Surface Fitting and Representation By Using 2D Least Squares Method in CAD Applications

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Abstract

This paper presents a general method for automatic surface fitting from scattered range data and describes the implementation of three methods for fitting surfaces: linear, quadratic and cubic. It uses a modified 2D least squares method to fitting, reconstructing and modeling several surfaces and statistical criteria to compare the three approaches. The comparison is performed using a mathematically defined data as real data obtained from the proposed models.

The method can be used in a variety of applications such as reverse engineering, automatic generating of a CAD model, etc, and it has proven to be effective as demonstrated by a number of examples using real data from mathematical functions (sine, cosine, exponential and cubic). By applying the proposed surface fitting model the standard deviation was found to be (0.04-0.26), (0.02-0.07) and (0.0-0.12) mm for linear, quadratic and cubic fitting models respectively.

Keywords: Surface Representation, Surface Fitting, Computer Graphics, Data Analysis, Least Square Fitting

استكمال و تمثيل السطوح باستخدام طريقة اصغر المربعات الثنائية في تطبيقات التصميم المعان بالحاسوب

الخلاصــة

يقدم هذا البحث طريقة عامة لاستكمال وتمثيل السطوح الهندسية باستخدام طريقة اصغر المربعات للتحليلات العددية وتطويرها لجعلها ملائمة للتطبيقات الهندسية, وقد تم اعتماد ثلاث طرق للاستكمال وهي الخطية والثنائية والتكعيبية لتوليد بيانات السطوح ومقارنة النتائج مع البيانات الحقيقية للسطوح المقترحة في هذا البحث حيث تم التطبيق لاربعة سطوح ممثلة بـ دالة الجيب و دالة الجيبتمام و دالة أسية و دالة تكعيبية وتمت المقارنة من خلال التحليل الاحصائي للبيانات و اظهرت النتائج فاعلية الطريقة المطورة المقترحة لتمثيل السطوح في تطبيقات الهندسية العكسية و التصميم المعان بالحاسوب وغيرها من التطبيقات, وعند مقارنة النتائج المختلفة وجد ان قيمة الانحراف المعياري تراوحت (0.04-0.26) , (0.02-0.07) و (0.0–0.12) ملم لدوال الاستكمال الخطي والثنائي والتكعيبي على التوالي.

Introduction

Digital surface representation from a set of three-dimensional data samples is an important issue of computer graphics that has applications in different areas of study such as engineering, geology, geography, meteorology, medicine, etc. . The digital model allows important information to be stored and analyzed without the necessity of

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working directly with the real surface [1] and [2].
Methods of approximating a continuous target function using finite measurements of the function have been extensively studied since ancient times [3]. These methods include interpolation schemes and regression schemes. In numerical schemes for solving differential equations, approximation schemes are used to reconstruct the continuous solution from its value on a finite set of grid points. Polynomial interpolation is an essential part of spectral methods and finite element methods [4] and [5].

The main objective of this work is the comparison of different methodologies to model surfaces from a set of three dimensional data samples using 2D least squares method. The basic structure used to represent the surface is the rectangular regular network, whose rectangle vertices are the sample points.

This work presents three methods for rectangular surface fitting: linear, quadratic and cubic by using a modified 2D least squares method. It also gives a visual representation and a statistical analysis of the three methods using mathematically defined functions.

Statistical analysis of the grid models:

In order to perform a statistical analysis of the surfaces rendered by the three proposed 2D least squares fitting approaches, we compared them with the original surfaces. This was achieved by comparing the regular rectangular grids created by the 2D least squares for surface fitting method with the real grids. For each point of a regular rectangular grid we can calculate the error function R_f defined as the difference between the real elevation of the function Z_f and the estimated elevation Z_i in that point. The error function is defined in equation -1-.
If there are n points representing the surface, then the average A_v, variance V_r, and standard deviation S_d of the error function E_f can be evaluated according to the equations -2-,3- and -4- [2],[7].
In this paper sine, cosine, exponential, and cubic functions data file have been used as source of real data Z_f on each point of the grid. For several tested surfaces the value of Z_i in each point of the grid, has been estimated using the linear, quadratic and cubic 2D least squares fitting models.

