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بعض طرق التدرج المترافق لإزالة الضوضاء في التصوير الطبي

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الملخص:

في هذه الورقة، تمت صياغة مشكلة إزالة الضوضاء في التصوير الطبي كمشكلة تحسين غير مقيدة مع دالة هدف سلسة. هناك العديد من الطرق لحل مشكلة التحسين غير المقيدة هذه. كواحدة من الطرق الشهيرة، يمكن حلها من خلال طرق التدرج المترافق ذات الخصائص المرغوبة والذاكرة المنخفضة والتقارب العالمي القوي في الأبعاد العالية. باستخدام العديد من الصور الطبية، تبين أن طرق التدرج المترافق قوية جدًا وفعالة لإزالة الضوضاء النبضية، خاصة من حيث نسبة الإشارة إلى الضوضاء القصوى (PSNR).

كلمات مفتاحية : التدرج المترافق ، الضوضاء ، التصوير الطبي

Some conjugate gradient methods for denoising medical imaging

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Abstract:

In this article, denoising medical imaging problem is converted to an unconstrained optimization problem in which the objective function is smooth. There are several algorithms to solve it. As one of the famous methods, conjugate gradient (CG) methods can be solving the unconstrained optimization problem. CG methods have strong convergence properties and small memory for high dimensions problems. Using some medical images, we show that the CG are very efficient for denoising impulse noise. Finally, to compare the amount of noise reduction, we use terms of the peak signal to noise ratio (PSNR).

Keywords: conjugate gradient, noise, medical imaging

1. Introduction

Denoising medical imaging has been investigated by several researchers, see [1,9]. Denoising medical imaging methods are different. These methods have the following properties:

(i) The noise removal rate from images with noise.

(ii) The required CPU time for implementation.

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In application, removing noise from medical images is very important. If we have high-resolution medical images, then diagnosis and treatment of the disease will be easy. In medical images, there are Gaussian noise, Rayleigh noise and Poisson noise [11]. So far are provided many methods to remove noise from medical images. The median filter (MF) method has been used in most of these methods [1]. The MF method replaces noisy pixels by the median value of them. If the noise of the images is low then MF method is suitable.

2. Denoising medical imaging problem

In [2], researchers have been suggested two-step algorithms to denoising medical imaging. Firstly, the noisy pixels are recognized by the median filter (MF) method. Secondly, these noises are removed by solving an unconstrained optimization problem. Let $t \in R^{m\times n}$ indicating the original image. We define the index set as follows

$$T_{mn} = \{(i,j) \mid i = 1,2,...,m,\ j = 1,2,...,n\}.$$

Also, z is the image with noise and $\tilde{z}$ is the image obtained by MF method. Now, we will correct the pixel $t_{i,j}$ using four closest neighborhoods $t_{i-1,j}$, $t_{i,j-1}$, $t_{i,j+1}$ and $t_{i+1,j}$. Let $[k_{\min}, k_{\max}]$ be dynamic distance from noisy image t. Using the MF method, we express $t_{i,j}$ with $1-\alpha-\beta$ probability in which $0 \le \alpha \le 1$, $0 \le \beta \le 1$. We define the set of indices

$$M = \{(i,j) \in T_{mn} \mid \tilde{z}_{i,j} \ne z_{i,j},\ z_{i,j} = k_{\min}\ or\ k_{\max}\},$$

and

$$M^C = \{(i,j) \in T_{mn} \mid (i,j) \notin M\}.$$

Therefore, the noisy pixels are identified by the median filter (MF) method in the first step. For $(i,j) \in M$ let $t_{i,j}$ be the noisy pixel, $c = |M|$ and

