(Yamini, et al.,2014)

Let $(\Lambda; *, 0_\Lambda)$ and $(\theta; \Delta, 0_\theta)$ be B – algebras. A mapping $f: \Lambda \rightarrow \theta$ is named **homomorphism** if: $f(r * p) = f(r)\Delta f(p), \forall r, p \in \Lambda$.

Definition (2.5):(Yamini, et al.,2014)

For any homomorphism $f: \Lambda \rightarrow \theta$ the set $\{r \in \Lambda / f(r) = 0_\theta\}$ is titled the **kernel of f** , signified by $\ker(f)$, the set $\{f(r) / r \in \Lambda\}$ is named the **image of f** symbolized by $\text{Im}(f)$.

Definition (2.6): (Yamini, et al.,2014)

Assume $f: \Lambda \rightarrow \theta$ be a mapping of B – algebras and η be a fuzzy set of θ . The map η^f is the **pre – image of η under f** if $\eta^f(r) = \zeta(f(r))$, $\forall r \in \Lambda$.

Definition (2.7): (Yamini, et al.,2014)

Assume Λ be a B – algebra, ζ be a fuzzy subset of Λ and $T = 1 - \sup \{\zeta(r) / r \in \Lambda\}, \acute{a} \in [0, T]$. A mapping $\zeta_\acute{a}^T: \Lambda \rightarrow [0, 1]$ is known as a **fuzzy $\acute{a}$ – translation of ζ** if it holds:

$$\zeta_\acute{a}^T = \zeta(r) + \acute{a}, \forall r \in \Lambda.$$

Definition (2.8): (Prasanna, et al.,2018)

Assume ζ be a fuzzy subset of Λ and $\tilde{n} \in [0, 1]$. A

mapping $\zeta_\tilde{n}^T: \Lambda \rightarrow [0, 1]$ is known as a **fuzzy $\tilde{n}$ – multiplication of ζ** if it fulfils $\zeta_\tilde{n}^M = \tilde{n}\zeta(r), \forall r \in \Lambda$.

Definition (2.9): (Prasanna, et al.,2018)

Suppose that ζ_1 and ζ_2 be a fuzzy subsets of Λ . If $\zeta_1(r) \leq \zeta_2(r)$, for all $r \in \zeta$. Then we take ζ_2 is a **fuzzy extension of ζ_1** . By means of this description of $\acute{a}$ – translation, we recognize that $\zeta_\acute{a}^T(r) \geq \zeta(r), \forall r \in \Lambda$.

Definition (2.10):(Prasanna, et al.,2018)

Assume ζ be a fuzzy subset of Λ , $\acute{a} \in [0, T]$ and $\tilde{n} \in [0, 1]$. A mapping $\zeta_{\tilde{n}\acute{a}}^{MT}: \Lambda \rightarrow [0, 1]$ is understood to be a **fuzzy magnified – $\tilde{n}\acute{a}$ – translation of ζ** if it fulfils $\zeta_{\tilde{n}\acute{a}}^{MT}(r) = \tilde{n}\zeta(r) + \acute{a}, \forall r \in \Lambda$.

Remark(2.11):

In the next part of our research we will symbolize to B – algebra $(\tilde{U}; *, 0)$ by $\tilde{U}$.

3- A Fuzzy ANS – ideal of B – algebra

Definition (3.1):

A fuzzy ideal γ of Ω is named a **fuzzy ANS – ideal of $\tilde{U}$** if it holds the situation : $\forall \mathfrak{d}, \mathfrak{z}, \mathfrak{s} \in \Omega$, if $\tilde{a}(\mathfrak{d}) \geq \sup(\min\{\tilde{a}(\mathfrak{s}), \tilde{a}(\mathfrak{z})\})$, where $\mathfrak{d} = \mathfrak{s} * \mathfrak{z}$.

