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Product of Two Fuzzy Normed Spaces and its Completion

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ABSTRACT
The aim of this paper is to prove that the Cartesian product of two complete fuzzy normed space is again a complete fuzzy normed space. Also to prove that the Cartesian product of two complete fuzzy inner product spaces is a complete fuzzy inner product space.

Keywords: fuzzy normed spaces, fuzzy inner product spaces.

الضرب لفضائيين معياريين ضبابيين وتمام هذا الفضاء

الخلاصة
في هذا البحث برهنا الضرب الديكارتي لفضائيين معياريين ضبابيين تامين هو أيضاً فضاء معياري ضبابي تام ، وكذلك برهنا الضرب الديكارتي لفضائيين جداء داخلي ضبابيين تامين هو فضاء جداء داخلي ضبابي تام.

INTRODUCTION
In [6] , Kider J.R and Sabre R.I introduced the definition of fuzzy inner product space .We shall prove that the Cartesian product $(X \times Y, \langle\cdot,\cdot\rangle(.))$ of two complete fuzzy inner product spaces $(X,\langle.,.\rangle(.)_1)$ and $(Y,\langle\cdot,\cdot\rangle(.)_2)$ is complete fuzzy inner product space

BASIC CONCEPTS
Definition 1.1:[2]
Let X and Y be any two sets, the Cartesian product is denoted by X×Y and is defined by X×Y={(x,y)| $x \in X, y \in Y$ }.

Definition 1.2 :[5],[1]
A fuzzy point P in X is a fuzzy set with membership function.

$$P(y) = \begin{cases} \alpha, & if\, y = x \\ 0, & otherwise \end{cases}$$

For all y in Xwhere 0<α<1. P is said to have support x and value α (x is fixed point).We denote this fuzzy point by x_a or (x,α).

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Two fuzzy point x_a and y_d are said to be distinct if and only if x≠y.

Definition 1.3:[5],[3]

Let X be a vector space over a real or complex field K .Let $\| . \|(.) : P(x) \to R^{+} \cup \{0\}$ is a function which assigns to each $x_a \in P(X)$, $a \in (0,1)$ a non-negative real number $\| x \|(a)$ such that

(FN1) $\| x \|(a)$ =0 if and only if , x=0.

(FN2) $\| l\, x \|(a) = |l| \| x \|(a)$ for all $l \in K$.

(FN3) $\| x + y \|(a) \le \| x \|(a) + \| y \|(a)$.

(FN4) If $0 \le s \le a < 1$,then $\| x \|(a) \le \| x \|(s)$, and there exists $0 < a_n < a$ such that $\lim_{n \to \infty} \| x \|(a_n) = \| x \|(a)$..

Then $\| . \|(.)$ is called a fuzzy norm and $(X, \| . \|(.))$ is called a fuzzy normed space.

Definition 1.4:[5]

A fuzzy sequence $\{(x_n, a_n)\}$ in a fuzzy normed space $(X, \| . \|(.))$ is said to be fuzzy convergent to x_a in X where $a \in (0,1)$ if $\lim_{n \to \infty} \| x_n - x \|(s) = 0$ where $s = \min\{a, a_1, ...\}$; x_a is called the fuzzy limit of $\{(x_n, a_n)\}$ and we write $\lim_{n \to \infty} (x_n, a_n) = (x, a)$ or ,simply , $(x_n, a_n) \to (x, a)$.

Definition 1.5:[5]

A fuzzy sequence $\{(x_n, a_n)\}$ in a fuzzy normed space $(X, \| . \|(.))$ is said to be fuzzy Cauchy if for every $e > 0$ there is an integer $M > 0$ such that $\| x_m - x_n \|(s) < e$ for every $m, n > M$

Where $s = \min\{a_m, a_n\}$.

Definition 1.6:[5]

A fuzzy normed space $(X, \| . \|(.))$ is said to be fuzzy complete if every fuzzy Cauchy sequence $\{(x_n, a_n)\}$ fuzzy converge to a fuzzy vector x_a in X where $a \in (0,1)$.

Definition 1.7:[6],[4]

A fuzzy inner space on H, where H is a vector space over the field K(where K is either R or C) is a mapping of $H \times H$ into the field K, that is with every pair of fuzzy vectors x_a, y_b there is associated a scalar Which written $\langle x_a, y_b \rangle$ or $\langle x, y\rangle(l)$.