$s = [s_{i,j}]_{(i,j)\in M} \in R^c$ is the noisy pixels. Let ζ_ι be an edge-preserving function and

$$\pi^1_{i,j} = \sum_{(m,n)\in V_{i,j}\backslash M} \zeta_\iota(s_{i,j} - z_{m,n}),$$

$$\pi^2_{i,j} = \sum_{(m,n)\in V_{i,j}\cap M} \zeta_\iota(s_{i,j} - s_{m,n}).$$

Now, we solve the unconstrained optimization problem for impulse noise removal with non-smooth objective function [10]

$$\min_{s\in \quad c} \quad \chi_\iota(u)=\sum_{(i,j)\in M}\left|s_{i,j}-z_{i,j}\right|+\frac{\lambda}{2}\sum_{(i,j)\in M}\left(2\pi^1_{i,j}+\pi^2_{i,j}\right),$$

in which $\lambda>0$ is called regularization parameter. In generally, the function ζ_ι has the following properties:

(1) ζ_ι has two continuous derivatives,

(2) $\zeta_\iota''>0,$

(3) ζ_ι is even function.

In this paper, we use the edge-preserving function $\zeta_\iota(s)=\sqrt{s^2+\iota}$ where $\iota>0$ is a parameter. The absolute value term in $\chi_\iota(s)$ is non-smooth. Therefore, we remove the non-smooth term and obtain a smooth function. Hence, we get the objective function as follows

$$G_\iota(s)=\lambda\sum_{(i,j)\in M}\left(\pi^1_{i,j}+\frac{1}{2}\pi^2_{i,j}\right).$$

There are several optimization methods to solve

$$\min_{s\in \quad c} G_\iota(s), \qquad (1)$$

where $G_i: R^c \to \mathrm{R}$ and for the initial point $S_0 \in R^c$ the following sequence can be obtained

$$s_{k+1}=s_k+\alpha_k d_k, \qquad k=0,1,2,\ldots$$

Note that α_k is the step length and d_k is a descent direction. In other words

$$g_k^T d_k<0$$

and $g_k=\nabla G_\iota(s_k)$.

3. Conjugate gradient methods

Conjugate gradient methods are suitable algorithms to solve the unconstrained optimization problems with smooth objective function. These methods have three important properties [7]:

(1) They require low memory,
(2) Strong local and global convergence properties,
(3) They are suitable for high-dimensional problems because they do not need to store any matrices.

In conjugate gradient methods for impulse noise removal, the direction is computed by

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$$d_k = \begin{cases} -\nabla G_\iota(s_k), & k = 0, \\ -\nabla G_\iota(s_k) + \beta_k d_{k-1}, & k \geq 1, \end{cases}$$

where the conjugate gradient parameter β_k is a real number. However, different types of conjugate gradient methods are obtained based on different CG parameters. Some of them are

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}}, \quad (\text{Hestenes and Stiefel})\ [6]$$

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \quad (\text{Fletcher and Reeves})\ [4]$$

$$\beta_k^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T y_{k-1}}, \quad (\text{Dai and Yuan})\ [3]$$

$$\beta_k^{HZ} = \beta_k^{HS} - 2\|y_{k-1}\|^2 \frac{d_{k-1}^T g_k}{\left(d_{k-1}^T y_{k-1}\right)^2}, \quad (\text{Hager and Zhang})\ [5]$$

$$\beta_k^{PR} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2}, \quad (\text{Polyak and Ribiere})\ [8]$$

in which $y_{k-1} = g_k - g_{k-1}$. Also $\|\cdot\|$ is the Euclidean norm. In conjugate gradient methods, we compute the descent direction and then obtain the step length α_k by solving one-dimensional problem. The inexact step length satisfies in strong Wolfe conditions [7]

$$\begin{cases} G_\iota(s_{k+1}) < G_\iota(s_k) + c_1 \nabla G_\iota(s_k)^T d_k, \\ \left|\nabla G_\iota(s_{k+1})^T d_k\right| \leq -c_2 \nabla G_\iota(s_k)_k^T d_k. \end{cases}$$

where $0 < c_1 < c_2 < 1$.