Example (3.2) :

Assume $\Omega = \{0, \mathfrak{u}, \mathfrak{p}, \mathfrak{i}\}$ be a set with the next table :

*	0	$\mathfrak{u}$	$\mathfrak{p}$	$\mathfrak{i}$
0	0	$\mathfrak{i}$	$\mathfrak{p}$	$\mathfrak{u}$
$\mathfrak{u}$	$\mathfrak{u}$	0	$\mathfrak{i}$	$\mathfrak{p}$
$\mathfrak{p}$	$\mathfrak{p}$	$\mathfrak{u}$	0	$\mathfrak{i}$
$\mathfrak{i}$	$\mathfrak{i}$	$\mathfrak{p}$	$\mathfrak{u}$	0

Then Ω is a B – algebra. Describe a fuzzy set $\tilde{a}: \Omega \rightarrow [0, 1]$ by

$$\tilde{a}(\mathfrak{d}) = \begin{cases} 0.9 & \mathfrak{d} = 0 \\ 0.5 & \mathfrak{d} = \mathfrak{p} \\ 0.1 & \mathfrak{d} \in \{\mathfrak{u}, \mathfrak{i}\} \end{cases}$$

Then $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proposition (3.3) :

Assume $\tilde{a}$ is a fuzzy subset of Ω . If the fuzzy $\acute{a}$ – translation $\tilde{a}_\acute{a}^T(\mathfrak{d})$ of $\tilde{a}$ is a fuzzy ANS – ideal for some $\acute{a} \in [0, T]$, then $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proof :

Assume $\mathfrak{d}, \mathfrak{z}, \mathfrak{s} \in \tilde{U}, \acute{a} \in [0, T]$

$\therefore \tilde{a}_\acute{a}^T$ is a fuzzy ANS – ideal of Ω

$\therefore \tilde{a}_\acute{a}^T(\mathfrak{d}) \geq \sup(\min\{\tilde{a}_\acute{a}^T(\mathfrak{s}), \tilde{a}_\acute{a}^T(\mathfrak{z})\})$, where $\mathfrak{d} = \mathfrak{s} * \mathfrak{z}$
 $\tilde{a}(\mathfrak{d}) + \acute{a} \geq \sup(\min\{\tilde{a}(\mathfrak{s}) + \acute{a}, \tilde{a}(\mathfrak{z}) + \acute{a}\})$

$$\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}) + \alpha$$

$$\tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})$$

$\therefore \tilde{a}(b)$ is a fuzzy ANS – ideal of Ω . ■

Proposition (3.4) :

If $\tilde{a}$ is a fuzzy ANS – ideal Ω and $\alpha \in [0, T]$, then the fuzzy α – translation $\tilde{a}_\alpha^T(b)$ of $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proof :

Let $b, z, s \in \Omega$ and $\alpha \in [0, T]$, when $T = 1 - \sup\{\tilde{a}(b)/b \in \Omega\}$

$\therefore \tilde{a}$ is a fuzzy ANS – ideal of Ω

$\therefore \tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})$, when $b = s * z$

$$\tilde{a}_\alpha^T(b) = \tilde{a}(b) + \alpha, \quad \forall b \in \Omega$$

$$\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}) + \alpha$$

$$\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{a}(s) + \alpha, \tilde{a}(z) + \alpha\})$$

$$\tilde{a}_\alpha^T(b) \geq \sup(\min\{\tilde{a}_\alpha^T(s), \tilde{a}_\alpha^T(z)\})$$

$\therefore \tilde{a}_\alpha^T(b)$ is a fuzzy ANS – ideal of Ω . ■

Proposition (3.5):

Assume $f: \Omega \rightarrow \emptyset$ be a homomorphism of B – algebras.

If $\tilde{a}$ is a fuzzy ANS – ideal of $\emptyset \rightarrow \tilde{a}^f$ is a fuzzy ANS – ideal of Ω **Proof :**

Assume $b, z, s \in \Omega$

$$\tilde{a}^f(b) = \{\tilde{a}(f(b)) : \forall b \in \Omega\}$$

$$\tilde{a}(f(b)) \geq \sup(\min\{\tilde{a}(f(s)), \tilde{a}(f(z))\}), \text{ where } b = s * z$$

$$\tilde{a}^f(b) \geq \sup(\min\{\tilde{a}^f(s), \tilde{a}^f(z)\})$$

$\therefore \tilde{a}^f(b)$ is a fuzzy ANS – ideal of Ω . ■

Proposition (3.6) :

If $\tilde{a}$ is a fuzzy ANS – ideal of Ω and $\tilde{n} \in [0, 1]$, then the fuzzy $\tilde{n}$ – multiplication $\tilde{a}_{\tilde{n}}^M(b)$ of $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proof :