Where $l = \min\{a, b\}, a, b \in (0,1]$ and is called the fuzzy inner product of x_a *and* y_b such that for all fuzzy vectors x_a, y_b, z_s with $l = \min\{a, b, s\}$ and scalar r we have:

$$\text{(FIP1)}\ \langle x+z, y\rangle(l) = \langle x, y\rangle(l) + \langle z, y\rangle(l)\ .$$

$$\text{(FIP2)}\ \langle rx, y\rangle(l) = r\langle x, y\rangle(l)$$

$$\text{(FIP3)}\ \langle x, y\rangle(l) = \overline{\langle y, x\rangle}(l)$$

$$\text{(FIP4)}\ \langle x, x\rangle(a) \geq 0\ \ and\ \ \langle x, x\rangle(a) = 0 \Leftrightarrow x = 0$$

(FIP5) $If\ \ 0 < b \leq a < 1$ then $\langle x, x\rangle(a) \leq \langle x, x\rangle(b)$ and then there exists $0 < a_n < a$ such that $\lim_{n\to\infty}\langle x, x\rangle(a_n) = \langle x, x\rangle(a)$

COMPLETION OF CARTESIAN PRODUCT OF TWO FUZZY NORMED SPACES

In this section we shall prove that the product of two fuzzy normed spaces is also fuzzy normed space.Also we prove that the product of two complete fuzzy normed spaces $(X, \|.\|(.)_1)$ and $(Y, \|.\|(.)_2)$ is complete fuzzy normed space.

Theorem2.1:

If $(X, \|.\|(.)_1)$ and $(Y, \|.\|(.)_2)$ are two fuzzy normed spaces then $(X \times Y, \|(.,.)\|(.))$ is a fuzzy normed space by defining

$$\|(x, y)\|(a) = \|x\|(a)_1 + \|y\|(a)_2$$

Proof:

Let (xε,y)3X×Y and λ3K

(FN1) $\|(x, y)\|(a)$=0↔$\|x\|(a)_1 + \|y\|(a)_2$=0↔ $\|x\|(a)_1 = 0$ and $\|y\|(a)_2 = 0$ ↔ x=0 and y=0 ↔ (x,y)=(0,0)

(FN2) $\|l\ (x, y)\|(a) = \|(lx, ly)\|(a)$

$$=\|l\ x\|(a)_1 + \|\ ly\|(a)_2$$

$$=|\ l\ |\|x\|(a)_1 + |\ l\ |\|y\|(a)_2$$

$$= | l | [\| x \| (a)_1 + \| y \| (a)_2]$$
$$= | l | \| (x,y) \| (a)$$

(FN3) $\left\| (x,y) + (x_1 + y_1) \right\| (a) = \| (x + x_1, y + y_1) \| (a)$

$$= \| x + x_1 \| (a)_1 + \| y + y_1 \| (a)_2$$
$$\leq \| x \| (a)_1 + \| x_1 \| (a)_1 + \| y \| (a)_2 + \| y_1 \| (a)_2$$
$$= \| x \| (a)_1 + \| y \| (a)_2 + \| x_1 \| (a)_1 + \| y_1 \| (a)_2$$
$$= \| (x,y) \| (a) + \| (x_1, y_1) \| (a)$$

(FN4) If $0 \leq s \leq a < 1$, then $\| x \| (a)_1 \leq \| x \| (s)_1$ and $\| y \| (a)_2 \leq \| y \| (s)_2$ so $\| (x,y) \| (a) \leq \| (x,y) \| (s)$ also there exists $0 < a_n < a$ such that

$$\lim_{n \to \infty} \| x \| (a_n)_1 = \| x \| (a)_1 \text{ and}$$
$$\lim_{n \to \infty} \| y \| (a_n)_2 = \| y \| (a)_2 \text{ which implies that}$$
$$\lim_{n \to \infty} \| (x,y) \| (a_n) = \| (x,y) \| (a)$$

Thus $(X \times Y, \| (.,.) \| (.))$ is a fuzzy normed space.

