

Image Coding and Decoding by Using Wavelet Transform

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ABSTRACT

Human perception of image is, in part, reliant on detection of formant frequencies and their transitions times. Subjects utilize formant transition length to categorize looks. Image coding and decoding by wavelet is an active area of image research and this done by take the wavelet transform to send the information the frequency domain ,impossible to recognize responsible to the filters of the program and in the same manner but in the reverse form take the inverse wavelet transform and return to the time domain .

تشفير وفك التشفير للصورة باستخدام التحويل الموجي

الخلاصة

الحس البشري او الادراك للصورة يعتمد جزئيا على الترددات الخاصة بكل اجزاء الصورة. ان موضوع البحث هو لتصنيف و تشفير الرؤية الترددية للصورة وفك التشفير باستخدام الـ wavelet ، ومحور البحث يتلخص بعمل التحويل الموجي للصورة بحيث للبحث القدرة على تشفير الصورة بتحويل مفاهيمها الى مجال التردد عند ذلك من المستحيل التعرف عليها لانها بكل بساطة اصبحت على شكل اخر من المعلومات لا تمثل سوى ردة فعل الفلاتر المستخدمة لتلك الصورة و البرنامج اثبت صحة انجاز بنسبة 95% و عندما نعمل فك شفرة اي باراجاع الصورة يكون ذلك بامراتر المعلومات على فلاتر اخرى لها القدرة على استنباط المعلومة بمجال الزمن المحصور بالمنطقة المنخفضة التردد LL ثم عرضها.

INTRODUCTION

A Simple image coding system generally consists of two stages. Firstly, the purely time series data filtered and mapped into a domain more suited to coding. In order to obtain reasonable image coding the transformed signal must have both localized time and frequency resolution, i.e. the ability to capture non-stationary aspects of image such as stop-constants. The windowed FFT process used by many image coding systems provides excellent information on formant frequencies

and slower transitions; however, it has serious deficiencies in representing stop consonants and other key looks used by humans in image coding.

In this research; a statistical wavelet-based analysis of image coding using wavelet decomposition of the covariance matrix of the wavelet coefficients was proposed.

Many researchs and researchers believe that the excellent time-frequency resolution will provide a good representation of image. The abstract nature information obtained in the wavelet transform domain is overcome by using covariance analysis on the wavelet coefficients to determine the best representation of image characteristics [3].

Applications

Practical applications for image coding are obviously various kinds of security systems. Image can serve as a key for any security objects, and it is not so easy in general to lose or forget it. Another important property of image is that it can be transmitted by internet, for example. This provides an ability to automatically code the image and provide access to security objects by telephone. Nowadays, this approach begins to be used for telephone credit card purchases and bank transactions [3].

WAVELET TRANSFORM

Introduction

Wavelets provide convenient sets of basis functions for function spaces, like the Fourier basis consisting of sine and cosine functions. The wavelets are better suited to represent functions that are localized both in time and frequency (it requires fewer terms than the Fourier analysis). In particular, wavelets enable us to represent functions with sharp spikes or edges with fewer terms. This is important in many applications, such as image and data coding, multi-resolution analysis. How does one construct wavelets? One starts from the basic “mother wavelet” $\psi(t)$ and generates the basis by dilation and translation in time $\psi(at-b)$. For discrete wavelets, $a = 2^j$ and $b = k$, where j and k are integers. The dilation underlies the a hierarchical representation of the data set, which is the basis of the so-called multi-resolution analysis [1].

There exists a large selection of wavelet families depending on the choice of the mother wavelet. To limit the choice, we typically enforce some desirable properties such as orthogonality, compactness of support, rapid decay, and smoothness. But the optimal choice of the wavelet basis will depend on the application at hand [2].

Wavelet theory can be divided into the following main categories:

1. Continuous wavelet transforms
2. Discrete wavelet transforms

The Continuous Wavelet Transform

The Continuous Wavelet Transform (CWT) is provided by equation (1), where $x(t)$ is the signal to be analyzed. $\Psi(t)$ is the mother wavelet or the basis function. All the wavelet functions used in the transformation are derived from the mother wavelet through translation (shifting) and scaling (dilation or coding).

$$X_{wt}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) - \Psi^t\left(\frac{1-\tau}{s}\right) dt. \dots (1)$$

The mother wavelet used to generate all the basis functions is designed based on some desired characteristics associated with that function. The translation parameter τ is related to the location of the wavelet function as it is shifted through the signal. Thus, it corresponds to the time information in the Wavelet Transform. The scale parameter s is defined as $(1/\text{frequency})$ and corresponds to frequency information. Scaling either dilates expands or compresses a signal. Large scales (low frequencies) dilate the signal and provide detailed information hidden in the signal, while small scales (high frequencies) compress the signal and provide global information about the signal. Notice that the Wavelet Transform merely performs the convolution operation of the signal and the basis function. The above analysis becomes very useful as in most practical applications, high frequencies (low scales) do not last for a long duration, but instead, appear as short bursts, while low frequencies (high scales) usually last for entire duration of the signal [3].

