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Control Design for the Balancing of an Automated Three-Link Gymnastics Robot Based Discrete -Time Linear Quadratic Regular Technique

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Abstract

This paper proposes how to balance a non-linear triple inverted pendulum, which is represented by a gymnastic robot in the vertical plane. This robot simulates a human acrobat. This type of robot is installed on ball bearings and connected to a high bar that rotates freely. It is made of three joints; the first joint is idle with no actuator (not powered). The second joint is active. As for the third joint, it is active. The first joint presents a major challenge in how to control the balance of the three-link gymnastic robot. To achieve that goal, Discrete Linear Quadratic Regular (DLQR) and Linear-quadratic-Gaussian (LQG) control theory is used and implemented on the time discrete controller by using the mathematical model of the robot. The simulation results showed the difference between LQR and LQG control systems to balance and control the gymnastics robot system. Finally, the LQR is better where superior results for the three links (first, second, third) were obtained in terms of overshot (23.37%, 176.8%, 73.125%), rise time (0.05013s, 0.07519s, 0.02506s), and setting time (3.208s, 3.233s, 4.16s) compared to the than the LQG.

Keywords: gymnastic robot; Inverted Triple Pendulum; Discrete Linear Quadratic Regular (DLQR); Control of balance; Linear-quadratic-Gaussian (LQG).

1. Introduction

In this paper, how to control the balancing of a non-linear and unstable three-link gymnastics robot in the required vertical position is discussed. The gymnastics robot mimics the human acrobat. The gymnastics robot consists of three joints; the first joint is idle because it is not powered. The second and third joints are active [1, 2, 3, 4]. In [5], a dual model based on an inverted pendulum with two degrees of freedom to control the balance is proposed. In [6], it is proposed how to install a passive dual-link gymnastics robot that simulates a human acrobat in the vertical position. In [7], the authors investigated the stability of non-linear systems and how to stabilise the Dof-3 inverted pendulum system. In [8], the author demonstrates the use of the LQR control system in discrete-time systems. The system described in [9] is a non-linear and unstable double inverted pendulum performance analysis and LQR controller design to reduce settling time. In [10], the authors investigated how to compare LQR and PID controllers based on the response to a perturbation rejecter for an inverted pendulum with two wheels using Euler-Lagrangian principles. In addition to this, research models have been proposed about the balancing of the inverted pendulum, which is considered one of the difficult tasks as different control units were used to achieve this, such as LQR, PID [11] [12] [13] [14]. The system described in [15] is a model of an unstable inverted triple pendulum using Lagrangian equations and designing an LQR controller to achieve stability in the vertical position of the pendulum. There is much research on the performance of the non-linear inverted pendulum and on trying to stabilise it in the vertical position by designing an LQR controller [16] [17] [18] [19]. The author in [20] used the LQR and LQG controllers to control the double inverted pendulum, where the LQR has a good performance. In [21], the spatial inverted pendulum was designed and controlled with two degrees of freedom through the LQR controller, which works best in terms of maintaining the balance of the system.

The research proposed in this paper builds on the work reported by the authors in [1], where they discuss a study of how to balance a three-link robot. Their approach was based on how the DLQR and LQG controllers were designed with the initial angle. In this paper, the design of the DLQR controller and LQG controller to balance the three-link gymnastics robot in the vertical plane has been implemented. Weighting matrices Q and R were selected by using (DLQR and LQG) for the mathematical model of the gymnastics robot for optimal control to meet the desired purpose without spending a large of energy to maintain the performance of the gymnastics robot. All studies were performed by simulation using MATLAB software and mfile.

Following the overview in the introduction, the sections of the paper are constituted as follows. The second section describes the gymnastics robot system and its mathematical model. In Section 3, the DLQR is designed. In Section 4, the LQG is designed. In Section 5, the simulation results are analysed. Finally, Section 6 concludes the work and describes future works.

2. Description of the model and the system

The three-link gymnastic robot is a model that is similar to human gymnastics. It is mounted on ball bearings and connected to a high bar that rotates freely. The robot consists of three joints; the first joint is raw and inactive, which is the elbow and wrist joint. The second joint is the head, neck and trunk, and it is active. As for the third joint, it is active, and it is a combination of the knee and ankle joints.