Methodology

The methodology of this work used to analyze and to compare the different approaches for surface fitting by using modified 2D least squares models, can divide into the following five steps:
1. Definition of the input data point set.
2. Construction of least squares models.
3. Surface fitting for a mathematically defined function.
4. Surface fitting by using 2D least squares.
5. Statistical analysis for rectangular regular grid models.

Definition of the input control point set

The first step for modeling surfaces is the definition of the input data point set that will be used to reconstruct the surfaces. This sample set must be representative of the phenomenon to be modeled [2],[7]. To compare the three initial approaches for surface fitting, linear, quadratic and cubic, four different patterns have been chosen.

The first pattern of comparison is the following mathematically defined function that will be called sine function as defined in equation -5-.

The second pattern of comparison is the following mathematically defined function that will call cosine function defined in equation -6-.

The third pattern of comparison is the following mathematically defined function that will called exponential function defined in equation -7-.

Finally, the fourth pattern of comparison is the following mathematically defined function that will call cubic function defined in equation -8-.

These test functions have continuity of degree greater than 0 that are smooth functions, are adequate to represent artificial shapes like mechanical parts.

Construction of a least squares model

The method of least squares assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared (least square error) from a given set of data [8],[9].

Suppose that the data points are $(x_1, y_1),(x_2, y_2),..........,(x_n, y_n)$, where (x) is the independent variable and (y) is the dependent variable. The fitting curve $f(x)$ has the deviation (d) from each data point, i.e. as in equation -9-.

According to the method of least squares, the best fitting curve has the property that shown in equation -10- [4].

The Linear Least-Squares

The least-squares linear method uses a straight line according to equation -11-[8],[10].

to approximate the given set of data, $(x_1, y_1),(x_2, y_2),..........,(x_n, y_n)$, where $n \geq 2$

The best fitting curve $f(x)$ has the least square deviation, i.e. equation -12-

To obtain the least square deviation, the unknown coefficients (a_0) and (a_1) must yield zero first derivatives as shown in equation -13- and -14-.

The unknown coefficients (a_0) and (a_1) can therefore be obtained.

Surface fitting using 2D least squares

Multiple regression estimates the dependent variables which may be affected by more than one control parameter (independent variables) or there may be more than one control parameter being changed at the same time [8],[11].

The assumption that the z component of the data is functionally dependent on the x- and y- components represented as in equation -15-.

For a given data set $(x_1, y_1, z_1),(x_2, y_2, z_2),..........,(x_n, y_n, z_n)$, where $n \geq 3$, the best fitting curve $f(x)$ has the least square error, i.e. equation -16-.

To obtain the least square error, the unknown coefficients a_0 , a_1 and, a_2 must yield zero first derivatives as in equations -17-,-18- and -19-.Expanding the above equations, we obtained the equations -20-,-21- and -22-

The unknown coefficients a , a_1, and a_2 can be obtained by solving the equations -20-,-21- and -22-.

In matrix form the equations -20-,-21- and- 22- can be represented as in equation -23-.

The general polynomial equation of n[th] degree can be represented as shown in equation -24- [8].

The difference between surface elevation and $Z_f(x_i,y_i)$ gives the residual $R(x_i,y_i)$ as illustrated in equation -25-.

In general form the equation of 2D least square for surface fitting can be represented as equation -26-.

The two independent variables x and y and one dependent variable z in the quadratic relationship case can be represented in equation -27- [7],[9].

The best value of $(a_0,a_1,a_2,a_3,a_4,a_5)$ are determined by the setting the sum of the square of the residual error as in equation -28-.

Differentiating each variable each coefficient $(a_0,a_1,a_2,a_3,a_4,a_5)$ and then equal to zero for min error.

The quadratic 2D least squares equation for surface fitting can be compacted in a matrix form as shown in the first matrix form.

