

Principal Component Analysis Based Wavelet Transform

Dr. Hana'a M. Salman

Control Engineering & Information System Department, University of Technology/Baghdad

Email: salmanhana2007@yahoo.com

Received on: 2/11/2011 & Accepted on: 5/4/2012

ABSTRACT

The principal component analysis (PCA) is a valuable statistical means, implemented in time domain that has found application in many fields such as face recognition and image compression, and is a common technique for finding patterns in data of high dimension. This paper investigates the ability to implement PCA in frequency domain, by using the wavelet transform (WT), and evaluate its effectiveness based on face recognition as a means to find patterns in data. The basic idea of frequency domain implementation of the PCA refers to the correlation implementation using wavelet transform.

The Min-max is invoked to increase wavelet based eigenface robustness to variations in facial geometry and illumination. Two face images are contrast in terms of their correlation distance. A threshold is used to restrict the impostor face image from being identified. Experimental results point up the effectiveness of a new method in either using varying (noisy images, unknown images, face expressions, illumine, and scales).

Keywords: PCA, Wavelet Transform, Eigenface, Correlation, Min-max.

وسيلة التحليل الاولى على اساس التحويل الويفي

الخلاصة

وسيلة التحليل الاولى هي احدى وسائل الاحصائية القيمة، والتي وجدت العديد من التطبيقات في فروع معرفة متعددة مثل تميز الوجه وضغط الصور، بالاضافة الى كونها الوسيلة المستعملة لايجاد انماط في البيانات ذات الابعاد المتعددة. في هذا البحث، تمت دراسة امكانية بناء وسيلة التحليل الاولى في الوقت الترددي باستعمال التحويل الويفي وتقييم فعاليتها كسمات الوجه الطيفية لتمييز الاشخاص. التحويل من تمثيل محددات الوجه في فضاء الحالي الى محددات الوجه الطيفية في الفضاء الترددي، تم من خلال استخدام الارتباط المبني باستعمال تحويل الويفي السريع. وجعل قيمها سوية باستخدام طريقة الاصغر-الاكبر، لتزيد من كفاءة محددة الوجه الطيفية اتجاه التنوع في هندسة الوجة والاضاءة. اي صورتان للوجه، تعتبران مختلفتان بالاعتماد على المسافة الارتباط. تم استخدام حد لمنع الصور الدخيلة من ان تميز داخل المنظومة. نتائج الاختبارات عبرت عن مدى فعالية الطريقة الجديدة المطبقة في حالة الانواع المختلفة لـ (انواع الضوضاء، صور الوجه الدخيلة، تعابير الوجه، الاضاءة، والاحجام).

INTRODUCTION

PCA is a statistical method under the broad title of factor analysis that acts as a linear map from data space to feature space with minimum mean square error. It is eminent for its aptitude in feature extraction/selection in pattern recognition, noise reduction in signal processing and de-correlation. Based on the covariance matrix of (training) samples, eigenvectors of the covariance matrix, which are orthogonal, are found and sorted in descending order according to their importance, i.e. the magnitude of corresponding eigenvalues [1].

Transforming from original space, data can be effectively represented by a subspace of fewer dimensions, i.e. PCA, with the essential information retained such that mean-squared error is optimized and is equal to the sum of variances of truncated elements [1].

Eigenface is a basic method in signal process, being significant in the fields of data fusion, automatic control, signal inspection, system recognition, image compression and biomedicine. The rapid development of wavelet analysis technology, leads to broader space application of eigenface technology.

Many research on face identifications based on PCA, from these:

Mao uses PCA and Nearest Neighbor Classification (NN) with 85% recognition rate or Minimum Distance Classification (MD) with 77% recognition rate. Sim, uses PCA and Convolution Network (CN) with 78% recognition rate, while Lawrence, and find it 83%. Su uses PCA, FFT, and Linear Discriminate Analysis (LDA), and Radial Basis Function Network (RBFN) with 97% recognition rate. Zhao uses PCA, and Linear Discriminate Anal (LDA), with recognition rate of 92%. Phrasal uses PCA with recognition rate of 95%. Huang uses PCA, and Back propagation (BP), with recognition rate of 92% [2]. Hanaa M. Salman implements the PCA in frequency domain using FFT as a feature vector, with identification rate of 100% [3].