Presume that $b, z, s \in \Omega$, $\tilde{n} \in [0, 1]$ and $\tilde{a}$ is a fuzzy ANS – ideal of Ω

$$\therefore \tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}), \quad \text{where } b = s * z$$

$$\tilde{n}\tilde{a}(b) \geq \tilde{n}[\sup(\min\{\tilde{a}(s), \tilde{a}(z)\})]$$

$$\tilde{n}\tilde{a}(b) \geq [\sup(\min\{\tilde{n}\tilde{a}(s), \tilde{n}\tilde{a}(z)\})]$$

$$\tilde{a}_{\tilde{n}}^M(b) \geq \sup(\min\{\tilde{a}_{\tilde{n}}^M(s), \tilde{a}_{\tilde{n}}^M(z)\})$$

$\therefore \tilde{a}_{\tilde{n}}^M(b)$ is a fuzzy ANS – ideal of Ω . ■

Theorem (3.7) :

Assume $\tilde{a}$ is a fuzzy subset of Ω . If the fuzzy $\tilde{n}$ – multiplication $\tilde{a}_{\tilde{n}}^M(b)$ is a fuzzy ANS – ideal of Ω for some $\tilde{n} \in [0, 1]$, then $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proof :

presume $b, z, s \in \Omega$, $\tilde{n} \in [0, 1]$

Since $\tilde{a}_{\tilde{n}}^M(b)$ is a fuzzy ANS – ideal

$$\tilde{a}_{\tilde{n}}^M(b) \geq \sup(\min\{\tilde{a}_{\tilde{n}}^M(s), \tilde{a}_{\tilde{n}}^M(z)\})$$

$$\therefore \tilde{a}_{\tilde{n}}^M(b) = \tilde{n}\tilde{a}(b)$$

$$\tilde{n}\tilde{a}(b) \geq \sup(\min\{\tilde{n}\tilde{a}(s), \tilde{n}\tilde{a}(z)\})$$

$$\tilde{n}\tilde{a}(b) \geq \tilde{n}[\sup(\min\{\tilde{a}(s), \tilde{a}(z)\})]$$

$$\tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})$$

$\therefore \tilde{a}(b)$ is a fuzzy ANS – ideal of Ω . ■

Notice (3.8) :

We will apply the definition of the fuzzy magnified, we mentioned it in definition [2.10] in B – algebra Ω and we will use it in the following theorem.

Theorem (3.9) :

If $\tilde{a}$ is a fuzzy ANS – ideal of Ω , $\alpha \in [0, T]$ and $\tilde{n} \in [0, 1]$, then the fuzzy magnified $\tilde{n}\alpha$ – translation $\tilde{a}_{\tilde{n}\alpha}^{MT}(b)$ of $\tilde{a}$ is a fuzzy ANS – ideal of Ω .

Proof :

suppose $b, z, s \in \Omega$ and $\alpha \in [0, T]$, $\tilde{n} \in [0, 1]$

Since $\tilde{a}(b)$ is a fuzzy ANS – ideal of Ω

$\Rightarrow \tilde{a}_\alpha^T$ is a fuzzy ANS – ideal of Ω (By theorem (3.6))

And $\tilde{a}_{\tilde{n}}^M(b)$ is a fuzzy – ideal of Ω (By theorem (3.7))

$$\therefore \tilde{a}_\alpha^T(b) \geq \sup(\min\{\tilde{a}_\alpha^T(s), \tilde{a}_\alpha^T(z)\})$$

$$\tilde{n}\tilde{a}_\alpha^T(b) \geq \tilde{n}[\sup(\min\{\tilde{a}_\alpha^T(s), \tilde{a}_\alpha^T(z)\})]$$

$$\tilde{n}\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{n}\tilde{a}(s) + \alpha, \tilde{n}\tilde{a}(z) + \alpha\})$$

$$\tilde{a}_{\tilde{n}\alpha}^{MT}(b) \geq \sup(\min\{\tilde{a}_{\tilde{n}\alpha}^{MT}(s), \tilde{a}_{\tilde{n}\alpha}^{MT}(z)\})$$

$\therefore \tilde{a}_{\tilde{n}\alpha}^{MT}(b)$ is a fuzzy ANS – ideal of Ω . ■

Theorem (3.10) :

presume $\tilde{a}$ is a fuzzy subset of Ω . If the fuzzy magnified $\tilde{n}\alpha$ – translation $\tilde{a}_{\tilde{n}\alpha}^{MT}(b)$ is a fuzzy ANS – ideal of Ω , then $\tilde{a}(b)$ is a fuzzy ANS – ideal of Ω .