The Discrete Wavelet Transform

The Discrete Wavelet Transform (DWT) involves choosing scales and positions based on powers of two, so called dyadic scales and positions. The mother wavelet is rescaled or dilated, by powers of two and translated by integers.

The Discrete Wavelet Transform provides high redundancy as far as the reconstruction of the signal is concerned. This redundancy, on the one hand, requires a significant amount of computation time and resources. The Discrete Wavelet Transform, on the other hand, provides sufficient information both for analysis and synresearch of the original signal, with a significant reduction in the computation time [3]. DWT employs two sets of functions, called scaling functions and wavelet functions, as below:

$$\phi_{j,k}(x) = 2^{j/2} \phi(2^j x - k) \quad (2)$$

$$\psi_{j,k}(x) = 2^{j/2} \psi(2^j x - k) \quad (3)$$

For different integer values of j and k . Integer k represents translation of the wavelet function and is an indication of time or space in wavelet transform. Integer j , however, is an indication of the wavelet frequency or spectrum shift and generally referred to as scale [1].

Wavelet Transform (WT)

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelets cut up data into different frequency components, and then analyze each component with a resolution matched to its scale. Instead of fixing

the time and the frequency resolution Δt and Δf , both of them can be varied in time-frequency plane in order to obtain a multiresolution analysis [4].in terms of filter bank terminology, the frequency responses of the analysis filters are regularly spaced in a logarithmic scale, as shown in Figure (1).

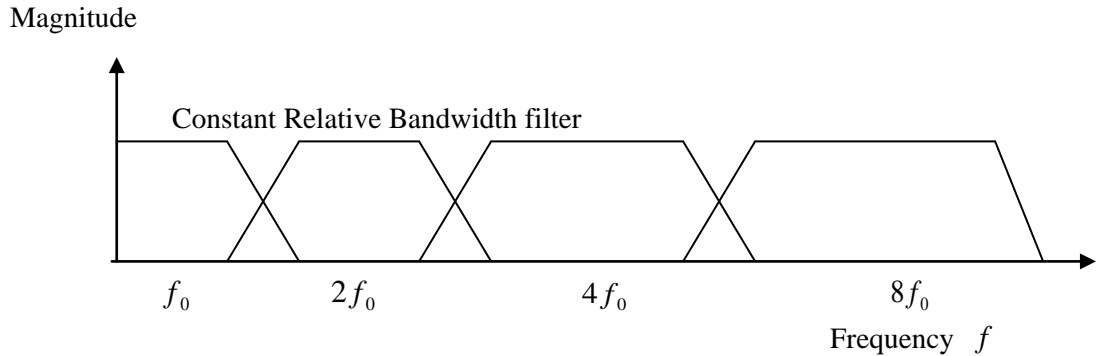


Figure (1) Division of the Frequency Domain for the WT.

The way that the time-frequency plane is resolved in this approach as shown in Figure (2):

