T-Semi α-Operator

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ABSTRACT

In this paper we have defined the T-operator ,T-semi a -operator and we studied the relation among them, and we define T-semi a -open, T-semi a -regular, and monotone operator, also we defined (T,L) semi a -conations ,(T,L) strongly semi a -continues ,(T,L)semi α-homeomorphism ,(T,L) strongly semi a -homeomorphism and we study the relation among them .

شبه α _ المؤثر T

الخلاصة

في هذا البحث قدمنا تعريف المؤثر T وشبه a المؤثر T ودراسة العلاقة بينهم وأيضا قدمنا تعار يف لأنواع من المؤثرات, شبه a المؤثر T المفتوح وشبه a المؤثر المنتظم وأيضا تعرفنا على شبه *(T,L) a المستمر و شبه a (T,L)المستمر بقوه و شبه (T,L) a التشاكل و درسنا العلاقة بينهم .*

INTRODUCTION

In[6]Kashara introduced the concept of an operator associated with a topology t of a space X as follow: let (X,t)be a topological space and B a subset of X , let T be a function from t to P(x) i.e. $T : t \to P(x)$,we say that T is an (S)T induces an operator $T_B : t_B \to P(B)$ such that $T_B(U \cap B) = T(U) \cap B$ for every $U \in t$, where t_B is the relative topology on B.[3]In this paper, we will use these concepts to introduce and study the concepts of T- semi a -operator and semi a -continuous, we prove several theorems concerning these space, which are similar to those proved T semi a -operator. Throughout this paper, all spaces X and Y are topological spaces and t, Φ set of all semi a open sets. In section 2 we perfect the definition T-semi a -operator ,T- semi a - open ,T- semi a -regular ,T-semi a -monotone and we study the relation among them and T-operator, T- a -operator, T-open,T- a -open ,T-regular ,T- a - regular, T- monotone , T- a -monotone and T-semi pre operator ,In section 3 we present the definition of (T,L) semi a -continues ,(T,L)strongly semi a -continues,(T,L) semi a -

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homeomorphism ,(T,L)strongly semi a -continuous , also we study the relation among them and (T,L) continuous, (T,L) a -continuous , (T,L)homeomorphism (T,L) a -homeomorphism .

T-SEMI A-OPERATOR

***Definition* :**Let (X,t) be a topological space and B a subset of X ,Let T be a function from t to P(x) i.e. $T:t \to P(x)$ .we say that T is an a operator associated with t if the following condition holds :

(O) $U \subseteq T(U)$ for each $U \in t$.

We say that the operator α- T associated with t is stable with respect to $B \subseteq X$ if the following condition holds:

(S) T induces an operator $T_B : t_B \to P(B)$ such that $T_B(U \cap B) = T(U) \cap B$ for every $U \in t$, where t_B is relative topology on B.[1]

Definition :Let (X,t) be a topological space and B a subset of X ,Let T be a function from t to P(X) i.e. $T:t \to P(X)$ .we say that T is a semi α-operator associated with t if the following condition holds :

(O) $U \subseteq T(U)$ for each $U \in t$.

We say that the semi α- operator T associated with t is stable with respect to $B \subseteq X$ if the following condition holds:

(S) T induces a semi α- operator $T_B : t_B \to P(B)$ such that $T_B(U \cap B) = T(U) \cap B$ for every $U \in t$, where t_B is relative topology on B.

Example :

Let X={a,b,c}, t={ f ,X,{a},{a,b},{a,c},{b,c},{c},{b}} if $T:P(X) \to P(X)$ defined by T(A)=Ker(A) where Ker(A) is the intersection of all semi- α-open sets that contain a , then T is an semi-α-operator associated with t .

Theorem 1 : Every T-operator is T-semi α-operator.

Proof : suppose that (X,t) is a topological space and $T:t \to P(X)$ is operator i.e. $U \subseteq T(U), \forall U \in t$ and Since (every open set is semi α-open set) [5]. Then $U \subseteq T(U), \forall U \in t$,Thus , T is semi α-operator.

Theorem 2 : Every T-α-operator is T-semi α-operator.

***Proof*:** suppose that (X,t) is a topological space and $T:t \to P(x)$ is α-operator i.e. $U \subseteq T(U), \forall U \in t$ and Since (every α-open set is semi α-open set) [2]

$\therefore U \subseteq T(U)$ for every semi-α-open set U .

$\therefore$ T is semi α-operator.

Corollary : Every T-semi α-operator is T-semi pre –operator.

Proof : suppose that (X,t) is a topological space and $T: t \to P(x)$ is semi α-operator i.e. $U \subseteq T(U)$, $\forall U \in t$ and Since (every semi α-open set is semi preopen set)[5].

$\therefore$ $U \subseteq T(U)$ for every semi-preopen set U .

$\therefore$ T is semi per-operator .

Definition:

Let (X,t) be a topological space and T-be an semi α-operator of t , A subset A of X is said to be T- semi-α-open if for each $x \in A$ there exists a semi - α - open set U containing x s.t $T(U) \subseteq A$.

