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Uniform Step Loading of a Partial Interaction Composite Beam

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ABSTRACT

In the present paper, a one-dimensional finite element model for the analysis of composite beams of partial interaction is constructed. This model was verified against some analytical results available in the literature and achieved very good agreement with the natural frequencies and the time histories it was compared to. Then it was utilised to analyse partial interaction composite beams under the effect of uniform step loads and provided important information about the expected dynamic amplification factors, which turned out to be particularly high, and the effects of the linear stiffness ratio of the interface and the boundary conditions of the lower layer of the beam. The results, in particular, showed that even for extreme cases the orders of magnitude of the slip and the corresponding uplift remain the same. This pointed out an important finding that the uplift in the researched context, at least, is not negligible as it is widely assumed in the literature.

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1. Introduction

The structural performance of composite beams of partial interaction has been in the research interest in structural engineering since the pioneering work of Newmark (1951) in the 1950s in which he pointed out essential theoretical and experimental aspects of the subject in the static loading realm. Later on, many researchers expanded on that pioneering work and bifurcated it into several trends, some of which are analytical, some are numerical and the others are completely experimental (Plum and Horne, 1975; Hirst and Yeo, 1980; Oven *et al.*, 1997; Taig and Ranzi, 2015; Turmo *et al*., 2015; Liu *et al*., 2016; Nguyen and Hjiaj, 2016; Bedon and Fragiacomo, 2019; Wang *et al*., 2020; Martinelli, 2021; Sun *et al*., 2022).

The appearance of works in the literature that dealt with the dynamics of composite beams of partial interaction came much later. One pioneering work of those is that of Girhammar and Pan (1993) who extended the usual separation of variables analysis to the governing equation of the vibrations of a composite beam of partial interaction. They presumed that the only flexibility of the interface is the shear (or the tangential) one, neglecting any effects of the uplift (or the transverse) flexibility of the interface. This particular trend of neglecting the effects of uplift was, seemingly, inherited from the mainstream research dealing with the static of those composite beams.

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It persisted for most parts of the two compartments of the relevant literature; the static and the dynamic ones. This neglect seems to depend on no reliable experimental or heuristic basis, however. On the contrary, inspecting the dominant form of constructing an interface in civil engineering practice reveals that the two flexibilities should be of the same order of magnitude. This is the case since those flexibilities are usually derived from the tensile and the shear strains of the same number of connectors which are of the same order of magnitude. Furthermore, experimental evidence exists that the transverse flexibility could be as low as half of the shear counterpart (Balakrishnan, 1963). This is in direct contradiction with the prevailing assumption of neglecting the effect of transverse flexibility since that requires the presumption that this particular flexibility is infinite, or practically infinite, at least. One notable exception to this trend is that of Adekola (1968) who presented a force-based extension of Newmark's original theory that took uplift into account and delineated its importance and true order of magnitude in some static loading conditions.

The application of the finite element model constructed in the present work to the particular problem of a uniform step loading of those composite beams revealed the large extent to which the usual assumption of negligibility of the uplift might be erroneous.

2. The Finite Element Model

In the present section, a one-dimensional finite element model is constructed to simulate the action of composite beams with partial interaction. Then, the use of this model is going to demonstrate that such a one-dimensional representation composed of only two types of linear elements is capable of reproducing the corresponding results previously obtained for the vibrations, uplift, and internal actions of composite beams of partial interaction. In particular, detailed conclusions are made about the performance of such composite beams under uniform step loads.

To simulate the material and the mechanical properties of a two-layer partial interaction composite beam, two types of finite elements are needed. The first, of course, is some choice of beam elements. The choice would certainly be affected by the particular implementation of the present FEM model. In this work, the model was implemented on ANSYS and it was found appropriate to use BEAM188 for the beam elements and COMBIN14 for the spring elements. Both elements are endowed by the programmers of ANSYS with much more capabilities than those immediately needed for the present work. Those extra capabilities were turned off and the remaining representations were linearly elastic springs in the two directions with no added damping or masses and simple beam elements that are capable of Timoshenko beam properties for short beams only while behaving practically like Euler-Bernoulli beams for slenderer beams.

Irrespective of boundary conditions, the composite beam is to be discretised into two distinct layers of the beam element of choice on top of each other in a node-by-node format as shown in Fig. 1 below. Those two layers are meant to simulate the beam action introduced by the two layers of the composite beam. At each layer, a different modulus of elasticity and different geometrical properties of area and moment of inertia are assigned in the FEM software of choice to characterise that layer completely. Those two layers of beam elements are situated exactly at the centroidal axes of those layers and as such would be separated vertically by the eccentricity characterising the composite beam. In the practice of implementing this model on ANSYS, it was found that building the eccentricity into the ANSYS representation is problematic. A workaround, however, was to introduce it later in the section data.

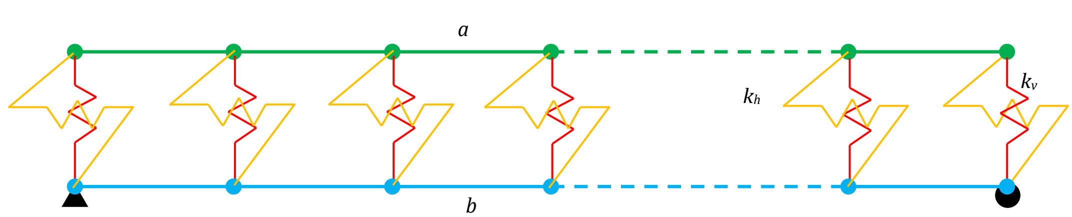


Fig. 1 A one-dimensional finite element model of a partial interaction composite beam

The interface between those two layers of beam elements is represented by two sets of linear springs, a set of horizontal and another of vertical springs. The horizontal springs are meant to represent horizontal or shear stiffness of the interface, while the vertical springs are meant to represent the transverse counterpart. All springs are to be attached at the nodes of the beam elements and the value of each spring is given by

$$k_h = \frac{K_s L}{n+1} \tag{1}$$

$$k_v = \frac{K_n L}{n+1} \tag{2}$$

where k_h and k_v are the values of each horizontal and vertical linear spring, respectively. The symbols K_s, and K_n stand for the linear shear and transverse stiffness values of the interface, respectively; while L stands for the length of the beam and n stands for the number of beam elements used to discretise each layer.

In order to show the validity and robustness of the FEM model presented herein, verification against the results of Shen *et al*. (2011) is conducted.

Shen *et al*.'s (2011) example used in the present work for verification purposes consists of a four metre T-composite beam as illustrated in Fig. 2 below.