In same sequence two independent variables x and y and one dependent variable z in cubic relationship case can be represented as in equation -29-[9],[12].

The best value of $(a_0,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9)$ are determined by the setting the sum of the square of the residual error as shown in equations -30-,-31- and -32-.

The cubic 2D least squares equation for surface fitting can be compacted in a matrix form as shown in the second matrix form.

These compact matrices for linear, quadratic and cubic of 2D least square for surface fitting models have been manipulated and implemented in the Matlab V.7 programming language in a WINDOWS XP operational system environment to reconstruct and represent several surfaces as shown in next sections.

Surface fitting for mathematically definedfunctions: Engineering surfaces can be modeled and rendered in several ways [13],[14]. In this work the linear, quadratic and cubic 2D least squares for surface fitting models were compared using the sine, cosine, exponential and cubic function. To perform this task, four rectangular grids (6x6, 8x8, 10x10 and 15x15 points) were constructed using (z) values calculated from the models, from the linear, quadratic and from cubic. In addition, four more grids were created representing the difference between the linear, quadratic and cubic with the original surface. Figures 1,2,3 and 4 show the perspective view of rectangular grids fitted by linear, quadratic and cubic 2D least squares of the four different functions. They also show the difference between those models and the model defined by the original functions as shown in the figure (1), (2), (3) and (4).

Statistical analysis for rectangular regular grid models : The statistical analysis for 2D least square fitting models (linear, quadratic and cubic) of four different real functions (sine, cosine, exponential and cubic) , according to the equations (1, 2 and 3) were performed and illustrated in the tables (1, 2, 3 and 4) respectively as shown in tables (1),(2),(3) and (4).

Results

Table 1 contains the results of the statistical analysis, in mm, of the error function defined by the linear , quadratic and cubic 2D least squares fitting models. The sample set was obtained from the sine function. Table 2 presents the statistical results, analysis, in mm, of the error function defined by the linear and quadratic and cubic fittings. The sample set was obtained from the cosine function, while the result of statistical analysis that obtained from the exponential function is illustrated in table 3. Finally , table 4 represents the result of the statistical analysis of the cubic function in mm.

A visual analysis of the 2D least square surface fitting models (sine, cosine, exponential and cubic) are illustrated in the figures 1, 2, 3 and 4, respectively.

Analysis leads to the following notes:

- An already predicted result is that the linear fitting is computationally more efficient than the quadratic and cubic fitting. This is because of the number of calculations required for each approach. For the quadratic approach, the necessity to calculate the derivatives in the samples creates a significant time overhead, as well as and more in cubic.
- Table 1 shows that an increase in the number of input samples will not improve the accuracy of the models. However, satisfactory results, depending on the requirements, can be obtained after reducing the sample set. This reduction saves memory space and can increase the speed of the programs designed to create the digital models.

From Table 1 the quadratic and cubic fitting models have the same statistical results and perform better visualization than the linear fitting for samples chosen from the sine function, this because the sine surface has continuity greater than 0.

- As shown in Table 2, statistical difference was found between the linear, quadratic and cubic fitting approaches to model cosine function, the function is continuous and it is satisfactory to use a quadratic fitting instead of high degree .
- Table 3 shows that the decrease in the number of input samples data leads to more accurate models, while the cubic interpolator gives the best results
- Table 4 shows that the cubic fitting have perfect results and the generated fitted surface model coincided with the original surface, while the increase in the number of input samples data leads to accurate models
- The cubic fitting model can be successfully used to represent most of engineering surfaces in CAD applications, but he major problem seems to be the definition of the appropriate parameters x and y to best represent the variations of the real surface.

A visual analysis of the figures (1, 2, 3 and 4) leads to the following considerations:

- From Figures 1, 2, 3 and 4, the greater the number of data set points give better appearance of the final modeled surface. In addition, these figures show the differences between the fitted models and the model defined by the mathematical functions, and give an idea of the error distribution along the surface.
- The figures also show that, for the same sample set of the functions, the model fitted by

quadratic and cubic fittings are smoother than the model fitted by a linear .

