



# Developing a Prediction Model of Present Serviceability Index Using Fuzzy Inference System

**Maher Mahmood<sup>\*1</sup>, Nazhon Ismael Khaleel<sup>2</sup>**

<sup>1</sup> Civil Engineering Department, Engineering College, University Of Anbar, Ramadi, Iraq

<sup>2</sup> College of Education for women, University Of Anbar, Ramadi, Iraq

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## ABSTRACT

Pavement maintenance and rehabilitation prioritization are conducted based on the accessibility of overall measures for evaluating the condition of each section in the pavement network. Regularly, the pavement condition of each section has been evaluated by some common condition indicators. One of the most important indicators is the present serviceability index (PSI) which is adapted to depict the functional performance regarding ride quality. The main aim of this study is to develop a prediction model of ride quality for flexible pavement using the fuzzy logic technique. The data of input variables are extracted from the database of Long-Term Pavement Performance (LTPP). The research involved 36 pavement sections with 319 data samples for pavement networks of different states in the USA. The ride quality measure which is PSI estimated by the AASHTO equation represents the output variable, whereas patching area, cracking length, slope variance, and rut depth are considered input variables. The results showed that the fuzzified model of ride quality prediction has a decent accuracy with a high determination coefficient. In addition, based on the testing results, the developed prediction model showed a strong accuracy to predict the ride quality index.

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## 1. Introduction

The highway which is the biggest capital asset of a nation has a significant role in local and national economic and social well-being. Pavement is a fundamental component of highway infrastructure. The cumulative traffic volumes, heavier loads, frequent adverse climatic conditions, and poor reinstatement after excavating by public services companies cause a substantial deterioration in the pavement and loss in its levels of services or serviceability. Moreover, increased deterioration of pavement, increasing maintenance needs, and limited allocation of resources make the maintenance task of pavement networks more challenging (Chen, Flintsch, & Al-Qadi, 2004).

The concept of pavement management has been raised to manage and organize the actions involved in attaining excellent conditions and performance with limited funds (Karan, M.A., Haas, R. and Walker, 1981). In response to the increased demands for pavement maintenance with limited availability of funds, the interest in evolving a

\* Corresponding author. Tel.: +00964-7832783421.

E-mail address: [maher.mahmood@uoanbar.edu.iq](mailto:maher.mahmood@uoanbar.edu.iq)

management concept to improve the application of pavement construction and preservation resources has increased hence, the particular element of this method related to the pavement is called pavement management system (PMS) (Rada, Perl, & Witczak, 1986; Tavakoli, Lapin, & Ludwig Figueroa, 1992).

The evaluation of current pavement conditions is the initial and main task of effective pavement management systems (N. Bandara and M. Gunaratne, 2001). Selecting the priorities for pavement preservation is based on the availability of a general indicator for evaluating the pavement condition. Commonly, pavement condition for each section is evaluated by some condition indicators such as the Pavement condition index (PCI), which is a common index applied to describe the distress extent, and the present serviceability index (PSI), which is a popular indicator employed to represent the functional performance regarding ride quality (Shoukry, Samir N., David R. Martinelli, n.d.). The relationship between pavement performance or serviceability and all types of distresses is not well-defined. However, there is general agreement that the pavement's capability to carry traffic volumes smoothly and safely is negatively affected by the development of visible distress. The rating technique of pavement performance or serviceability presents a systematic way that considers the composite effects of different types of distresses and/or their extent and severity (Terzi, 2007).

Prediction of pavement performance or serviceability is necessary for estimating the preservation budget at the network level. In addition, pavement performance prediction is also essential for selecting the most cost-effective repair action at the project level (N. Bandara and M. Gunaratne, 2001). Prediction modeling of pavement deterioration is not only based on pavement design, construction, climatic effect, traffic volume, and subgrade condition, but it is also based on the extent of some distresses such as cracking, surface roughness, rut depth, or combinations of these distresses (Madanat, Prozzi, & Han, 2002).