Proof :

Suppose $b, z, s \in \Omega$, $\alpha \in [0, T]$, $\tilde{n} \in [0, 1]$

Since $\tilde{a}_{\tilde{n}\alpha}^{MT}(b)$ is a fuzzy ANS – ideal of Ω

$$\tilde{a}_{\tilde{n}\alpha}^{MT}(b) \geq \sup(\min\{\tilde{a}_{\tilde{n}\alpha}^{MT}(s), \tilde{a}_{\tilde{n}\alpha}^{MT}(z)\})$$

$$\therefore \tilde{a}_{\tilde{n}\alpha}^{MT}(b) = \tilde{n}\tilde{a}(b) + \alpha$$

$$\tilde{a}_{\tilde{n}\alpha}^{MT}(z) = \tilde{n}\tilde{a}(z) + \alpha, \text{ and}$$

$$\tilde{a}_{\tilde{n}\alpha}^{MT}(s) =$$

$$\tilde{n}\tilde{a}(s) + \alpha$$

$$\tilde{n}\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{n}\tilde{a}(s) + \alpha, \tilde{n}\tilde{a}(z) + \alpha\})$$

$$\tilde{n}\tilde{a}(b) + \alpha \geq \sup(\min\{\tilde{n}\tilde{a}(s), \tilde{n}\tilde{a}(z)\}) + \alpha$$

$$\tilde{n}\tilde{a}(b) \geq \sup(\min\{\tilde{n}\tilde{a}(s), \tilde{n}\tilde{a}(z)\})$$

$$\tilde{n}\tilde{a}(b) \geq \tilde{n}[\sup(\min\{\tilde{a}(s), \tilde{a}(z)\})]$$

$$\tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}), \quad \text{where } b = s * z$$

$\therefore \tilde{a}(b)$ is a fuzzy ANS – ideal of $\tilde{U}$. ■

Theorem (3.11) :

Suppose that $\tilde{e}(b)$ and $\tilde{a}(b)$ be two fuzzy ANS – ideals of Ω , then $\tilde{e}(b) \cap \tilde{a}(b)$ is a fuzzy ANS – ideal of $\tilde{U}$ where

$$\tilde{e}(b) \cap \tilde{a}(b) = \min\{\tilde{e}(b), \tilde{a}(b)\}$$

Proof :

$$\because \tilde{e}(b) \cap \tilde{a}(b) = \min\{\tilde{e}(b), \tilde{a}(b)\}.$$

And since $\tilde{e}(b)$ and $\tilde{a}(b)$ are fuzzy ANS – ideals of $\tilde{U}$.

$$\tilde{e}(b) \cap \tilde{a}(b) \geq$$

$$\min\{\sup(\min\{\tilde{e}(s), \tilde{e}(z)\}), \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})\},$$

Where $b = s * z$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq$$

$$\min\{\sup(\min\{\tilde{e}(s), \tilde{e}(z)\}), \min\{\tilde{a}(s), \tilde{a}(z)\}\}$$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq$$

$$\min\{\sup(\min\{\tilde{e}(s), \tilde{a}(s)\}), \min\{\tilde{e}(z), \tilde{a}(z)\}\}$$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq \min\{\sup(\min\{\tilde{e}(s), \tilde{a}(s)\}), \sup(\min\{\tilde{e}(z), \tilde{a}(z)\})\}$$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq$$

$$\sup(\min\{\min\{\tilde{e}(s), \tilde{a}(s)\}, \min\{\tilde{e}(z), \tilde{a}(z)\}\})$$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq \sup(\min\{\tilde{e}(s), \tilde{a}(s)\}, \min\{\tilde{e}(z), \tilde{a}(z)\})$$

$$\tilde{e}(b) \cap \tilde{a}(b) \geq \sup(\min\{\tilde{e}(s), \tilde{e}(z)\}, \min\{\tilde{a}(s), \tilde{a}(z)\})$$

$$\therefore \tilde{e}(b) \cap \tilde{a}(b) \text{ is a fuzzy ANS – ideal of } \tilde{U}. \blacksquare$$

Theorem (3.12) :

