Theoretical Evaluation of Required Plastic Work in Bulging of Statically loaded Plate

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ABSTRACT

Finding the amount of plastic deformation into metallic plates,metallicstructures, ships frame, bodies of different types of cylinders and reservoirsbodies as a result of exposure tostaticor dynamic forceis so necessary todetermine the extent ofdistortionand destructionhappening intheseobjects and this is one of the main factors in design calculations to determine the dimensions and thicknesses of these bodies. Derivation anew theoreticalmodelto calculate theamount of work and forcerequired to bulge the sheet metalisa very important targetbecause of the lack of a theoretical relations to calculate the work of plastic deformation with good approximationoridenticalto exact required work and force; or vice-versa to find the amount of plastic deformation as a result of a specific work or specific force.This was donein this research where a theoretical model has been derivedthrough the expense of work and force necessary to generatebulging in sheet metal. This theoretical model has been adopted on the mildsteel plates which are of wide application in practical life. Results obtained from the theoretical model were compared with those obtained from tests conducted onsimply supported circular steel plates made from low carbon steel ,milled steel , of a (280) mm diameter and (1,2, 3, 4,5, 6, 8,10) mm thickness andexposed to central force applied by a flat ended punch of 10 mm diameter. The applying force was increased gradually till it caused plastic deformation in the sheet, and then reached the state of shear of the disk, which is locatedunder the punch. Results comparison appear matchorsubstantial convergence between the practical resultsand those obtained from the theoretical model to find the work and for ceit takes to plastic deformation in the tested plates.

التقييم النظري للشغل اللون اللازم لانبعاج صفيحة محملة استاتيكيا

الخلاصة

لايجاد مقدار التشويه اللدن في الصفائح المعدنية , البنايات المعدنية , هياكل السفن ,واجسام الانواع المختلفة من الاسطوانات والخزانات نتيجة تعرضها لقوة استاتيكية او ديناميكية يعد ضروري جدا لايجاد مدى التمزق والتدمير الحاصل في هذه الاجسام وهو من عوامل الحسابات التصميمية الرئيسية لتحديد سمك هذه الاجسام .

ان اشتقاق موديل نظري جديد لحساب مقدار الشغل والقوة اللازمة لانبعاج الصفائح المعدنية يعد هدف مهم وذلك لوجود نقص في العلاقات الرياضية لايجاد الشغل اللازم للتشويه اللدن بتقريب دقيق

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او مطابق للشغل الفعلي اللازم لذلك .او على العكس بالعكس ايجاد التشويه اللدن الناتج عن شغل محدد او قوة مسلطة . وهذا ما قد تم في هذا البحث اشتقاق موديل رياضي من خلال الشغل المصروف والقوة اللازمة لاحداث الانبعاج المتولد في الصفيحة المعدنية . تم تبني هذا الموديل النظري على صفائح الفولاذ الطري واللذي هو ذو تطبيقات واسعة في الحياة العملية.
تمت مقارنة نتائج النموذج النظري مع النتائج التي اجريت على صفائح فولاذية دائرية بسيطة الاسناد (حرة النهايات) مصنوعة من الفولاذ قليل الكاربون, فولاذ طري, بقطر (٢٨٠)ملم وسمك(١،٢،٣،٤،٥،٦،٨،١٠)ملم معرضة الى قوة مركزية بواسطة مكبس ذو نهاية مستوية بقطر (١٠)ملم . القوة المسلطة في زيادة مستمرة حتى تسبب تشويها لدنا في الصفائح والى حد حالة قص القرص الواقع تحت المكبس .
مقارنة النتائج اظهرت مطابقة او تقارب كبير بين التنائج العملية والحسابات الناتجة من الموديل النظري لايجاد الشغل والقوة اللازمة لاحداث تشويه لدن في الصفائح التي اجريت التجارب عليها .

INTRODUCTION

An important area in the field of large deformation of circular plate is concerned with what is known as bulging .The present work is concerned with simply supported circular plate statically loaded by concentrated load at the center .The main important work is consumed in the bulging plastic work, and about (10-20 %) elastic work is recovered after releasing out the load, and about(3 %) is consumed as a friction work between punch head surface and plate, also it includes the friction work between punch side surface and hole surface[1-3]. Ohashi, Y.Murakami,S.(1966),[4]noticed that the extent of the region of plasticity is unsymmetricaldue to the membrane tensionimposed on the bending stresses, theory and experimentwere in a good agreement, theoretical results of their work were numerically evaluated. The references[5-9] show that the hoop strain equal to the radial strain, while the other normal strain equal to the negative value of the sum for both radial and hoop strain in bulged simply supported circular plate dynamic and statically loaded. Ansel C. Ugural [10],introduced the analyticalsolution, of the problem of circular plate loaded by uniformly distributed load and loaded at the center for clamped edge and simply supported edge. The value of the maximum deflection at plate center for free edge central loaded plate was given by equation(a)below:

$$\text{Wmax}=\frac{PR^2(3+u)}{16\pi \text{D}(1+\text{u})} \qquad \text{... (a)}$$