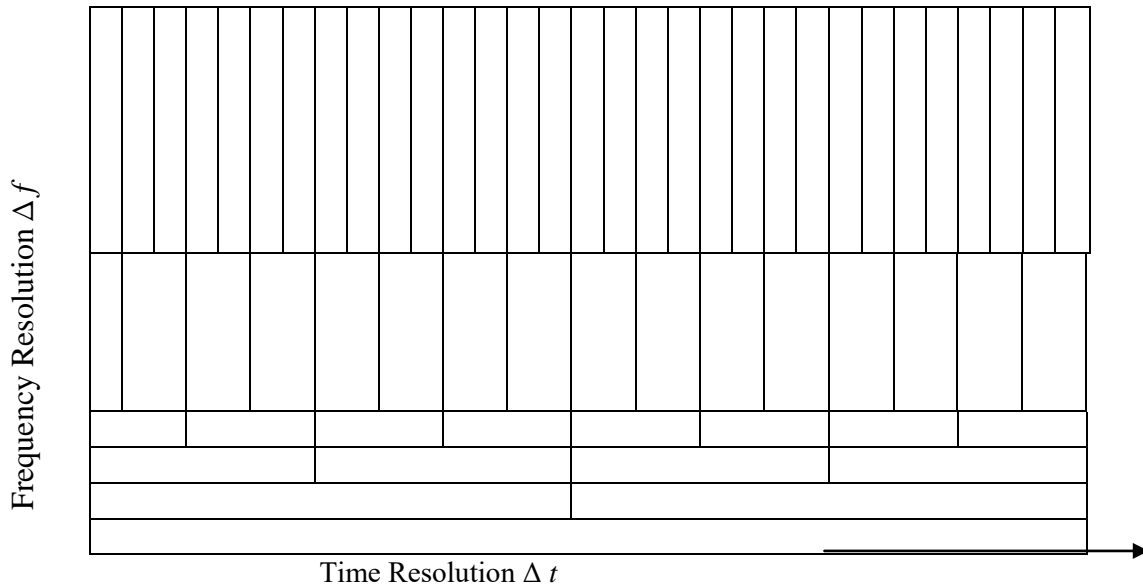


Figure (2) Time – Frequency Resolution of WT.

Wavelet transform has been found very useful for the time-scale representation and has been widely used in signal processing and computer vision [3]. The most interesting dissimilarity between those kinds of transforms is that individual wavelet

function are localized in space. This localization feature makes many function and operators using wavelets “sparse” when transformed into the wavelet domain and results in a number of useful applications, such as data coding, detection features in images, and removing noise from time series [4]. A signal or function $x(t)$ can often be better analyzed, described, or processed if expressed as a linear decomposition by:

$$x(t) = \sum_k a_k \Psi_k(t) \quad \dots (4)$$

Where k is integer index for the finite sum, a_k are real-valued expansion coefficients, and $\{\Psi_k(t)\}$ are a set of real-valued functions of t called the expansion set.

For the wavelet expansion, a two parameter system is constructed such that equation (2.4) becomes:

$$x(t) = \sum_k \sum_j a_{j,k} \Psi_{j,k}(t) \quad \dots (5)$$

This wavelet expansion is in terms of two indices, the time translation index k and the scaling index j , where both j and k are integer indices and the $\Psi_{j,k}(t)$ are the wavelet expansion basis function. [3].

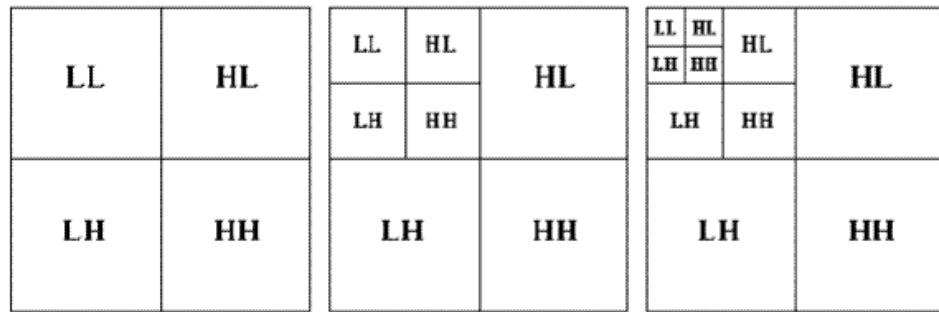
Wavelet Analysis Properties

Wavelet expansions and wavelet transforms have proven to be very efficient and effective in analyzing a very wide class of signals and phenomena. The properties that give this effectiveness are [5]:

1. The sizes of wavelet expansion coefficients $a_{j,k}$ in equation (3) drop off rapidly with j and k for a large class of signals. This property is called unconditional basis and it is why wavelets are so effective in signal and image coding, denoising, and detection.
2. The wavelet expansion allows a more accurate local description and separation of signal characteristics. A Fourier coefficient represents a component that lasts for all time and, therefore, temporary events must be described by a phase characteristic that allows cancellation or reinforcement over large time periods. A wavelet expansion coefficient represents a component that is itself local and is easier to interpret. The wavelet expansion may allow a separation of components of a signal that overlap in both time and frequency.
3. Wavelets are adjustable and adaptable. Because there is not just one wavelet, they can be designed to fit individual applications. They are ideal for adaptive system that adjusts them to suit the signal.
4. The generation of wavelets and the calculation of the discrete wavelet transform are well matched to the digital computer. There are no derivatives or integrals, just multiplication and additions-operations that are basic to a digital computer.

Wavelet Transform in Multi-dimensions

A wavelet transform of a d-dimensional array is most easily obtained by transforming the array sequentially on its first index and then on its second index, and so on. The situation is exactly like that for multidimensional FFTs. An immediate application of the two-dimensional transform is image coding. It is generally better than Fourier transform see Figure (3) [5].



