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The Application of Weyl Module in the Skew-partition (5,5)/(u,0) when u=1,2,3

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Abstract

My work included this survey the rows resolution of Weyl module and specify terms and the exactness according to skew partition $(5,5)/(u,0)$ when $u = 1,2,3$ where we relied on the contracting homotopy.

Keywords: Weyl module; exactness; Skew-partition; divided power.

تطبيق مقاس وايل في التقسيم المائل(u , 0) / (5,5)/ عندما u=1,2,3

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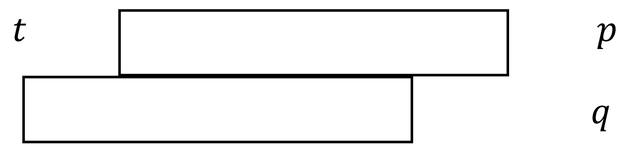
الخلاصة

تضمن عملي هذا مراجعة تحلل صفوف مقاس وايل وتحديد حدود ذلك التحلل وبرهان انه تام وفقا الى التجزئة المنحرفة بالاعتماد على التوافق الهوموتوبي. $u = 1,2,3$ $(5,5)/(u,0)$ عندما

الكلمات المفتاحية: مقاس وايل؛ تام ؛ تجزئة منحرفة ؛ قوة مقسمة.

1. Introduction

Let F be a free R-module and D_nF be divided power of degree n, in the [2], the author described it to us the Weyl module $K_{\lambda/\mu}F$ when $K_{\lambda/\mu}F$ = Im($d'_{\lambda/\mu}$), $d'_{\lambda/\mu}$:DF $\to \wedge F$ which takes the following image



$$\sum D_{p+k} \otimes D_{q-k} \xrightarrow{\boxdot} D_p \otimes D_q \xrightarrow{d'_{\lambda/\mu}} K_{\lambda/\mu} \longrightarrow 0$$

We used letter place, in order to get

$$\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(p+k)} \\ 2^{(q-k)} \end{matrix}\right) \xrightarrow{\partial_{21}^{(k)}} \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{matrix}\right) \longrightarrow$$

$$\sum_w \left(\begin{matrix} w_{(1)} \\ w'w_{(2)} \end{matrix}\middle| \begin{matrix} (t+1)'(t+2)' \dots (p+t)' \\ 1'2'3' \dots q' \end{matrix}\right)$$

$$w \otimes w' \in D_{p+k} \otimes D_{q-k} \quad , \square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$$

And

$$d'_{\lambda/\mu} = \partial_{q'2} \dots \partial_{1'2} \partial_{(p+t)'1} \dots \partial_{(t+1)'1}$$

Particularly, □ move items $x \otimes y$ of $D_{p+k} \otimes D_{q-k}$ to $\sum x_p \otimes x'_k y$, see[3]. The graded algebra $D(Z_{21})$ operates on $M = \sum D_{p+k} \otimes D_{q-k} = \sum M_{q-k}$, where M the graded left A-module, whith $w = Z_{21}^{(k)} \in A$ and $v \in D_{\beta_1} \otimes D_{\beta_2}$, we will have it:

$$w(v) = Z_{21}^{(k)}(v) = \partial_{21}^{(k)}(v)$$

$$M_{\bullet} : 0 \to M_{q-t} \xrightarrow{\partial_s} \dots \to M_l \xrightarrow{\partial_s} \dots M_1 \xrightarrow{\partial_s} M_0$$

Bar(M, A; S, •), S = {x}.

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_l}$$

$$\sum_{k_1 \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \dots x Z_{21}^{(k_l-1)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_{l-1}}$$

$$\dots \xrightarrow{d_1} \sum_{k_i \geq 0} Z_{21}^{(t+k)} x D_{p+t+|k|} \otimes D_{q-t-k} \xrightarrow{d_0} D_p \otimes D_q \text{ , with } |k| = \sum k_i.$$

In the source [4],the author studied the exactness towards (8,7), In [5] the author presented the exactness for partition (8,6)/(2,t) .In [6] the author presented the exactness for partition (9,7)/(S,0).As for my work this , I presented the exactness for partition (5,5)/ (u, 0) in cases u = 1,2,3.