Assume $\tilde{e}(b)$, $\tilde{a}(b)$ be two fuzzy ANS – ideals of Ω , then

$\tilde{e}(b) \cup \tilde{a}(b)$ is a fuzzy ANS – ideal of $\tilde{U}$. wherever $\tilde{e}(b) \cup$

$$\tilde{a}(b) = \max\{\tilde{e}(b), \tilde{a}(b)\}.$$

Proof :

$$\because \tilde{e}(b) \text{ and } \tilde{a}(b) \text{ are fuzzy ANS – ideals of } \tilde{U}.$$

$$\text{And } \tilde{e}(b) \cup \tilde{a}(b) = \max\{\tilde{e}(b), \tilde{a}(b)\}$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq$$

$$\max\{\sup(\min\{\tilde{e}(s), \tilde{e}(z)\}), \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})\}$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq$$

$$\max\{\sup(\min\{\tilde{e}(s), \tilde{e}(z)\}), \sup(\min\{\tilde{a}(s), \tilde{a}(z)\})\}$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq \max\{\sup(\min\{\tilde{e}(s), \tilde{a}(s)\}), \sup(\min\{\tilde{e}(z), \tilde{a}(z)\})\}$$

$$(\tilde{e}(s), \min\{\tilde{e}(z), \tilde{a}(z)\})\}$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq$$

$$\max\{\sup(\min\{\tilde{e}(s), \tilde{a}(s)\}), \sup(\min\{\tilde{e}(z), \tilde{a}(z)\})\}$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq$$

$$\sup(\max\{\min\{\tilde{e}(s), \tilde{a}(s)\}, \min\{\tilde{e}(z), \tilde{a}(z)\}\})$$

$$\tilde{e}(b) \cup \tilde{a}(b) \geq \sup(\min\{\tilde{e}(s), \tilde{a}(s)\}, \min\{\tilde{e}(z), \tilde{a}(z)\})$$

$$\tilde{e}(b) \cup \tilde{a}(b) = \sup(\min\{\tilde{e}(s), \tilde{e}(z)\}, \min\{\tilde{a}(s), \tilde{a}(z)\})$$

$$\therefore \tilde{e}(b) \cup \tilde{a}(b) \text{ is a fuzzy ANS – ideal of } \tilde{U}. \blacksquare$$

Proposition (3.13):

If $\tilde{a}_a^T(b)$ is a fuzzy ANS – ideal of Ω , then it is a fuzzy extension to the fuzzy ANS – ideal $\tilde{a}(b)$ of $\tilde{U}$.

Proof :

suppose $b, z, s \in \tilde{U}$, $a \in [0, T]$

$$\because \tilde{a}_a^T(b) = \tilde{a}(b) + a$$

Since $\tilde{a}_a^T(b)$ is a fuzzy ANS – ideal of $\tilde{U}$

$$\tilde{a}_a^T(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}) + a, \text{ where } b = s * z$$

$$\text{And } \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}) + a \geq \sup(\min\{\tilde{a}(b), \tilde{a}(z)\})$$

$$\therefore \tilde{a}(b) \leq \tilde{a}_a^T(b). \text{ By the definition (2.9)}$$

$$\therefore \tilde{a}_a^T \text{ is a fuzzy extension of } \tilde{a}(b). \blacksquare$$

Proposition (3.14) :

If $\tilde{a}_a^T(b)$ is a fuzzy ANS – ideal of $\Omega \rightarrow \tilde{a}_n^T(b)$ is a fuzzy extension to the fuzzy ANS – ideal $\tilde{a}_n^M(b)$.

Proof :

Assume $b, z, s \in \tilde{U}$, $a \in [0, T]$, $n \in [0, 1]$ and $T = 1 -$

$$\sup\{\tilde{a}(b)/b \in \tilde{U}\}$$

$$\because \tilde{a}_a^T \text{ is a fuzzy ANS – ideal}$$

$$\therefore \tilde{a}_a^T(b) \geq \sup(\min\{\tilde{a}_a^T(s), \tilde{a}_a^T(z)\}), \text{ where } b = s * z$$

$$\tilde{a}_a^T \geq \sup(\min\{\tilde{a}_a^T(s), \tilde{a}_a^T(z)\}) \geq \tilde{a}(b)$$

$$\tilde{a}(b) > \tilde{a}_n^M(b) \geq \sup(\min\{\tilde{a}_n^M(s), \tilde{a}_n^M(z)\})$$

$$\therefore \tilde{a}(b) \geq \sup(\min\{\tilde{a}_n^M(s), \tilde{a}_n^M(z)\})$$

$$\sup(\min\{\tilde{a}_a^T(s), \tilde{a}_a^T(z)\}) \geq \sup(\min\{\tilde{a}_n^M(s), \tilde{a}_n^M(z)\})$$

$$\therefore \tilde{a}_n^M(b) \leq \tilde{a}_a^T(b). \text{ By the definition (2.9)}$$

$$\therefore \tilde{a}_a^T \text{ is a fuzzy extension to the } \tilde{a}_n^M. \blacksquare$$