Proposition 2.2:

If $\{(x_n, a_n)\}$ is a sequence in the fuzzy normed space $(X, \| . \| (.)_1)$ converges to x in X and $\{(y_n, a_n)\}$ is a sequence in the fuzzy normed space $(Y, \| . \| (.)_2)$ converges to y in Y then $\{(x_n, y_n), a_n\}$ is a sequence in X×Y converges to (x_a, y_a) in $(X \times Y, \| (.,.) \| (.))$ where $a = \min\{a_n | n \in N\}$.

Proof:

By theorem 2.1 , $(X \times Y, \| (.,.) \| (.))$ is a fuzzy normed space. Since $(x_a, a_n) \to x_a$ and $(y_a, a_n) \to y_a$

So $\lim_{n \to \infty} \| x_n - x \| (a) = 0$ and $\lim_{n \to \infty} \| y_n - y \| (s) = 0$

Where $s = \min\{a, a_n | n \in N\}$ .So $\lim_{n\to\infty} \| (x_n, y_n) - (x, y) \| (s) =$
$\lim_{n\to\infty} \| x_n - x \| (a) + \lim_{n\to\infty} \| y_n - y \| (s) = 0 + 0 = 0$
Thus $\{(x_n, y_n), a_n\}$ converges to (x_a, y_a).

PROPOSITION 2.3:
If $\{(x_n, \alpha_n)\}$ is Cauchy sequence in(X, $\|.\|(.)_1$) and $\{(y_n, \alpha_n)\}$ is Cauchy sequence in (Y, $\|.\|(.)_2$) then $\{(x_n, y_n), \alpha_n\}$ is Cauchy sequence in (X×Y, $\|(.,.)\|(.)$).

Proof:
By theorem 2.1 ,X×Y is a fuzzy normed space. since $\{(x_n, \alpha_n)\}$ and $\{(y_n, \alpha_n)\}$ *are* Cauchy sequences then for each given ε>0 there is a positive constant M such that $\| x_m - x_n \| (\sigma)_1 < \frac{\varepsilon}{2}$ and $\| y_m - y_n \| (\sigma)_2 < \frac{\varepsilon}{2}$ for each m,n >M and σ=min$\{\alpha_n | n \in N\}$. Now for each m,n >M $\| (x_m, y_m) - (x_n, y_n) \|(\sigma) =$
$\| (x_m - x_n, y_m - y) \| (\sigma)$
$\| x_m - x_n \| (\sigma)_1 + \| y_m - y_n \| (\sigma)_2 = \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$

Thus $\{(x_n, y_n), \alpha_n\}$ is Cauchy sequence in (X×Y, $\|(.,.)\|(.)$).

Theorem 2.4:
If (X, $\|.\|(.)_1$) and (Y, $\|.\|(.)_2$) are complete fuzzy normed spaces then (X×Y, $\|(.,.)\|(.)$) is a complete fuzzy normed space.

Proof:
By theorem 2.1 , (X×Y, $\|(.,.)\|(.)$) is a fuzzy normed space.Let $\{(x_n, y_n), \alpha_n\}$ *be a* Cauchy sequence in X×Y that is for any given ε>0 there is M>0 such that $\| (x_m, y_m) - (x_n, y_n) \|(\sigma) < \varepsilon$ which implies that
$\| x_m - x_n \| (\sigma)_1 + \| y_m - y_n \| (\sigma)_2 < \varepsilon$ so that
$\| x_m - x_n \| (\sigma)_1 < \varepsilon$ and $\| y_m - y_n \| (\sigma)_2 < \varepsilon$ that is $\{(x_n, \alpha_n)\}$
is Cauchy in (X, $\|.\|(.)_1$) and $\{(y_n, \alpha_n)\}$ is Cauchy in (Y, $\|.\|(.)_2$).But (X, $\|.\|(.)_1$) and (Y, $\|.\|(.)_2$)are complete fuzzy normed spaces , so there is x_α in X and y_α in Y such that $\{(x_n, \alpha_n)\}$ converges to x_α *and* $\{(y_n, \alpha_n)\}$ converges to y_α *that is* $\lim_{n\to\infty} \| x_n - x \| (\alpha)_1 = 0$ and

$\lim_{n\to\infty} \| y_n - y \| (\alpha)_2 = 0$.

Now

$\lim_{n\to\infty} \| (x_n, y_n) - (x, y) \| (\alpha) = \lim_{n\to\infty} \| x_n - x \| (\alpha)_1 + \lim_{n\to\infty} \| y_n - y \| (\alpha)_2 = 0 + 0 = 0$.