2. The resolution of Weyl module according to (5,5)/ (1, 0):

$$M_0 = D_4 \otimes D_5$$

$$M_1 = Z_{21}^{(2)} x D_6 \otimes D_3 \oplus Z_{21}^{(3)} x D_7 \otimes D_2 \oplus Z_{21}^{(4)} x D_8 \otimes D_1 \oplus Z_{21}^{(5)} x D_9 \otimes D_0$$

$$M_2 = Z_{21}^{(2)} x Z_{21} x D_7 \otimes D_2 \oplus Z_{21}^{(3)} x Z_{21} x D_8 \otimes D_1 \oplus Z_{21}^{(2)} x Z_{21}^{(2)} x D_8 \otimes D_1 \oplus Z_{21}^{(4)} x Z_{21} x D_9 \otimes D_0 \oplus Z_{21}^{(2)} x Z_{21}^{(3)} x D_9 \otimes D_0 \oplus Z_{21}^{(3)} x Z_{21}^{(2)} x D_9 \otimes D_0$$

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$M_3 = Z_{21}^{(2)} x Z_{21} x Z_{21} x D_8 \otimes D_1 \oplus Z_{21}^{(3)} x Z_{21} x Z_{21} x D_9 \otimes D_0$
$\oplus Z_{21}^{(2)} x Z_{21} x Z_{21}^{(2)} x D_9 \otimes D_0 \oplus Z_{21}^{(2)} x Z_{21}^{(2)} x Z_{21} x D_9 \otimes D_0$

$M_4 = Z_{21}^{(2)} x Z_{21} x Z_{21} x Z_{21} x D_9 \otimes D_0$

We will get the following complex

$$M_4 \xrightarrow{\partial_x} M_3 \xrightarrow{\partial_x} M_2 \xrightarrow{\partial_x} M_1 \xrightarrow{\partial_x} M_0$$

id s_3 id s_2 id s_1 id s_0 id

$$M_4 \xrightarrow{\partial_x} M_3 \xrightarrow{\partial_x} M_2 \xrightarrow{\partial_x} M_1 \xrightarrow{\partial_x} M_0$$

$$S_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(4)}2^{(k)} \\ 2^{(5-k)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(4+k)} \\ 2^{(5-k)} \end{matrix}\right) & ; if\ k = 2,3,4,5 \\ 0 & ; if\ k \le 1 \end{cases}$$

$S_1 : M_1 \longrightarrow M_2$

$$S_1\left(Z_{21}^{(k+1)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k)}2^{(m)} \\ 2^{(4-k-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} Z_{21}^{(k+1)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(4-k-m)} \end{matrix}\right) & ; if\ m = 1,2,3 \\ 0 & ; if\ m = 0 \end{cases}$$

$S_2 : M_2 \longrightarrow M_3$

$$S_2\left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)}2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) & ; if\ m = 1,2, \\ 0 & ; if\ m = 0 \end{cases} \quad ;\ \text{for } |k| = k_1 + k_2$$

$S_3 : M_3 \longrightarrow M_4$

$$S_3\left(Z_{21}^{(k_{1+1})} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)}2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)\right)$$

$$= \begin{cases} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases} ; \text{for } |k| = k_1 + k_2 + k_3$$

$$S_0 \partial_{\varkappa} \left(Z_{21}^{(k+1)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix} \right) \right) = S_0 \partial_{21}^{(k+1)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix} \right)$$

$$= \binom{k+1+m}{m} Z_{21}^{(k+1+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(4-k-m)} \end{matrix} \right),$$

and

$$\partial_{\varkappa} S_1 \left(Z_{21}^{(k+1)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix} \right) \right) = \partial_{\varkappa} \left(Z_{21}^{(k+1)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(4-k-m)} \end{matrix} \right) \right)$$

$$= - \binom{k+1+m}{m} Z_{21}^{(k+1+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(4-k-m)} \end{matrix} \right) + Z_{21}^{(k+1)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix} \right)$$

$$= Z_{21}^{(k+1)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(4-k-m)} \end{matrix} \right)$$