Teng H. Hsu [11], used equation (b) below to find the deflection at any point on a simply supported circular plate loaded at the center:

$$w=\frac{3p(1-u^2)}{4\pi Et^3}\left[\frac{(3+u)}{(1+\text{u})}\left(R^2-r^2\right)+2\pi r^2\ \text{Ln}\frac{r}{R}\right] \qquad \text{... (b)}$$

JunuthulaNarasimha Reddy [12] discussed the derivation of the formula of simply supported plate loaded at the center given in equation(c) below:

$$\text{Wmax}=\frac{P\ a^2}{16\pi \text{D}}\frac{(3+u)}{(1+\text{u})} \qquad \text{.....(C)}$$

$$;\quad \text{D}=\frac{Et^3}{12(1-u^2)} \qquad \text{... (d)}$$

where: a: circular plate radius[mm] ;u:poisson'sratio; E:modulus of elasticity[Gpa]; R: outer plate radius[mm]; r: radius at any point[mm] ; t: plate thickness[mm]; Wmax: plate central bulging[mm]; w:plate bulging[mm];P: central load[N]. The references [9-13] illustrated different equations dealt with this subject, all these equations either it can be applied on the range of elastic limits or with a very small permanent deflection.

A comparison between the experimental results of circular platebehavior's for both cases statically and dynamically loaded were explained in reference [14, 15,16]. Zaid, A.I.O., [17], from his experimental results found that, the representative-stress, σ^{-}, for mild steel fitted by Swift law: $\sigma^{-} = A(B + \overline{\in})^{n}$

Where: A and B are constants depend on material characteristics ; n: strain-hardening exponent .

The equation derived by him in his research gives a good agreement between experimental and theoretical results which proportional between (70- 80%).

In this research aquasi-static mathematical model is developed to evaluate the requiredtotal plastic work. The limiting case of this condition occurs when the small diameter of flat ended punch penetrate and shear a circular disc. The large amount of energy that is introduced into a plate when it is statically loaded usually distributed itself within the material and much of the absorbed energy appears in the form of elastic- plastic deformation of the plate. Shearing a plug from the plate, friction between punch head and plate surface, also punch sides with whole surface are the other work consumed.

THEORETICALCONSIDERATION

A solid circular plate with outer effective radius R and thickness ho is considered in this study. The plate is subjected to gradually increasing central concentrated load and it is assumed to be edges freely supported. The solution is assumed that the plate movement is in the direction of the geometric axis of symmetry, this being also the direction of punch movement. Conditions through theplate thickness are considered to be non-uniform. Referring to Figure (1), in the central zonedirection of the geometric axis of symmetry, this being also thedirection of punch movement. Conditions through the plate thickness are considered to be non-uniform. Referring to Figure (1), in the central zone bounded by R=r , the platedeforms to a conical shape ,and where the bulging reachesIts maximumvalue(w_0)atPunched point.

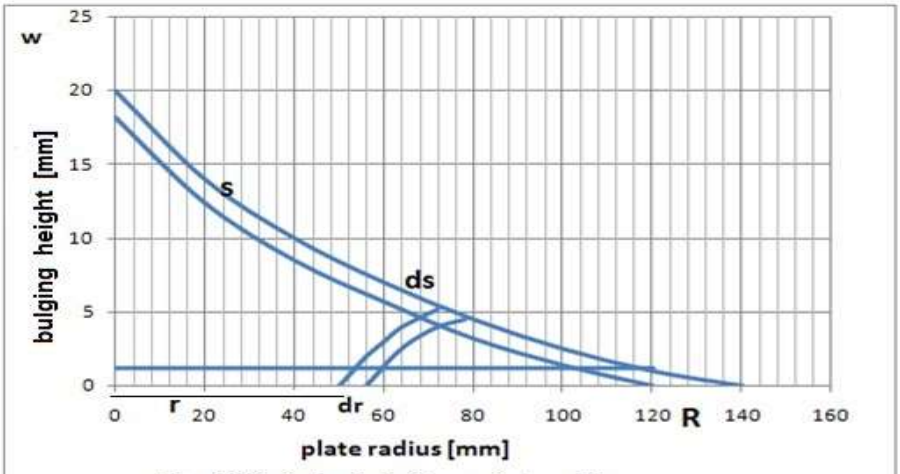

Fig. (1) bulging height vs. plate radius

Figure (1) bulging height vs.plate radius.