Thus $\{(x_n, y_n), \alpha_n\}$ converges to (x_α, y_α) in X×Y , therefore (X×Y, $\| (.,.) \| (.)$) is a complete fuzzy normed space.

Theorem 2.5:

If (X×Y, $\| (.,.) \| (.)$) is a fuzzy normed space, then (X,$\|.\| (.)_1$) and (Y,$\|.\| (.)_2$)are fuzzy normed spaces by defining $\| x \| (\alpha)_1 = \| (x,0) \| (\alpha)$ and $\| y \| (\alpha)_2 = \| (0,y) \| (\alpha)$.

Proof:

Let xεX and λεK

(FN1) $\| x \| (\alpha)_1 = 0 \leftrightarrow \| (x,0) \| (\alpha) = 0 \leftrightarrow (x,0) = (0,0) \leftrightarrow x = 0$

(FN2) $\| \lambda x \| (\alpha)_1 = \| (\lambda x, 0) \| (\alpha) = |\lambda| \| (x, 0) \| (\alpha) = |\lambda| \| x (\alpha)_1$

(FN3) $\| x + x_1 \| (\alpha)_1 = \| (x + x_1, 0) \| (\alpha)$

$\leq \| (x,0) \| (\alpha) + \| (x_1, 0) \| (\alpha)$

$= \| x \| (\alpha)_1 + + \| x_1 \| (\alpha)_1$

(FN4) If $0 \leq s \leq a < 1$ then

$\| x \| (\alpha)_1 = \| (x,0) \| (\alpha) \leq \| (x,0) \| (\sigma) = \| x \| (\sigma)_1$. Also there exists $0 < \alpha_n < \alpha$ such that

$\lim_{n\to\infty} \| x \| (\alpha_n)_1 = \lim_{n\to\infty} \| (x_n, 0) \| (\alpha_n)$

$= \| (x,0) \| (\alpha)$

$= \| x \| (\alpha)_1$.

Thus (X,$\|.\| (.)_1$) is a fuzzy normed space . Similarly we can prove that (Y,$\|.\| (.)_2$) is a fuzzy normed space.

Theorem 2.6:

If (X×Y, $\| (.,.) \| (.)$) is a complete fuzzy normed space, then (X,$\|.\| (.)_1$) and (Y,$\|.\| (.)_2$) are complete fuzzy normed spaces .

Proof:

By theorem 2.5 , (X,$\|.\| (.)_1$) and (Y,$\|.\| (.)_2$)are fuzzy normed spaces

Let$\{(x_n, \alpha_n)\}$ be a Cauchy sequence in (X,$\|.\| (.)_1$) then $\{((x_n, 0), \alpha_n)\}$ is Cauchy sequence in X×Y.But X×Y is complete fuzzy normed space

That is there is $(x_\alpha, 0)\ in$ X×Y such that $\{((x_n, 0), \alpha_n))\}\ converges\ to\ (x_\alpha, 0)$.

Now $\lim_{n\to\infty} \| x_n - x \| (\alpha)_1 = \lim_{n\to\infty} \| (x_n - x, 0) \| (\alpha) = 0$.

That is $(X, \|.\| (.)_1)$ is a complete fuzzy normed space . similarly we can prove that $(Y, \|.\| (.)_2)$is a are complete fuzzy normed space.

COMPLETION OF CARTESIAN PRODUCT OF TWO fuzzy inner product spaces

In this section we shall use the definition of fuzzy inner product space appeared [6] to prove that the product of two fuzzy inner product spaces is also fuzzy inner product space.

Theorem 3.1 :

If $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ and $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ are fuzzy inner product spaces then $(X \times Y, \langle\cdot,\cdot\rangle(.))$ is a fuzzy inner product space by defining

$$\langle(x_1 y_1), (x_2 y_2)\rangle(\alpha) = \langle x_1, x_2\rangle(\alpha)_1 + \langle y_1, y_2\rangle(\alpha)_2$$

Proof:

Let $(x_1 y_1)(\sigma), (x_2 y_2)(\beta), (x_3, y_3)(\delta)$ $X \times Y$ and α= min {σ , β , σ }