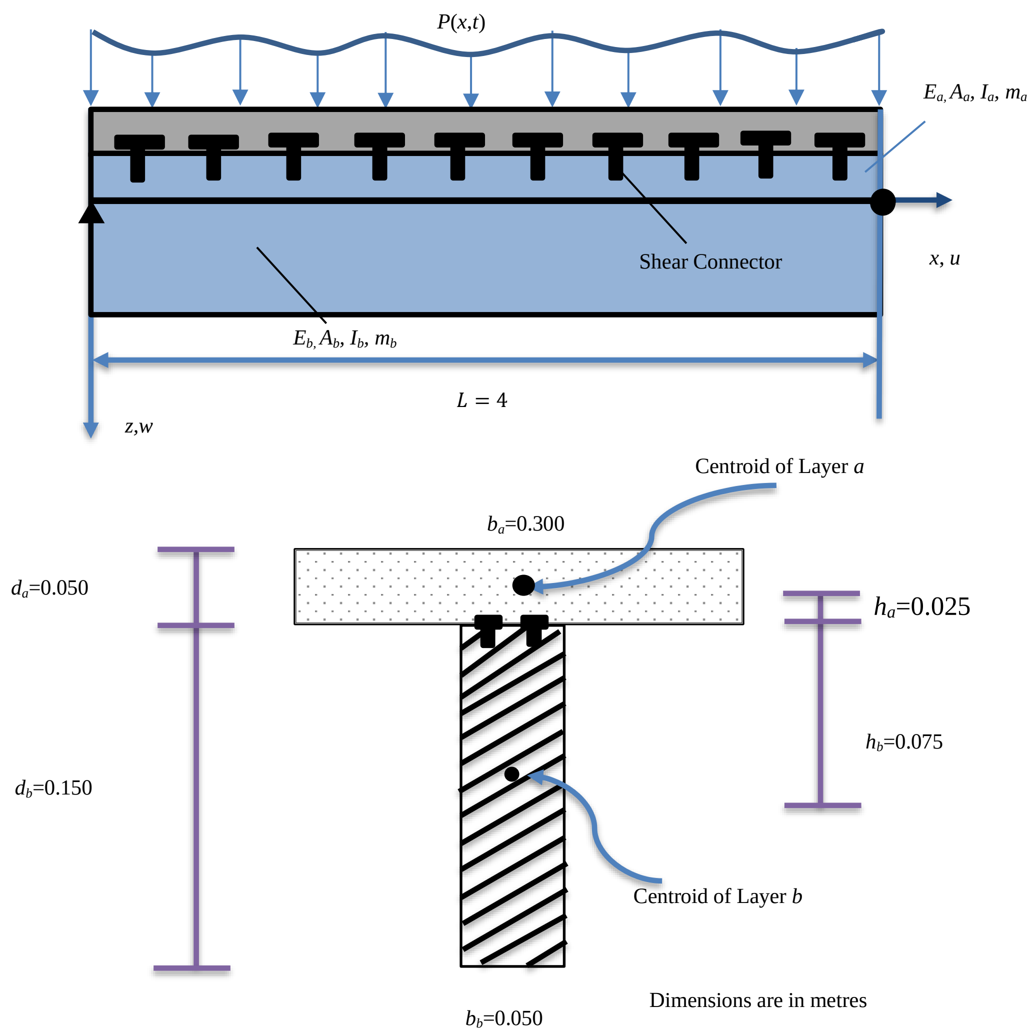


Fig. 2 Shen *et al*.'s (2011) example

The linear masses of the example are m_a = 36 kg/m and m_b = 3.75 kg/m, while the moments of inertia are I_a = 3.125×10^{-6} m^4 and I_b = 14.0625×10^{-6} m^4. The moduli of elasticity of the two layers were E_a = 12×10^9 N/m^2 and E_b

$= 8\times10^9$ N/m^2. The shear modulus K_s of the interface was taken to be 50 MN/m. In Shen *et al*.'s (2011) work, the transverse modulus was not taken into account, setting the curvatures and the deflections of the two layers of the beam equal everywhere.

In linear analysis, it is always possible, at least in an implicit manner to express the response of a physical system as a sum of products each of which is a single variable function of the independent variables of the problem.

More specifically, if $u(x,t)$ is the response of a system, say u is a displacement of a vibrating beam, for instance, then there is an infinite set of real-valued functions $\{\phi_i(x)\}$ and another set $\{\psi_i(t)\}$ for i=1,2,3,… such that

$$u(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\psi_i(t) \tag{3}$$

Equation 3 may reduce to a finite sum in particular cases. Together with the linearity of the governing equation, equation 3 could be exploited to construct an efficient solution of the problem at hand.

In the jargon of the theory of vibration of mechanical systems, $\phi_i(x)$ are known as mode shapes, while $|\psi_i(t)|$ are known as the natural frequencies of the system.

A quality of those mode shapes known as the orthogonality of mode shapes could be used to further increase the efficiency of the constructed solution. The orthogonality property of the mode shapes is expressed by

$$\int_0^L \phi_k(x)\phi_m(x) = 0 \quad for\ all\ k \neq m \tag{4}$$

Although the current FEM model of composite beams with partial interaction does not express the physics of the problem directly in terms of a set of governing equations, it certainly is underlain by such a set. At any rate, the FEM model or the underlying governing set of differential equations should relate $u_a(x,t)$, $u_b(x,t)$, $w_a(x,t)$, and $w_b(x,t)$ as a set of dependant field variables to the independent spatial, x, and the temporal variable, t. Where $u_a(x,t)$, and $u_b(x,t)$ are the axial displacements along the centroids of the layers a and b, respectively, while $w_a(x,t)$, and $w_b(x,t)$ are the corresponding transverse counterparts.

The mode superposition method for the constructed FEM model would rely on the plausible constructability of suitable compositions similar to 3. In particular, the existence and the numerical reliability of the sets of functions $\left\{\phi_{u_{a_i}}(x)\right\}$, $\left\{\phi_{u_{b_i}}(x)\right\}$, $\left\{\phi_{w_{a_i}}(x)\right\}$, $\left\{\phi_{w_{b_i}}(x)\right\}$, $\left\{\psi_{u_{a_i}}(t)\right\}$, $\left\{\psi_{u_{b_i}}(t)\right\}$, $\left\{\psi_{w_{a_i}}(t)\right\}$, $\left\{\psi_{w_{b_i}}(t)\right\}$ such that the following compositions apply, could be presumed

$$u_a(x,t) = \sum_{i=1}^{\infty} \phi_{u_{a_i}}(x)\psi_{u_{a_i}}(t) \tag{5}$$

$$u_b(x,t) = \sum_{i=1}^{\infty} \phi_{u_{b_i}}(x)\psi_{u_{b_i}}(t) \tag{6}$$

$$w_a(x,t) = \sum_{i=1}^{\infty} \phi_{w_{a_i}}(x)\psi_{w_{a_i}}(t) \tag{7}$$

$$w_b(x,t) = \sum_{i=1}^{\infty} \phi_{w_{b_i}}(x)\psi_{w_{b_i}}(t) \tag{8}$$

However, Equations 5-8, above, are only directly usable when there exists a governing system of differential equations that is explicitly separable. In numerical work, like in the FEM, only combinations of the pure modes are calculable.

For validation purposes, a comparison of the natural frequencies of the modes as computed by the present FEM analysis and that of Shen *et al*. (2011) revealed good agreement as is shown in Table 1 below.