Out sidethis zone, the shape is logarithmic and the bulging at any radius (r)may be approximated in terms of the maximum central bulging (w_0) as follows [16-17]:

$$w = w_0 e^{\frac{-ar}{R}} \qquad \text{... (1)}$$

where: α is a function of material type . And for tested mild steel = 1.6, it was calculated numerically from the system of bulging curves.
From Figure (1) and Figure (A-1) appendix (A) it can be shown that:

$$ds^2 = dw^2 + dr^2; \quad \frac{ds}{dr} = \sqrt{1 + \left(\frac{dw}{dr}\right)^2} \qquad \text{...(2)}$$

Differentiation eq.(1). Then substituting the result in Eq.(2) gives that:

$$\frac{ds}{dr} = \sqrt{1 + \left(\frac{a^2 \;\; wo^2}{R^2}\right) e^{\frac{-2ar}{R}}} \qquad \text{... (3)}$$

The binomial expansion of equation (3) for three terms only is:

$$\frac{ds}{dr} = 1 + \frac{1}{2}\left(\frac{a^2}{R^2}\right) wo^2 e^{\frac{-2ar}{R}} + \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \left(\frac{a^2}{R^2}\right) wo^4 e^{\frac{-4ar}{R}}$$

;and by neglecting the small terms of order higher than $e^{\frac{-2ar}{R}}$, it can be proved that [16-17]:

$$\frac{ds}{dr} = 1 + \frac{1}{2}\left(\frac{a^2 \;\; wo^2}{R^2}\right) e^{\frac{-2ar}{R}} \qquad \text{… (4)}$$

Note: For more details of mathematical explanation see appendix (A)

2.1-The required plastic work:

The total work required for bulging the plate can be obtained as follows [6] :

$$Ep = \int_o^R \int_o^{\bar{\in}} \sigma^- d\,\bar{\in}\; dv \qquad \text{…. (5)}$$

The true stress – natural strain curve can be represented by anempirical equation of suitable form, such as ludwik power law, as follows:

$$\sigma^- = k \;\; \bar{\in}^n$$

Ep is the plastic work; $\sigma^-$is the equivalent effective stress
$\bar{\in}$ is the effective strain; k: a constant stress,
n is a strain – hardening exponent (n=0.28, for plate mild steel type(1020) according to SAE and AISI standa
dv is an incremental volume,($dv = 2\pi\ r\ dr\ h_o$) Therefore , equation (5) can be rewritten as follows :

$$Ep = \int_o^R \frac{k}{n+1} \bar{\in}^{(n+1)}\, dv \qquad \text{...(6)}$$

from the symmetry of the deformed plate , the natural hoop strain " $\varepsilon\theta$" can be assumed equal to the natural radial strain " εr", and therefore the natural thickness strain " εt " can be deduced from the equation of incompressibility , as

$$\varepsilon_t + \varepsilon\theta + \varepsilon_r = 0 \ldots\ldots.(7)$$

And the effective strain[6]:

$$\bar{\in} = \sqrt{\frac{2}{9}(\varepsilon t - \varepsilon\theta)^2\{ +(\varepsilon\theta - \varepsilon r)^2 + (\varepsilon r - \varepsilon t)^2\}} \ldots.. (8)$$

$\bar{\in}$=2 εr

; substituting the value of $\frac{ds}{dr}$ from the equation (4) gives:

$\bar{\in}$ =2 ln$\frac{ds}{dr}$;substituting the value of $\frac{ds}{dr}$ from the equation (4) gives:

$$\bar{\in} = 2\ln\left(1 + \frac{1}{2}\left(\frac{a^2 \quad wo^2}{R^2}\right)e^{\frac{-2ar}{R}}\right) \quad \ldots.(9)$$

For a small value of $\frac{1}{2}\left(\frac{a^2 \quad wo^2}{R^2}\right)e^{\frac{-2ar}{R}}$,gives that:

$$\bar{\in} = \left(\frac{a^2 \quad wo^2}{R^2}\right) e^{\frac{-2ar}{R}} \quad \ldots. (10)$$

Then the total plastic work is:

$$Ep = \frac{2\pi kho}{n+1}\int_o^R\left\{ \left(\frac{a^2 \quad wo^2}{R^2}\right) .e^{\frac{-2ar}{R}}\right\}^{(n+1)}.r .dr\ldots. (11)$$

$$Ep = \frac{2\pi kho}{n+1}\left(\frac{a^2 \quad wo^2}{R^2}\right)^{(n+1)}\int_o^R\left\{ e^{\frac{-2ar}{R}}\right\}^{(n+1)}. r. dr\ldots. (12)$$

The integration of the second part of equation (12):

$$\int_o^R\left\{ . e^{\frac{-2ar}{R}}\right\}^{(n+1)} . \ r \ . Dr =$$

$$=\left[\frac{R^2}{-2\alpha(n+1)}e^{-2(n+1)\alpha} - \frac{R^2}{-4\alpha^2(n+1)^2}e^{-2(n+1)\alpha} - \frac{R^2}{-4\alpha^2(n+1)^2}\right]$$

$$= \left[-\frac{R^2}{4\alpha^2(n+1)^2}\{2\alpha^2(n+1)e^{-2(n+1)\alpha} - e^{-2(n+1)\alpha} - 1\}\right] \quad \ldots. (13)$$

