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ORIGINAL STUDY

Liu-Type Estimator in Inverse Gaussian Regression Model Based on (r-(k-d)) Class Estimator

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ABSTRACT

When multicollinearity arises in the inverse Gaussian regression (IGR), there is a substantially unstable variance in the maximum likelihood estimator. Based on the (r-(k-d)) class estimation method, we present a novel Liu-type estimator in the IGR model in this study. The study examines the effectiveness of the suggested estimator and draws comparisons with alternative estimators. Based on simulation and real data results, the suggested estimate performs better than the other estimators in terms of mean squared error.

Keywords: (k – d) class estimator, Liu-type estimator, Inverse Gaussian regression model, (r – d) class estimator, (r – k) class estimator

1. Introduction

One of the most popular models for assessing real-world data is the inverse Gaussian regression model (IGR), which is especially useful in the fields of medical science, insurance, and health-care economics [8, 10, 23]. The IGR is employed when the response variable is favorably biased or not distributed normally. Consequently, the response variable in inverse Gaussian regression is assumed to have an inverse Gaussian distribution [11, 14, 15].

Similar to the linear regression model, the IGR also makes the assumption that there is no correlation between the regressors. In many applications of regression models, there is a natural link between the explanatory variables. Regression coefficient estimates are challenging to comprehend when correlations are large because estimation of the regression parameters becomes erratic [2, 7, 18–20].

As pointed out by Mackinnon and Puterman [18], when using the maximum likelihood (ML) technique to estimate the regression coefficients for the IGR, the estimated coefficients usually contain a high variance that results in low statistical significance [1, 4, 17]. In regression analysis, multicollinearity complicates the estimation of the values of each explanatory variable if the other predictors included in the model also have visible correlations with the dependent variable. In addition, the sample variance of the regression coefficients can influence prediction and inference.

Numerous strategies have been proposed as corrective measures to address the multicollinearity problem. The ridge estimator (RE), proposed by Hoerl and Kennard in 1970, has been demonstrated to be a good substitute for the ML estimator. The RE was employed by Mackinnon and Puterman [18] in his generalized linear models (GLM). Liu [16] suggested an additional solution to the collinearity problem, which is referred to as the Liu-type estimator. Additionally, this estimator was thoroughly examined in the literature and applied to a number of models that belong to GLMs.

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The Liu-Type estimator offers a connection between estimating parameters of the IGR model in presence of analytic complexities like the multicollinearity and small sample sizes. The experience shared in this paper can thus be judged as evidence of its efficacy in enhancing estimation accuracy across all fields of application. Future research may explore a direct comparison of the Liu-Type with other estimation methodologies to strengthen the role of the former as commonly used denoising techniques.

2. The inverse Gaussian regression model

The inverse Gaussian distribution is defined as

$$f(y,\theta,\eta) = \frac{1}{\sqrt{2\pi y^3\eta}} \exp\left[-\frac{1}{2y}\left(\frac{y-\theta}{\theta\sqrt{\eta}}\right)^2\right], \quad y>0,\ \theta,\eta>0 \tag{1}$$

Eq. (1) can re-write in terms of generalized linear models (GLM) as [3, 21]

$$f(y,\theta,\eta) = \frac{1}{\eta}\left\{-\frac{y}{2\theta^2}+\frac{1}{\theta}\right\} + \left\{-\frac{1}{2}\ln(2\eta y^3) - \frac{1}{2}\ln(\eta)\right\}, \tag{2}$$

The ML method is the basis of the IGR model estimation, the log probability function of the IGR is described as

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n}\left\{\frac{1}{\eta}\left[\frac{y_i\mathbf{x}_i^T\boldsymbol{\beta}}{2} - \sqrt{\mathbf{x}_i^T\boldsymbol{\beta}}\right] - \frac{1}{2\tau y_i} - \frac{\ln\eta}{2} - \ln(2\pi y_i^3)\right\}. \tag{3}$$

Then the ML estimator can be solved as

$$\frac{\partial\ell(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}} = \sum_{i=1}^{n}\frac{1}{2\theta}\left[y_i - \frac{1}{\sqrt{\mathbf{x}_i^T\boldsymbol{\beta}}}\right]\mathbf{x}_i = 0. \tag{4}$$

and then

$$\hat{\boldsymbol{\beta}}_{IGR} = \mathbf{Q}^{-1}\mathbf{X}^T\hat{\mathbf{W}}\hat{\mathbf{m}}, \tag{5}$$

where $\mathbf{Q} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X})$, $\hat{\mathbf{W}} = \mathrm{diag}(\hat{\mu}_i^3)$, the MSE equal

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{IGR}) = \eta\sum_{j=1}^{p}\frac{1}{\lambda_j}, \tag{6}$$

where λ_j is the eigenvalue of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix [22].