- Figure 4 reveals that we get a more natural looking surface by using cubic fitting, compared with linear and quadratic fittings, as well as the statistical analysis confirm this improvement.

Conclusions

The least square method finds the best-fit surface by minimizing the sum of the squared deviations between the input values and the calculated surface. Because this is a best-fit for the input points, typically the output surface does not match the original value at each input point. The Polynomial Order parameter controls the form of the polynomial equation, which in turn defines the complexity of the computed surface. A second-order (quadratic) polynomial equation defines a parabolic curved surface with only one sense of curvature (concave or convex). A third-order (cubic) equation allows one change in sense of curvature in any cross-section. Higher-order equations allow for increasing complexity.

To compare the surfaces fitted by linear, quadratic and cubic using 2D least square methods we implemented an algorithm to model three interpolators that fit linear, quadratic and cubic surfaces for several models. The algorithms were implemented in the Matlab programming language in a WINDOWS XP operational system environment. In addition, statistical tables were created to accomplish the visual and statistical analysis of the rendered models.

From the presented results it seem that the quality of a digital model depends on the type of surface, that's been modeled. A representation that is useful for engineering object modeling may not be suitable to the other representing forms. Quadratic fitting is recommended for accurate modeling of surfaces dominated by smoothing processes, where tend to require higher order continuity. Finally, cubic fitting is recommended for modeling surfaces change in sense of curvature and interest in visualization.

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$$R_{fi} = Z_{fi} - Z_i \;\ldots\ldots(1)$$

Where R_f is the difference between the real elevation of the function Z_f and the estimated elevation Z_i in that point .

$$A_v = \sum_{i=0}^{n} (R_{f_i} / n) \;\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$V_r = \left[(\sum_{i=0}^{n} R_{f_i}^2) - n * A_v^2 \right] / (n-1) \;\cdots(3)$$

$$S_d = \sqrt{V_r} \;\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Where A_v is the average **, *n*** represent the surface points, V_r represent the variance, and S_d represent standard deviation of the error function.

$$z = \sin(x^2 + y^2) \;\ldots\ldots\ldots\ldots(5)$$

$$z = \cos(x^2 + y) \;\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$z = (e^{\frac{-(5-x^2)}{2}}) - 0.1(e^{\frac{-(5-y^2)}{2}})(e^{\frac{-(5-y^2)}{2}}) \;\ldots\ldots\ldots\ldots(7)$$

$$z = 2x^3 - 3y^2 \;\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

$$d_1 = y_1 - f(x_1)\,,\; d_2 = y_2 - f(x_2)\,,\ldots\ldots,\; d_n = y_n - f(x_n) \;\ldots..(9)$$

Where ***d*** is the deviation between real values and fitted values for every point.

$$S = d_1^2 + d_2^2 + \ldots\ldots + d_n^2 = \sum_{i=1}^{n} [y_i - f(x_i)]^2 = \min \;\ldots\ldots\ldots..(10)$$

$$y_i = a_0 + a_1 x_i \;\ldots\ldots\ldots\ldots\ldots..(11)$$

where (x) is the independent variable and (y) is the dependent variable and (a_0) and (a_1) are unknown coefficients while all (x_i) and (y_i) are given.

$$S = \sum_{i=1}^{n}[y_i - f(x_i)]^2 = \sum_{i=1}^{n}[y_i - (a_0 + a_1 x_i)]^2 = \min \quad \ldots\ldots\ldots\ldots\ldots(12)$$

$$\frac{\partial S}{\partial a_0} = 2\sum_{i=1}^{n}[y_i - (a_0 + a_1 x_i)] = 0 \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$\frac{\partial S}{\partial a_1} = 2\sum_{i=1}^{n}x_i[y_i - (a_0 + a_1 x_i)] = 0 \ldots\ldots\ldots(14)$$

$$z = a_0 + a_1 x_i + a_2 y_i \ldots\ldots\ldots\ldots(15)$$

$$S = \sum_{i=1}^{n}[z_i - f(x_i, y_i)]^2 = \sum_{i=1}^{n}[z_i - (a_0 + a_1 x_i + a_2 y_i)]^2 = \min \ldots\ldots(16)$$