Neglect the terms of small values $\{2\alpha^2(n+1)e^{-2(n+1)\alpha} - e^{-2(n+1)\alpha}\}$

Then equation (13) will be as follows:= $\frac{R^2}{4\alpha^2(n+1)^2}$ …. (14)

Substitution of the value given in equation (14) into the original equation (12)as the final results of the integral term gives:

$$Ep = \frac{2\pi kho}{n+1}\left(\frac{a^2 \quad wo^2}{R^2}\right)^{(n+1)}. \frac{R^2}{4\alpha^2(n+1)^2} \quad \ldots. (15)$$

$$Ep = \frac{\pi kho}{2(n+1)^3}\left(\frac{a^2 \quad wo^2}{R^2}\right)^n wo^2 \quad \ldots. (16)$$

The sheer force Fs andwork Es:

The shear force Fs required to punch a circular plug of a radius (r_o) from a plate of a wall thickness (h)

$Fs= 2\pi r_o h_o \tau$……(17) The work required to shear a circular plug

$$Es= \int_0^{ho} 2\pi r_o h_o \tau$$

$$Es = \pi ro ho^2 \tau \qquad ...(18)$$

The total work ET:

$$Et=Ep+Es \qquad … (19)$$

3- Experimental results:

Experiments were conducted on cold rolling mild steel low carbon type (1020) according to the specifications of" SAE and AISI" standard. This type of steel has (232)MPa yield stress and (309) MPa ultimate tensile strength at (26%-28%) elongation. Circular plates of(1,2,3,4,5,6,8,10) mmplate thickness and (280) mm diameter are chosen according to the concluded results given in reference [14,15,16,17] which display that it can be aassumed that, the plate of (280) mm diameter is a semi-infinite plate, so that the bulging will not excess this value. The simply supported plates are loaded at their centers with concentrated load applied by a punch of (10) mm diameter of a cross head speed (2, 5, 10) mm/min.

Platesare deformed plastically and bulged in the direction of applied force displacement. Plates bulging (w_0) increases with the increasing of the load (F) till it reaches its maximum value when the punch sheared a disk from the loaded plate.

Figure (2) displays the results of experiments conducted on plates, represented by the plate central bulging heights(w_0)vs. plate thickness (h_o).Figure (3) shows the relation between the actual applied forces (F)

In flicted on the plates vs. plate thicknesses(h_o). All experimental results are given in table [1 and 2] [Appendix - A].As shown in Table (2), results of the experimental calculations of the required total work [Et.exp.] needed to bulging each plate, was calculated by finding the area under the curves obtained from experiments for each plate confined between the amount of force inflicted on the plate and bulging at the center of plate.

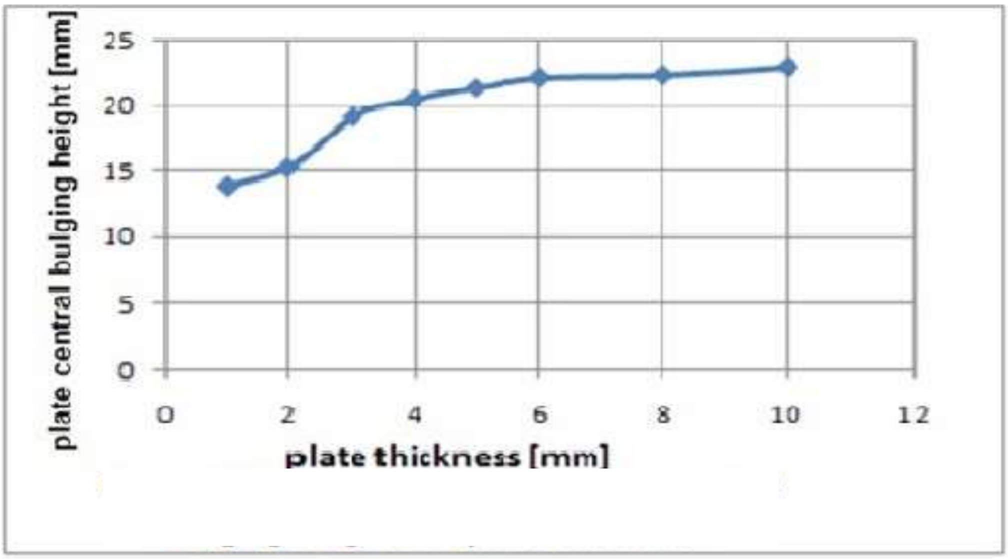

Figure (2) the relation between plate central bulging height and plate thickness.