(a) Single Level Decomposition (b) Two Level Decomposition (c) Three Level Decomposition

Figure (3) Image Decomposition Using Wavelet Transform.

Wavelet Families

There are a number of basis functions that can be used as the mother wavelet for Wavelet Transformation. Figure (4) illustrates some of the commonly used wavelet functions. Haar wavelet is one of the oldest and simplest wavelet. Therefore, any discussion of wavelets starts with the Haar wavelet. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and π . This is a very desirable property in some applications. The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer, Morlet and Mexican Hat wavelets are symmetric in shape. The wavelets are chosen based on their shape and their ability to analyze the signal in a particular application [6].

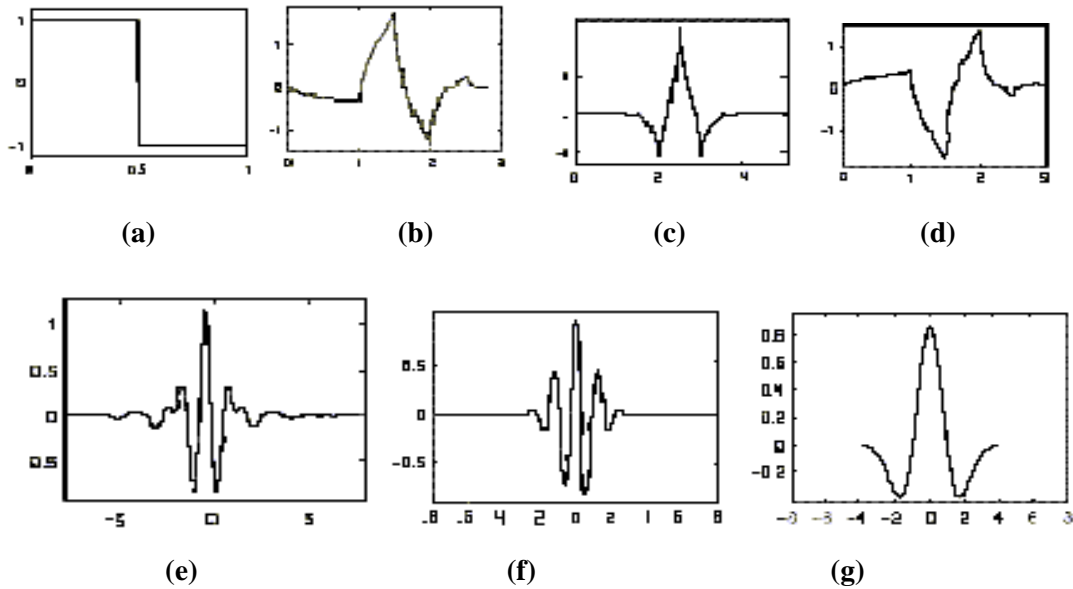


Figure (4) Wavelet Families (a) Haar (b) Dabechies4 (c) Coiflet (d) Symlet2 (e) Meyer (f) Morlet (g) Mexican Hat.

Implementation of wavelet transform

To explain the implementation of 2D discrete wavelet transform decomposition and reconstruction, numerous filters can be used to implement the wavelet transform. One of the commonly used is haar. It is separable, so they can be used to implement the wavelet transform by first convolving them with the rows and then the columns. The haar basis filters are:

$$\text{LOWPASS: } [1/\sqrt{2}] \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\text{HIGHPASS: } [1/\sqrt{2}] \begin{bmatrix} 1 & -1 \end{bmatrix}$$

The following matrix is used as input image example:
original image M (4,4).

$$m = \begin{array}{|c|c|c|c|} \hline 1 & 1 & 2 & 4 \\ \hline 1 & 2 & 4 & 1 \\ \hline 0 & 1 & 0 & 2 \\ \hline 1 & 2 & 3 & 4 \\ \hline \end{array}$$

A- Discrete wavelet transform decomposition

To start decomposition ,Low_ pass filter for decomposition was needed {Lo_d=[1 1]},and High_pass filter {Hi_d=[-1 1]}.