4. Numerical experiment

In this section, we compare the numerical results of conjugate gradient methods for denoising medical imaging. These methods are as follows:

- CGFR: CG algorithm by Fletcher and Reeves [4],
- CGPR: CG algorithm by Polak and Ribiere [8],
- CGHS: CG algorithm by Hestenes and Stiefel [6],
- CGDY: CG algorithm by Dai and Yuan [3],
- CGHZ: CG algorithm by Hager and Zhang [5],

All CG algorithms have run in MATLAB 2017(a) on a laptop Acer with a 1.7 GHz Intel Core i3 and 4 GB of memory. We use $\iota = 10^{-4}$, $c_1 = 10^{-4}$, and

$c_2 = 0.5$. Let $s^*_{i,j}$ be the pixel values of the original image and $s^r_{i,j}$ be the pixel values of the restored image. Then, to evaluate the images restoration, we use the peak signal to noise ratio [2]

$$\mathrm{PSNR} = 10\log_{10}\frac{255^2}{\frac{1}{mn}\sum_{i,j}\left(s^r_{i,j} - s^*_{i,j}\right)^2},$$

In all algorithms the condition of stopping is

$$\frac{\left|G_t(s_k) - G_t(s_{k-1})\right|}{\left|G_t(s_k)\right|} < 10^{-4}.$$

The test images are as follows:

- Lena image in size 256×256,
- Cameraman image in size 512×512,
- HeadCT image in size 512×512,
- CerebSagE image in size 512×512.

For these images, n_i is the total number of iterations, n_f is function evaluations, C_t is CPU times and PSNR have reported in the Tables 1-4. These tables show that CGPR and CGHS are better than other algorithms in n_i, n_f, C_t and PSNR, respectively.

Table 1: Total number of iterates (n_i).

Image	size	CGFR	CGPR	CGHS	CGDY	CGHZ
Lena	256×256	87	47	**39**	89	65
Cameraman	512×512	59	37	**33**	57	49
HeadCT	512×512	2416	**184**	253	1900	1010
CerebSagE	512×512	184	**45**	49	280	244

Table 2: Total number of function evaluations (n_f).

Image	size	CGFR	CGPR	CGHS	CGDY	CGHZ

Lena	256×256	357	299	**249**	368	358
Cameraman	512×512	267	242	**213**	272	264
HeadCT	512×512	4073	**1193**	1661	3908	5062
CerebSagE	512×512	751	**288**	317	1338	1239

Table 3: Total number of CPU times (C_t).

Image	size	CGFR	CGPR	CGHS	CGDY	CGHZ
Lena	256×256	10.04	8.14	**6.61**	10.15	10.06
Cameraman	512×512	46.84	42.31	**37.32**	47.78	46.48
HeadCT	512×512	687.76	**198.66**	279.08	666.74	852.87
CerebSagE	512×512	128.94	**49.14**	58.82	277.73	210.04

Table 4: Total number of PSNR.

Image	size	CGFR	CGPR	CGHS	CGDY	CGHZ
Lena	256×256	26.87	26.90	**26.93**	26.87	26.86
Cameraman	512×512	30.18	30.26	**30.27**	30.20	30.21
HeadCT	512×512	8.20	**13.85**	12.68	8.15	8.06
CerebSagE	512×512	28.52	**28.85**	28.83	28.44	28.46

5. Conclusion

In this paper, the denoising medical imaging problem is formulated as a smooth unconstrained optimization problem. Then, some CG algorithms are used to solve smooth unconstrained problem to denoising medical imaging. Also, the robust and efficient of conjugate gradient methods are shown using some standard medical images.