Theorem 3: Every T-open is T-semi α-open.

Proof: suppose that (X, t) is a topological space, and A is T-open in X

Let $x \in A$, since A is T-open set then there exists open set U in X s.t $x \in U$, and T (U) =A.

Since (every open set is semi α-open set) [5].Therefore, U is semi α-open Hence, A is T-semi α open.

Theorem 4: Every T-α-open is T-semi α-open .

Proof:

suppose that (X,t) is a topological space, and A is T - α-open in X

Let $x \in A$, since A is T-α-open set then there exist α -open set U in X s.t $x \in U$, and T(U)=A.Since [every α- open set is semi α-open set] [2]Therefore U is semi α-open Hence A is T-semi a -open.

Corollary : Every T-semi α-open is T-semi preopen .

Proof: suppose that (X,t) is a topological space, and A is T-semi α-open in X ,Let $x \in A$, since A is T-semi α-open set $\Rightarrow$ then there exist semi α open set `U in X s.t $x \in U$, and T(U)=A.

Since (every semi α -open set is semi preopen set). Therefore U is semi preopen and hence A is T-semi preopen.

Definition: Let (X,t) be a topological space, and T be an operator on t , we say that T is a regular operator if for every $x \in X$ and every pair U,V of open neighborhoods of x there exists an open neighborhood W of x such that $T(W) \subseteq T(U) \cap T(V)$ [7].

Definition: Let (X,t) be a topological space, and T be a semi α -operator on t , we say that T is regular semi α- operator if for every $x \in X$ and every pair U,V of semi α open neighborhoods of x there exists a semi α open neighborhood W of x such that $T(W) \subseteq T(U) \cap T(V)$.

Theorem 5: Every T- regular is T-semi α-regular.

Proof: suppose that (X, t) is a topological space, i.e. T is regular.

Then for each $x \in X$ and U, V pair open neighborhoods of x then there exists an open neighborhood W such that $T(W) \subseteq T(U) \cap T(V)$.

Since (every open neighborhood is semi-α open neighborhood).

Then U, V and W are semi α- open neighborhoods i.e. $T(W) \subseteq T(U) \cap T(V)$

Hence T is semi α-regular.
Theorem 6: Every T-α- regular is T-semi α-regular.
Proof: suppose that (X, t) is a topological space, i.e. T is α- regular
Then for each $x \in X$ and U, V pair α-open neighborhoods of x then there exists an α -open neighborhood W such that $T(W) \subseteq T(U) \cap T(V)$.
Since (every α open neighborhood is semi-α open neighborhood)
Then U, V and W are semi α- open neighborhoods i.e. $T(W) \subseteq T(U) \cap T(V)$.
Hence T is semi α-regular.
Corollary :_ Every T-semi α- regular is T-semi pre-regular .
Proof: suppose that (X, t) is a topological space, i.e. T is semi α -regular.
Then for each $x \in X$ and U, V pair α-open neighborhoods of x then there exists a semi α- open neighborhood W such that $T(W) \subseteq T(U) \cap T(V)$.
Since (every semi α open neighborhood is semi-preopen neighborhood).
Then U, V and W are semi preopen neighborhoods i.e. $T(W) \subseteq T(U) \cap T(V)$
Hence T is semi pre-regular.
Definition: Let (X, t) be a topological space, and T be semi α -operator associated with t, T is said to be monotone semi α- operator if for every pair of open sets U and V s.t $U \subseteq V$ then $T(U) \subseteq T(V)$.
Theorem 7: Every T- monotone is T-semi α- monotone.
Proof: suppose that (X, t) is a topological space and $T : t \to P(x)$ isan operator
i.e. for every pair of open sets U and V s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Since [every open set is semi α-open set]
Therefore Uand V are semi α-open sets s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Hence T is semi α- monotone
Theorem 8: Every T-α monotone is T-semi α- monotone.
Proof: suppose that (X, t) is a topological space and $T : t \to P(x)$ is α-operator i.e. for every pair of α-open sets U and V s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Since [every α-open set is semi α-open set]
Therefore Uand V are semi α-open sets s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Hence T is semi α- monotone.
Corollary: Every T-semi α- monotone is T-semi premonotone.
Proof: suppose that (X,t) is a topological space and $T : t \to P(x)$ is semi α-operator i.e. for every pair of semi α-open sets U and V s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Since (every semi α-open set is semi preopen set).
Therefore Uand V are semi- preopen sets s.t $U \subseteq V \Rightarrow T(U) \subseteq T(V)$.
Hence T is semi premonotone.

T-semi α -operator continuous map

Definition: Let (X,t,T) and (Y,Φ,L) be two operator topological spaces . then we say that the function $f:(X,t)\to(Y,\Phi)$ is a (T,L) continuous if $f^{-1}(V)$ is a T-open set in X for each L-open set U in Y .[4]

Definition: Let (X, t, T) and (Y,Φ, L) be two semi α-operator topological spaces . then we say that the function $f:(X,t)\to(Y,\Phi)$ is a (T,L) semi α-continuous if $f^{-1}(V)$ is a T- semi α open set in X for each L-open set U in Y.