Where a_0 , a_1 and, a_2 are unknown coefficients while all x_i, y_i , and z_i are given.

$$\frac{\partial S}{\partial a_0} = 2\sum_{i=1}^{n}[z_i - (a_0 + a_1 x_i + a_2 y_i)] = 0 \ldots\ldots(17)$$

$$\frac{\partial S}{\partial a_1} = 2\sum_{i=1}^{n}x_i[z_i - (a_0 + a_1 x_i + a_2 y_i)] = 0 \ldots(18)$$

$$\frac{\partial S}{\partial a_2} = 2\sum_{i=1}^{n}y_i[z_i - (a_0 + a_1 x_i + a_2 y_i)] = 0 \ldots\ldots(19)$$

$$\sum_{i=1}^{n}z_i = a_0\sum_{i=1}^{n}1 + a_1\sum_{i=1}^{n}x_i + a_2\sum_{i=1}^{n}y_i \ldots\ldots(20)$$

$$\sum_{i=1}^{n}x_i z_i = a_0\sum_{i=1}^{n}x_i + a_1\sum_{i=1}^{n}x_i^2 + a_2\sum_{i=1}^{n}x_i y_i \ldots\ldots\ldots(21)$$

$$\sum_{i=1}^{n}y_i z_i = a_0\sum_{i=1}^{n}y_i + a_1\sum_{i=1}^{n}x_i y_i + a_2\sum_{i=1}^{n}y_i^2 \ldots(22)$$

$$\begin{pmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum y_i^2 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \sum z_i \\ \sum x_i z_i \\ \sum y_i z_i \end{pmatrix} \ldots\ldots\ldots\ldots -23-$$

…..(23)

Where n represent the number of data set points

$$z_i = S(x_i, y_i) = \sum_{j=0}^{m}a_j\sum_{i=1}^{n}p_i(x_i, y_i) \ldots \quad \ldots\ldots(24)$$

Where a_j = a_0, a_1,…., a_m are coefficients to be determined by adjustment using 2D Least square method and p_j = p_1 , p_2, …., p_n are approximately chosen functions of x and y called the based function

At each data set point.

$$\sum_{i=1}^{n}R(x_i, y_i) = Z_f(x_i, y_i) - S(x_i, y_i) = \sum_{i=1}^{n}Z_f(x_{ni}, y_{ni}) - \sum_{j=0}^{m}a_j\sum_{i=1}^{n}p_i(x_i, y_i) \ldots\ldots -25-$$

….(25)

$$R^2 = Z_f^{\,2} - Z_i^{\,2} = minimum \ldots(26)$$

$$z = f(x, y) = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^{\,2} + a_4 x_i y_i + a_5 y_i^{\,2} \ldots\ldots -27-$$

….(27)

Where the ***x*** and ***y*** are independent variables, ***Z*** dependent is variable, and the $(a_0, a_1, a_2, a_3, a_4, a_5)$ there are unknown coefficients.

$$sr = \sum_{i=1}^{n}(z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2)^2 \quad \ldots(28)$$

$$\frac{dsr}{da_0} = -2\sum_{i=1}^{n}(z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2) = 0$$

