A New Approach on Decision Making for Multi-Objectives Problems

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ABSTRACT

In many real life problems, one is usually confronted with several objectives, which are in mutual conflict. In this paper, multi-objective decision making is obtained, based on a mathematical model by interpolating multi-objective functions with weights functions coefficients, instead of constant coefficients. Some theories and computational results are presented to point out the efficiency of our model and the implemented approach.

Keywords : linear programming , optimization , non-linear programming , multi-objective function.

أسلوب جديد في صناعة القرار لمسائل ذات دوال متعددة الأهداف

الخلاصة

هنالك العديد من البحوث التي تتعامل مع أستنباط ما يسمى بمجموعة الحلول الكفوءة (Efficient Solutions) ،لمشاكل حقيقية ذات أهداف متعددة ومتضاربة(mutual conflict) بعضها مع البعض .ففي هذا البحث تم أستنباط قرار متعدد الأهداف متأسسا على بناء نموذج رياضي وذلك بتكوين دالة هدف واحدة من خلال ربط الدوال ذات أهداف مختلفة وتحديد معامل لكل دالة هدف (Objective function) بدالة وزنيه (Weight Function) بدلا من وزن ثابت (Constant Coefficient).كذلك تم عرض بعض النظريات المتعلقة بالموضوع ،وأعطاء بعض المؤشرات لكفاءة أداء النموذج الرياضي والاسلوب المتبع من خلال النتائج العملية.

الكلمات المرشدة : البرمجة الخطية ، الامثلية ، البرمجة اللاخطية ، دالة متعددة الاهداف.

INTRODUCTION

In many real life – problems ,one is usually confronted with several objectives, which are in mutual conflict .Multi – objective programming, and their optimization methods are difficult to use because human subjectivity in an integral part of them .We cannot just formulate a model and leave it to an optimization expert to calculate an optimal solution. Many algorithms appeared in

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the literature have been designed to obtain solutions to decision problems which must accomplish with multiple objectives .Each algorithm has its own claim of power .Decision maker may not be able to select an appropriate procedure to support the decision making (DM).The lack of guidelines in the selection of Multi – Objectives Decision Making (MODM) algorithm is partially reflected in the fact that not many. Empirical tests have been reported in the literature various models applying them in a real – world setting. Applications which use only illustrative data may also mislead the practitioner to believe that the model may be practical in a wider setting. From the managerial point of view, there is a need to investigate which method would be better in what situations. Many of the recent works deals with the determination of efficient and methodologies are frequently developed in the theoretical sense without addressing the practically of solutions set, and with their utilization in solving problems. Number of researches effort in an area known as "Efficient Solution" is constructed "local efficiency " sets. A motivated some works are discussed in the context of "proper efficiency" ,see [2] & [6].

In this paper, multi – objective decision making is obtained, based on a mathematical model by interpolating multi – objectives functions with weights variable coefficients. Some theories and experimental results are presented to point out how efficiency our model and procedure is good.

The Problem

Multi – objective optimization problem can be stated as follows :

$$\left.\begin{array}{c} \text{Find } X = (x_1 , x_2 , \ldots , x_n)^T \text{ ,which;} \\ \text{maximize ; } f_1(X) , f_2 (X), \ldots, f_K(X) , \\ \text{subject to:} \\ g_i(X) \leq 0, \quad for\ i = 1,2, \ldots, I \end{array}\right\} \quad (1)$$

Where any or all of the functions $f_k(X)\ (k = 1,2, \ldots, K)$ & $g_i(X)\ (i = 1,2, \ldots, I)$ may be nonlinear.

Definition (1): A point ($X \in S$) is said to be efficient in S with respect to

$$f_k(X) , \ if\ \nexists \ \ Y \in S \ with \ \ f_k(Y) \geq f_k(X) .$$

The set of all such points ($X \in S$) is denoted by ;

$$\mathcal{E}(X, f_k) = \{ \ X \in S : \nexists \ \ Y \in S \ \ with \ \ f_k(Y) \geq \ f_k(X) \ \}$$

Model formulation

Multi – objectives decision making (MODM) procedures seek to obtain the "most preferred" of the feasible solutions across all the objectives which the decision maker wishes to optimize. Usually ,no solution can be found which allows concurrent optimization of all objectives ,because of the conflicting nature of the individual objective .For instance ,an objective related to reduction the manufacturing costs may conflict to an objective of maintain full employment.

Nearly ,all the literatures { see [5] ,[9] , [7] },who proposed the properties of different types of solution sets, with respect to linear combination of the original objective functions ,in which the coefficients are constants, denoted by (t_k) with

($0 \le t_k < 1$) and ($\sum_{k=1}^{K} t_k = 1$).