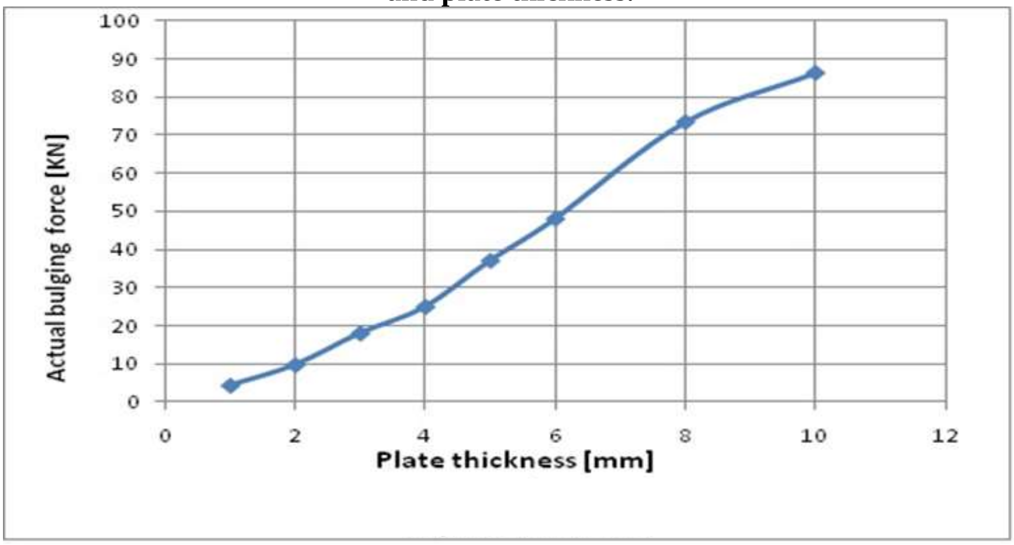

Figure (3) variation of actual bulging force vs.plate thicknesses.

Figure (4) gives a relationship between force inflicted by the punch cross head continuously with the offset to the bottom representing the bulge made in plate center. The relation between applied force (F) and bulge height (w_0) was plotted for all the tested plates, It is clear that the relation is linear. Figure (4) representing one model's of these results for plate thickness (3) mm.

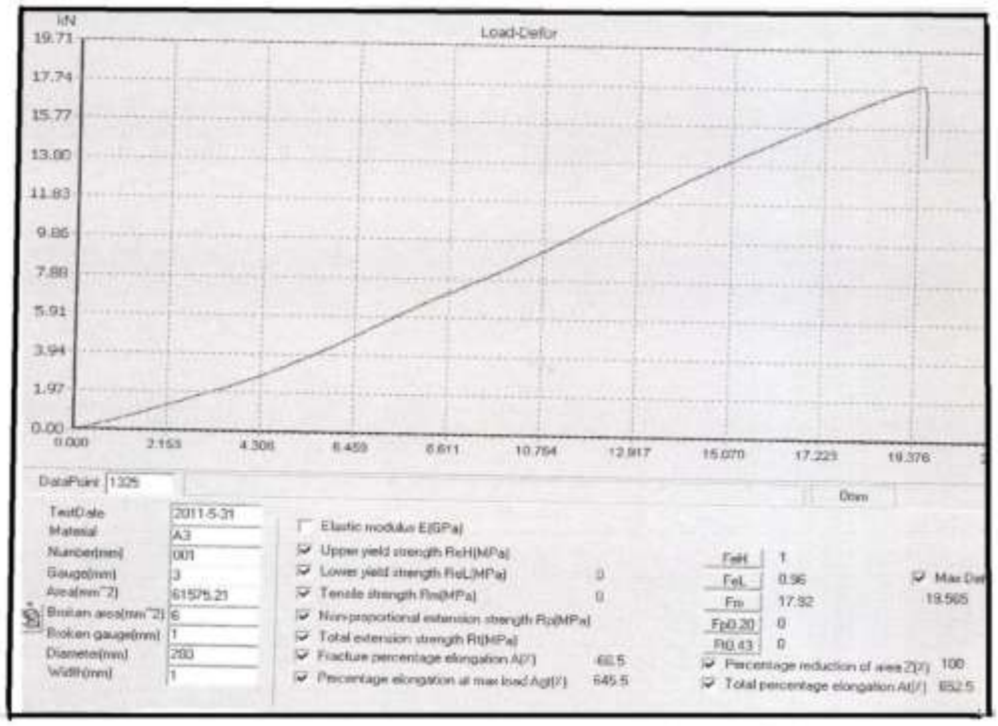

Figure (4) the values of central load (F) vs. plate bulging (w0) given from testing machine for plate ofathickness3 [mm].

Figure (5-a, b, c, d) displays the picture of final plate shapes after plates bulging by central load (F) for the plates (3, 5, 6, 10) [mm] respectively. The Figure also shows the punch inserted in plate holes.

(a)

(b)

(c)

(d)

Figure (5) a,b,c and d.

RESULTS AND DISCUSSION

Theoretical calculations based on the mathematical model that has been derived to calculate the labor force and the work needed to find a bulge in the total sum assigned to circular plate in a simple manner and exposed to the force at the center of plate has been found. Table (3) gives a comparison between the theoretical calculations and practical results obtained from experiments.