Using the so-called "state-space" method, Shen *et al*. (2011), also, solved the example appearing in Fig. 2 for the effects of a 1 N load moving across the beam at a constant speed of 10 *m*/*sec*. Here, histories of $u_a(x,t)$, $u_b(x,t)$, $w_a(x,t)$ and $w_b(x,t)$ are produced in Figs. 3 *a-e* for the time interval [0,0.4] *sec*, near the roller and at the mid-span for the purpose of comparing the present results to the those of Shen *et al*. (2011). Only $w_a(x,t)$ and $w_b(x,t)$ are actually of direct utility for the comparison since they are the only ones comparable to $w(x,t)$ that was computed by Shen *et al*. (2011). In fact, according to their assumptions $w_a(x,t) = w_b(x,t) = w(x,t)$ for all x and t.

Table 1- Comparison of the natural frequencies of Shen *et al.*'s (2011) example as given by their analysis and the present one

Shen *et al.* (2011) mode order	Frequency (rad/sec)	Frequency (Hz)	Current work mode order	Frequency (Hz)	Corresponding Shape	Discrepancy (%)
1	64.85	10.3212	1	10.187	1	1.30
2	210.65	33.52599	2	32.676	2	2.54
3	417.72	66.4822	3	63.577	3	4.37
4	692.22	110.1702	4	89.911	Spring Mode	×
5	1038.44	165.2729	5	100.95	4	8.37
6	1458.39	232.11	6	139.98	5	15.30

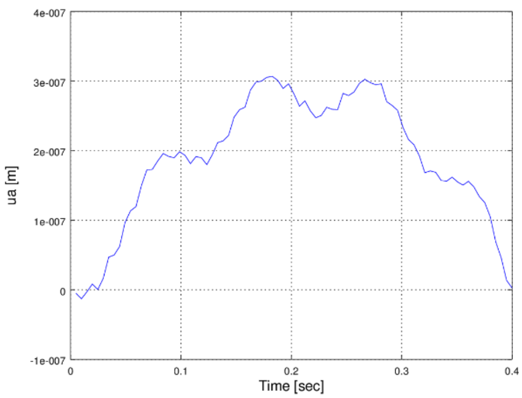


Fig. 3 *a* The variation of $u_a(t)$ near the roller support of Shen *et al.*'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m/sec* as computed by the present model

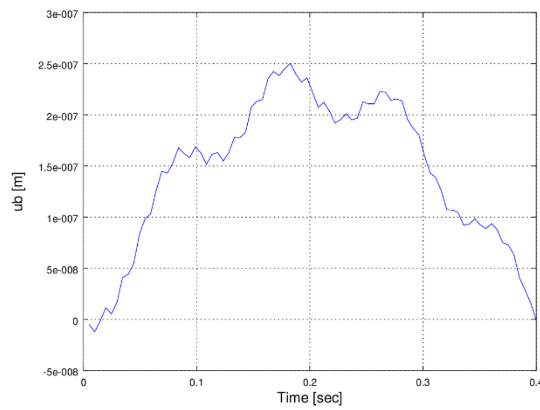


Fig. 3 *b* The variation of $u_b(t)$ near the roller support of Shen *et al.*'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m/sec* as computed by the present model

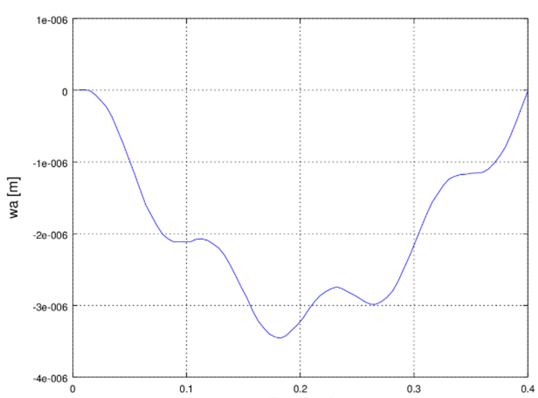


Fig. 3 *c* The mid-span $w_a(t)$ of Shen *et al.*'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m/sec* as computed by the present model

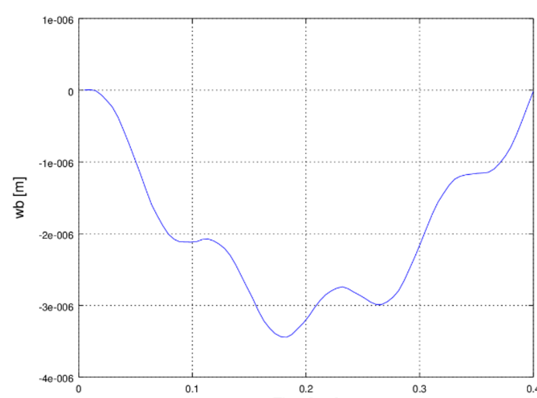


Fig. 3 *d* The mid-span $w_b(t)$ of Shen *et al.*'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m/sec* as computed by the present model

Figure 3 *e* below, on the other hand, give the definitive comparison of the results of the present work and those of Shen *et al*. (2011), for time histories. Finite element convergence of the present model was obtained with 80 beam elements and the corresponding spring elements. The spring elements of the model summed up to K_s = 50 MN/*m*, the value that has been presumed by Shen *et al*. (2011) for their *K*. An equal value of K_n was presumed here, although experimentally the real value could be as half as that (Balakrishnan, 1963).

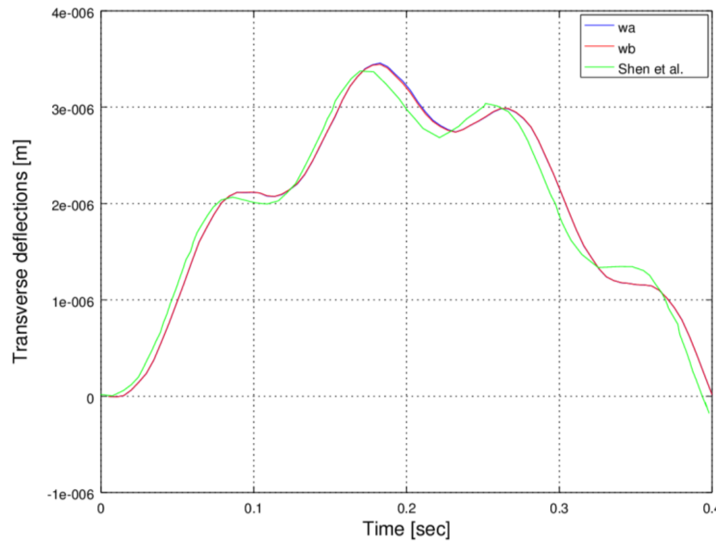


Fig. 3 *e* Comparison of mid-span deflection $w(t)$ of Shen *et al.*'s (2011) example to $w_a(t)$ and $w_b(t)$ given by the present model corresponding to a 1 Newton force crossing the beam at 10 *m/sec*