The inverse Gaussian ridge estimator (GRE) is defined:

$$\hat{\beta}_{\mathrm{GRE}} = (D+kI)^{-1}X'\hat{\mathrm{W}}\hat{\mathrm{z}} \tag{8}$$

Then,

$$cov\left(\hat{\beta}_{GRE}\right) = \tau D_k^{-1} D\, D_k^{-1} \tag{9}$$

$$\mathbf{b}_{GRE} = bias\left(\hat{\beta}_k\right) = -kD_k^{-1}\,\beta \tag{10}$$

The Inverse Gaussian Liu estimator (GLE) is known as

$$\hat{\beta}_{GLE} = T_d\ \hat{\beta}_{MLE} \tag{11}$$

$\mathbf{T}_d = (\mathrm{D}+\mathrm{I})^{-1}(\mathrm{D}+\mathrm{dI})$ calculated using the following formulas.

$$cov\left(\hat{\beta}_d\right) = \tau\ \mathbf{T}_d D^{-1}\mathbf{T}_d{}^T \tag{12}$$

$$\mathbf{b}_{GLE} = bias\left(\hat{\beta}_d\right) = -\left(1-d\right)(\mathrm{D}+\mathrm{I})^{-1}\,\beta \tag{13}$$

3. The proposed estimator

Baye and Parker [9] proposed a RE and principal components regression (PCR) estimator ((r – k) class estimator)).

$$\hat{\beta}_r(k) = T_r\left(T_r'X\grave{W}XT_r + kI_r\right)^{-1} T_r'\grave{X}y, \tag{14}$$

Kaçıranlar and Sakallıoğlu [12] proposed Liu estimation and PCR ((r-d) class estimator), which is defined as:

$$\hat{\beta}_r\,(d) = T_r\left(T_r'\grave{X}WXT_r + I_r\right)^{-1}\left(T_r'\grave{X}y + dT_r'\hat{\beta}_r\right) \quad 0<d<1, \tag{15}$$

where $\hat{\beta}_r = T_r(T_r'X\grave{W}XT_r)^{-1}T_r'\grave{X}y$ is PCR. $\hat{\beta}_r(d)$ is define as the. Alheety and Golam Kibria [6] proposed Liu estimator with the (r – k) class estimator.

In this paper we extend this estimator to IGR, the new estimation is known the (r - (k-d)) class estimator. Our new estimator is defined as

$$\hat{\beta}_r\,(k,d) = T_r\left(T_r'\grave{X}WXT_r + I_r\right)^{-1}\left(T_r'\grave{X}y + dT_r'\hat{\beta}_r(k)\right) \tag{16}$$

$k>0,\ -\infty<d<\infty.$

4. Comparison of the proposed estimators

An estimator's mean squares error matrix is defined as follows:

$$\mathrm{MSE}\left(\hat{\beta}\right) = \mathrm{E}\left(\hat{\beta}-\beta\right)^{\mathrm{T}}\left(\hat{\beta}-\beta\right)$$
$$= \mathrm{Var}\left(\hat{\beta}\right) + \left(\mathrm{Bias}\left(\hat{\beta}\right)\right)\left(\mathrm{Bias}\left(\hat{\beta}\right)\right)' \tag{17}$$

where $Bias(\hat{\beta}) = E(\hat{\beta}) - \beta$ is the bias of $\hat{\beta}$ and $\mathrm{Var}(\hat{\beta}) = E[(\hat{\beta}-E(\hat{\beta}))(\hat{\beta}-E(\hat{\beta}))']$ is a variation of β. When $MSE(\hat{\beta}_2) - MSE(\hat{\beta}_1)$ is a non-negative definite matrix, the e$\hat{\beta}_2$ is used.