Table[3].

ho[mm]	w[mm]	F[KN]exp.	F[KN]cal.	Es[J]cal.	Ep[J]cal	Ept[J]cal.	Et[J]exp.
1	13.9	4.2	3.68	2.8	22.77	25.58	28.8
2	15.3	9.6	9.07	11.21	58.23	69.44	73.44
3	19.56	17.92	18.89	25.22	156.21	181.155	174.72
4	20.5	24.88	28.4	44.48	246.32	291.16	271.5
5	21.4	37	39	70.07	327.5	397.57	393.3
6	23.15	48	49.9	100.9	504.38	605.28	644.5
8	23.2	73.4	71.2	179.39	676.243	855.633	819.8
10	23.86	86.32	96.26	280.3	826.77	1107.07	1029.8

The final bulging height at plate's center (w0) and the thickness of the plates (ho) were shown in Figure (2). This relation shows that, bulging height increases with the increasing of plate thickness with a high rate till (6) mm thickness then the rate of increasing will be decreased smoothly. While Figure(3) comprises the relation between the force applied by punch cross-head and plate thicknesses(ho).All the values of applied forces and plate central deflections(bulging of plate centers) were given from the final results of the tested plates of (1,2,3,4,5,6,8,10)mm thickness. These values are so clear in Figure (4), the value of the final bulging at plate center(w0) and the applied load (F) can be read directly from the highest point on the curve or from the printed table below the curve. Generally the relation between the applied force and plate bulging was approximately a linear relation which helps to find the area under the curve as the expended work required to deform the plate and shear a circular plug.

Figure (6) displays the relationship between the required applied force (F) and the value of plate thickness (ho). This figure shows a comparison between each of the practical results and the results obtained from thetheoretical model .The Figure illustrates good compatibility between these experimental and theoretical results .

It appears that approximately there is a linear relation between bulging forces and the thicknesses of plates. The little difference in linearity comes from difference in work hardening due to different stage numbers of rolling processes that were worked on each plate thickness type.

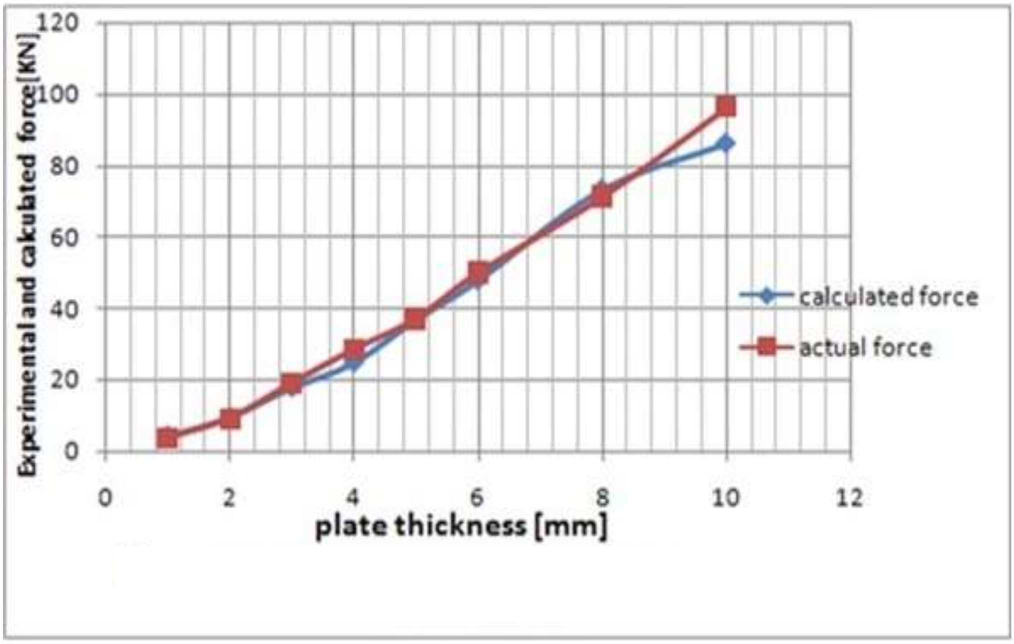

Figure (6) the relation between plate thickness and bulging.

The amount of work necessary for the over all bulgingprocesson the plates of different thickness in relation to the thickness of the plate were shown in Figure(7).The Figureshowsacomparisonbetweenthetheoreticalresultscalculatedfromthemathematical modelthat has been derived as it given in equation (19)and the practical results. The results show the amount of compatibility between the experimental results and theoretical calculations. The amount of differences in these results are up to (3-10%).