The schematic diagram of a three-link gymnastics robot is in Fig.1. The Lagrangian Equation derives the motion equations for the robot through the use of the schematic diagram. The time-continuous linear state was obtained by the presence of the gymnastics robot in the vertical position. Table 1 defines the Vocabulary of System Symbols.

The linearised continuous-time and state space representation of the system are obtained as [1]:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

Where the system matrix is defined as A, the input matrix is defined as B, the output matrix is defined as C, the feed matrix is defined as D, the state vector is defined as x, and the output vector is defined as y [1].

:

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 36.42 & 0.35 & -0.21 & -0.20 & 88.38 & 9.17 \\ -13.10 & 22.06 & 2.23 & 0.20 & -168.29 & 7.70 \\ -2.14 & 1.50 & 5.68 & 0.02 & 7.69 & -201.45 \end{bmatrix}, \quad B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -15.19 & -0.74 \\ 28.92 & -0.62 \\ -1.32 & 16.21 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad D=0, \qquad x=\begin{bmatrix} \theta_1 \\ \theta_2-\theta_1 \\ \theta_3-\theta_2 \\ \dot\theta_1 \\ \dot\theta_2-\dot\theta_1 \\ \dot\theta_3-\dot\theta_2 \end{bmatrix}, \quad u=\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \qquad y=\begin{bmatrix} \theta_1 \\ \theta_2-\theta_1 \\ \theta_3-\theta_2 \end{bmatrix}$$

Table 1. Vocabulary of the System Symbols.

Symbols	Vocabulary
l_n	length of n link
a_n	centre gravity of n link
m_n	mass of the n-link
i_n	Inertia is the moment of the n-link
θ_i	Measured angles between the n link and vertical line.

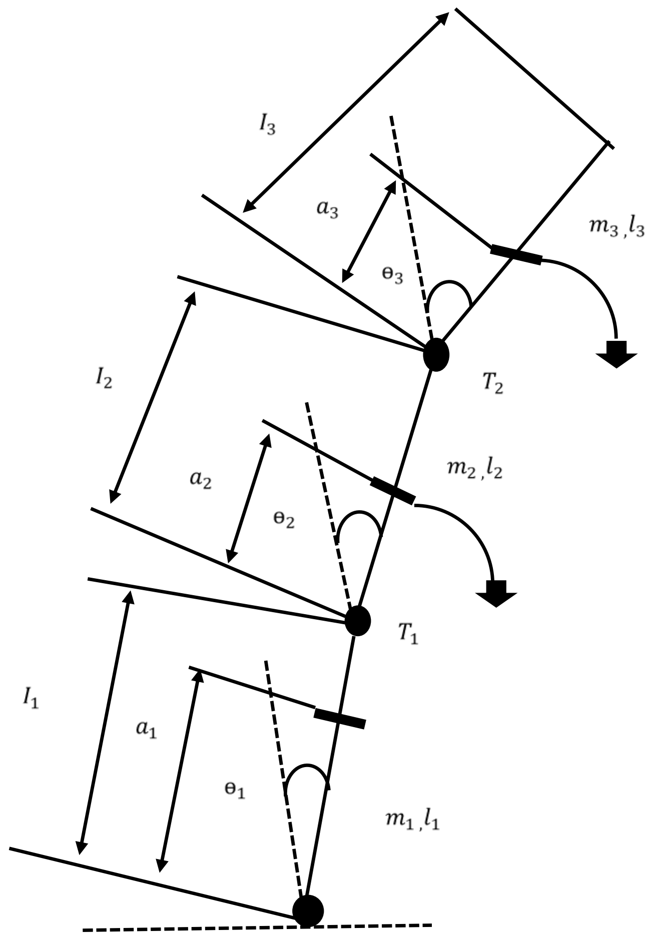

Figure.1 Schematic diagram of Robot Gymnastics

The eigenvalues are present through the continuous time model in a vertical position. There

[-166.8716 -203.1779 -5.4598 5.3762 0.1661 0.0270]

As noted above, the system is unstable because of the presence of three positive eigenvalues by estimating the continuous time model.