Step 1: By passing the lowpass filter on the rows of the original matrix M through the convolution process {a1=conv2(M,Lo_d)},the result is a matrix a1 (4,5):

$$aL = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 3 & 6 & 4 \\ \hline 1 & 3 & 6 & 5 & 1 \\ \hline 0 & 1 & 1 & 2 & 2 \\ \hline 1 & 3 & 5 & 7 & 4 \\ \hline \end{array}$$

Step 2: By applying the decimation by2 (down sampling) to the columns of the matrix a1 the result is a matrix aL (4,2):

$$aL = \begin{array}{|c|c|} \hline 2 & 6 \\ \hline 3 & 5 \\ \hline 1 & 2 \\ \hline 3 & 7 \\ \hline \end{array}$$

Step 3: By passing the lowpass filter on the columns of the matrix (aL) through the convolution process{a2=conv2(AL,Lo_d)}, the result is a matrix a2 (5,2) :

$$a2 = \begin{array}{|c|c|} \hline 2 & 6 \\ \hline 5 & 11 \\ \hline 4 & 7 \\ \hline 4 & 9 \\ \hline 3 & 7 \\ \hline \end{array}$$

Step 4: By applying the decimation by2 (down sampling) to the rows of the matrix a2 the result is a matrix LL (2,2):

$$LL = \begin{array}{|c|c|} \hline 5 & 11 \\ \hline 4 & 9 \\ \hline \end{array}$$

Step 5: By passing the highpass filter on the columns of the matrix (aL) through the convolution process the result is a matrix a3 (5,2) :

$$a_3 = \begin{array}{|c|c|} \hline -2 & -6 \\ \hline -1 & 1 \\ \hline 2 & 3 \\ \hline -2 & -5 \\ \hline 3 & 7 \\ \hline \end{array}$$

Step 6: By applying the decimation by 2 (down sampling) to the rows of the matrix a_3 , the result is a matrix LH (2,2):

$$LH = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline -2 & -5 \\ \hline \end{array}$$

Step 7: Again, by passing the highpass filter on the columns of the original matrix (M) through the convolution process, the result is a matrix a_4 (4,5):

$$a_4 = \begin{array}{|c|c|c|c|c|} \hline -1 & 0 & -1 & -2 & 4 \\ \hline -1 & -1 & -2 & 3 & 1 \\ \hline 0 & -1 & 1 & -2 & 2 \\ \hline -1 & -1 & -1 & -1 & 4 \\ \hline \end{array}$$

Step 8: By applying the decimation by 2 (down sampling) to the columns of the matrix a_4 , the result is a matrix a_H (4, 2):

$$a_H = \begin{array}{|c|c|} \hline 0 & -2 \\ \hline -1 & 3 \\ \hline -1 & -2 \\ \hline -1 & -1 \\ \hline \end{array}$$

Step 9: By passing the lowpass filter on the columns of the matrix (a_H) through the convolution process, the result is a matrix a_5 (5,2):

$$a5 =$$

0	-2
-1	1
-2	1
-2	-3
-1	-1

Step 10: By applying the decimation by2 (down sampling) to the rows of the matrix a5, the result is a matrix HL (2,2):

$$HL =$$

-1	1
-2	-3

Step 11: By passing the highpass filter on the columns of the matrix (aH) through the convolution process, the result is a matrix a6 (5,2):

$$a6 =$$

0	2
1	-5
0	5
0	-1
-1	-1

Step 12: By applying the decimation by2 (down sampling) to the rows of the matrix a6, the result is a matrix HH (2, 2):

$$HH =$$

1	-5
0	-1

The final result of 2_D DWT one level is a matrix X (4,4) by assembling the four subimages:

$$X =$$

5	11	-1	1
4	9	-2	-5
-1	1	1	-5
-2	-3	0	-1

The matrix(X) is represented the final result of discrete wavelet transformation (one level of decomposition).

Discrete wavelet transform reconstruction (wavelet inverse):

To start reconstruction, Low pass filter for reconstruction was needed {Lo_r=[1 1]}, and High_pass filter {Hi_r=[1 -1]} .

Step 1: By interpolation (up sampling) on the rows of the matrix LL the obtained result a matrix x1 (3,2):

$$x1 = \begin{array}{|c|c|} \hline 5 & 11 \\ \hline 0 & 0 \\ \hline 4 & 9 \\ \hline \end{array}$$

Step 2: By interpolation (up sampling) on the rows of the matrix LH the obtained result a matrix x2 (3,2):

$$x2 = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline 0 & 0 \\ \hline -2 & -5 \\ \hline \end{array}$$

Step 3: By passing the lowpass filter on the columns of the matrix (x1) through the convolution process, the result is a matrix u1 (4,2):

$$u1 = \begin{array}{|c|c|} \hline 5 & 11 \\ \hline 5 & 11 \\ \hline 4 & 9 \\ \hline 4 & 9 \\ \hline \end{array}$$

Step 4: By passing the highpass filter on the columns of the matrix (x2) through the convolution process, the result is a matrix u2 (4,2):

$$u2 = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline 1 & -1 \\ \hline -2 & -5 \\ \hline 2 & 5 \\ \hline \end{array}$$