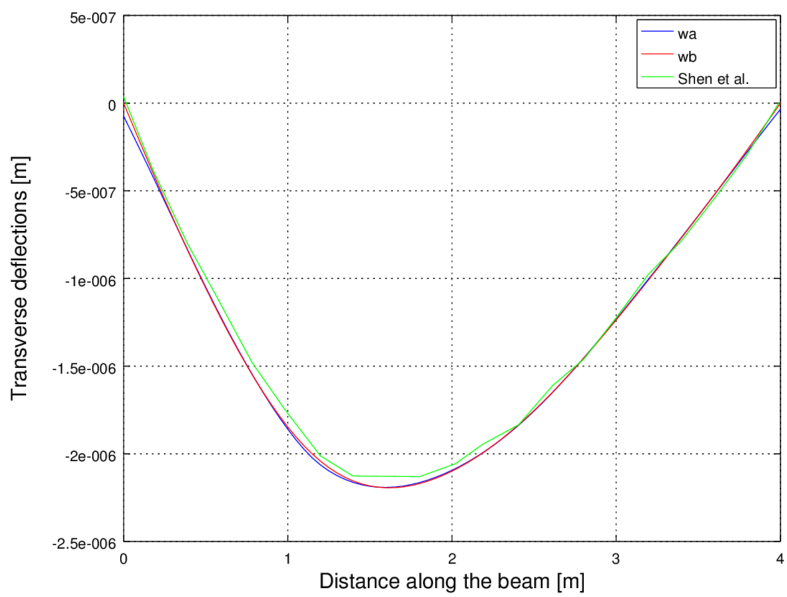


Fig. 3 *f* Comparison of transverse deflection $w(x)$ of Shen *et al.*'s (2011) example to $w_a(x)$ and $w_b(x)$ given by the present model corresponding to a 1 Newton force crossing the beam at 10 *m/sec*, exactly at 0.12 seconds

It could be noted, above, that the present model could reproduce the corresponding analytical result to a very good degree of agreement. The discrepancy is attributable to the effect of K_n that has been neglected altogether along with the corresponding effect of uplift in Shen *et al.*'s (2011) analysis, and indeed in most of the similar dynamic analyses. Both $w_a(t)$ and $w_b(t)$ actually appear in Figs. 3 *e* and 3 *f* above, but they are very close since the uplift is, usually, very small at mid-span.

The displacement fields of the slip and the uplift are as easily calculable using the present model and appear in Figs. 3 *g* and 3 *h* below for the same scenario. The absence of symmetry in the last two figures just reflects the absence of symmetry of the loading condition captured at the time value of 0.12 *secs*. Both slip and uplift distributions in this example reflect displacement fields of the same order of magnitude generally, furthermore, uplift, as usual, is much larger at the ends of this one-span beam, deepening the justification for questioning the neglect of uplift in most studies on the dynamics of composite beams or even the static ones.

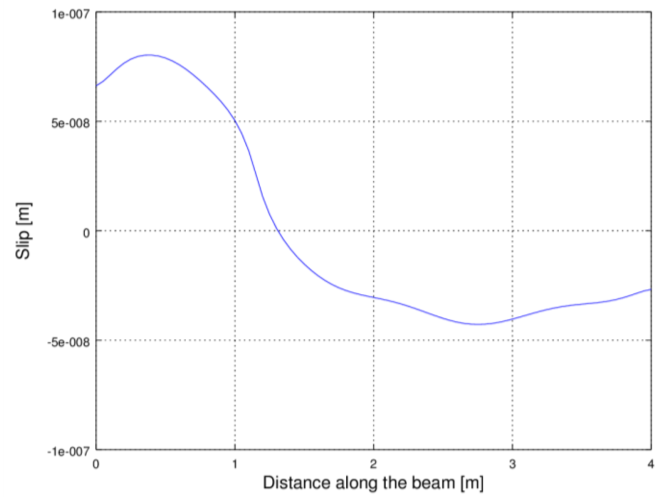


Fig. 3 *g* The slip field of Shen *et al*.'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m*/*sec* at time 0.12 *sec*, as given by the present model

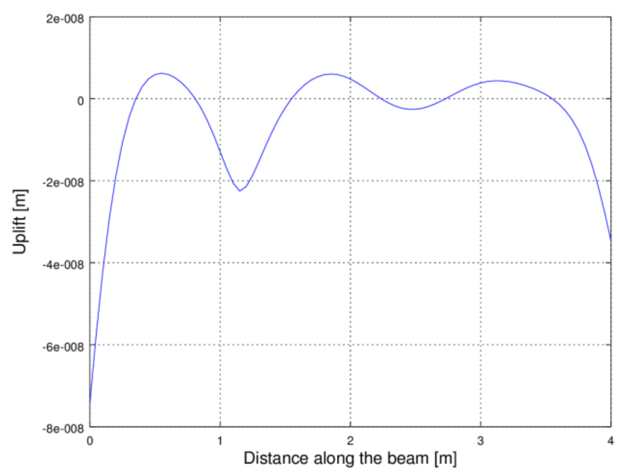


Fig. 3 *h* The uplift field of Shen *et al*.'s (2011) example corresponding to a 1 Newton force crossing the beam at 10 *m*/*sec* at time 0.12 *sec*, as given by the present model

3. Uniform Step Load

A uniform step load is a uniformly distributed load that is, suddenly, applied to the entire span of the beam and kept thereafter. In a sense, it gives the intuition of a more pressing agent than that of the corresponding uniform impulsive load, since it remains applied after the sudden instant of time at which it is applied to the structure. In this section, the numerical experiments of applying a 1 kN uniform step load to the beam section of Fig. 2 corresponding to different beam lengths and other parameters are reported. The more pressing nature of the step load is mostly obvious through extraordinarily large amplification factors.

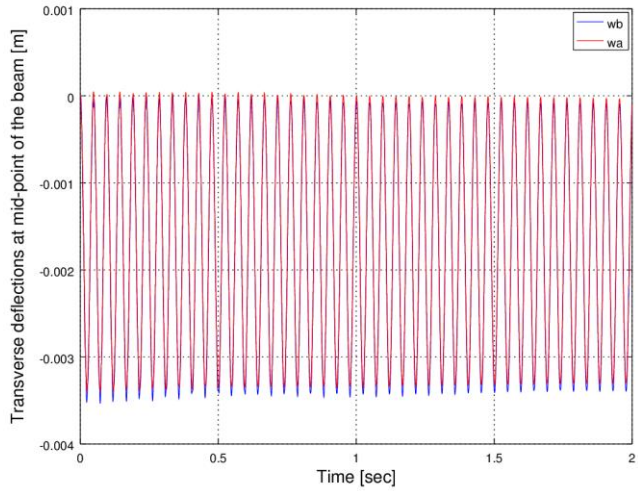


Fig. 4 *a* Dynamic mid-span transverse deflections of the numerical simply supported beam of 5 *m* length due to a uniform step 1 kN load, for a period of 2 seconds