The voltage input (u_1, u_2) to the DC motors in the second and third joints, y is the relative angle in the three joints 1, 2 and 3 [1]. The discrete-time model obtained by Equations 1 and 2 at sampling time equal to 0.025 s in Equations 3 and 4.

$$x(k+1)= A_d x(k) + B_d u(k) \quad (3)$$

$$y(k)= C_x(k) \quad (4)$$

After examining the stability of the system and knowing the eigenvalue of the model in the vertical plane at the discrete time [0.0154 0.0062 1.1439 0.8724 1.0042 1.0007], after knowing the eigenvalues, it reports that the system is unstable by observing that there are three values outside the circle. So the gymnastics robot can be controlled and observed.

3. DLQR Controller

DLQR is one of the ideal control methods used to stabilise and balance linear systems around the equilibrium point by presenting the gain matrix. K and DLQR are used in systems that need high performance [1]. The three-link robot can be shown with the image of the steady-state linear differential equations of the system in Equations 1 and 2. The A is known as the system matrix, the B is known as the input matrix, the C is known as the output matrix, the D is known as the feed matrix, the x is known as the state vector, and the y is known as the output vector [8].

The performance function is defined as:

$$J= \sum_{k=0}^{\infty}(x_k^T \; Q \; x_k \; + \; u_k^T \; R \; u_k \;) \quad (5)$$

The state feedback law is presented as

$$u(k)= -K\,x(k) \quad (6)$$

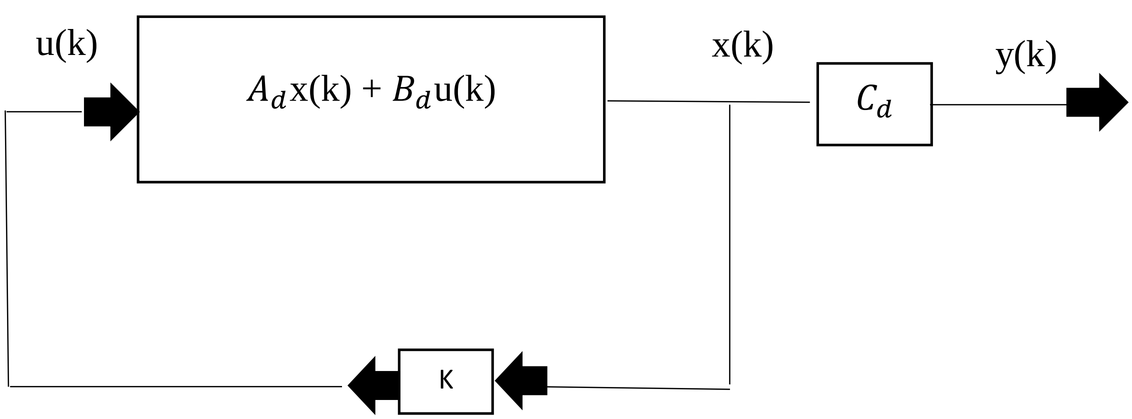

Figure 2. System Control Organization.

The main and traditional problem facing DLQR is how to find the Q and R matrices at a lower cost level. The weighting matrix Q and R found by trial and error in order to achieve stability requirements of the system and to improve the steady-state performance of the system, discrete time integration is used by introducing additional parameters of the system and putting them into the feedback law to improve the stability of the system. To obtain the integral of the error signal (e), one could use an integrator.

$$e(k)= y_r(k) - y(k) \tag{7}$$

Here

$$y_r(k)=\begin{bmatrix} y_{r_2} \\ y_{r_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} y(k) \tag{8}$$

The values of the second and third correlations are zeros to balance the system, and the symbol for the reference signal is as y_r. The expression for the discrete-time integral equations is as follows:

$$w(k+1)= w(k) + T_s\, e(k) \tag{9}$$

The cases of integrator are denoted by w. Fig.3 shows the block diagram of the closed-loop control system using an LQR controller.