In this paper, in order to construct an efficient solution set, we are finding more suitable values of these coefficients, by considering K optimum solutions points as the base points in constructing new weight coefficients as variables functions, denoted by $t_k(X)$,defined as

$$t_k(X) = \prod_{k \neq l} \left(\frac{X - X^{k*}}{X^{l*} - X^{k*}}\right) \qquad (2)$$

Therefore, a new multi-objective functions problem can be formulated as following:

$$F(X) = \sum_{k=1}^{K} t_k(X) f_k(X)$$

subject to:

$$\left.\begin{array}{l} g_i(X) \le 0, \quad for\ i - 1,2,\dots,I \\ t_k^l \le t_k(x) \le t_k^u \end{array}\right\} \qquad (3)$$

where (t_k^l & t_k^u) are the given lower and upper bounds of the weight function for{ $f_k(X)$ },and (X^{k*}) are the optimum points of { $f_k(X)$ }, which is unique vector and in practical problems, such a vector is always unfeasible (otherwise there would be no conflicts), but it is conceivable that the nearest feasible solution could be an acceptable compromise for the decision maker.

Let $S' = \{X \in R^n : l_k < X^k < u_k\}$, where

$l_k = \min_k \{X^k\}$, & $u_k = \max_k \{X^k\}$,then we would state the following definition;

Definition (2): A point ($X \in S'$) is said to be global efficient in (S') with respect to $F(X)$,if $\nexists$ $Y \in S'$,with $F(Y) \ge F(X)$,

$$G(S', F) = \{X \in S' : \nexists Y \in S', \text{with } F(Y) \ge F(X)\}$$

Therefore, the problem (1) can be reformulated as following;

Minimize $d_\alpha = \left(\sum_{k=1}^{K} \left| F(X^e) - f_k(X^{k*}) \right|^{\alpha} \right)^{\frac{1}{\alpha}}$

Subject to:

$$\left.\begin{array}{l} g_i(X^e) \le 0, \quad i = 1,\dots,I. \\ t_k^l \le t_k(X^e) \le t_k^u, \quad k = 1,\dots,K \\ \sum_{k=1}^{K} t_k(X^e) = 1 \end{array}\right\} \qquad (4)$$

Where, $1 \le \alpha \le \infty$, designates the norm in the objective space, X^e is the efficient solution of F(X) that minimized d_α, and X^{k*} is the optimum solution of the problem:

$$\left.\begin{array}{ll} \max f_k(X), & k=1,\dots,K \\ \textit{subject to}: & \\ g_i(X) \le 0, i = 1,\dots,l. & \end{array}\right\} \quad (5)$$

Theorem: $\mathcal{E}(S, f_k) \subseteq G(S', F)$

Proof: / Let $X \in G(S', F)$, that means $\exists Y \in S',\ s.t.\ F(Y) \ge F(X)$

And since { $\exists Y \in S$/} it means that we have either { $Y \in S'/S$ } (i,e $Y \notin S$) which is of no interest ,or $(Y \in S)$ $with$ $f_k(Y) \ge f_k(X)$,and that means $X \notin \mathcal{E}(S, f_k)$.

Corollary: $G(S', F)$ be independent of the ordering of the components of (F).

The Approach and Computational Results

The optimum value for each objective function $\{f_k(X^{k*})\}$ are calculated,then we solve the problem (5). The percentage of { d_α } (for certain value of α) is calculated. This can be demonstrated in the following steps:

1)_ Solve problem (5) for all k, k=1,…,K
2)_ Formulate F(X) from (2) and (3),
3)_ Solve problem (4), to find X^e.

In order to point out the efficiency of our model ,and the proposed approach man experiment is designed to include the following factors:

a - the number of objective $f_k(X)$,

b - the number of constrains $g_i(X) \le 0$, i=1,…,m

c - the number of decision variables $x_n(n = 1,2, \dots N)$

The problem sets generated for this research contain hypothetical situations with the restriction number of objectives ,constraints and decision variables. The result is presented in the table (1), which show as that an acceptable convergence on { X^e} is obtained , since % d_∞ represent the percentage variance between x^e and each optimum solution for $f_k(x)$

Discussion and Conclusions

Although there have been several decades of research in MODM, the reported successful applications are far less than what had been promised. Recent developments in computer technology bring new promises to the applications of MODM models. Specifically, the use of computer graphics may greatly facilitate the process of interactive decision making. As we mentioned before, there is a lack of guidelines in helping users to select the appropriate MODM algorithm .We found that the use of our model and the procedure is preferred when the accuracy of the solution is the critical factor in selecting an interactive multi – objectives decision making .In any case, the performance of our procedure is quite good.

We are suggested that, further development area is in creating software, which permit the application of such technique in stand – alone decision support system.

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Table (1) Computational Efficiency

No. of obj. functs.(j)	Problem size (N x m)	% d_α
2	8 x 12	0.08
3	13 x 17	0.10
4	14 x 19	0.12
5	18 x 25	0.15
6	22 x 30	0.19