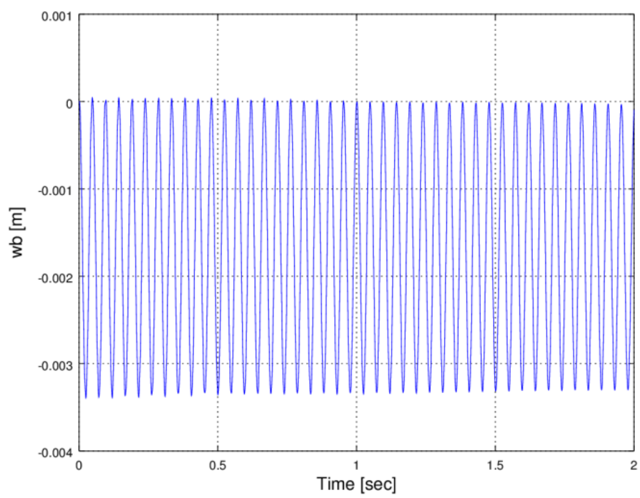
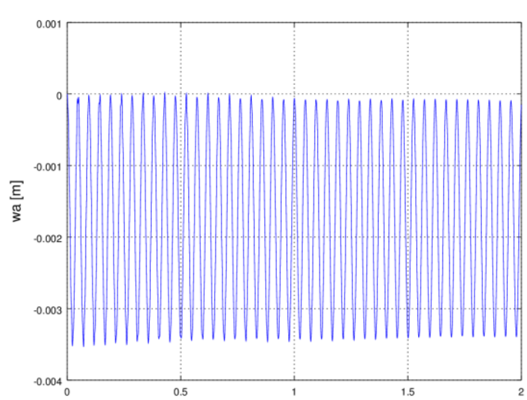
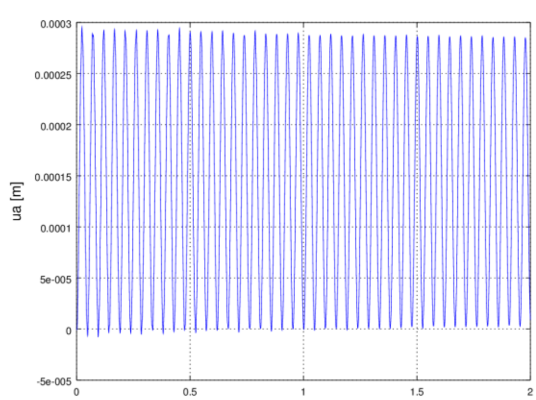


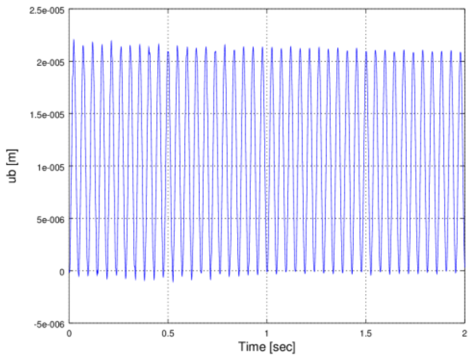
Fig. 4 *b* Dynamic mid-span $w_b(t)$ of the numerical simply supported beam of 5 *m* length due to a uniform step 1 kN load, for a period of 2 seconds



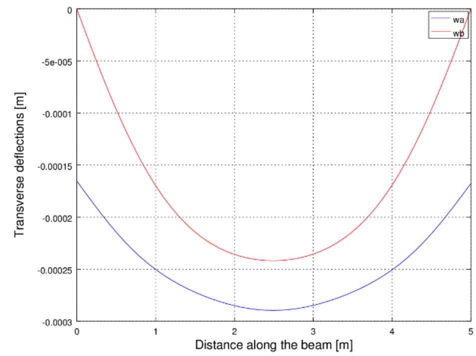
(*c*) $w_a(t)$ at mid span



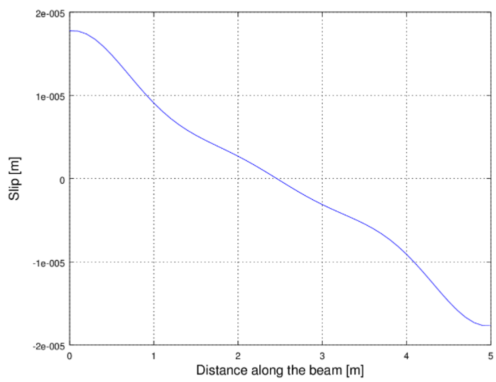
(*d*) $u_a(t)$ near the roller



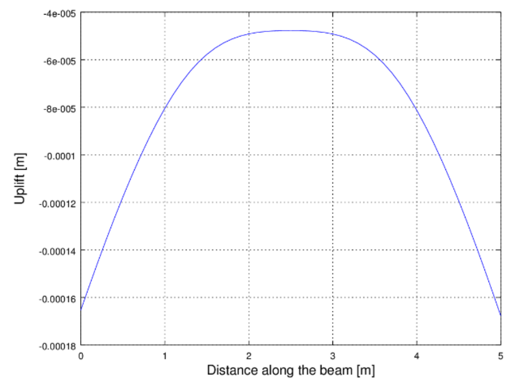
(*e*) $u_b(t)$ near the roller



(*f*) Transverse deflections at 0.1 seconds



(*g*) Slip at 0.1 seconds



(*h*) Uplift at 0.1 seconds

Fig. 4 *c-h* Dynamic responses of the numerical simply supported beam of 5 *m* length due to a uniform step 1 kN load

Already at 5 *m*, the effect of uplift is significant as is seen in Figs. 4 *f*, *g* and *h*. Comparing the *g* and *h* parts, in particular, shows that the uplift could be an order of magnitude larger than the slip, even at this relatively short length of the beam. Fig. 4 *a*, however, does not show that obviously because of the large dynamic amplification factor which is about 17.5 in this case.

A couple, more, numerical beams are analysed here to study the effects of varying the linear transverse modulus, K_n, under uniform step loading. The length of each of those two beams is 6 *m*.

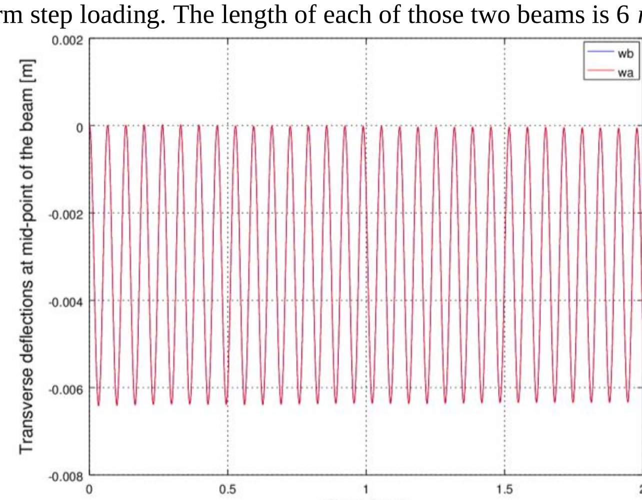


Fig. 5 *a* Dynamic mid-span transverse deflections of the numerical simply supported beam of 6 *m* length with $K_n = 10^{10}$ N/*m* and $K_s = 10^8$ N/*m* due to a uniform step 1 kN load, for a period of 2 seconds