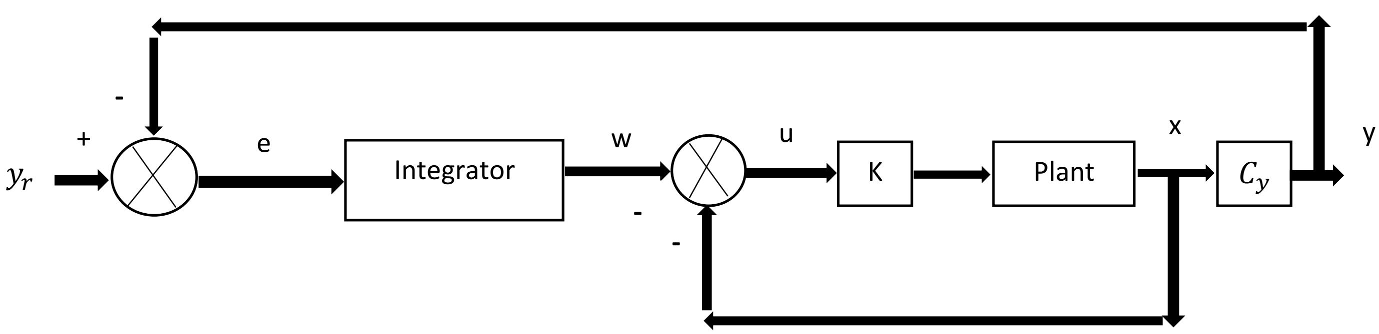

Figure 3. block diagram of close loop control system with using LQR controller.

The state feedback optimisation is used to design the controller for the first and second motors, so the feedback of the state and integral is shown as:

$$u(k)= -K \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \tag{10}$$

The weighting matrixes Q and R were selected as diagonal matrices:

$$Q = \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_6 \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$$

4. LQG Controller

LQG control is one of the most fundamental optimal control problems of control theory. Linear-quadratic-Gaussian (LQG) control is a modern state-space technique for designing optimal dynamic regulators. The LQG controller is a combination of the linear quadratic regulator (LQR) and the Kalman filter, which is a linear quadratic estimator (LQE). The method also holds good for linear time-invariant systems as well as linear time-variant ones. The state equation of the system in the presence of is as follows:

$$\dot{x} = Ax + Bu + w(t) \tag{11}$$

$$y = Cx + Du + v(t) \tag{12}$$

Where the A is known as the system matrix, the B is known as the input matrix, the C is known as the output matrix, the D is known as the feed matrix, the x is known as the state vector, the y is known as the output vector, the w (t) is the system noise, and v (t) is the measurement noise. For more important information about the equations of the LQG can be found in [20]. The block diagram of the LQG controller is shown in Fig.4. We use the Kalman filter to reduce the noise and combine it with the LQR controller:

LQG = LQR + Kalman Filter

$$\begin{cases} \dot{\hat{x}} = A\hat{x}(t) + Bu(t) + L\,(y(t) - \hat{y}(t) \\ \hat{y}(t) = C\,\hat{x}(t) \end{cases} \tag{13}$$

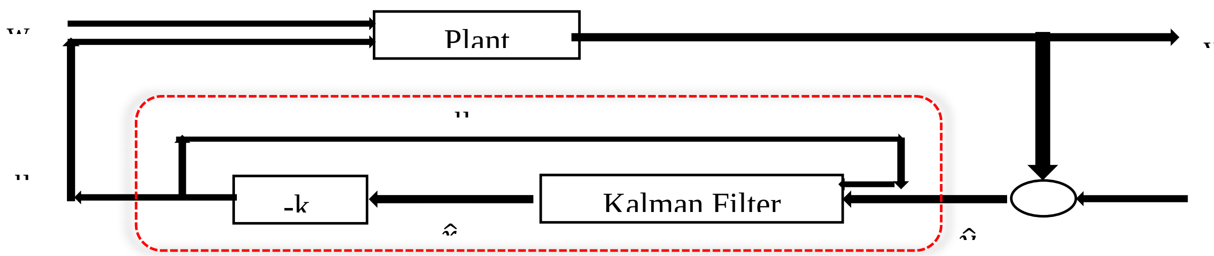

Figure 4. Block diagram of LQG controller.

5. The simulation results

In this paper, a model of a three-jointed gymnastics robot is designed, and DLQR and LQG controllers control its balance. This is in order to investigate the ability of the DLQR and LQG controllers to balance the gymnastic robot in the vertical position with the best values of the Q and R matrices. Good selections are found through extensive trials in Table 2:

Table 2. The Tuned Parameter of the DLQR and LQG Controller.