Theorem 9: every (T, L) continuous map is (T, L) semi α-continuous map.

Proof: Let $f:(X,t)\to(Y,\Phi)$ be (T,L) continuous map ,where Tand L are operator of X and Y respectively i.e. Let V be any L open set in Y $\Rightarrow f^{-1}(V)$ is T-open in X ,by Theorem 3 $\Rightarrow f^{-1}(V)$ is T-semi α open .

Therefore f is (T, L) is semi α-continuous.

Theorem 10: Every (T, L) α-continuous map is (T, L) semi α-continuous map.

Proof: Let $f:(X,t)\to(Y,\Phi)$ be (T,L) α-continuous map ,where Tand L are α-operator of X and Y respectively i.e. Let V be any L open set in Y $\Rightarrow f^{-1}(V)$ is T-α open in X ,by Theorem 4 $\Rightarrow f^{-1}(V)$ is T-semi α-open .

There fore (T, L) is semi α-continuous.

Corollary: Every (T, L) semi α-continuous map is (T,L) semi precontinuous map.

Proof: Let $f:(X,t)\to(Y,\Phi)$ be (T, L) semi α-continuous map, where T and L are semi α-operator of X and Y respectively i.e. Let V be any L open set in Y $\Rightarrow f^{-1}(V)$ is T-semi α open in X, by Corollary of theorem 8 $\Rightarrow f^{-1}(V)$ is T-semi preopen.

There fore (T, L) is semi precontinuous.

Definition: Let (X, t, T) and (Y,Φ, L) be two semi α-operator topological space then we say that function $f:(X,t)\to(Y,\Phi)$ is (T,L) strongly semi α-continuous if $f^{-1}(V)$ is a T- semi α-open set in X for each L-α semi-open set U in Y.

Theorem 11: Every (T, L) strongly semi α-continuous map is (T, L) semi α-continuous map.

Proof: suppose that $f:(X,t)\to(Y,\Phi)$ be (T, L) strongly semi α-continuous map. Let V be any L- open set in Y by theorem 3 $\Rightarrow$ V is semi α -open.

Since f is strongly semi α -continuous $\Rightarrow f^{-1}(V)$ is T-semi α open.

There fore (T, L) is semi α-continuous.

Theorem 12: Every (T, L) strongly semi –pre continuous map is (T, L) semi -precontinuous map.

Proof: suppose that $f:(X,t)\to(Y,\Phi)$ be (T, L) strongly semi precontinuous map. Let V be any L open set in Y by theorem 3 and Corollary of theorem 3 $\Rightarrow$ V is semi pre open.

Since f is strongly semi precontinuous $\Rightarrow f^{-1}(V)$ is T-semi preopen.

There fore (T, L) is semi-pre continuous .

Definition: A function f is called a (T, L) homeomorphism if

1- f is a bijective function.

2- f is (T, L) continuous function .

3- f^{-1} is a (T,L) continuous function [4].

Definition: A function f is called a (T,L) semi α- homeomorphism if

1- f is a bijective function

2- f is (T, L) semi α-continuous function

3- f^{-1} is a (T,L) semi α continuous - function.

Theorem13: Every (T,L) homeomorphism map is (T,L) semi-α-homeomorphism map.

Proof: suppose that $f:(X,t)\to(Y,\Phi)$ be (T,L) homeomorphism map i.e. f is bijective and f is (T,L)continuous function ,also f^{-1} is a (T,L) continuous function. Since by Theorem 9 $\Rightarrow$ f is (T,L)semi α- continuous function ,also f^{-1} is a (T,L) semi α- continuous function.

Hence (T,L) is semi-α- homeomorphism map.

Theorem 14: Every (T,L) α-homeomorphism map is (T,L) semi-α-homeomorphism map.

Proof: suppose that $f:(X,t)\to(Y,\Phi)$ be (T,L) α-homeomorphism map i.e. f is bijective and f is (T,L)α-continuous function ,also f^{-1} is a (T,L) α-continuous function. Since by Theorem 10 $\Rightarrow$ f is (T,L)semi α- continuous function ,also f^{-1} is a (T,L) semi α- continuous function .Hence, (T,L) is semi-α- homeomorphism map.

Definition:A function f is called (T,L) strongly semi α- homeomorphism if

1- f is a bijective function

2- f is (T,L) strongly semi α- continuous

3- f^{-1} is a (T,L) strongly semi α- continuous

Theorem 15: Every (T,L) strongly semi α- homeomorphism map is (T,L) semi α- homeomorphism map.

Proof: suppose that $f:(X,t)\to(Y,\Phi)$ be (T,L) strongly semi α-homeomorphism map. i.e. f is bijective ,f and f^{-1} are(T,L) strongly semi α-continuous by Th13 $\Rightarrow$ f and f^{-1} are (T,L) semi α- continuous

Hence (T,L) is semi α- homeomorphism map.

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