The objective of this paper is to present the eigenface implementation using wavelet Transform, as proposed means for human identification. A threshold is used to restrict the impostor face image from being identified. The experimental results point up the effectiveness of a new method in either of using different (noisy images, unknown images, face expressions, illumines, and scales) .

In the following subsections, background knowledge is presented in section 2. Secondly, the proposed eigenface based wavelet transform is in Sections 3. In Section 4, Experiment results, followed by a conclusions in Section 5.

BACKGROUND

Discrete Wavelet Transform

For continual signal $f(t) \in L^2(R)$, the wavelet transforms (namely the decomposition formula of signals) and wavelet inversion (namely the reconstruction of signals) of signal are [4]:

$$W_{\Psi}(f)(a,b) = \frac{1}{\sqrt{a}} \int f(t) \overline{\Psi\left(\frac{t-b}{a}\right)} dt, \quad \dots\dots (1)$$

$$f(t) = \frac{1}{C_v} \int_0^{+\infty} \frac{da}{a^2} \int_{-\infty}^{+\infty} W_{\Psi}(f)(a,b) \frac{1}{\sqrt{a}} \overline{f\left(\frac{t-b}{a}\right)} dt, \quad \dots\dots (2)$$

While a , is a contraction factor, b , is a translation factor, $\frac{1}{\sqrt{a}}$ is the normalized coefficient of the wavelet base, Ψ and f are dual wavelet base, usually $a > 0$. When applied in practice, a , and b is separated as $\{a_m, b_n | m, n \in \mathbb{Z}\}$ and discrete wavelet transform and wavelet inversion are [4]:

$$W_{\Psi}(f)(a_m, b_n) = \frac{1}{\sqrt{a_m}} \int f(t) \overline{\Psi\left(\frac{t-b_n}{a_m}\right)} dt, \quad \dots\dots (3)$$

$$f(t) = \sum_{m,n \in \mathbb{Z}} W_{\Psi}(f)(a_m, b_n) \overline{f(a_m, b_n, t)} \quad \dots\dots (4)$$

Correlation Implementation Using WT

Let X , and Y be data sets such that, correlations based WT is defined as: take WT of X , and WT of Y , multiply one resulting transform by the complex conjugate of the other, and inverse transform the result product such as [4]:

$$\text{Corr}(X, Y) = \text{IWT}(\text{WT}(X) * \text{WT}(Y)), \quad \dots\dots (5)$$

Eigenface

Eigenface or the Principle Component Analysis (PCA) is a statistical measurement method, which operates in the linear domain and can be used to reduce the dimensionality of an image. A face image can be viewed as vectors and represented in matrix form. This method can be described as follows [5]:

Let I denote a $n_1 \times n_2$ gray scale image. Then represent it by means of a vector, $x = n_1 \times n_2$ which can be seen as a point in R^n . When performing PCA on these vectors, the eigenvectors obtained from the sample covariance matrix are called *Eigenfaces*. Here are the steps to computing these Eigenfaces [5]:

1. Obtain face images I_1, I_2, \dots, I_M (training faces).
2. Represent every image I_i as a vector x_i .

$$3. \text{ Compute the average face } \mathbf{y} = \frac{1}{M} \sum_{i=1}^M \mathbf{x}_i \quad \dots (6)$$

$$4. \text{ Subtract the mean face } \mathbf{j}_i = \mathbf{x}_i - \mathbf{y}_i, \quad \dots (7)$$

$$5. \text{ Compute the covariance matrix } \mathbf{C} = \frac{1}{M} \sum_{n=1}^M \mathbf{j}_n \mathbf{j}_n^T = \mathbf{A} \mathbf{A}^T \quad \dots (8)$$

6. Compute the eigenvectors u_i of $\mathbf{A} \mathbf{A}^T$:

6.1 Consider matrix $\mathbf{A} \mathbf{A}^T$ as an $M \times M$ matrix.

6.2 Compute the eigenvectors v_i of $\mathbf{A} \mathbf{A}^T$ such that

$$\mathbf{A}^T \mathbf{A} \mathbf{v}_i \Rightarrow \mathbf{m}_i \mathbf{v}_i \Rightarrow \mathbf{A} \mathbf{A}^T \mathbf{A} \mathbf{v}_i \Rightarrow \mathbf{m}_i \mathbf{A} \mathbf{v}_i \Rightarrow \mathbf{C} \mathbf{u}_i = \mathbf{m}_i \mathbf{u}_i \text{ Where } \mathbf{m}_i = \mathbf{A} \mathbf{v}_i, \dots (9)$$