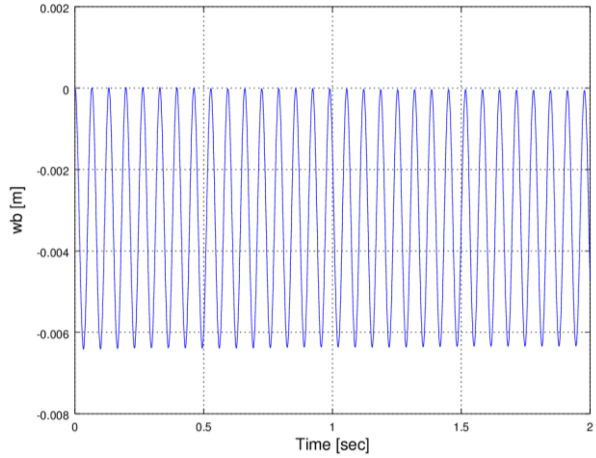
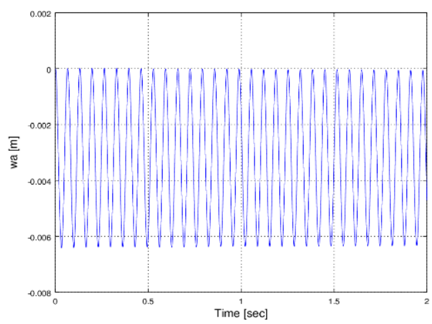
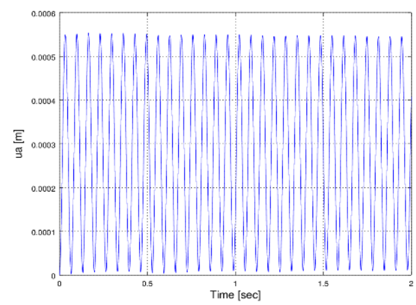


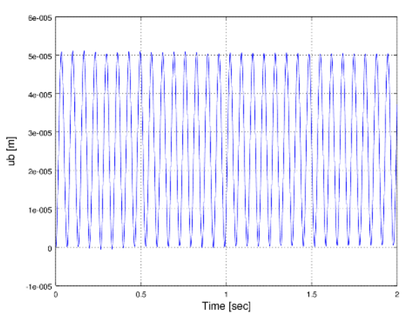
Fig. 5 *b* Dynamic mid-span $w_b(t)$ of the numerical simply supported beam of 6 *m* length with K_n = 10^{10} N/*m* and K_s = 10^8 N/*m* due to a uniform step 1 kN load, for a period of 2 seconds



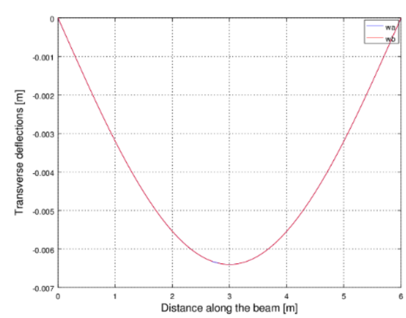
(*c*) $w_a(t)$ at mid span



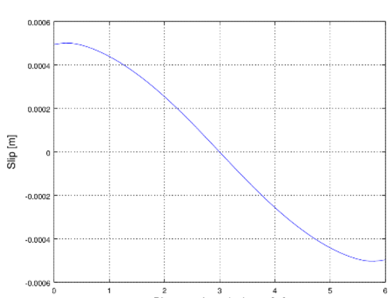
(*d*) $u_a(t)$ near the roller



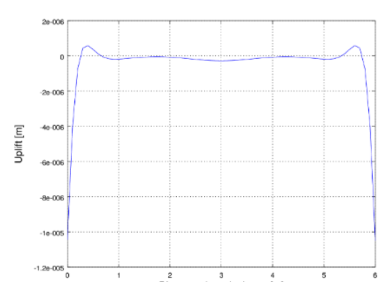
(*e*) $u_b(t)$ near the roller



(*f*) Transverse deflections at 0.1 seconds



(*g*) Slip at 0.1 seconds



(*h*) Uplift at 0.1 seconds

Fig. 5 *c-h* Dynamic responses of the numerical simply supported beam of 6 *m* length with K_n = 10^{10} N/*m* and K_s = 10^8 N/*m* due to a uniform step 1 kN load

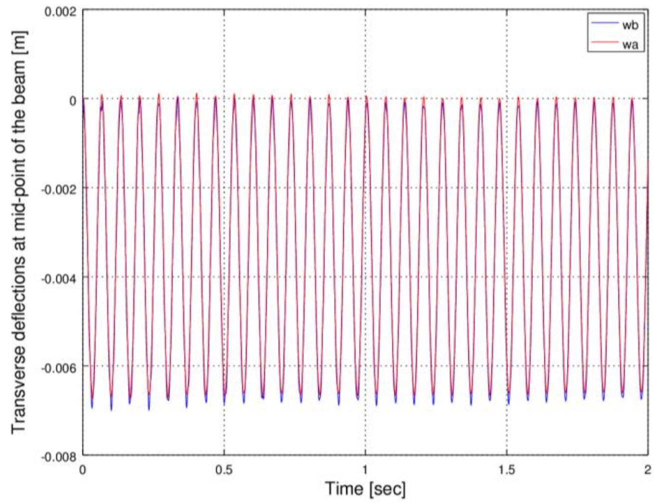


Fig. 6 *a* Dynamic mid-span transverse deflections of the numerical simply supported beam of 6 *m* length with K_n = 0.5×10^8 N/*m* and K_s = 10^8 N/*m* due to a uniform step 1 kN load, for a period of 2 seconds

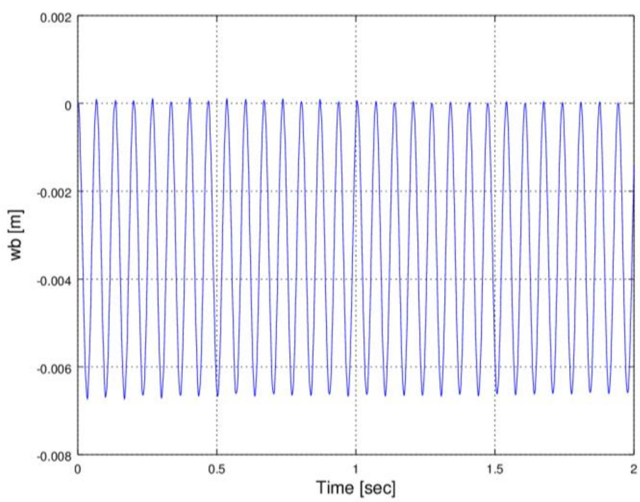
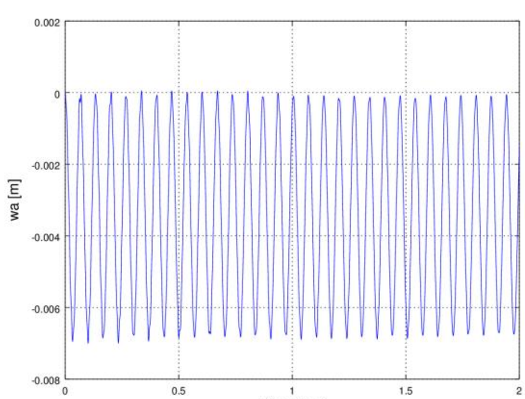
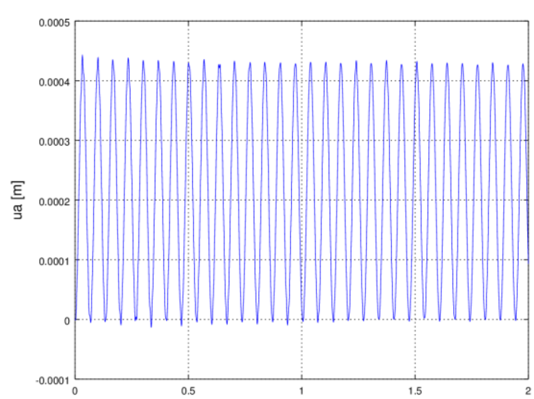