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$$\frac{dsr}{da_5} = -2\sum_{i=1}^{n} x_i y_i^2 (z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2) = 0$$

$$\sum_{i=1}^{n}\begin{pmatrix} n & x_i & y_i & x_i^2 & x_i y_i & y_i^2 \\ x_i & x_i^2 & x_i y_i & x_i^3 & x_i^2 y_i & x_i y_i^2 \\ y_i & x_i y_i & y_i^2 & x_i^2 y_i & x_i y_i^2 & y_i^3 \\ x_i^2 & x_i^3 & x_i^2 y_i & x_i^4 & x_i^3 y_i & x_i^2 y_i^2 \\ x_i y_i & x_i^2 y_i & x_i y_i^2 & x_i^3 y_i & x_i^2 y_i^2 & x_i y_i^3 \\ y_i^2 & x_i y_i^2 & y_i^3 & x_i^2 y_i^2 & x_i y_i^3 & y_i^4 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \sum_{i=1}^{n}\begin{pmatrix} z_i \\ x_i z_i \\ y_i z_i \\ x_i^2 z_i \\ x_i y_i z_i \\ y_i^2 z_i \end{pmatrix}$$

$$\sum_{i=1}^{n}\begin{pmatrix} n & x_i & y_i & x_i^2 & x_i y_i & y_i^2 & x_i^2 y_i & x_i y_i^2 & x_i^3 & y_i^3 \\ x_i & x_i^2 & x_i y_i & x_i^3 & x_i^2 y_i & x_i y_i^2 & x_i^3 y_i & x_i^2 y_i^2 & x_i^4 & x_i y_i^3 \\ y_i & x_i y_i & y_i^2 & x_i^2 y_i & x_i y_i^2 & y_i^3 & x_i^2 y_i^2 & x_i y_i^3 & x_i^3 y_i & y_i^4 \\ x_i^2 & x_i^2 & x_i^2 y_i & x_i^4 & x_i^3 y_i & x_i^2 y_i^3 & x_i^4 y_i & x_i^3 y_i^2 & x_i^5 & x_i^2 y_i^3 \\ x_i y_i & x_i^2 y_i & x_i y_i^2 & x_i^3 y_i & x_i^2 y_i^2 & x_i y_i^3 & x_i^3 y_i^2 & x_i^2 y_i^3 & x_i^4 y_i & x_i y_i^4 \\ y_i^2 & x_i y_i & y_i^3 & x_i^2 y_i^2 & x_i y_i^3 & y_i^4 & x_i^2 y_i^3 & x_i y_i^4 & x_i^3 y_i^2 & y_i^5 \\ x_i^2 y_i & x_i^3 y_i & x_i^2 y_i^2 & x_i^4 y_i & x_i^3 y_i^2 & x_i^2 y_i^3 & x_i^4 y_i^2 & x_i^3 y_i^3 & x_i^5 y_i & x_i^2 y_i^4 \\ x_i y_i^2 & x_i^2 y_i^2 & x_i y_i^3 & x_i^3 y_i^2 & x_i^2 y_i^3 & x_i y_i^4 & x_i^3 y_i^3 & x_i^2 y_i^4 & x_i y_i^5 & x_i^4 y_i^2 \\ x_i^3 & x_i^4 & x_i^3 y_i & x_i^5 & x_i^4 y_i & x_i^2 y_i^3 & x_i^5 y_i & x_i^4 y_i^2 & x_i^6 & x_i^3 y_i^3 \\ y_i^3 & x_i y_i^3 & y_i^4 & x_i^2 y_i^3 & x_i y_i^4 & y_i^5 & x_i^2 y_i^4 & x_i y_i^5 & x_i^3 y_i^3 & y_i^6 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix} = \sum_{i=1}^{n}\begin{pmatrix} z_i \\ x_i z_i \\ y_i z_i \\ x_i^2 z_i \\ x_i y_i z_i \\ y_i^2 z_i \\ x_i^2 y_i z_i \\ x_i y_i^2 z_i \\ x_i^3 z_i \\ y_i^3 z_i \end{pmatrix}$$

$$z = f(x, y) = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 x_i y_i + a_5 y_i^2 + a_6 x_i^2 y_i + a_7 x_i y_i^2 + a_8 x_i^3 + a_9 y_i^3$$

……(29)

$$sr = \sum_{i=1}^{n} (z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2 - a_6 x_i^2 y_i - a_7 x_i y_i^2 - a_8 x_i^3 - a_9 y_i^3)^2$$