(FIP1)

$$\langle(x_1 y_1) + (x_2 y_2), (x_3, y_3)\rangle(\alpha) = \langle(x_1 + x_2, y_1 + y_2), (x_3, y_3)\rangle(\alpha)$$

$= \langle x_1 + x_2, x_3\rangle(\alpha)_1 + \langle y_1 + y_2, y_3\rangle(\alpha)_2$

$= \langle x_1, x_3\rangle(\alpha)_1 + \langle x_2, x_3\rangle(\alpha)_2 + \langle y_1, y_3\rangle(\alpha)_2 + \langle y_2, y_3\rangle(\alpha)_2$

$= \langle(x_1 y_1), (x_3 y_3)\rangle(\alpha) + \langle(x_2 y_2), (x_3 y_3)\rangle(\alpha)$

(FIP2)

For any c≠ 0 ε K

$\langle c(x_1 y_1), (x_2 y_2)\rangle(\alpha) = \langle(cx_1 cy_1) + (x_2 y_2)\rangle(\alpha)$

$= \langle cx_1, x_2\rangle(\alpha)_1 + \langle cy_1, y_2\rangle(\alpha)_2$

$= c\langle x_1, x_2\rangle(\alpha)_1 + c\langle y_1, y_2\rangle(\alpha)_2$

$= c\,\langle(x_1 y_1), (x_2 y_2)\rangle(\alpha)$

(FIP3) $\langle(x_1 y_1), (x_2 y_2)\rangle(\alpha) = \langle x_1, x_2\rangle(\alpha)_1 + \langle y_1, y_2\rangle(\alpha)_2$

$= \langle x_2, x_1\rangle(\alpha)_1 + \langle y_2, y_1\rangle(\alpha)_2$

$= \langle(x_2 y_2), (x_1 y_1)\rangle(\alpha)$

(FIP4) Since $\langle x_1, x_1\rangle(\alpha)_1 \geq 0\ and\ \langle y_1, y_1\rangle(\alpha)_2 \geq 0$ so

$\langle(x_1 y_1), (x_1 y_1)\rangle(\alpha) \geq 0$ and

$\langle(x_1 y_1), (x_1 y_1)\rangle(\alpha) = (0,0) \leftrightarrow \langle x_1, x_1\rangle(\alpha)_1 + \langle y_1, y_1\rangle(\alpha)_2 = 0$

$\leftrightarrow x_1 = 0 \; and \; y_1 = 0 \;\leftrightarrow\; (x_1 y_1) = (0,0)$

(FIP5) If $0 < \beta \le \alpha < 1$ then $\langle x_1, x_1\rangle(\alpha)_1 \le \langle x_1, x_1\rangle(\beta)_1$ and $\langle y_1, y_1\rangle(\alpha)_2 \le \langle y_1, y_1\rangle(\beta)_2$ which implies that
$\langle (x_1 y_1), (x_1 y_1)\rangle(\alpha) \le \langle (x_1 y_1), (x_1 y_1)\rangle(\beta)$
Also there exists $0 < \alpha_n < \alpha$ *such that* $\lim_{n\to\infty}\langle (x_1 y_1), (x_1 y_1)\rangle(\alpha_n)$
$= \lim_{n\to\infty}\langle x_1, x_1\rangle(\alpha_n)_1 + \lim_{n\to\infty}\langle y_1, y_1\rangle(\alpha_n)_2$
$= \langle x_1, x_1\rangle(\alpha)_1 + \langle y_1, y_1\rangle(\alpha)_2 = \langle (x_1 y_1), (x_1 y_1)\rangle(\alpha)$
Thus $(X \times Y, \langle\cdot,\cdot\rangle(.))$ is a fuzzy inner product space.

PROPOSITION 3.2:
If $\{(x_n, \alpha_n)\}$ *is a sequence in the fuzzy inner product* $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ Converges to x_α in X and $\{(y_n, \alpha_n)\}$ is a sequence in the fuzzy inner product $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ Converges to y_α in Y then $\{(x_n, y_n), \alpha_n\}$ is asequence in X×Y Converges to $(x, y)(\alpha)$ in X×Y .