6.3 Compute the M best eigenvectors of $\mathbf{A} \mathbf{A}^T$:

$$\mathbf{m}_i = \mathbf{A} \mathbf{v}_i \quad \dots (10)$$

7. Keep only K eigenvectors.

Normalization

A feature is normalized by scaling its values so that they fall within a small-specified range, such as 0.0 to 1.0. For distanced-based methods, normalization helps prevent features with initially large ranges from outweighing features with initially smaller ranges. Min-max normalization performs a linear transformation on the original data. Suppose that \min_a and \max_a are the minimum and the maximum values for feature A . Min-max normalization maps a value v of A to v' in the range $[\min_{new}, \max_{new}]$ by computing:

$$v' = ((v - \min_a) / (\max_a - \min_a)) \times (\max_{new} - \min_{new}) + \min_{new}, \dots (11)$$

Eigenface Matching

Let X and Y are two spectral eigenface feature vectors where, $x_i \in X$, $y_i \in Y$, $i=1, \dots, n$. to calculate the degree of association, a correlation distance is defined as [6]:

$$R = 1 - r, \quad \dots (12)$$

Where r is the linear correlation coefficient which is given by the formula [6]:

$$r(X, Y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad \dots (13)$$

Where \bar{x} , is the mean of the vector X is, \bar{y} is the mean of the vector Y .

The correlation distance determines the genuine or forged query sample; it is easy to verify the input pattern by a pre-defined threshold value T . If the value R is smaller than threshold T , then the owner of query sample is claimed to be individual X . Otherwise, the query sample is classified as a forged pattern.

Threshold Selection

In any face identification system, it is essential to pick a suitable threshold (T), for good performer's results. To this end, an approach based on intra-class and inter-class information collected from the Enrolment database. The intra-class (D) measures the distances between images of the same individual, therefore it gives an indication of how similar the images of the same individual are. The intra- distance is defined as:

$$d_{ik}^{ik'} = (1 - r(\Omega_{ik}, \Omega_{ik'})), \text{ Where } i \in I, k' \in K, \text{ and } K \neq K' \dots(14)$$

The inter-class (P): The distances between the images of an individual are measured against the images of other individuals in the Enrolment database; hence it gives an indication on how different each image of an individual is when compared to images of other individuals in the Enrolment database. The inter-distance is defined as:

$$p_{ik}^{ik'} = (1 - r(\Omega_{ik}, \Omega_{jl})), \text{ Where } j \in I, i \neq j, \text{ and } l, k \in K \dots(15)$$

A threshold (T) is then calculated from intra-class and inter-class information as described in [7]. The estimation of threshold (T) depends mainly on the number of images per individual in Enrolment, therefore as in [7], every individual should have at least 4 images for Enrolment database. The algorithm for the maximum intra-class, and the minimum inter-class calculation is described as:

Input: $I = \{1, \dots, I'\}$, where I' = number of individuals, $K = \{1, \dots, K'\}$, where K' = number of images per individual, F_{ik} = normalized Face image feature vector, $i \in I$, and $k \in K$

Ω = feature matrices of training images for each image

Output: D_{max} = Maximum intra-class, and P_{min} = minimum inter-class

Process:

Step1: Do while the ($F_{ik} \neq 0$)

Step2: Compute the intra distances, and the inter distances

Step3: End of while

Step4: Store the intra-class (D), and the inter-class (P) in ascending order

Step5: Compute D_{max} , and P_{min} .

Step6: End.

Eigen face Identification Process

An unknown query face image can be represented as a linear combination of the best K Eigenfaces of the obtaining eigenvectors for a given dataset. In face identification, the eigenfaces are used once again in order to compute a distance from the query image in the face space, as presented in Fig (1). The algorithm is summarized below:

Input: an unknown image vector x

Output: matching result

Process:

$$\text{Step1. Compute } \mathbf{j} = \mathbf{x} - \mathbf{y}, \quad \dots\dots(16)$$

$$\text{Step2. Compute } \mathbf{j}^{\wedge} = \sum_{i=1}^k w_i \mathbf{u}_i \text{ where } w_i = \mathbf{u}_i^T \mathbf{j}, \quad \dots\dots(17)$$

$$\text{Step3. Compute } D_e = \|\mathbf{j} - \mathbf{j}^{\wedge}\|, \quad \dots\dots(18)$$

Step4. If $D_e < T$, then x is a face, and D_e is the distance from face space

THE PROPOSED WAVELET BASED EIGENFACE FEATURE

The proposed idea of applying the WT in the implementation for Eigenface is by use WT in the implementation of the correlation, as an alternative of conventional ideas of converting the intensity of the image face data into the spectral domain, followed by applying the Eigenface. The proposed idea is named as the wavelet based eigenface feature.