It became clear $S_0 \partial_{\varkappa} + \partial_{\varkappa} S_1 = id_{M_1}$

$$S_1 \partial_{\varkappa} \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= S_1 \left(- \binom{|k|+1}{k_2} Z_{21}^{|k|+1} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + Z_{21}^{(k_1+1)} x\, \partial_{21}^{(k_2)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= - \binom{|k|+1}{k_2} Z_{21}^{|k|+1} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + \binom{k_2+m}{m} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right),$$

and

$$\partial_{\varkappa} S_2 \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right) =$$

$$\partial_{\varkappa} \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= \binom{|k|+1}{k_2} Z_{21}^{|k|+1} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) -$$

$\binom{k_2+m}{m} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) +$

$Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right),$

For $|k| = k_1 + k_2$

It became clear $S_1 \partial_\varkappa + \partial_\varkappa S_2 = id_{M_2}$

$$S_2 \partial_\varkappa \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= S_2 \left(\binom{k_1+k_2+1}{k_2} Z_{21}^{(k_1+k_2+1)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) - \binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+k_3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + Z_{21}^{(k_1+1)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= \binom{k_1+k_2+1}{k_2} Z_{21}^{(k_1+k_2+1)} x Z_{21}^{(k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) - \binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + \binom{k_3+m}{m} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right),$$

and

$$\partial_\varkappa S_3 \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right) = \partial_\varkappa \left(Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) \right)$$

$$= - \binom{k_1+k_2+1}{k_2} Z_{21}^{(k_1+k_2+1)} x Z_{21}^{(k_3)} \varkappa Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + \binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) -$$

$$\binom{k_3+m}{m} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right) + Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \partial_{21}^{(m)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix} \right)$$

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$= -\binom{k_1+k_2+1}{k_2} Z_{21}^{(k_1+k_2+1)} x Z_{21}^{(k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) +$

$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2+k_3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right) -$

$\binom{k_3+m}{m} Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$

$+ Z_{21}^{(k_1+1)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(4-|k|-m)} \end{matrix}\right)$,for $|k| = k_1+k_2+k_3$.

It became clear $S_2\partial_\varkappa + \partial_\varkappa S_3 = id_{M_3}$

3. The resolution of Weyl module according to $(5,5)/(2,0)$:

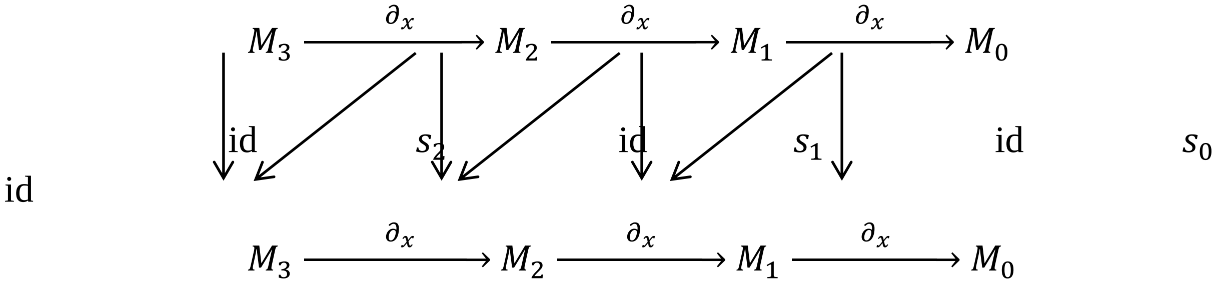
$M_0 = D_3 \otimes D_5$

$M_1 = Z_{21}^{(3)} x D_6 \otimes D_2 \oplus Z_{21}^{(4)} x D_7 \otimes D_1 \oplus Z_{21}^{(5)} x D_8 \otimes D_0$