Step 5: Find the summation of the two matrices (u1) and (u2), {a1=u1+u2} the result is a matrix a1 (4,2):

$$a1 = \begin{array}{|c|c|} \hline 4 & 12 \\ \hline 6 & 10 \\ \hline 2 & 4 \\ \hline 6 & 14 \\ \hline \end{array}$$

Step 6: By interpolation (up sampling) on the rows of the matrix HL the obtained result a matrix x3 (3,2):

$$X3 = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline 0 & 0 \\ \hline -2 & -3 \\ \hline \end{array}$$

Step 7: By interpolation (up sampling) on the rows of the matrix HH the obtained result a matrix x4 (3, 2):

$$X4 = \begin{array}{|c|c|} \hline 1 & -5 \\ \hline 0 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$$

Step 8: By passing the lowpass filter on the columns of the matrix (x3) through the convolution process, the result is a matrix u3 (4, 2):

$$u3 = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline -1 & 1 \\ \hline -2 & -3 \\ \hline -2 & -3 \\ \hline \end{array}$$

Step 9: By passing the highpass filter on the columns of the matrix (x4) through the convolution process, the result is a matrix u4 (4, 2):

$$u4 = \begin{array}{|c|c|} \hline 1 & -5 \\ \hline -1 & 5 \\ \hline 0 & -1 \\ \hline 0 & 1 \\ \hline \end{array}$$

Step 10: Find the summation of the two matrices (u3) and (u4), {a2=u3+u4} the result is a matrix a2 (4,2)

$$a2 = \begin{array}{|c|c|} \hline 0 & -4 \\ \hline -2 & 6 \\ \hline -2 & -4 \\ \hline -2 & -2 \\ \hline \end{array}$$

Step 11: By interpolation (up sampling) on the columns of the matrix a1 the obtained result a matrix x5 (4, 3):

$$x5 = \begin{array}{|c|c|c|} \hline 4 & 0 & 12 \\ \hline 6 & 0 & 10 \\ \hline 2 & 0 & 4 \\ \hline 6 & 0 & 14 \\ \hline \end{array}$$

Step 12: By interpolation (up sampling) on the columns of the matrix a2 the obtained result a matrix x6 (4, 3):

$x_6 =$

0	0	-4
-2	0	6
-2	0	-4
-2	0	-2

Step 13: By passing the lowpass filter on the rows of the matrix (x_5) through the convolution process, the result is a matrix u_5 (4,4):

$u_5 =$

4	4	12	12
6	6	10	10
2	2	4	4
6	6	14	14

Step 14: By passing the highpass filter on the rows of the matrix (x_6) through the convolution process, the result is a matrix u_6 (4, 4):

$u_6 =$

0	0	-4	4
-2	2	6	-6
-2	2	-4	4
-2	2	-2	2

Step 15: Find the summation of the two matrices (u_5) and (u_6), $\{u=u_5+u_6\}$ the result is a matrix u (4,4):

$u =$

4	4	8	16
4	8	16	4
0	4	0	8
4	8	12	16

Step 16: The final result of reconstruction is a matrix y (4,4) after dividing the elements of matrix u by four:

$y =$

1	1	2	4
1	2	4	1
0	1	0	2
1	2	3	4

Software Implementation

First: Run the matlab file about coding the image as in Figure (5):

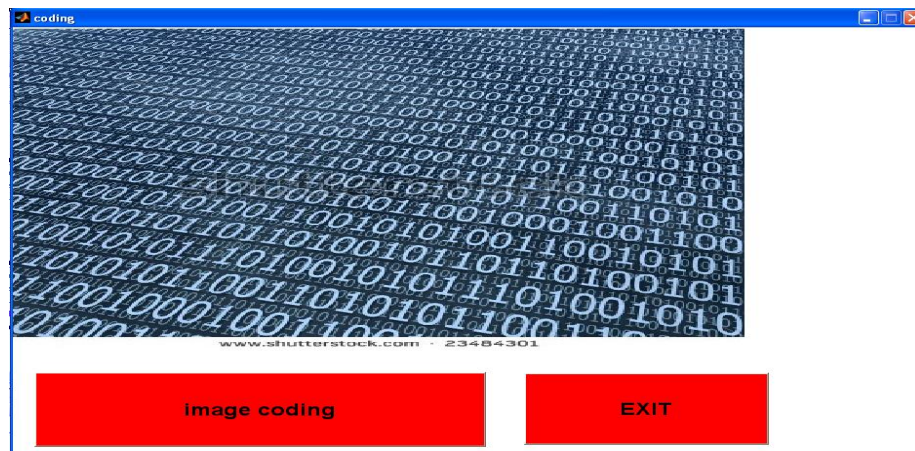


Figure (5) Matlab File Image.