4.1. Comparison between our proposed estimator and (r – d) estimator

The $\hat{\beta}_r(k,d)$ and $\hat{\beta}_r(d)$ are known as:

$$\mathrm{MMSE}\left(\hat{\beta}_r\left(k,d\right)\right)$$
$$= \tau\ T_r D_r^{-1}\left(1\right)\left(I_r + dD_r^{-1}\left(k\right)\right) T_r' D_r^{-1} T_r \left(I_r + dD_r^{-1}\left(k\right)\right) D_r^{-1}\left(1\right) T_r' + \left(T_r D_r^{-1}\left(1\right)\left(I_r + dD_r^{-1}\left(k\right) T_r' D_r T_r\right) T_r' + T_{p-r}' T_{p-r}\right)\beta\beta'\left(T_r D_r^{-1}\left(1\right)\left(I_r + dT_r' D_r T_r D_r^{-1}\left(k\right)\right) T_r' + T_{p-r}' T_{p-r}\right). \tag{18}$$

$$\mathrm{mse}\left(\hat{\beta}_r\left(k,d\right)\right) = \sum_{i=1}^{r}\frac{\tau\lambda_i\ \left(\lambda_i+k+d\right)^2 + \ \left(\lambda_i+k-d\lambda_i\right)^2\alpha_i^2}{\left(\lambda_i+k\right)^2\ \left(\lambda_i+1\right)^2} + \sum_{i=p-r}^{p}\alpha_i^2 \tag{19}$$

where $D_r^{-1}(1) = (\Lambda_r - I_r)$.

$$\mathrm{mse}\left(\hat{\beta}_r\left(k,d\right)\right) - \mathrm{mse}\left(\hat{\beta}_r\left(d\right)\right) = \mathrm{d}^2\sum_{i=1}^{r}\frac{\left(\lambda_i\alpha_i^2+\tau\right) + \left(\lambda_i^2 - \ \left(\lambda_i+k\right)^2\right)}{\lambda_i\left(\lambda_i+k\right)^2\ \left(\lambda_i+1\right)^2} + 2dk\sum_{i=1}^{r}\frac{\left(\alpha_i^2-\tau\right)}{\left(\lambda_i+k\right)\ \left(\lambda_i+1\right)^2}$$

Let $M_1 = \sum_{i=1}^{r}\frac{(\lambda_i\alpha_i^2+\tau)+(\lambda_i^2-(\lambda_i+k)^2)}{\lambda_i(\lambda_i+k)^2(\lambda_i+1)^2}$ and $M_2 = k\sum_{i=1}^{r}\frac{(\alpha_i^2-\tau)}{(\lambda_i+k)(\lambda_i+1)^2}$ then,

$$\mathrm{mse}\left(\hat{\beta}_r\left(k,d\right)\right) - \mathrm{mse}\left(\hat{\beta}_r\left(d\right)\right) = d^2M_1 + 2dM_2$$

$$\mathrm{mse}\left(\hat{\beta}_r\left(k,d\right)\right) - \mathrm{mse}\left(\hat{\beta}_r\left(d\right)\right) = d^2M_1 + 2dM_2 = d(dM_1 + 2M_2)$$

so, $\mathrm{mse}(\hat{\beta}_r(k,d)) - \mathrm{mse}(\hat{\beta}_r(d))$ will be positive when $\mathrm{d} < 0$ and $dM1 + 2M2 < 0$. In this case,

$$dM_1 + 2M_2 < 0 \Leftrightarrow dM_1 < -2M_2 \Leftrightarrow d\left(-M_1\right) > 2M_2$$
$$\Leftrightarrow d > \frac{2M_2}{\left(-M_1\right)} = d^* > 0.$$

So, $\mathrm{mse}(\hat{\beta}_r - (k,d)) - \mathrm{mse}(\hat{\beta}_r(d)) > 0$ for $0 < d < d^*$.

$(\hat{\beta}_r(k,d)) - \mathrm{mse}(\hat{\beta}_r(d)) < 0$. This inequity, will be held when $d < 0$ and, $dM_1 + 2M_2 > 0 \Leftrightarrow d < d^* > 0$. So, $\mathrm{mse}(\hat{\beta}_r(k,d)) - \mathrm{mse}(\hat{\beta}_r(d)) < 0$ for < 0 *or* $d < d^*$.

at the same method, when $M_2 < 0$, $\mathrm{mse}(\hat{\beta}_r(k,d)) - \mathrm{mse}(\hat{\beta}_r(d)) > 0$ for $d^* < d < 0$.