$M_2 = Z_{21}^{(3)} x Z_{21} x D_7 \otimes D_1 \oplus Z_{21}^{(4)} x Z_{21} x D_8 \otimes D_0 \oplus Z_{21}^{(3)} x Z_{21}^{(2)} x D_8 \otimes D_0$

$M_3 = Z_{21}^{(3)} x Z_{21} x Z_{21} x D_8 \otimes D_0$

We will get the following complex



$S_0: D_3 \otimes D_5 \longrightarrow \sum_{k>0} Z_{21}^{(k+2)}\, x\, D_{3+k} \otimes D_{5-k}$

$S_0\left(\left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(3)} 2^{(k)} \\ 2^{(5-k)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k+2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(3+k)} \\ 2^{5-k)} \end{matrix}\right) & ; if\ k = 3,4 \\ 0 & ; if\ k \le 2 \end{cases}$

$S_1: \sum_{k>0} Z_{21}^{(k+2)}\, x\, D_{6+k} \otimes D_{3-k} \longrightarrow Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x D_{5+k} \otimes D_{3-k}$ such that:

$S_1\left(Z_{21}^{(k+2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(3-k-m)} \end{matrix}\right)\right) = \begin{cases} Z_{21}^{(k+2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(3-k-m)} \end{matrix}\right) & ; if\ m = 1,2 \\ 0 & ; if\ m = 0 \end{cases}$;

For $|k| = k_1 + k_2$

$S_2 : \sum_{k_i>0} Z_{21}^{(k_1+2)}\, x\, Z_{21}^{(k_2)} x \mathfrak{D}_{5+|k|} \otimes \mathfrak{D}_{3-|k|} \longrightarrow$
$Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x Z_{21}^{(k_3)} x \mathfrak{D}_{5+|k|} \otimes \mathfrak{D}_{3-|k|}$

such that:

$$S_2\left(Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix}\right)\right) =$$

$$\begin{cases} Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(3-|k|-m)} \end{matrix}\right) & ; if\ m = 1 \\ 0 & ; if\ m = 0 \end{cases};$$

For $|k| = k_1 + k_2$

$$S_0 \partial_\varkappa \left(Z_{21}^{(k+2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(3-k-m)} \end{matrix}\right)\right) = S_0 \partial_{21}^{(k+2)} \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5)} 2^{(k+m)} \\ 2^{(3-k-m)} \end{matrix}\right)$$

$$= \binom{k+2+m}{m} Z_{21}^{(k+2+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(3-k-m)} \end{matrix}\right),$$

and

$$\partial_\varkappa S_1 \left(Z_{21}^{(k+2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(3-k-m)} \end{matrix}\right)\right) = \partial_\varkappa \left(Z_{21}^{(k+2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(3-k-m)} \end{matrix}\right)\right)$$

$$= -\binom{k+2+m}{m} Z_{21}^{(k+2+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(3-k-m)} \end{matrix}\right) + Z_{21}^{(k+2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(3-k-m)} \end{matrix}\right)$$

It became clear $\quad S_0\, \partial_\varkappa + \partial_\varkappa S_1 = \mathrm{id}_{M_1}$

$$S_1 \partial_\varkappa \left(Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix}\right)\right)$$

$$= S_1 \left(-\binom{|k|+2}{k_2} Z_{21}^{|k|+2} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix}\right) + \right.$$
$$\left. Z_{21}^{(k_1+2)} x \partial_{21}^{(k_2)} \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix}\right)\right)$$

$$= -\binom{|k|+2}{k_2} Z_{21}^{|k|+2} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(3-|k|-m)} \end{matrix}\right) +$$
$$\binom{k_2+m}{m} Z_{21}^{(k_1+2)} x Z_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(3-|k|-m)} \end{matrix}\right),$$

and

$$\partial_\varkappa S_2 \left(Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix}\right)\right) =$$

$$\partial_\varkappa \left(Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix}\middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(3-k-m)} \end{matrix}\right)\right)$$