A literature review shows that there are some studies about developing ride quality prediction models. In this regard, deterministic prediction models were developed for the flexible pavement to forecast the serviceability status of PSR (Abaza, 2004) and PSI (Al-Khateeb & Khadour, 2020)(Aleadelat, Saha, & Ksaibati, 2018). In addition, several studies attempted to determine the relation between ride quality and pavement condition index PCI (Bryce, Boadi, & Groeger, 2019; Mok & Smith, 1997) and also the correlation between PSI and international roughness index (IRI) (Gulen, Woods, Weaver, & Anderson, 1994). Moreover, artificial intelligence techniques such as artificial neural networks (Oladapo Samson Abiola, 2014; Shah, Jain, Tiwari, & Jain, 2013; Terzi, 2007), fuzzy regression (Pan, Ko, Yang, & Hsu, 2011), and data mining (Terzi, 2006) were applied to estimate pavement serviceability index.

Based on the literature review, the modeling of ride quality prediction has not yet attracted much attention. Moreover, the majority of condition prediction models for flexible pavement at the network were developed to estimate the overall pavement condition/distresses index like PCI. A few researchers tried to develop prediction models of the ride quality but without considering the nonlinearity and also the uncertainties in condition data. Generally, pavement condition data gathered through the manual survey by experts is usually associated with uncertainty and subjectivity. Therefore, this research is trying to address the subjectivity and uncertainty by developing a prediction model of the ride quality using the artificial intelligence technique. The main purpose of this research is to develop a prediction model of the present serviceability index using a fuzzy inference system. The pavement serviceability index as ride quality measure is considered an output variable, while patches, cracking length, slope variance, and rut depth are considered as input variables.

## 2. Present Serviceability Index PSI

Present Serviceability Index (PSI) was established as a concept by Cary and Irick in 1960 (18). In addition, the present serviceability rating (PSR) was established at the AASHO Road Test to present a reliable technique for evaluating the performance of a pavement segment based on the opinion of road users. There are two ways to estimate the original PSR either by a panel of road users or by applying the PSI (Carey Jr & Irick, 1960).

PSI is predominantly an indicator of pavement functional performance or rides quality. It is estimated based on a quantitative scale ranging from 0 to 5 with 0 being the worst quality of ride and 5 being the excellent ride quality. For flexible pavement, it can be estimated using equation 1 which is a function of cracking and patching, rutting, and slope variance (Fwa, 2006; Hall & Correa Muñoz, 1998):

$$PSI = 5.03 - 1.91 \log(1 + SV) - 0.01\sqrt{C + P} - 1.38RD^2 \quad (1)$$

Where: *PSI* = present serviceability index,

$SV = 10^6 \times$  population variance of slopes measured at 1-ft intervals,

$$SV = \sum (Y_i - Y_{\text{mean}})^2 / n \quad (2)$$

$Y_i$  = individual measured slope,

$Y_{\text{mean}}$  = mean of measured slopes,

$n$  = number of slope measurements,

$C$  = cracking length, linear feet per 1,000 ft<sup>2</sup>,

$P$  = patching area, square feet per 1,000 ft<sup>2</sup>, and

$RD$  = mean rut depth of the pavement, in.

### 3. Long-Term Pavement Performance (LTPP) Data

As one of the key study areas of the Strategic Highway Research Program (SHRP), Long Term Pavement Performance (LTPP) program was established to gather data on pavement performance. LTPP which is one of the main data sources of pavement performance was sponsored and achieved by both the Federal Highway Administration (FHWA) and SHRP (Elkins, Schmalzer, Thompson, Simpson, & Ostrom, 2011).

The LTPP program is a huge research project including two main kinds of studies, the General Pavement Studies (GPS) and the Specific Pavement Studies (SPS). In addition, it includes some minor studies to discover specific pavement with information that is essential to pavement conditions. The LTPP program observers and gathers data on pavement conditions on entire in-service sections. The gathered data comprise seven modules of data: Inventory, Monitoring, Traffic, Materials Testing, Climatic, Maintenance, and Rehabilitation (Elkins et al., 2011).