Also, $\mathrm{mse}(\hat{\beta}_r(k,d)) - \mathrm{mse}(\hat{\beta}_r(d)) < 0$ for $d > 0$ *and* $d > d^*$.

Theorem 4.1:

(a) When $\sum_{i=1}^{r}\frac{(\alpha_i^2-\tau)}{(\lambda_i+k)\ (\lambda_i+1)^2} > 0$ *then:*

(1) $\mathrm{mse}(\hat{\beta}_r(k,d)) > \mathrm{mse}(\hat{\beta}_r(d))$ *for* $0 < d < d^*$.

(2) $\mathrm{mse}(\hat{\beta}_r(k,d)) < \mathrm{mse}(\hat{\beta}_r(d))$ *for* < 0 *or* $d < d^*$.

(b) When $\sum_{i=1}^{r}\frac{(\alpha_i^2-\tau)}{(\lambda_i+k)\ (\lambda_i+1)^2} < 0$ *then:*

(1) $\mathrm{mse}(\hat{\beta}_r(k,d)) > \mathrm{mse}(\hat{\beta}_r(d))$ *for* $d^* < d < 0$.

(2) $\mathrm{mse}(\hat{\beta}_r(k,d)) < \mathrm{mse}(\hat{\beta}_r(d))$ *for* > 0 *and* $d > d^*$.

Where

$$d^* = \frac{2k\sum_{i=1}^{r}\frac{\left(\hat{\alpha}_i^2-\tau\right)}{\left(\lambda_i+k\right)\ \left(\lambda_i+1\right)^2}}{\sum_{i=1}^{r}\frac{\left(\ \left(\lambda_i+k\right)^2-\lambda_i^2\right)\left(\lambda_i\alpha_i^2+\tau\right)}{\lambda_i\ \left(\lambda_i+1\right)^2\left(\lambda_i+k\right)^2}}$$

4.2. Comparison between our estimator and (r – k) Class estimator

$$\mathrm{MSE}\left(\hat{\beta}_r\left(k\right)\right) = \tau\ T_r D_r^{-1}\left(k\right)\Lambda_r D_r^{-1}\left(k\right) T_r' + \left[T_r D_r^{-1}\left(k\right)\Lambda_r T_r' - I_p\right]\beta\beta'\left[T_r D_r^{-1}\left(k\right)\Lambda_r T_r' - I_p\right] \tag{22}$$

$$\mathrm{mse}\left(\hat{\beta}_r\left(k\right)\right) = \mathrm{mse}\left(\hat{\beta}_r\left(k,1-k\right)\right) = \sum_{i=1}^{r}\frac{\tau\lambda_i + k^2\alpha_i^2}{\left(\lambda_i+k\right)^2} + \sum_{i=p-r}^{p}\alpha_i^2 \tag{23}$$

Theorem 4.2:

When

$$d = \frac{\sum_{i=1}^{r}\lambda_i\left(\alpha_i^2-\tau\right)/\left(\lambda_i+k\right)\left(\lambda_i+1\right)^2}{\sum_{i=1}^{r}\lambda_i\left(\lambda_i\alpha_i^2-\tau\right)/\left(\lambda_i+k\right)^2\left(\lambda_i+1\right)^2}$$

$$\mathrm{mse}\left(\hat{\beta}_r\left(k,d\right)\right) \le \mathrm{mse}\left(\hat{\beta}_r\left(k\right)\right).$$

5. Simulation research

In this section, we use a Monte Carlo simulation experiment to test our proposed estimator's efficiency under various degrees of multicollinearity.