Proposition (3.15) :

If $\tilde{a}(b)$ is a fuzzy ANS – ideal of $\Omega \rightarrow$ it is a fuzzy extension to the fuzzy ANS – ideal $\tilde{a}_n^M(b)$.

Proof :

Assume $b, z, s \in \tilde{U}$, $n \in [0, 1]$

Since $\tilde{a}(b)$ is a fuzzy ANS – ideal of $\tilde{U}$

$$\tilde{a}(b) \geq \sup(\min\{\tilde{a}(s), \tilde{a}(z)\}), \text{ where } b = s * z$$

And $\tilde{a}_n^M(b)$ is a fuzzy ANS – ideal of Ω .

$$\therefore \tilde{a}_n^M(b) \geq \sup(\min\{\tilde{a}_n^M(s), \tilde{a}_n^M(z)\})$$

Since $n \in [0, 1]$

$$\sup(\min\{\tilde{a}(s), \tilde{a}(z)\}) \geq \sup(\min\{\tilde{a}_n^M(s), \tilde{a}_n^M(z)\})$$

$$\therefore \tilde{a}(b) \geq \tilde{a}_n^M(b). \therefore \text{By definition (2.9)}$$

a fuzzy ANS – ideal $\tilde{a}(b)$ is a fuzzy extension to the fuzzy ANS – ideal $\tilde{a}_n^M(b)$. ■

Proposition (3.16):

If $\tilde{a}_a^T(b)$ is a fuzzy ANS – ideal of B – algebra $(\tilde{U}; *, 0) \rightarrow$ it is a fuzzy extension to the fuzzy ANS – ideal $\tilde{a}_{na}^{\hat{0}}(b)$.

Proof :

Assume $\mathfrak{d}, \mathfrak{z}, \mathfrak{s} \in \mathfrak{U}$, $\mathfrak{a} \in [0, T]$ and $\mathfrak{n} \in [0, 1]$

Since $\tilde{\mathfrak{a}}(\mathfrak{d}) \leq \tilde{\mathfrak{a}}_{\mathfrak{a}}^{\hat{0}}(\mathfrak{d})$, (by theorem (3.13))

$\because \tilde{\mathfrak{a}}_{\mathfrak{n}}^M(\mathfrak{d}) \leq \tilde{\mathfrak{a}}(\mathfrak{d})$, (by theorem (3.15))

$\mathfrak{n}\tilde{\mathfrak{a}}(\mathfrak{d}) \leq \tilde{\mathfrak{a}}(\mathfrak{d})$

$\mathfrak{n}\tilde{\mathfrak{a}}(\mathfrak{d}) + \mathfrak{a} \leq \tilde{\mathfrak{a}}(\mathfrak{d}) + \mathfrak{a}$

$\therefore \tilde{\mathfrak{a}}_{\mathfrak{n}\mathfrak{a}}^{M\hat{0}}(\mathfrak{d}) \leq \tilde{\mathfrak{a}}_{\mathfrak{a}}^{\hat{0}}(\mathfrak{d})$. By definition (2.9)

$\therefore \tilde{\mathfrak{a}}_{\mathfrak{a}}^{\hat{0}}(\mathfrak{d})$ is a fuzzy extension to the $\tilde{\mathfrak{a}}_{\mathfrak{n}\mathfrak{a}}^{M\hat{0}}(\mathfrak{d})$. ■

4-Conclusion:

In this paper , fuzzy ANS – ideal of B – algebra are introduced with same of their useful properties investigated . The relationships between fuzzy $\mathfrak{a}$ – translation , fuzzy $\mathfrak{n}$ – multiplications fuzzy extension and fuzzy magnified and fuzzy ANS-ideal have constructed . It's our hope that this work would become another foundations for further study of the theory of B – algebra .

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