Proof:
The proof is similar to the prove of proposition 2.2 by using
$\| x \| (\alpha)_1 = [\langle x, x\rangle(\alpha)_1]^{1/2}$ *and* $\| y \| (\alpha)_2 = [\langle y, y\rangle(\alpha)_2]^{1/2}$

Proposition 3.3 :
If $\{(x_n, \alpha_n)\}$ is a Cauchy sequence in the fuzzy inner product $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ and $\{(y_n, \alpha_n)\}$ is Cauchy sequence in the fuzzy inner product $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ then $\{(x_n, y_n), \alpha_n\}$ is a Cauchy sequence in $(X \times Y, \langle\cdot,\cdot\rangle(.))$.

Proof:
The proof is similar to the prove of proposition 2.3 by using
$\| x \| (\alpha)_1 = [\langle x, x\rangle(\alpha)_1]^{1/2}$ and $\| y \| (\alpha)_2 = [\langle y, y\rangle(\alpha)_2]^{1/2}$.

Theorem 3.4:
If $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ and $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ are complete fuzzy inner product spaces then $(X \times Y, \langle\cdot,\cdot\rangle(.))$ is a complete fuzzy inner product space .

Proof:
The proof is similar to the prove of theorem 2.4 by using the fact
$\| (x_1, y_1) \| (\alpha) = [\langle (x_1, y_1), (x_1, y_1)\rangle(\alpha)]^{1/2}$.

Theorem 3.5:
If $(X \times Y, \langle\cdot,\cdot\rangle(.))$ is a fuzzy inner product space then $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ is a fuzzy inner product space and $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ is a fuzzy inner product space
By defining $\langle x_1, x_2\rangle(\alpha)_1 = \langle(x_1,0),(x_2,0)\rangle(\alpha)$ and
$\langle y_1, y_2\rangle(\alpha)_1 = \langle(0,y_1),(0,y_2)\rangle(\alpha)$.

Proof:
Let $x_1, x_2, x_3 \in X$
(FIP1)
$\langle x_1 + x_2, x_3\rangle(\alpha)_1 = \langle(x_1+x_2,0),(x_3,0)\rangle(\alpha)$
$\langle(x_1,0),(x_3,0)\rangle(\alpha) + \langle(x_2,0),(x_3,0)\rangle(\alpha)$
$= \langle x_1, x_3\rangle(\alpha)_1 + \langle x_2, x_3\rangle(\alpha)_1$

(FIP2)
$\langle cx_1, x_2\rangle(\alpha)_1 = \langle(cx_1,0),(x_2,0)\rangle(\alpha)$
$= c\,\langle(x_1,0),(x_2,0)\rangle(\alpha)$
$= c\,\langle x_1, x_2\rangle(\alpha)_1$ for all $0 \neq c$ K
(FIP3)
$\overline{\langle x_1, x_2\rangle(\alpha)_1} = \overline{\langle(x_1,0),(x_2,0)\rangle(\alpha)}$
$= \langle(x_2,0),(x_1,0)\rangle(\alpha) = \langle x_2, x_1\rangle(\alpha)$

(FIP4)
If $0 < \beta \leq \alpha < 1$ then
$\langle x_1, x_1\rangle(\alpha) = \langle(x_1,0),(x_1,0)\rangle(\alpha)$
$\leq \langle(x_1,0),(x_1,0)\rangle(\beta) = \langle x_1, x_1\rangle(\beta)$
And there exists $0 < \alpha_n < \alpha$ such that
$\lim_{n\to\infty}\langle x_1, x_1\rangle(\alpha_n)_1 = \lim_{n\to\infty}\langle(x_1,0),(x_1,0)\rangle(\alpha_n)_1$
$= \langle(x_1,0),(x_1,0)\rangle(\alpha)$
$= \langle x_1, x_1\rangle(\alpha)$
Thus $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ is a fuzzy inner product space. Similarly we can prove that $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ is a fuzzy inner product space.

Theorem 3.6:
If $(X \times Y, \langle\cdot,\cdot\rangle(.))$ is a complete fuzzy inner product space then $(X, \langle\cdot,\cdot\rangle(\cdot)_1)$ and $(Y, \langle\cdot,\cdot\rangle(\cdot)_2)$ are complete fuzzy inner product spaces.

Proof:
The proof is similar to the prove of theorem 2.3 by using the fact

$\| x \| (\alpha)_1 = [\langle x, x\rangle(\alpha)_1]^{1/2}$ and $\| y \| (\alpha)_2 = [\langle y, y\rangle(\alpha)_2]^{1/2}$

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