The correlations can be computed by using the WT as follows: WT the two data sets, multiply one resulting transform by the complex conjugate of the other, and inverse transform the product.

Hence, by evaluating this cross correlation, a speed up ratio can be obtained comparable to conventional Eigenface. Let I denote a $n_1 \times n_2$ gray scale image. We then can represent it by means of a vector, $x = n_1 \times n_2$, which can be seen as a point in R^n . Here are the steps to computing these Eigenfaces:

1. Obtain face images I_1, I_2, \dots, I_M (training faces).

2. Represent every image I_i as a vector x_i .

3. Compute the average face $\mathbf{y} = \frac{1}{M} \sum_{i=1}^M x_i$

4. Subtract the mean face $\mathbf{j}_i = x_i - \mathbf{y}$

5. Compute the covariance matrix $\mathbf{C} = IWT(WT(\mathbf{j})WT(\mathbf{j}^T)) = \mathbf{A}\mathbf{A}^T$

6. Compute the eigenvectors u_i of $\mathbf{A}\mathbf{A}^T$:

6.1 Consider matrix $\mathbf{A}\mathbf{A}^T$ as a $M \times M$ matrix.

6.2 Compute the eigenvectors v_i of $\mathbf{A}\mathbf{A}^T$ such that:

$$\mathbf{A}^T \mathbf{A} v_i \Rightarrow m_i v_i \Rightarrow \mathbf{A} \mathbf{A}^T v_i = m_i v_i \Rightarrow C v_i = m_i v_i$$

$$\text{Where } m_i = \mathbf{A} v_i$$

6.3 Compute the M best eigenvectors of $\mathbf{A}\mathbf{A}^T$: $m_i = \mathbf{A} v_i$

7. Keep only K eigenvectors.

EXPERIMENT RESULTS

The proposed algorithm is tested using, a set of faces, full between April 1992 and April 1994 at the Olivetti Research Laboratory (ORL) in Cambridge, UK. The set is consisting of 10 different images of 40 distinct individuals. The images were taken at

different times, varying illuminance, facial expressions “open/closed eyes, smiling/non-smiling” and facial details “glasses/no-glasses, a sample of the used images is in Fig (2). All the images are taken against a dark homogeneous background and the individuals are in up-right, frontal position “with acceptance for some side movement”. The files are in PGM format, with a size of each image is 92×112 , 8-bit grey levels. The Enrolment Database consists of 160 normalized spectral eigenfaces, 4 for each individual. A threshold T is calculated for the enrolment database, to prevent unknown individual from being identified and found to be equal to 10.50×10^7 .

The rest 360 normalized wavelet based eigenface, 6 for each individual, is used in the identification phase; the results are depicted in Table (1). The result of any identification goes in one of four situations. A correct identification: identify an individual already registered in the enrollment database. A correct refusal: refuse an individual not registered in the enrolment database. A wrong acceptance: accept an imposter not registered in the enrolment database, or to identify imposter as someone in the database incorrectly. Wrong refusal: refuse a genuine user registered in the enrolment database, or to identify the genuine users as unknown incorrectly.

The performance of the wavelet based eigenface approach is study under different conditions either from the 360 rest images or out of it, as depicted in bellow subsections, and is found it is a good method for Human face identification.

Identification With Different Head Tilts: The robustness of the wavelet based eigenface identification algorithm to head tilt is studied, with different head tilts either left-oriented or right-oriented, top-orientation, and down-orientation as shown in Fig (3(a-e)).

Identification with Varying Luminance: The robustness of the wavelet based eigenface identification algorithm to head tilt is studied, as depicted in Fig (4(a-c)) with face images moved by 45 degrees and the other with light moved by 90 degrees.