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$= \binom{|k|+2}{k_2} Z_{21}^{(|k|+2)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|+m)} \\ 2^{(3-|k|-m)} \end{matrix} \right) -$

$\binom{k_2+m}{m} Z_{21}^{(k_1+2)} x Z_{21}^{(k_2+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(6+|k|+m)} \\ 2^{(3-|k|-m)} \end{matrix} \right) +$

$Z_{21}^{(k_1+2)} x Z_{21}^{(k_2)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+|k|)} 2^{(m)} \\ 2^{(3-|k|-m)} \end{matrix} \right),$

For $|k| = k_1 + k_2$.

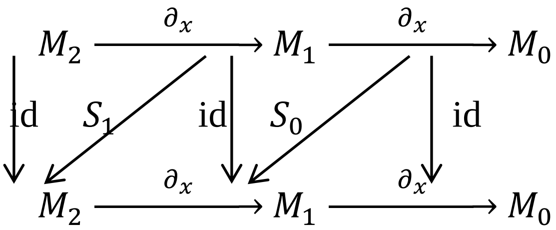
It became clear $S_1 \partial_{\varkappa} + \partial_{\varkappa} S_2 = id_{M_2}$

4. The resolution of Weyl module according to $(5,5)/(3,0)$:

$M_0 = D_2 \otimes D_5$

$M_1 = Z_{21}^{(4)} x D_6 \otimes D_1 \oplus Z_{21}^{(5)} x D_7 \otimes D_0$

$M_2 = Z_{21}^{(4)} x Z_{21} x D_7 \otimes D_0$



$S_0 : D_2 \otimes D_5 \longrightarrow \sum_{k>0} Z_{21}^{(k+3)} x \, D_{2+k} \otimes D_{5-k}$

$S_0 \left(\left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^2 \, 2^{(k)} \\ 2^{(5-k)} \end{matrix} \right) \right) = \begin{cases} Z_{21}^{(k+3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(2+k)} \\ 2^{5-k)} \end{matrix} \right) & ; if \ k = 4 \\ 0 & ; if \ k \leq 3 \end{cases}$

$S_1 \left(Z_{21}^{(k+3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(2-k-m)} \end{matrix} \right) \right) = \begin{cases} Z_{21}^{(k+3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(2-k-m)} \end{matrix} \right) & ; if \ m = 1 \\ 0 & ; if \ m = 0 \end{cases}$

$S_0 \partial_{\varkappa} \left(Z_{21}^{(k+3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(2-k-m)} \end{matrix} \right) \right) = S_0 \partial_{21}^{(k+3)} \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5)} 2^{(k+m)} \\ 2^{(2-k-m)} \end{matrix} \right)$

$= \binom{k+3+m}{m} Z_{21}^{(k+3+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(2-k-m)} \end{matrix} \right),$

$\partial_{\varkappa} S_1 \left(Z_{21}^{(k+3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(2-k-m)} \end{matrix} \right) \right) = \partial_{\varkappa} \left(Z_{21}^{(k+3)} x Z_{21}^{(m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(2-k-m)} \end{matrix} \right) \right)$

$= -\binom{k+3+m}{m} Z_{21}^{(k+3+m)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k+m)} \\ 2^{(2-k-m)} \end{matrix} \right) + Z_{21}^{(k+3)} x \left(\begin{matrix} w \\ w' \end{matrix} \middle| \begin{matrix} 1^{(5+k)} 2^{(m)} \\ 2^{(2-k-m)} \end{matrix} \right)$

It became clear $S_0 \partial_{\varkappa} + \partial_{\varkappa} S_1 = id_{M_1}$

We obtained from 2,3 and 4 that $\{S_0, S_1, S_2, S_3\}, \{S_0, S_1, S_2\}$ *and* $\{S_0, S_1\}$ have a contracting homotopy meaning the complex above are exact [8].

5. Conclusions

With my effort ,I concluded that the complex are exact in $(5,5)/\,(\mathrm{u},0)$ when $\mathrm{u} = 1,2,3$.

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