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	R_1	R_2
0.3571	35.8123	196.0674	0.1506	0.2637	0.8199	0.1476	0.0525

The values of the feedback matrix are found using the DLQR console, and the behaviour of the Gymnast robot in the balancing stage is simulated according to the values of the feedback matrix K.

$$\begin{bmatrix} -805.8957 & -343.7738 & -43.4913 & -147.4247 & -77.4386 & -9.6284 \\ -139.8597 & -62.1544 & 47.7933 & -25.5718 & -13.4333 & -1.3890 \end{bmatrix}$$

The eigenvalues of a closed loop of the system are as follows:

$$[0.0145 \quad 0.0057 \quad 0.9369 \quad 0.8894 \quad 0.8735 \quad 0.8734]$$

After knowing the eigenvalues, notice that the system is stable because the eigenvalues are all within the unit circle.

The values of the feedback matrix are found using the LQG console, and the behaviour of the Gymnast robot in the balancing stage is simulated according to the values of the feedback matrix K

$$\begin{bmatrix} -808.6979 & -345.0606 & -43.6418 & -147.4347 & -77.7070 & -9.6617 \\ -140.6609 & -62.5299 & 47.7494 & -25.7206 & -13.5117 & -1.3988 \end{bmatrix}$$

The eigenvalues of a closed loop of the system.

$$[0.0148 \quad 0.0059 \quad 0.9365 \quad 0.8894 \quad 0.8724 \quad 0.8744]$$

The steady-state simulation results are shown in Fig.5, 6, and 7. This shows the ability of the DLQR controller to balance the three-link gymnastics robot in the vertical position when the initial condition values are as follows $[\Theta_1, \Theta_2, \Theta_3]$ = [1 -0.9 1]. The DLQR controller response and the systems output specifications are presented in Table 3.