Identification with Varying Head Scale: The robustness of the wavelet based eigenface identification algorithm studied, with a medium head scale and the other with a small one, as shown in Fig (5(a-c)).

Identification With Different Face Expression: The robustness of the wavelet based eigenface identification algorithm is studied, with different head face expression either smiling, eye move, with glasses, and smiling with glasses as depicted in Fig (6(a-e)).

Identification with Different Noise Type and Level: The robustness of the wavelet based eigenface identification algorithm over noise image, with different noise types as shown in Fig (7(a-c)). Table (2) represents the parameters of the used varying types of noise. Table (3) represents the result of the identification.

Identification with Unknown Face Images: The robustness of the wavelet based eigenface identification algorithm is studied, with different unknown face images for

boys and girls with glass and without, with varying faces expressions, as shown in Fig(8(a-d)). The result of the identification is depicted in Table (4).

CONCLUSIONS

This paper investigates the ability of implementing the eigenfaces in the frequency domain by using the WT as a means for Human identification. The ORL face images are used to evaluate the performance of new proposed algorithm, which is implemented by using Matlab 7 as programming language. The proposed eigenface based WT system is investigated and it found that:

1. The used of min-max approach as a normalization method for the result feature vector is to remove the outlier in the enrolment database.
2. The benefit of using a threshold is to prevent the impostor from being identified.
3. One of the major advantages of wavelet based eigenfaces recognition approach is the ease of implementation. Furthermore, no knowledge of geometry or specific feature of the face is required, and only a small amount of work is needed regarding preprocessing for any type of face images.
4. The robustness of the wavelet based eigenfaces identification algorithm to head tilt is studied, using images depicted in Figure (3(a-e)).
5. The robustness of the wavelet based eigenfaces identification algorithm to varying luminance is studied using images as depicted in Figure(4(a-c)).
6. The robustness of the wavelet based eigenfaces identification algorithm studied with varying head scale as depicted in Figure (5(a-c)).
7. The robustness of the wavelet based eigenfaces identification algorithm studied with different face expression as depicted in Figure (6(a-e)).
8. A different noise types as in depicted in Table (3) are used to investigate the application of wavelet based eigenface in different noise levels, and the method gives a success of wavelet based eigenface in different noise levels, and the method gives a success identification of 100%. Table (4) depicts the effectiveness of the proposed method.
9. The robustness of the wavelet based eigenfaces identification algorithm is studied, with different unknown face images for boys and girls with glass and without , with varying faces expressions as in Figure(8(a-d)), with identification of 100% as in Table(5).

REFERENCES

- [1] CHEUNG, King Hong; KONG, Wai Kin; YOU, Jane; ZHANG, David, "On Effective Palmprint Retrieval for Personal Identification", Department of Computing, the Hong Kong University, 2003.
- [2] Tsuen Lee, "Hybrid Approaches for Face Recognition Using Principal Component Analysis", Thesis Proposal For the degree of Master of Science in Computer Science, At Southern Connecticut State University, January 2004.

- [3] Dr. Hana'a M. Salman, "Spectral Eigenface Representation for Human Identification", Department of Computer Science & Information System, University of Technology, Baghdad, Iraq, 2008-03-17
- [4] Zuojin Li, Weiren Shi, Kai Wang, Xiaowei Sun, Zhi Zhong, "New Correlation Analysis Method for Nonstationary Signal", ZUOJIN LI, College of Automation, Chongqing University, ISSN: 1790-0832 1677 Issue 12, Volume 5, December 2008.
- [5] Khalifa, O. O. *, M. Yusuf, and S. Gagula , "Face Recognition Using PCA Wavelet ,
- [6] Tee C., Andrew T., Michael G., and David N., "Palmprint Recognition with PCA and ICA", Faculty of Information Sciences and Technology, Multimedia University, Melaka, Malaysia, Palmerston North, November 2003 /22/7
- [7] Ronny T., Wanquan L.SenjianAn and Svetha V., "Face Recognition based on Ordinal Correlation", Auditing and Performance Evaluation", www.computing.edu.au/~svetha/.../tjahyadi_lu_an_venkatesh_ijisa06.pdf.
- [8] AT & T. (2002), the Database of Faces, AT & T Laboratories Cambridge. Retrieved [ftp://whitechapel.media.mit.edu/pub/images/](http://whitechapel.media.mit.edu/pub/images/)



Figure (1) Sample Eigenfaces for a given database [8].