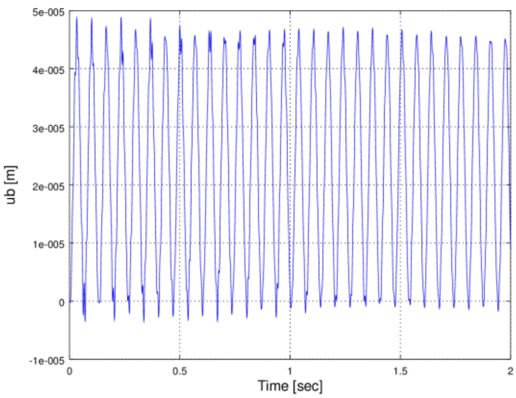
Fig. 6 *b* Dynamic mid-span $w_b(t)$ of the numerical simply supported beam of 6 *m* length with K_n = 0.5×10^8 N/*m* and K_s = 10^8 N/*m* due to a uniform step 1 kN load, for a period of 2 seconds



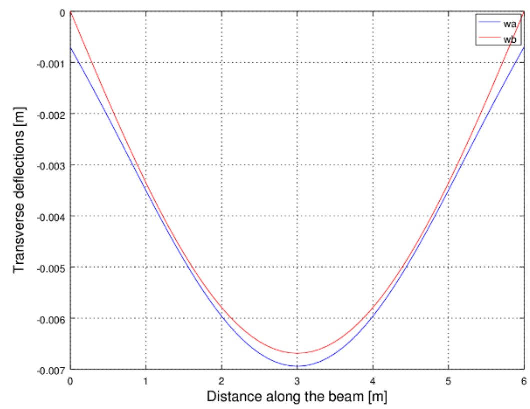
(*c*) $w_a(t)$ at mid span



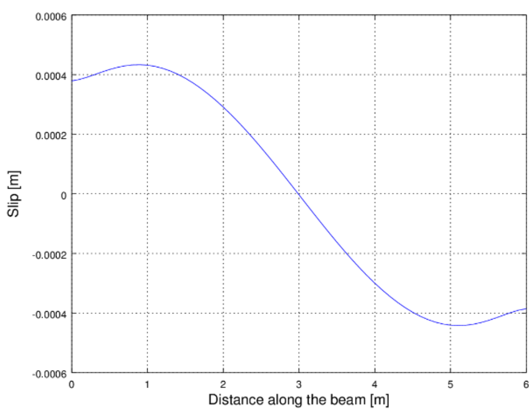
(*d*) $u_a(t)$ near the roller



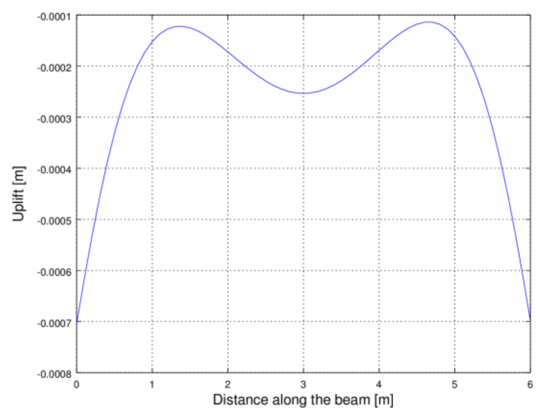
(*e*) $u_b(t)$ near the roller



(*f*) Transverse deflections at 0.1 seconds



(*g*) Slip at 0.1 seconds



(*h*) Uplift at 0.1 seconds

Fig. 6 *c-h* Dynamic responses of the numerical simply supported beam of 6 *m* length with K_n = 0.5×10^8 N/*m* and K_s = 10^8 N/*m* due to a uniform step 1 kN load

Letting the K_n/K_s go to infinity makes the current model behaves as if the uplift is not taken into account, and the usual assumption of neglecting K_n returns valid, contrary, of course, to the experimental evidence which suggests realistic K_n/K_s as low as 1/2. This is reflected, weakly, however, in Figs. 5 *g* and *h* where the slip is only less than twice the uplift despite a K_n/K_s ratio of a hundred. The quality of very large amplification factors remains valid, also, recording for the case of Fig. 5 an amplification factor of 19.7, which is quite large.

Figure 6 shows, on the other hand, that adopting a realistic K_n/K_s ratio keeps the property of very high amplification factors as is or even more pronounced by registering a DAF of 20.6. Furthermore, the stark nature of the importance of uplift returns very strongly when comparing Figs. 6 *g* and *h*, that is when it becomes manifest that the uplift could be as much as two times the slip.

All relevant boundary conditions are satisfied throughout Figs. 5 and 6. In particular, Figs. 5 *g* and 6 *g* indicate satisfying the zero derivative boundary conditions for both axial deformation fields. Furthermore, the expected relative magnitudes of axial deformations are also kept as usual. For instance, comparing Fig. 6 *d* to Fig. 6 *e* reveals that for the entire period of analysis, $u_a(t)$ near the roller remained an order of magnitude of the corresponding value of $u_b(t)$. This is explained by the more stringent constraint of the lower layer that is connected to a pin from one end, while the upper layer is free to move horizontally except for the horizontal spring constraint.

For the purpose of keeping the presentation as concise as possible, only one numerical beam model is tested under the different boundary conditions of fixing both ends of the lower layer under step load conditions.

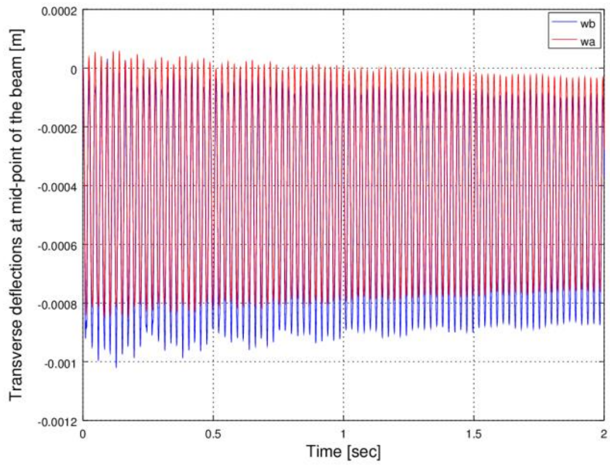


Fig. 7 *a* Dynamic mid-span transverse deflections of the numerical beam of 5 *m* length, the lower layer of which was fixed at both ends due to a uniform step 1 kN load, for a period of 2 seconds