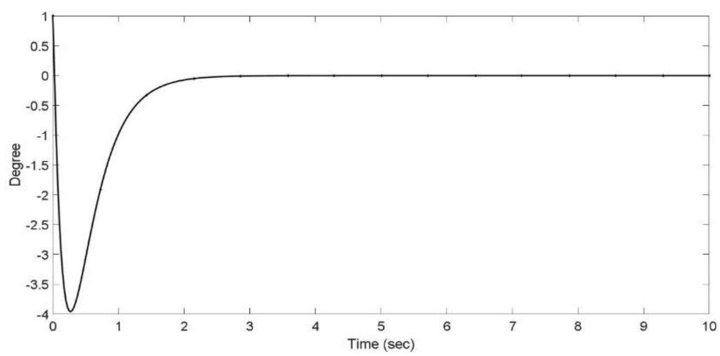

Figure 5. Simulated angular position Θ_1

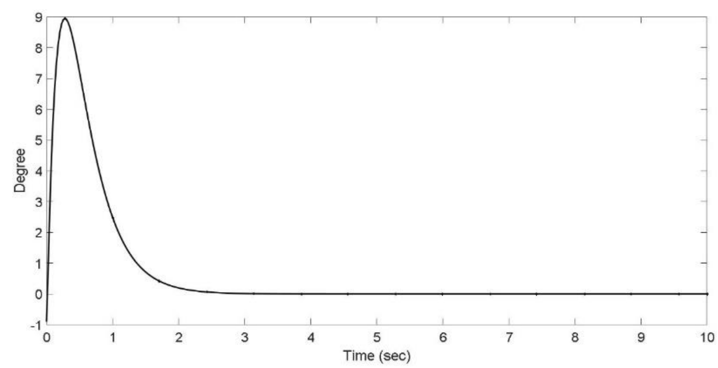

Figure 6. Simulated angular position Θ_2

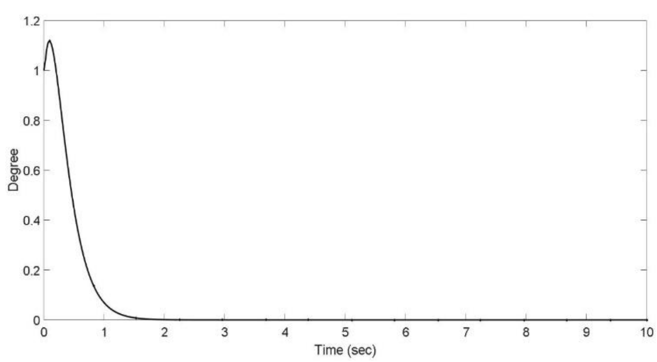

Figure 7. Simulated angular position Θ_3

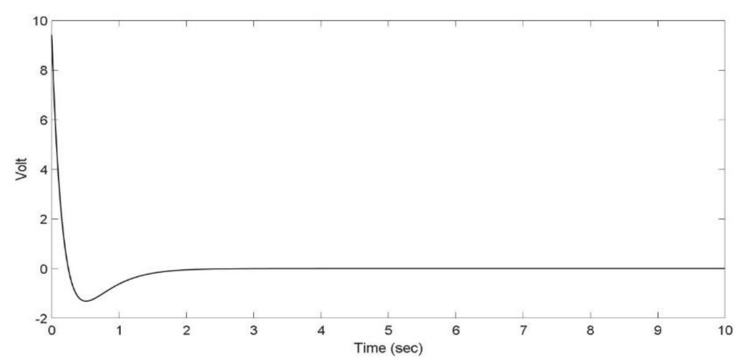

Figure 8. Simulated control action applied to motors1

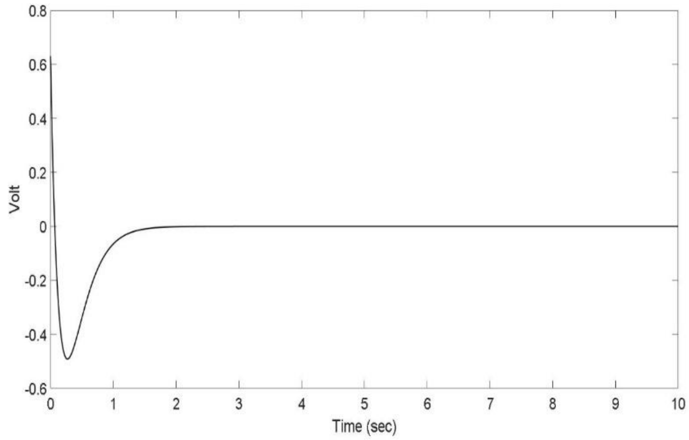

Figure 9. Simulated control action applied to motor 2.

Table 3. Control Performance of relative angular positions θ_1, θ_2, θ_3 of DLQR

	θ_1	θ_2	θ_3
Rise Time (tr)	0.05013s	0.07519s	0.02506s
Setting time (ts)	3.208s	3.233s	4.16s
Overshoot (os)	23.37%	176.8%	73.125%
Peak Time (tp)	3.958s	8.951s	1.118s

The steady-state simulation results are shown in Fig.10, 11, and 12. This shows the ability of the LQG controller to balance the three-link gymnastics robot in a vertical position. The LQG controller response and the systems output specifications are presented in Table 4.