In this research, cracking length, patching area, rut depth, and slope variance are collected from the monitoring module of the LTPP data. The raw historical information of 36 sections is extracted from the Access file of the monitoring data. The total data samples of 36 sections for flexible pavement are 319 historical data samples.

**Table 1- Sample of data used in the model development.**

State Code	SHRP_ID	Survey Date	Cracking Length ft/1000ft <sup>2</sup>	Patch Area ft <sup>2</sup> /1000ft <sup>2</sup>	Rut Depth (in.)	Slope Variance	PSI
1	0102	25-08-1994	0.00	0.00	0.20	0.19	4.8
1	0102	17-04-1996	0.01	0.00	0.24	0.05	4.9
1	0102	30-10-1997	0.05	0.00	0.24	0.10	4.9
1	0102	21-11-1998	0.00	0.00	0.28	1.03	4.3
1	0102	23-08-2001	0.03	0.00	0.43	1.29	4.1
1	0102	08-02-2002	0.07	0.00	0.43	16.01	2.4
1	0102	09-04-2003	0.20	0.00	0.63	53.74	1.2
1	0102	24-02-2004	0.17	3.38	0.28	2.68	3.8
17	0607	23-06-1995	0.17	0.00	0.04	0.10	4.9
17	0607	05-12-2001	1.25	0.00	0.08	0.43	4.7
17	0607	16-09-2009	0.00	0.07	0.08	6.09	3.4

### 4. Methodology

Initially, the historical data of long-term pavement performance (LTPP) for various states in the USA modules are considered to build a Fuzzified prediction model of ride quality for flexible pavement. For various years, the gathered data are cracking length, patch area, rut depth, and slope variance for some pavement sections. Some input variables require to do manipulation and computation before developing the prediction model, and therefore the amounts of input variables are extracted from the LTPP survey. Data of cracking length and patch area are obtained from the LTPP section of the manual distress survey. In addition, The data of rut depth are extracted

from the LTPP section of the transverse profile. Finally, to estimate slope variance, the longitudinal profile data are extracted from the High-Speed Survey section of LTPP taken at 150 mm intervals. Then, the slope variance is calculated for each data sample using equation 2. After finalizing the data of input variables, the output representing the existing present serviceability index is determined by using equation.1.

#### 4.1. Model Development

For flexible pavement, the PSI prediction model is developed using a fuzzy-based system (FIS). Based on equation 1, the slope variance, cracking length, patching area, and rut depth are considered as FIS inputs, and a calculated PSI is considered as the FIS output. Then, the developed prediction model is formulated by employing Fuzzy Inference System Professional (FISPro.) using the numerical data of LTPP. This software is an automatic learning technique, that was developed employing the C++ language and also using Java (Guillaume, Charnomordic, & Lablee, 2013).

#### 4.2. Fuzzy Inference System (FIS)

A fuzzy inference system is one of the most prevalent techniques applied in classification and prediction problems. It is an approach that explicates the magnitudes of the input variables and, based on predefined rules, assigns magnitudes to the output variable. This approach can represent the knowledge in the If–Then rules form, presenting the thinking mechanism in human-reasonable expressions. Moreover, it is capable to gather linguistic information from human professionals and combining it with numerical information. It is also capable to estimate difficult nonlinear functions with uncomplicated models (Dehzangi, Zolghadri, Taheri, & Fakhrahmad, 2007).

##### 4.2.1. Membership Functions Generation

"In fuzzy theory, the fuzzy set  $A$  of universe  $X$  is defined by the function  $\mu_A(x)$ , called the membership function of set  $A$ " (Negnevitsky, 2002).

$$\mu_A(x): X \rightarrow [0, 1]$$

" Where  $\mu_A(x) = 1$  if  $x$  is totally in  $A$ ;  $\mu_A(x) = 0$  if  $x$  is not in  $A$ ;  $0 < \mu_A(x) < 1$  if  $x$  is partly in  $A$ " (Negnevitsky, 2002).

