On Strong Periodic Ring

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Abstract

In this work we shall introduce a definition of strong periodic ring and we prove some properties of this type of rings.

1**. Introduction :** The concept of ring is very important concept in Algebra [1],[2] . We give a definition of periodic ring where a ring is called periodic if every subring is periodic that is there exist r∈ R such that rS=S for S subring of a ring R. And we give a definition of a strong periodic ring if rS=S for every r∈ R .

We find the intersection of two strong periodic subring is a gain strong periodic subring .we prove if R is a strong periodic ring and I be ideal of R then R/I is strong periodic ring .

2**. Basic definition and remarks:**

In this section ,we recall the basic definition needed in this work

2.1 Definition [3]

Let R and S be two ring then a ring homomorphism

F:R →S is a mapping for all $r_1, r_2 \in$ R we have

F(r_1+r_2) = F(r_1) +F(r_2)

F(r_1 . r_2) = F(r_1).F(r_2)

2.2 **Definition [3]**

Homomorphism function f:R →S where R,S be two rings is epimorphism if for each s $\in S$ there exists $r \in R$ such that f(r)=s .

2.3 **Definition [4]** : quotient ring

Let R be a ring and I be ideal of R then R/I is called quotient ring where

R/I={a + ;a ∈ R} and we define +, . on R/I by

$(a+ \quad)+_{R/I} (b+ \quad) = (a+b) +_{R/I} I$

$(a+ \quad) \cdot_{R/I} (b+ \quad) = (a.b)+_{R/I} I$

In this section ,we recall a new definition and some example

3.1 **Definition :** A subring S of a ring R is said to be periodic subring if there exist $r \in R$; rS=S where r is called period elements.

3.2 **Example :** Z_4= {0,1,2,3,}, let S= {0,2}

Let r=3∈ Z_4 ; 3S=S so S is periodic subring of R.

3.3 **Definition:** A ring R is called a periodic ring if every subring of R is periodic subring.

3.4 **Example :** Z_8 is periodic ring.

3.5 **Definition :** A subring S of a ring R is called strong periodic if rS=S for every $r \in R$.

3.6 **Definition** : A ring R is called strong periodic if every subring of R is a strong periodic.

3.7 **Example :**

1) {0} is a strong periodic subring of any ring since r{0}={0} for every $r \in R$.
2) Z_2 ,Z_3 ,Z_4 are all strong periodic rings .

3.8 **Remark:**

It is clear that by definition (3-3) and definition (3.6) every strong periodic ring is periodic but the converse is not true for example

3.9 **Example :**

Let $R = (Z_{12}, +_{12}, \cdot_{12})$ and let S1={0,2,4,6,8,10} is a subring of R then for every $r \in R$,rS1 = S1 then S1 is a strong periodic subring of R, also S2= [0,3,6,9] is a periodic subring but not strong subring since 10 S2≠ S2.

3.10 **Example :**

Let $R = (M_2(\mathbb{R}), +, .)$ where $\mathbb{R}$ be the set of all real numbers . let $S=\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} ; a,b \in \mathbb{R}, +, .\}$ be subring of $M_2(\mathbb{R})$ and since there is r= $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \in R$ such that rS=S ,So S is a periodic subring and since there is r= $\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} \in R$ but r$S \neq S$

S is not strong periodic subring.

3.11 **Remark:** It is clear that Z_{12} is periodic ring but the ring in Example (3.10) is not periodic ring since

S=$\{\begin{bmatrix}a & b\\ c & 0\end{bmatrix}$; **a, b, c** $\in \mathbb{R}$, +,.) is a subring of $M_2(\mathbb{R})$but not periodic subring of $M_2(\mathbb{R})$ **.**

In this section ,we shall prove several theorems concerning of strong periodic ring.

4.1 **Theorem:** The intersection of any two strong periodic subring is a gain strong periodic subring.

Proof:

Let R be a ring and S_1,S_2 are strong periodic subring there for each r$\in$ R ,rS_1=S_1 and rS_2=S_2 then r($S_1 \cap S_2$)=rS_1∩rS_2=S_1∩S_2

Thus $S_1 \cap S_2$ is a strong periodic subring.

4.2 **Cor:** let R be periodic ring and A,B be two subring of R then A ∩B is periodic subring if A and B are periodic subring which have the same period.

Proof: observeliy .

4.3 **Theorem :** Let R be a periodic ring and let A and B be two subring of R such that A$\subseteq$ B ,if B is strong periodic ring then A be periodic ring.

Proof :

TO prove $\forall$ S $\subseteq$ A ; rS =S $\forall$ r$\in$ R

$\forall$ S$\subseteq$ A $\subseteq$ B so S$\subseteq$ B but B is strong periodic ring so rS=S $\forall$ r$\in$ R.

4.4 **Remark :** It is Clear that every ideal is periodic subring.

4.5 Theorem : If R is strong periodic ring and I be ideal of R then R /I is strong periodic ring .

Proof:

Let S $\subseteq$ R and S be strong periodic subring of R, To prove $\frac{S}{\square}$ is strong periodic subring of R / I

Let r+ I $\in$ R /I for each r $\in$ R to prove

(r + I) **.** (S+ I) =S+ I

(r + I) **.** (S+ I) =rS + I = S+ I since S be strong periodic ring.

4.6 **Theorem:** Let R , S be two rings and f be epimorphism s.t f:R→S , if R is strong periodic ring then S is a strong periodic ring.

Proof:

Let W be a subring of S, then f^{-1}(W) is a subring of R
But R is strong periodic ring so for each r∈ R we have
rf^{-1}(W)=f^{-1}(W)   or   f(r)W=W
Thus S is a strong periodic ring.

4.7 **Theorem :**

Let F: R →S be epimorphism and S be strong periodic ring then R is strong periodic ring.

Proof :

Let A be a subring of R
Then f(A) is a subring of S
But S is a strong periodic ring so for each s∈ S we have
sf(A)=f(A)   or f^{-1}(s)A=A
Thus, R is a strong periodic ring.

References

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