Second: press the button Image Coding to select the image as shown in Figure (6):

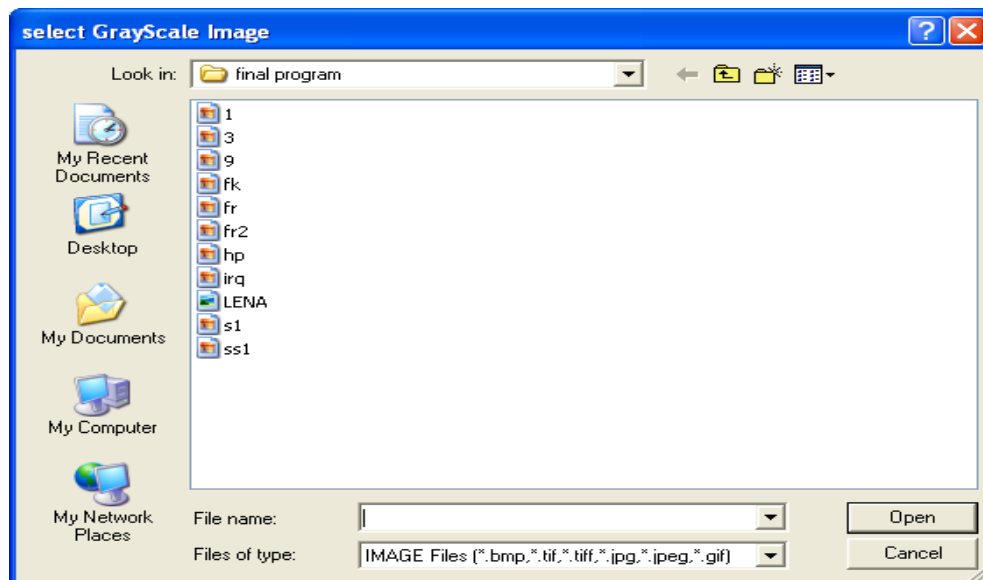


Figure (6) Selecting the Image.

Third: Take the code of the image that saved on the file named db.wk1, the coding process is illustrated as in Figure (7):

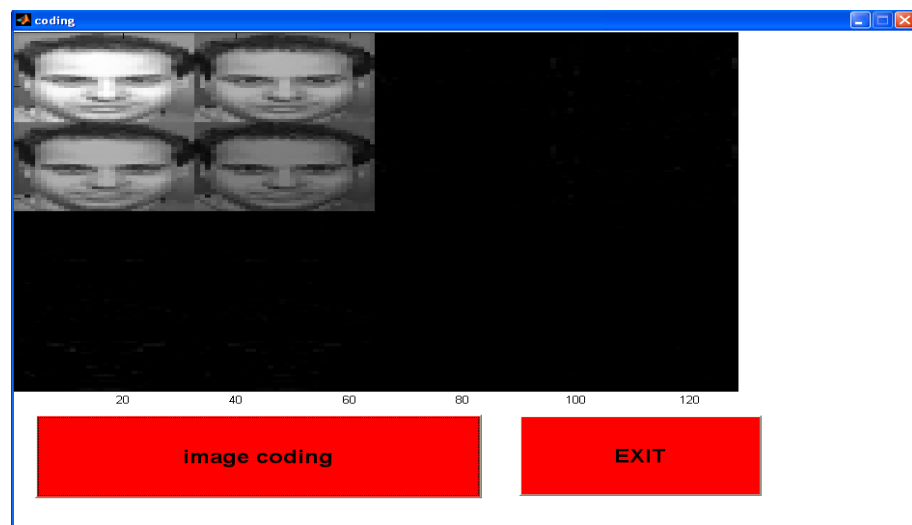


Figure (7) The Coding Process.

See that the image is shown to be in the square of LL and more accurate at second level multi-wavelet.

Forth: Run the matlab program named imdecoding as shown in Figure (8):