The membership degree matches the membership function  $\mu_A(x)$  for a component  $x$  of set  $A$ , and its quantity is between 0 and 1. This function has a graphical display that describes how each value in the range of the variable is mapped to the membership degree or a quantity between 0 and 1. There are different graphical representations like Gaussian, trapezoidal, triangular, etc. (Negnevitsky, 2002). The membership function can be estimated in two ways which are acquisitions of knowledge and numerical data. There are several techniques for creating membership functions from numerical data for input and output variables such as the data clustering algorithm. The clustering role is to classify massive numerical information set to natural clusters of information to create a short display of a model's behavior. It split the numerical information set into several subsets of information. There is similarity just inside a subset and not between the subsets (Naik, 2004).

One of the most common clustering algorithms used to generate membership functions is  $k$ -means clustering or C-means clustering. The main idea of this approach is that  $k$  initial cluster centers or means are randomly selected. After many generations, these initial cluster centers are improved in such a way that they demonstrate the data centers as far as possible. The main shortcoming of this clustering technique is that the cluster value is steady; after  $k$  is chosen there will permanently be  $k$  cluster centers. To avoid this issue, the  $k$ -means clustering technique can remove the rest clusters. A cluster center could be eliminated if it does not have adequate examples. The problem of choosing the number of the initial cluster is still unaddressed, but selecting a large enough  $k$  is the best strategy to address this issue (Naik, 2004).

##### 4.2.2. Fuzzy Rule Generation

The fuzzy rules generation is the next key part of FIS. To generate fuzzy rules, there are two ways which are expert knowledge and numerical data (Nelles, Fischer, & Muller, 1996). The main challenge of fuzzy rule

generation in high-dimensional problems is the difficulty to create all potential rules concerning all antecedent combinations. FISPro, the software can address this issue and also design fuzzy systems based on numerical data (Guillaume et al., 2013). The most popular method to produce fuzzy rules is Wang & Mendel's method (Wang & Mendel, 1992). This method requires the generation of the membership functions for each input and output variable. The fuzzy rules can be created automatically from numerical data. The first step is to generate one rule for every information couple of the training set (Guillaume et al., 2013). The next is the definition of the  $i^{\text{th}}$  couple fuzzy rule

*IF  $x_1$  is  $A_1^i$  AND  $x_2$  is  $A_2^i$  ... AND  $x_p$  is  $A_p^i$  THEN  $y$  is  $C^i$*

"The fuzzy sets  $A_1^i$  are those for which the match degree of  $X_1^i$  is maximum for each input variable  $j$  from pair  $i$ . The fuzzy set  $C^i$  is the one for which the match degree of the estimated output,  $y$ , is maximum" (Guillaume et al., 2013).

## 5. Results

The development of PSI prediction model-based FIS includes two stages which are the generation of membership functions and fuzzy rules. In the first stage, the membership functions of input variables are created based on the k-means clustering method from numerical data by using FISPro. Software. Based on this clustering method, a large number of clusters is selected to generate membership functions, hence this clustering method eliminates the excess number of clusters. Therefore, three triangular membership sets (low, moderate, and high) are created automatically from numerical data for each input as shown in Fig. 1, Fig. 2, Fig. 3, and Fig. 4. Moreover, the four triangular functions of membership are generated automatically for PSI as shown in Fig. 5. The x-axis of these figures represents the cracking length, patching area, rut depth, slope variance, and PSI, whereas the vertical axis is a membership value that has a range between 0 and 1.

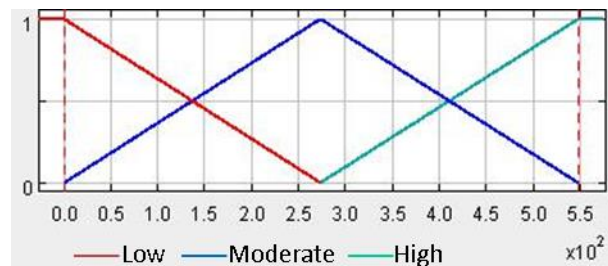


Fig. 1 Membership functions for cracking length (ft/1000ft²).