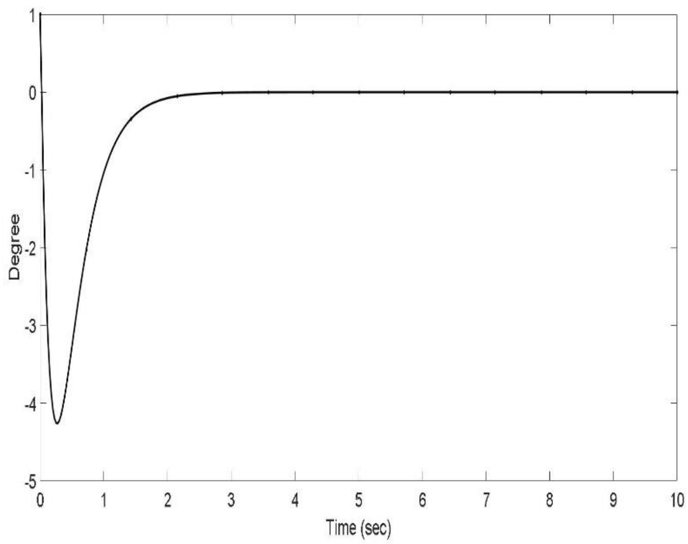

Figure 10. Simulated angular position Θ_1

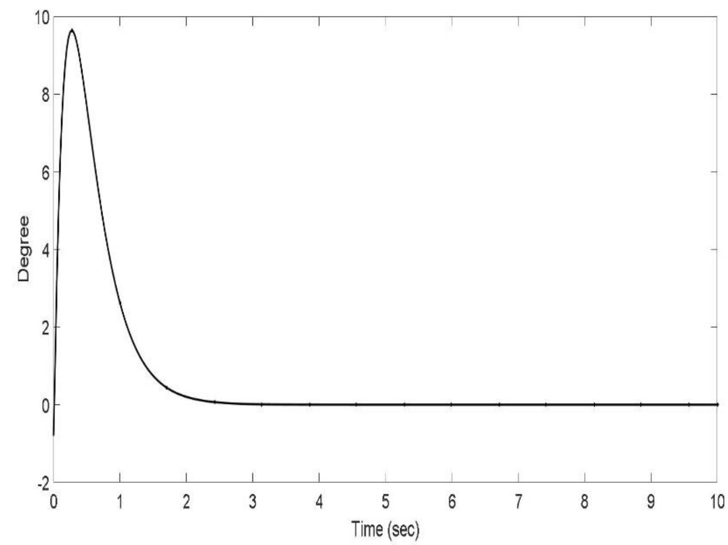

Figure 11. Simulated angular position Θ_2

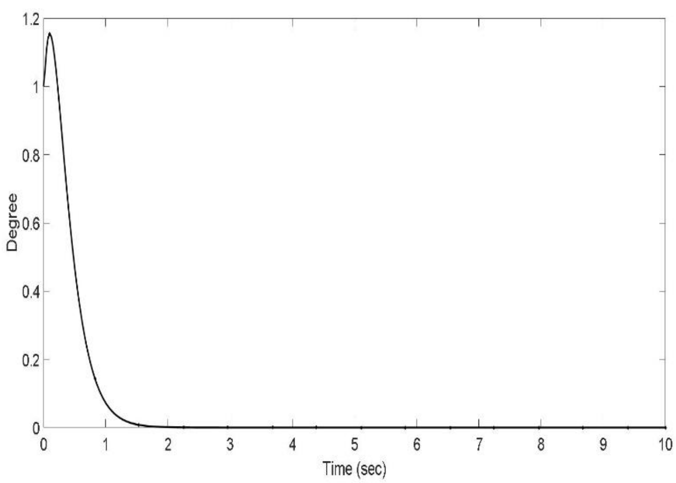

Figure 12. Simulated angular position Θ_3

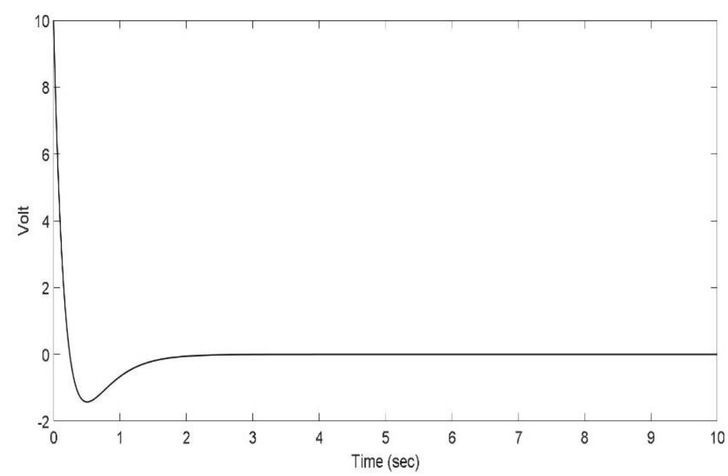

Figure 13. Simulated control action applied to motors1

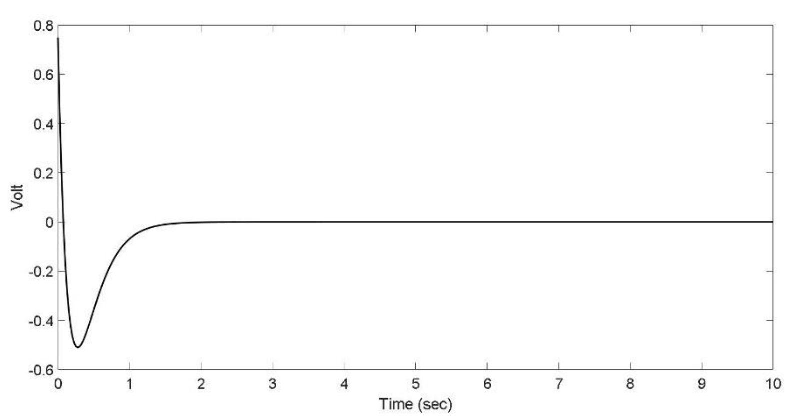

Figure 14. Simulated control action applied to motor 2.