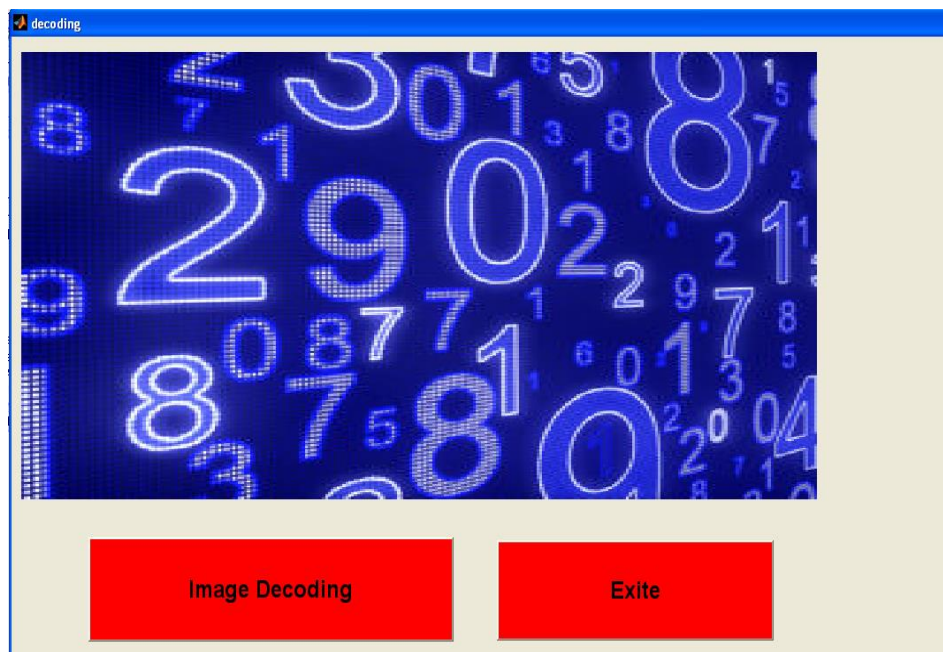


Figure (8) Matlab Program imdecoding.

Fifth: only press the button Image Decoding then show the genion image as in Figure (9):

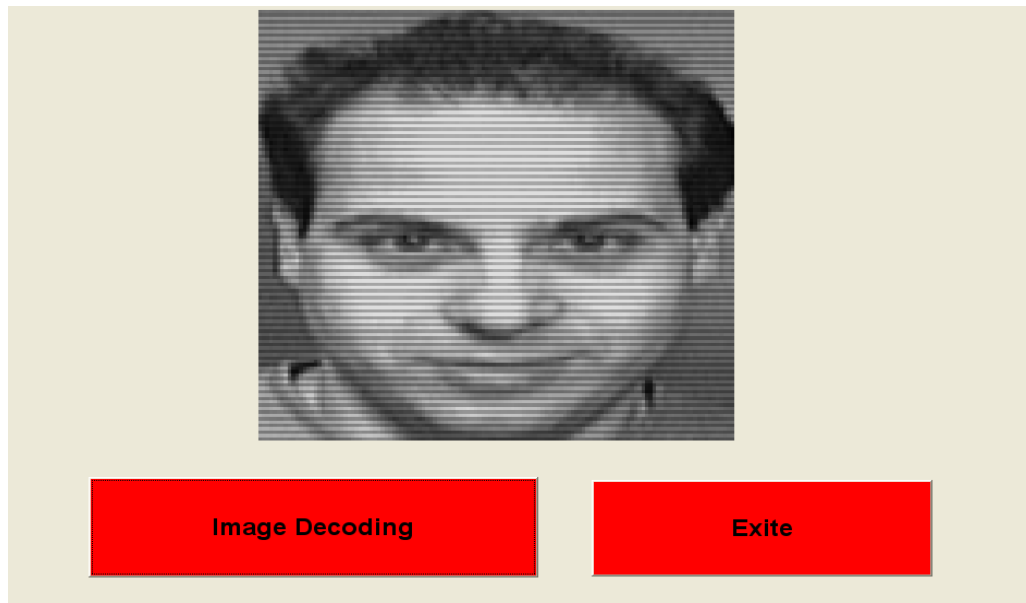


Figure (9) The Decoding Process.

CONCLUSION AND CONTRIBUTION

The demand for coding technology increases every year in parallel with the increase in aggregate bandwidth for the transmission of image and video signals. As a result, the Wavelet-based approach plays an important role in the scheme of things as Perceptual coding of image signals found its way to a growing number of consumer applications.

The foremost criterion for image coding technology is to achieve a certain signal quality at a given bit-rate as this directly translates to cost savings by getting a higher coding ratio at the same quality of service. Wavelet-based coding is claimed to be more efficient at low bit rates but are actually less successful than discrete cosine transform (DCT) -based systems in achieving good efficiency at near-transparent coding ratios.

Computational complexity also limits the algorithmic implementation of a codec. As a result, algorithmic delay becomes an important constraint especially for two-way communications applications. In that respect, it is notable that Wavelet coding does require more computational power than DCT-based coding.

Research Contribution is that the entire implemented scheme performs reasonably well with an average fidelity and with much less computational burden. In addition, using wavelets, the coding ratio can be easily varied, while most other coding techniques have fixed coding ratios.

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