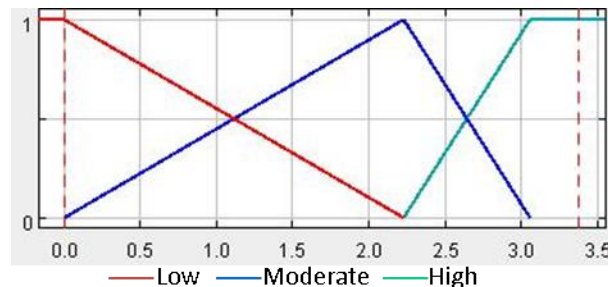


Fig. 2 Membership functions for Patching area(ft²/1000ft²).

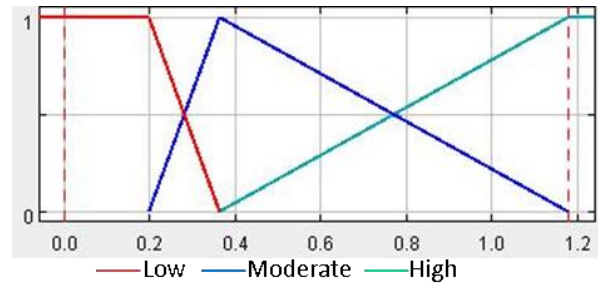


Fig. 3 Membership functions for rut depth (in.).

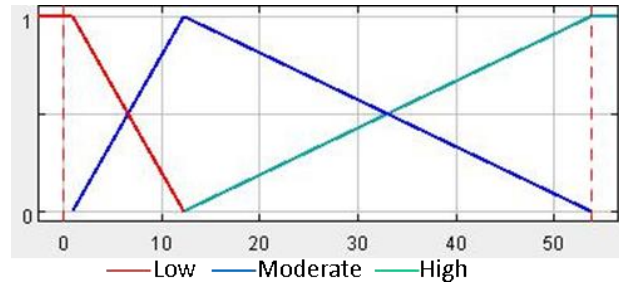


Fig. 4 Membership functions for slope variance.

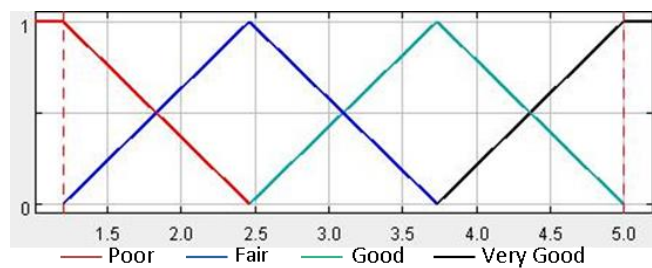


Fig. 5 Membership functions for PSI.

The fuzzy rules are created naturally by training from the data samples which are about 80% of the total data samples. The ten fuzzy rules are shown in **Table 2**. Based on this figure, the ride quality of pavement will be poor if the degree of distresses and slope variance are high.

Table 2 - Fuzzy If-Then rules.

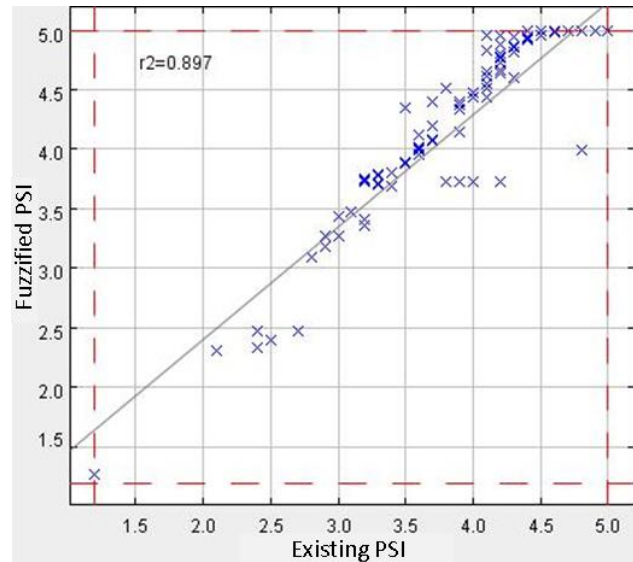
Rule No.	If "Cracking Length" is ...and "Patch Area" is ....and ....				Then "PSI" is ..
	Cracking Length	Patch Area	Rut Depth	Slope Variance	PSI
1	H	L	H	L	Fair
2	L	L	M	M	Fair
3	L	M	M	M	Fair
4	L	L	M	H	Poor
5	L	L	L	M	Fair
6	L	H	L	L	Good
7	L	M	L	L	Good
8	L	M	M	L	Good
9	L	L	M	L	Very Good
10	L	L	L	L	Very Good