The small amount of differences come from neglecting the terms of small values in equation (13) to make the equation derived for this model in a simple form. Also the small deviation of plates center from the applied forces lead to little differences in bulging values in spite of that all the possible procedure are considered through the experiments. The very small differences in work hard eningexponent (n) and the variation of plastic deformation values of plates result from the continuous rolling processes done to thinning plate thicknesses lead to give a little variation in plate bulging through the test. These small differences in the values of plates bulging affect in the force and plastic work required to bulging the tested plates. This deviation clear in the points related to the plates of thickness (6 mm and 10 mm) shown in Figure (7). That is why all the tested specimens give approximately the same yields and ultimate tensile stresses with small different values in plastic strain (26% -28%) in spite of that, all the plates of the same chemical composition analysis and treated with the same heat treatments according to the specifications of" SAE and AISI" standard for cold rollingmild steel low carbon type (1020).

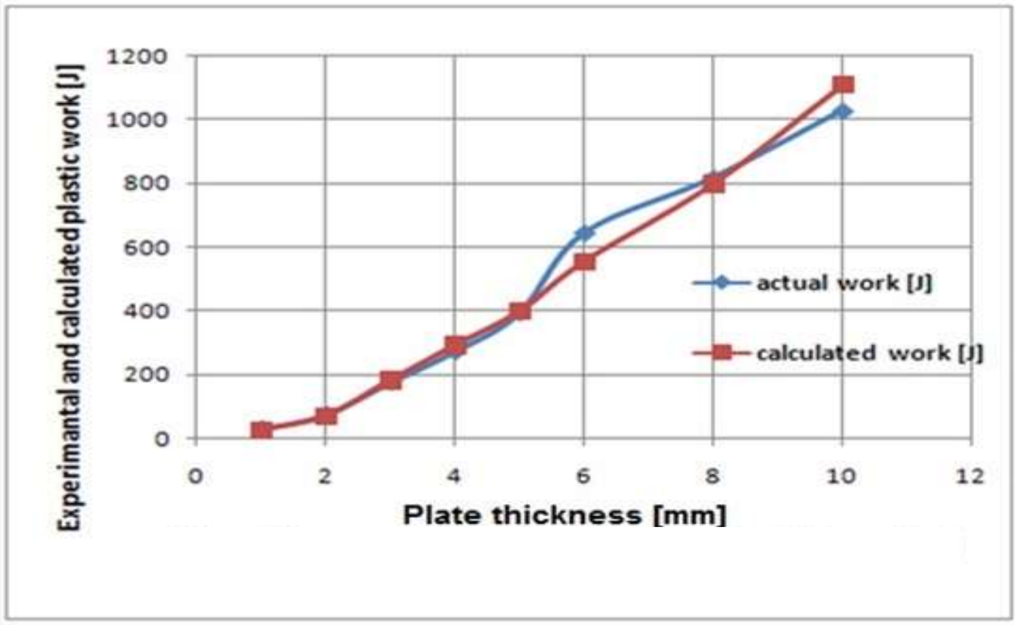

Figure (7) a comparison between actual and theoretical work.

CONCLUSIONS

1-In case of a simply supported plate statically loaded, plate bulging increases with increasing plate thickness.

2-The experimentalresults showa linear relationshipbetween the amount ofworkand force neededfor the process ofbulgingvice-versa the amount ofplastic deformationmade in thecenter ofplate. This wasthe relationshipof all theplatesthathad been tested. Also, thecalculated resultsof thetheoretical model showed the same behavior.

3- The errors in the calculating results given from the theoretical models results of small differences of work hardening of plates and a little deviation of plate centers from the exact center of punch heads.

4 -The few differencesbetweenexperimental results andresultscalculated from the theoretical modelwere caused by thenegligence offriction between theplateandthepunch head and side surfaceas well as thenegligence ofsmall amounts terms inequation(13).

5-The calculatedresults fromthetheoretical model whichwasderivedto calculate theforce andthe workrequired for theplatebulging are ofgreat convergencewith the resultsobtained from theexperimentsfor allmild steelplatesthathad been tested.

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Appendix – A

To find the value of the effective strain$\bar{\epsilon}$ =2 ln$\frac{ds}{dr}$,the value of the incrementrate in the radial direction ($\frac{ds}{dr}$) must be founded due to plate bulging. This value can be found as follows:

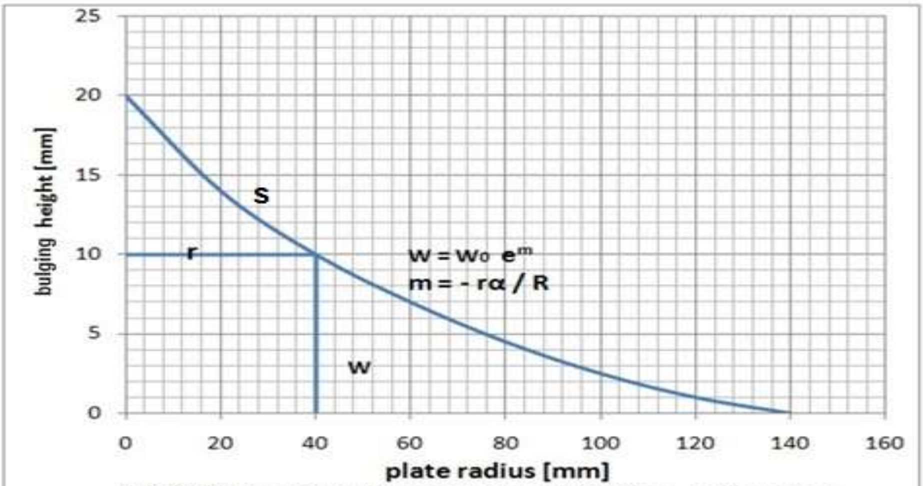

Fig (A-1) The distribution of bulging height vs. plate radius

From Figure (A-1) it can be shown that:

$dw^2 + dr^2$; $\frac{ds}{dr} = \sqrt{1 + (\frac{dw}{dr})^{2}}$ …..(A-1)

ds^2=

Differentiation eq.(1) gives: eq.(A-3)

$\frac{dw}{dr}$ = - …..(A-2) $w_0 e^{\frac{-ar}{R}} \frac{\alpha}{R}$

$$\frac{dw^2}{dr^2} = \frac{a^2}{r^2} wo^2 e^{\frac{-2ar}{R}}$$ …… (A-3)

Substituting the value of$\frac{dw^2}{dr^2}$ in eq.(A-3) in eq.(A-1)gives that:

$$\frac{ds}{dr} = \sqrt{1 + (\frac{a^2 \ wo^2}{R^2})\ e^{\frac{-2ar}{R}}}$$ …. (A-4)

Appendix - B

The total plastic work as it was given in eq. (12):

$$Ep = \frac{2\pi kho}{n+1} \left(\frac{a^2 \quad wo^2}{R^2} \right)^{(n+1)} \int_o^R \{ e^{\frac{-2\,ar}{R}} \}^{(n+1)} . r. dr \quad \text{.... (B-1)}$$

The integration of the second part of equation (B-1) :

$$\int_o^R \{ \; . \; e^{\frac{-2\,ar}{R}} \}^{(n+1)} \; . \; r \; . \; dr =$$

Let $c = \frac{-2\,(n+1)\alpha}{R}$ gives:

$$\int_o^R e^{c.r} \; . \; r \; . \; dr = [\, R . \frac{1}{c} \; . \; e^{c.R} \,] - \int_o^R . \frac{1}{c} e^{c.r} \; . \; dr \quad \text{..... (B-2)}$$

$$= [\, R . \frac{1}{c} \; . \; e^{c.R} \,] - [\, \{ (1/c^2) e^{c.R} \} - (1/c^2) \,] \quad \text{...(B-3)}$$

$$= [\, R \frac{1}{\frac{-2\,(n+1)\alpha}{R}} e^{\frac{-2\,(n+1)\alpha}{R} \cdot R} \,] - [(\frac{1}{\frac{-2\,(n+1)\alpha}{R}})^2 e^{\frac{-2\,(n+1)\alpha}{R} R}] - [\, (\frac{1}{\frac{-2\,(n+1)\alpha}{R}})^2 \,]$$

$$= [\frac{R^2}{-2\alpha\,(n+1)} e^{-2\,(n+1)\alpha} - \frac{R^2}{-4\alpha^2\,(n+1)^2} e^{-2\,(n+1)\alpha} - \frac{R^2}{-4\alpha^2\,(n+1)^2} \,] \quad \text{...(B-4)}$$

$$= [- \frac{R^2}{4\alpha^2\,(n+1)^2} \{\, 2\alpha^2\,(n+1) e^{-2\,(n+1)\alpha} - e^{-2\,(n+1)\alpha} - 1 \} \quad \text{....(B-5)}$$

Neglect the terms of small values :

$$\{\, 2\alpha^2 (n+1) e^{-2\,(n+1)\alpha} - e^{-2\,(n+1)\alpha} \}$$

From equation (B-5),it will be as follows:

$$= [- \frac{R^2}{4\alpha^2\,(n+1)^2} \{-1\}] \quad = \frac{R^2}{4\alpha^2\,(n+1)^2} \quad \text{....(B-6)}$$

Substitution of the value given in equation(B-6) into the original equation (12) as the final results of the integral term givesthe final equation of the plastic work as it was given in eq. (15and16).

Appendix -C

The experimental results of the final central bulging of plates ($w_{0)}$, applied load (F)and the work required(E) vs. plate thicknesses (h_o) are given in Table[1 and 2].

Table [1]

h_o[mm]	w_0[mm]	F[KN] Exp.
1	13.9	4.200
2	15.3	9.6
3	19.37	17.92
4	20.5	24.88
5	21.4	37
6	23.15	48
8	23.2	73.4
10	23.86	86.32

Table [2]

h_o[mm]	Et[J] Exp.	F[KN] Exp.
1	28.800	4.200
2	73.440	9.600
3	174.72	17.92
4	271.50	24.88
5	393.30	37.00
6	644.50	48.00
8	819.80	73.40
10	1029.8	86.32