Figure (2) A sample of the ORL Face Images



Figure (3) (a-e): Face images in varying head tilts.

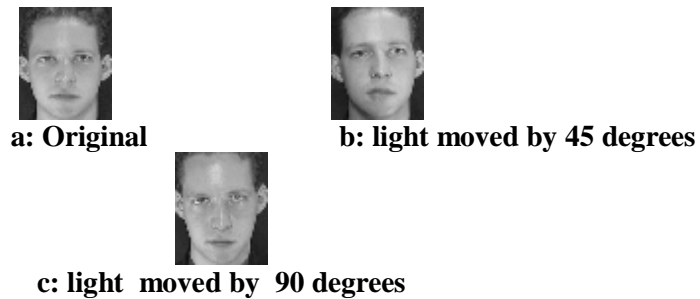
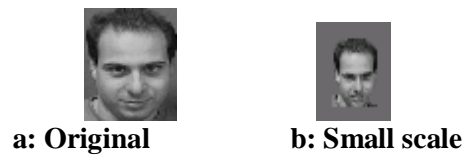


Figure (4) (a-c): Face Image in varying luminance.



c: Big scale

Figure (5) (a-c): A face image in varying scales.

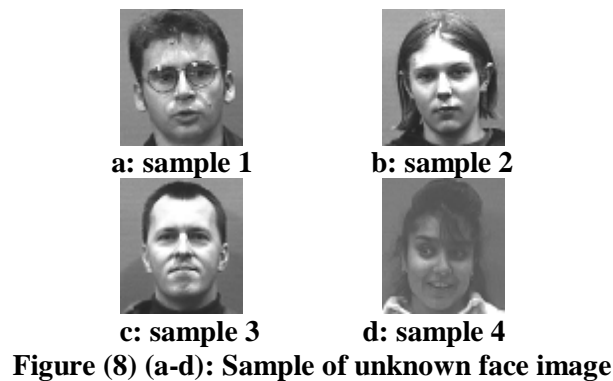
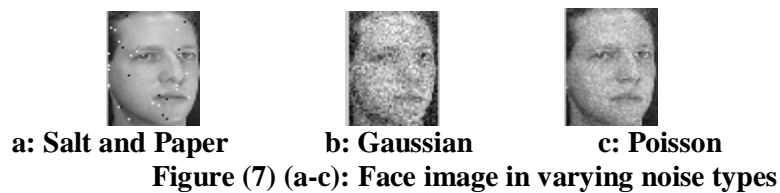
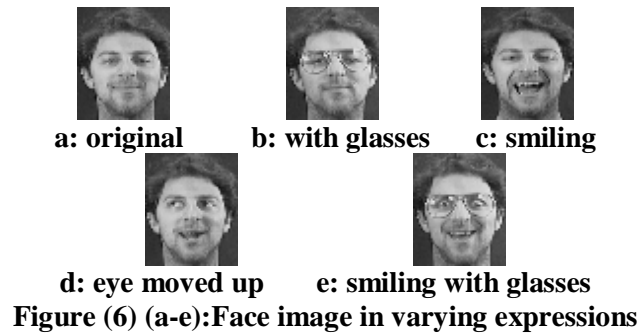


Table (1) The results of identification

No. of test images	No. Of Correct identifications	No. Of Correct Refuse identifications	No. Of Rung Acceptance Identifications	No. of Range Refuse identifications
360	360	0	0	0

Table (2) Noise types parameters

Slot and Paper	Gaussian	Poisson
Noise Density = 0.01	Mean = 0, Variance = 0.01	-

Table (3) The result of identification

No. Of test images	No. Of Correct identifications	No. Of Correct Refuse identifications	No. Of Rung Acceptance identifications	No. of Range Refuse identifications
30	30	0	0	0

Table (4): Identification of unknown individuals

No. Of test images	No. of Correct identifications	No. of Correct Refuse identifications	No. Of Rung Acceptance identifications	No. of Rung Refuse Identifications
30	0	30	0	0

Principal Component Analysis (PCA)
 Nearest Neighbor Classification (NN) Minimum Distance Classification (MD)
 Convolution Network (CN)
 Fast Fourier Transform (FFT),
 Linear Discriminate Analysis (LDA),
 Radial Basis Function Network (RBFN) Linear Discriminate Analysis (LDA),
 Back propagation (BP)