L: Low, M: Moderate, H: High

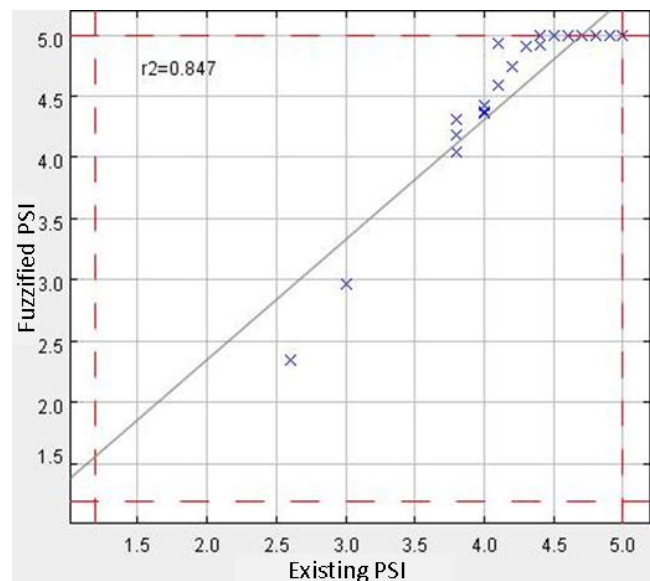


After creating fuzzy rules and membership functions, the developed prediction model is examined for training data sets by determining the performance of the fuzzy ride quality index for flexible pavement as shown in Figure 6. This figure illustrates the relationship between the Fuzzified PSI and existing PSI calculated by equation 1. It can be recognized that a correlation of nearly 90% was attained for 80% of 319 data samples. For testing the accuracy of the Fuzzified ride quality model, the remaining 20% of the total data samples are considered as shown in Figure 7. Based on this figure, this means that the Fuzzified PSI model has a strong accuracy to predict ride quality index.

Table 3 shows the determination coefficient, the mean absolute error (MAE), and the root mean square error (RMSE) to demonstrate the agreement level of the PSI quantities in both training and testing data sets. It can be seen that there is negligible change in the performance of the Fuzzified PSI prediction model.



**Figure 6: The model performance of the Fuzzified ride quality prediction for training data set.**



**Figure 7: The model performance of the Fuzzified ride quality prediction for testing data set.**

**Table 3- Performance model comparison between training data and testing data.**

	<b>R<sup>2</sup></b>	<b>RMSE</b>	<b>MAE</b>
<b>Training Data</b>	0.897	0.357	0.312
<b>Testing Data</b>	0.847	0.37	0.327

## 6. Conclusions

To generate new ride quality prediction model for flexible pavement, the fuzzy inference system has been applied. The four variables which are the most effective parameters on pavement ride quality (cracking length, patching area, rut depth, slope variance) are employed as input variables to estimate PSI. The results indicate a strong correlation between the Fuzzified PSI and the PSI determined by the equation 1. Therefore, the Fuzzified prediction model can be applied by highway authorities to predict ride quality for flexible pavement.

Based on the results, there is no significant need to make further improvement for Fuzzified ride quality model. However, it is possible to obtain extra strong correlation by using extra data samples. As future work, the developed prediction model of pavement ride quality could be also improved by using Gaussian membership functions rather than the simplest triangular membership function.

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