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Table 4. Control Performance of relative angular positions θ_1, θ_2, θ_3 of LQG

	θ_1	θ_2	θ_3
Rise Time (tr)	0.05020s	0.07523s	0.02509s
Setting time (ts)	3.222s	3.237s	4.21s
Overshoot (os)	23.56%	177.6%	74.225%
Peak Time (tp)	4.458s	9.751s	1.124s

It noted that when using DLQR, the first motor in Fig.8 did not spend a large amount of energy to maintain the robot's performance satisfactorily and, therefore, did not reach saturation. The second motor shown in Figure 9 is still in the range, and when using the LQG controller, the first motor in Figure 13 spends a large amount of energy to maintain the robot's performance satisfactorily and, therefore, reach saturation. The second motor shown in Figure 14 is still in the range.

In comparing the simulation results with the results reported in [1], The system succeeded in obtaining a satisfactory response, as explained in Fig.5, 6,and 7, without spending a large of energy to maintain the performance of the gymnastics robot in a satisfactory manner when using DLQR.

5. Conclusions

In this paper, the problem of controlling the balance of a gymnastic robot has been identified. DLQR control and LQG, an optimal control technique to make the optimal control decisions, have been implemented to control the balance of the gymnastic robot, and the controllers showed the optimal values used to exceed the angle positions of the three joints through which the robot can be stabilised in the vertical position in an appropriate period. The DLQR has a better performance than the LQG controller because the first motor in the LQG controller spends a large amount of energy to maintain the robot's performance satisfactorily and, therefore, reach saturation. Future work involves using different optimisation algorithms to obtain the optimal values for the weight matrix Q and control matrix R.

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التحكم في موازنة روبوت الجمباز الآلي ثلاثي الوصلات من خلال استخدام تقنية التربيع الخطي للوقت المنفصل

الخلاصة: : يقترح هذا البحث كيفية موازنة البندول الثلاثي المقلوب غير الخطي، والذي يمثله روبوت جمبازي في المستوى الرأسي. هذا الروبوت يحاكي البهلوان البشري. يتم تثبيت هذا النوع من الروبوتات على محامل كروية ومتصلة بقضيب مرتفع يدور بحرية. وهي مصنوعة من ثلاث مفاصل. المفصل الأول خامل بدون مشغل (غير مزود بالطاقة). المفصل الثاني نشط. أما المفصل الثالث فهو نشط. يمثل المفصل الأول تحديًا كبيرًا في كيفية التحكم في توازن روبوت الجمباز ثلاثي الوصلات. ولتحقيق هذا الهدف، تم استخدام نظرية التحكم الخطية التربيعية المنتظمة (DLQR) والخطية التربيعية الغوسية (LQG) وتنفيذها على وحدة التحكم المنفصلة للوقت باستخدام النموذج الرياضي للروبوت. أظهرت نتائج المحاكاة الفرق بين أنظمة التحكم LQR وLQG لتحقيق التوازن والتحكم في نظام الروبوت الجمباز. أخيرًا، يعتبر LQR أفضل حيث تم الحصول على نتائج متفوقة للروابط الثلاثة (الأولى والثانية والثالثة) من حيث التجاوز (23.37%، 176.8%، 73.125%)، وزمن الصعود (0.05013 ثانية، 0.07519 ثانية، 0.02506 ثانية)، وضبط الوقت (3.208 ثانية، 3.233 ثانية، 4.16 ثانية) مقارنة بـ LQG.

الكلمات الدالة: روبوت الجمباز; البندول الثلاثي المقلوب; منظم تربيعي خطي منفصل (LQR) ;. السيطرة على التوازن ;التحكم الخطي التربيعي الغوسي (LQG).