5.1. Simulation design

The yi of n observations from IGR is generated as, yi ∼ Inverse Gaussian (μ_i, τ); where $\tau = (0.7, 1.2, 3)$ and $\mu i = \exp(x_i{}^T\beta)$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)$ with $\sum_{j=1}^{p} \beta_j^2 = 1$ [13]. The $x_i^T = (x_{i1}, x_{i2}, \ldots, x_{in})$ have been generated from

$$\mathbf{x_{ij}} = \left(1-\rho^2\right)^{1/2} \boldsymbol{w_{ij}} + \rho \boldsymbol{w_{ip}} \quad \boldsymbol{i} = 1, 2, \ldots, \boldsymbol{n}, \quad \boldsymbol{j} = 1, 2, \ldots, \boldsymbol{p}$$

where w_{ij}'s are independent standard normal pseudo-random numbers. Three representative sample sizes of 50, 100, and 150 are considered. Furthermore, p = 4 and p = 8 are used with the degrees of correlation of $\rho = (0.90,\ 0.95,\ 0.99)$. The average MSE is calculated as:

$$\text{MSE}\left(\hat{\beta}\right) = \frac{1}{1000} \sum_{i=1}^{1000} \left(\hat{\beta}-\beta\right)^T \left(\hat{\beta}-\beta\right)$$

5.2. Results of the simulation

The averaged MSE for each combination of components is shown in Tables 1 to 3. The best value of the averaged MSE is bolded. From the tables, we can infer the following conclusions:

Table 1. Averaged MSE when $\tau = 0.7$.

n	p	ρ	IGR	r-k	r-d	r-(k-d)
50	4	0.90	4.229	3.988	3.649	**3.535**
		0.95	4.273	4.038	3.699	**3.585**
		0.99	4.539	4.304	3.965	**3.851**
	8	0.90	4.343	4.108	3.769	**3.655**
		0.95	4.393	4.158	3.819	**3.705**
		0.99	4.659	4.424	4.085	**3.971**
100	4	0.90	3.981	3.746	3.407	**3.293**
		0.95	4.031	3.796	3.457	**3.343**
		0.99	4.297	4.062	3.723	**3.609**
	8	0.90	4.107	3.866	3.527	**3.413**
		0.95	4.151	3.916	3.577	**3.463**
		0.99	4.417	4.182	3.843	**3.729**
150	4	0.90	3.93	3.695	3.356	**3.242**
		0.95	3.98	3.745	3.406	**3.293**
		0.99	4.246	4.011	3.672	**3.558**
	8	0.90	4.05	3.815	3.476	**3.363**
		0.95	4.1	3.865	3.526	**3.412**
		0.99	4.366	4.131	3.792	**3.678**

Table 2. Averaged MSE when $\tau = 1.2$.

n	p	ρ	IGR	r-k	r-d	r-(k-d)
50	4	0.90	4.12	3.885	3.546	**3.432**
		0.95	4.169	3.934	3.595	**3.481**
		0.99	4.436	4.201	3.862	**3.748**
	8	0.90	4.24	4.005	3.666	**3.552**
		0.95	4.289	4.054	3.715	**3.6**
		0.99	4.556	4.321	3.982	**3.868**
100	4	0.90	3.878	3.643	3.304	**3.19**
		0.95	3.928	3.692	3.353	**3.239**
		0.99	4.194	3.959	3.62	**3.506**
	8	0.90	3.998	3.763	3.424	**3.31**
		0.95	4.048	3.813	3.474	**3.36**
		0.99	4.314	4.079	3.74	**3.626**
150	4	0.90	3.827	3.592	3.253	**3.139**
		0.95	3.876	3.641	3.303	**3.188**
		0.99	4.143	3.908	3.569	**3.455**
	8	0.90	3.947	3.712	3.373	**3.259**
		0.95	3.996	3.761	3.423	**3.308**
		0.99	4.263	4.028	3.689	**3.575**

Table 3. Averaged MSE when $\tau = 3$.

n	p	ρ	IGR	r-k	r-d	r-(k-d)
50	4	0.90	3.747	3.512	3.173	**3.059**
		0.95	3.796	3.561	3.222	**3.108**
		0.99	4.063	3.828	3.489	**3.375**
	8	0.90	3.867	3.632	3.293	**3.179**
		0.95	3.916	3.681	3.342	**3.227**
		0.99	4.183	3.948	3.609	**3.495**
100	4	0.90	3.505	3.27	2.931	**2.817**
		0.95	3.555	3.319	2.98	**2.866**
		0.99	3.821	3.586	3.247	**3.133**
	8	0.90	3.625	3.39	3.051	**2.937**
		0.95	3.675	3.44	3.101	**2.987**
		0.99	3.941	3.706	3.367	**3.253**
150	4	0.90	3.454	3.219	2.88	**2.766**
		0.95	3.503	3.268	2.93	**2.815**
		0.99	3.77	3.535	3.196	**3.082**
	8	0.90	3.574	3.339	3.241	**2.886**
		0.95	3.623	3.388	3.05	**2.935**
		0.99	3.89	3.655	3.316	**3.202**