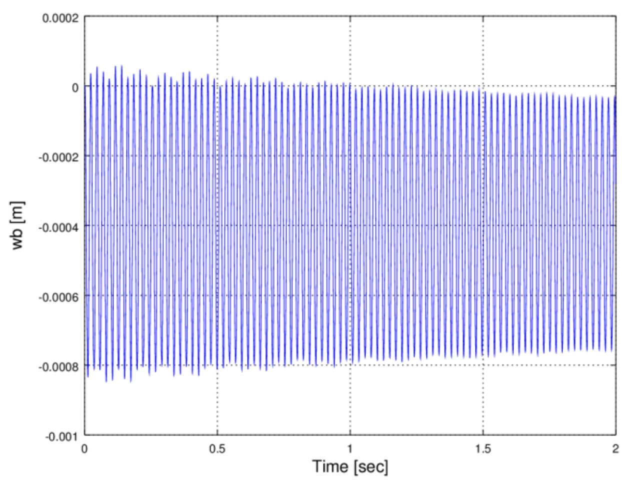
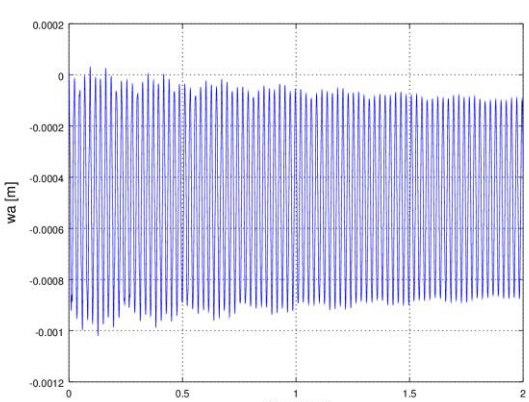
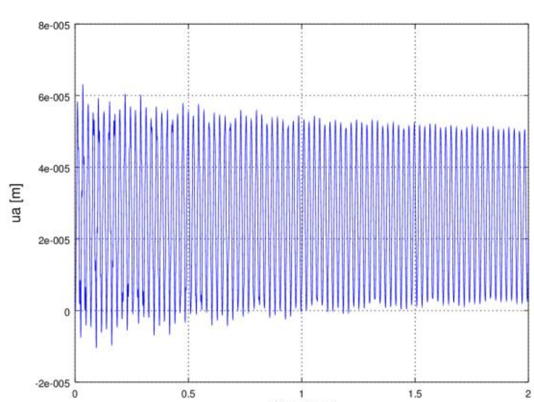


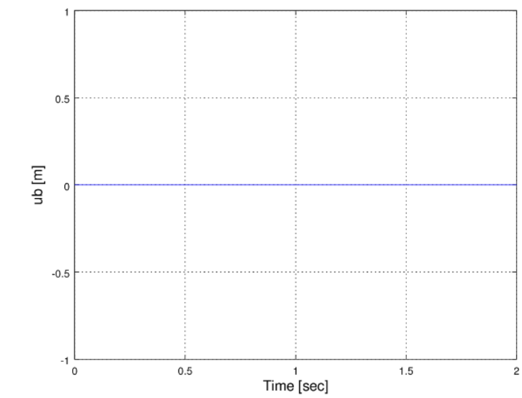
Fig. 7 *b* Dynamic mid-span $w_b(t)$ of the numerical simply supported beam of 5 *m* length the lower layer of which was fixed at both ends due to a uniform step 1 kN load, for a period of 2 seconds



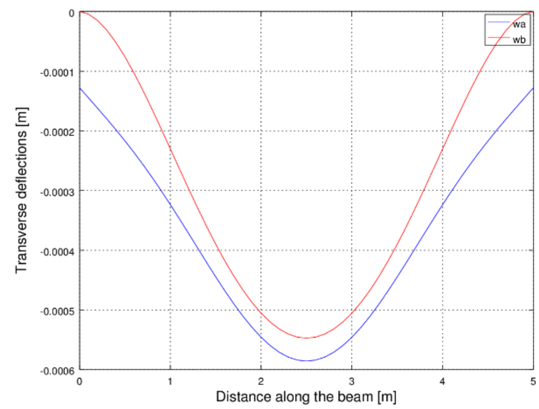
(*c*) $w_a(t)$ at mid span



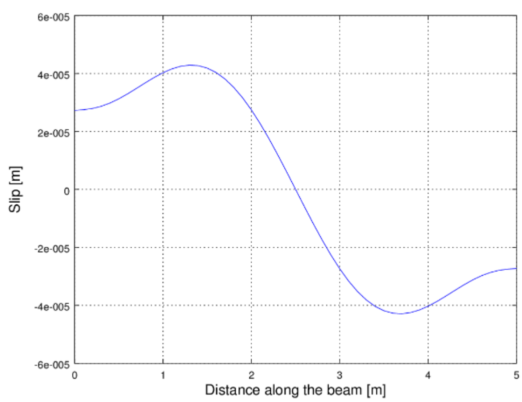
(*d*) $u_a(t)$ near the roller



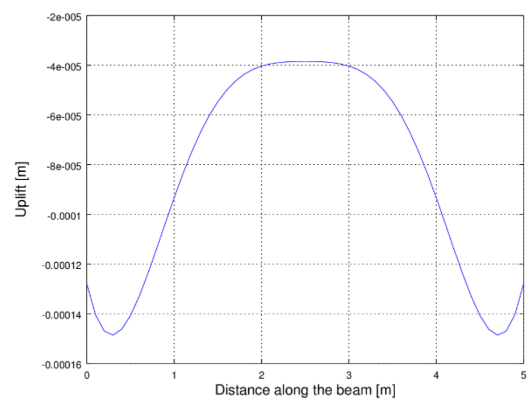
(e) $u_b(t)$ near the roller



(f) Transverse deflections at 0.1 seconds



(g) Slip at 0.1 seconds



(h) Uplift at 0.1 seconds

Fig. 7 *c-h* Dynamic responses of the numerical beam of 5 *m* length, the lower layer of which was fixed at both ends due to a uniform step 1 kN load

Comparing Fig. 4 *a* to 7 *a*, one could observe that the effect of fixing the two ends of the lower layer is quite manifest to the degree of reducing the dynamic amplification factor to a mere 30% of the original. This is, of course, due to the increase in overall rigidity produced by fixating the two ends of the lower layer of the beam. The use of other than transverse deflections to quantify the dynamic load factor is also possible, in principle. In this case, the use of $u_a(t)$ to quantify the dynamic load factor leads to the more stringent result of reducing the dynamic amplification factor to about 20% of the original due to the change in boundary conditions. The larger DAF still governs and the transverse displacements are more suitable to compute it, as is customary.

4. Conclusions

In the present paper, a successful one-dimensional finite element model for the analysis of composite beams of partial interaction was introduced. Verification of the results of this model to some of the relevant published literature has proved the veracity of this model and its implementation.

Specific application of the present model to the uniform step loading of such beams leads to a number of important conclusions. Prominent among those is that the dynamic amplification factors for those composite beams taking the maximum transverse deflections as a basis are quite large.

More importantly, the contribution of uplift to the responses, which could be taken into account by introducing four displacement variables into the model, was found important. Certainly, it is not negligible, as is the usual conduct in the literature. This corroborated the works of the minority of researchers who took uplift into account in their works.

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