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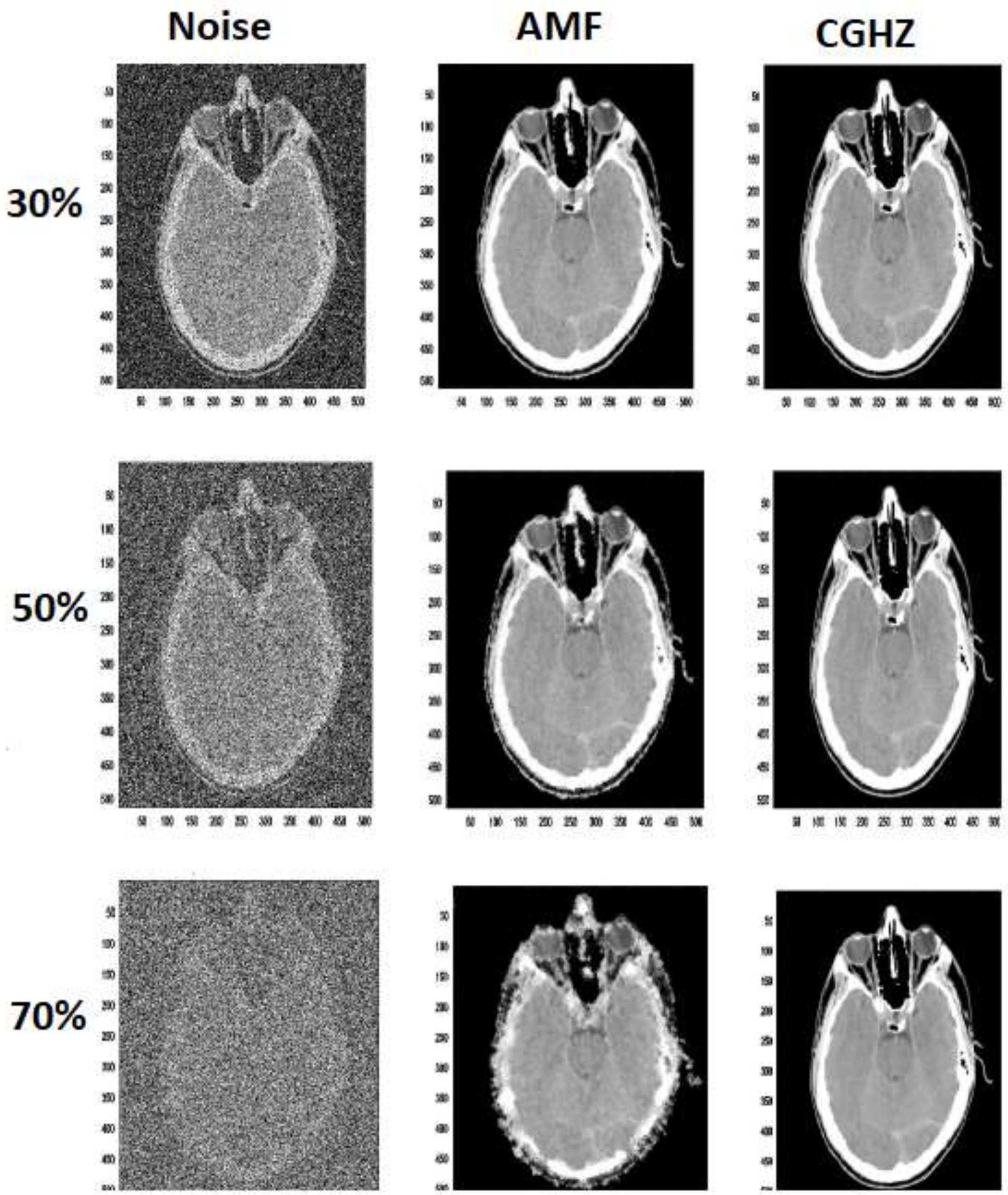

Figure 1: The noisy HeadCT image, the modified images with AMF method, the modified images with CGHZ method.