1) The results of the new estimator are the best in all the circumstances examined. Furthermore, at higher correlation coefficient values, r-(k-d) performs better.
2) In terms of MSE, Tables 1 to 3 demonstrate that r-(k-d) is the most efficient. In the second rank, the r-d estimator performs better than the IGR and the r-k estimators. Moreover, multicollinearity has a significant impact on the IGR estimator's efficiency, which is the lowest among the r-k, r-d, and r-(k-d) estimators.
3) Upon increasing the number of explanatory variables from four to eight, it becomes evident that the MSE experiences a detrimental impact as their values rise. Furthermore, with regard to sample size, the MSE values decrease with increasing n, independent of ρ and p values.

4) It is evident that the MSE values fall as the dispersion parameter, τ, increases.

6. The application

A chemical dataset was used to explain the r-(k-d) estimator's capacity in practical applications. This data with n = 212 and p = 10. The logarithm of the reciprocal of the minimum inhibitory concentration (MIC), which measures the antibiotic efficiency against Candida albicans in milligrams per milliliter, is known as pMIC. Although it shows how many chemical descriptors are used as explanatory variables [5].

The response variable is initially tested to see if it fits into the IG distribution using the Chi-square test. A p-value of 0.8704 and a result of 12.1056 were obtained from the test. This result demonstrates how well the IG a distribution fits this response variable. The approximate dispersion parameter is 0.0153. Second, following the fitting of the log link function and an approximate dispersion parameter of 0.063 to the IGR, the eigenvalues are 1.97×10^9, 3.74×10^6, 1.21×10^4, 1.34×10^3, 1.22×10^3, 1.07×10^3, 4.63×10^2, 2.08×10^1, 10.68, and 1.57.

The evaluated condition number $CN = \sqrt{\lambda_{max}/\lambda_{min}}$ of the data is 35422.83 suggesting the presence of a significant multicollinearity problem. The MSE values and approximate Inverse Gaussian regression coefficients for the IGR, r-k, k-d, and r-(k-d) estimators are shown in Table 4. As Table 4 illustrates, the r-(k-d) effectively reduces the value of the estimated coefficients.

Moreover, the decrease on the MSE is also evidently observed in favour of the r- (k-d). To be precise, the proposed estimator was about 36.174%, 23.621%, and 19.688% less than the IGR, r-k, and k-d estimator, respectively.

Table 4. The estimated IGR and MSE values.

	Estimators			
	IGR	r-k	k-d	r-(k-d)
X1	0.9148	0.8934	0.8745	0.8688
X2	1.9196	1.8982	1.8793	1.8736
X3	0.6957	0.6743	0.6554	0.6497
X4	−1.3663	−1.3877	−1.4066	−1.4123
X5	−1.9661	−1.9875	−2.0064	−2.0121
X6	−0.0743	−0.0957	−0.1146	−0.1203
X7	−1.3981	−1.4195	−1.4384	−1.4441
X8	−0.4238	−0.4452	−0.4641	−0.4698
X9	−1.3662	−1.3876	−1.4065	−1.4122
X10	2.8926	2.8712	2.8523	2.8466
MSE	3.7877	2.9539	2.8397	**2.1653**

7. Conclusion

The IGR model is used in modeling positive continuous data in which the response variable is inverse Gaussian. The Liu-Type estimator is a mollified form of traditional estimators where a new parameter is introduced with an aim of enhancing efficiency of estimation particularly under conditions of multicollinearity. In this study, we theoretically analyzed the (r-(k-d)) class estimator for the IGR model that we have proposed previously. The MSE has been used in the analysis to compare our suggested estimator with another estimator. Based on the simulation and real data evidence presented in detail, the suggested estimator outperforms the other estimators considering the MSE. For future work, the enhanced robustness against multicollinearity in IGR model can be handled. Further, applying (r-(k-d)) class estimator to other real-world datasets across various fields such as healthcare, finance, and environmental studies.

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