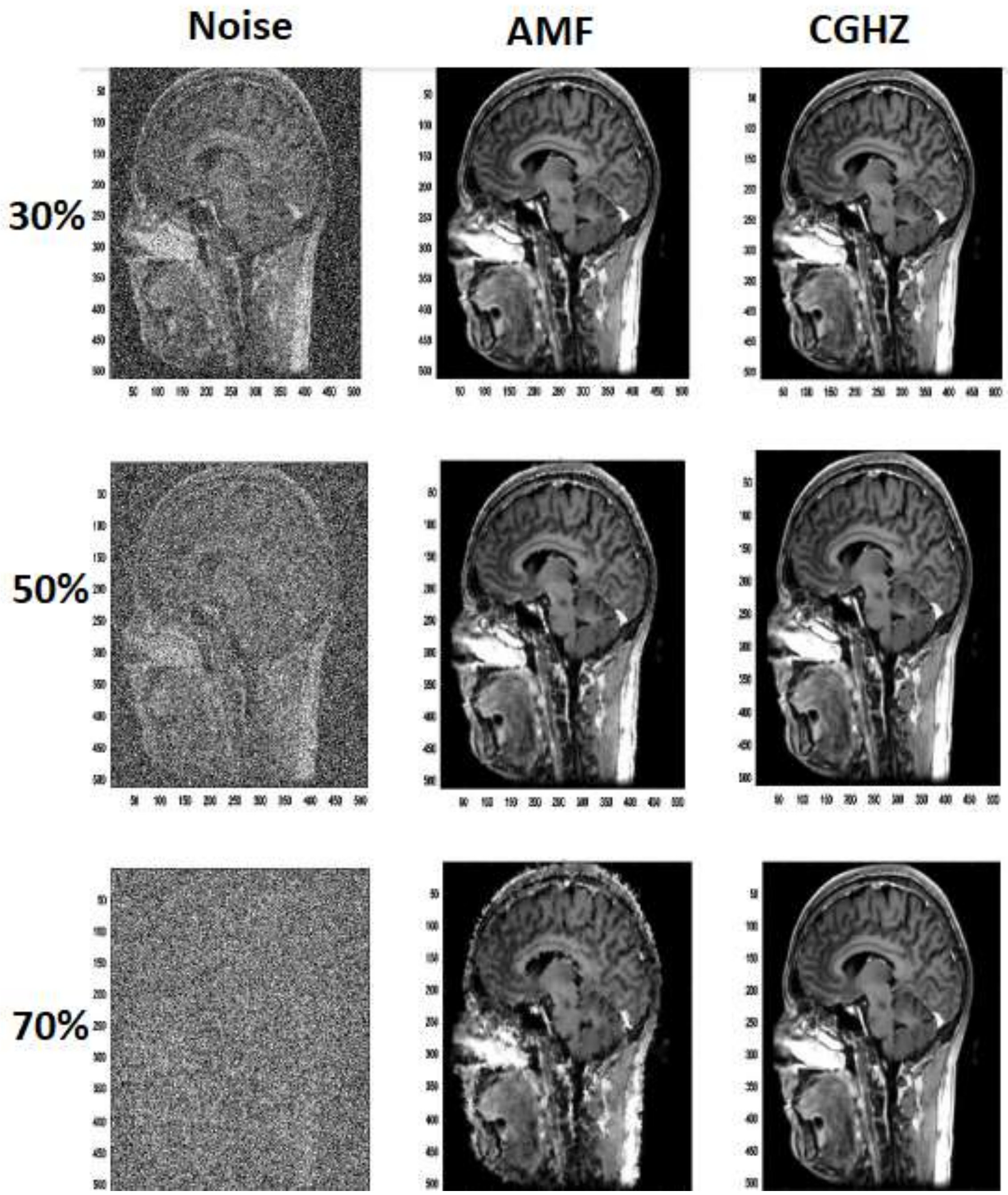

Figure 2: The noisy CerebSagE image, the modified images with AMF method, the modified images with CGHZ method.

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