….(30)

$$\frac{dsr}{da_0} = -2\sum_{i=1}^{n} (z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2 - a_6 x_i^2 y_i - a_7 x_i y_i^2 - a_8 x_i^3 - a_9 y_i^3) = 0$$

…(31)

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$$\frac{dsr}{da_9} = -2\sum_{i=1}^{n} x_i y_i^3 (z_i - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 x_i y_i - a_5 y_i^2 - a_6 x_i^2 y_i - a_7 x_i y_i^2 - a_8 x_i^3 - a_9 y_i^3) = 0$$

..(32)

Table 1.Statistical analysis of the error function for 2D least squares surface fitting models of the sine function

Fitting Order	Data set points (rectangular grid)	Average A_v	Variance V_r	Standard deviation S_d
$z = \sin(x^2 + y^2)$				
Linear	36	0.2601	0.0276	0.1660
Quadratic	36	0.1388	0.0033	0.0572
Cubic	36	0.1388	0.0033	0.0572
Linear	64	0.2636	0.0300	0.1732
Quadratic	64	0.1282	0.0048	0.0689
Cubic	64	0.1282	0.0048	0.0689
Linear	100	0.2636	0.0328	0.1810
Quadratic	100	0.1291	0.0040	0.0632
Cubic	100	0.1291	0.0040	0.0632
Linear	225	0.2634	0.0368	0.1919
Quadratic	225	0.1240	0.0049	0.0701
Cubic	225	0.1240	0.0049	0.0701

Table 2.Statistical analysis of the error function for 2D least squares surface fitting models of the cosine function.

Fitting Order	Data set points (rectangular grid)	Average A_v	Variance V_r	Standard deviation S_d
$z = \cos(x^2 + y)$				
Linear	36	0.2574	0.0294	0.1714
Quadratic	36	0.0482	0.0009	0.0307
Cubic	36	0.1865	0.0155	0.1243
Linear	64	0.2554	0.0258	0.1608
Quadratic	64	0.0474	0.0011	0.0331
Cubic	64	0.1757	0.0155	0.1244
Linear	100	0.2552	0.0241	0.1554
Quadratic	100	0.0476	0.0011	0.0326
Cubic	100	0.1749	0.0140	0.1183
Linear	225	0.2547	0.0230	0.1517
Quadratic	225	0.0469	0.0011	0.0332
Cubic	225	0.1693	0.0142	0.1192

Table 3.Statistical analysis of the error function for 2D least squares surface fitting models of the exp. function.

Fitting Order	Data set points (rectangular grid)	Average A_v	Variance V_r	Standard deviation S_d
$z = e^{\frac{-(5-x^2)}{2}} - 0.1(e^{\frac{-(5-y^2)}{2}})(e^{\frac{-(5-y^2)}{2}})$				
Linear	36	0.0884	0.0016	0.0402
Quadratic	36	0.0277	0.0004	0.0205
Cubic	36	0.0111	0.0001	0.0103
Linear	64	0.0883	0.0019	0.0431
Quadratic	64	0.0312	0.0006	0.0252
Cubic	64	0.0194	0.0002	0.0151
Linear	100	0.0904	0.0023	0.0475
Quadratic	100	0.0356	0.0009	0.0300
Cubic	100	0.0275	0.0004	0.0198
Linear	225	0.1004	0.0036	0.0603
Quadratic	225	0.0501	0.0018	0.0419
Cubic	225	0.0470	0.0010	0.0319

Table 4.Statistical analysis of the error function for 2D least squares surface fitting models of the cubic function.

Fitting Order	Data set points (rectangular grid)	Average A_v	Variance V_r	Standard deviation S_d
$z = 2x^3 - 3y^2$				
Linear	36	0.3365	0.0710	0.2665
Quadratic	36	0.0495	0.0006	0.0254
Cubic	36	0.0000	0.0000	0.0000
Linear	64	0.3188	0.0599	0.2447
Quadratic	64	0.0509	0.0006	0.0244
Cubic	64	0.0000	0.0000	0.0000
Linear	100	0.3066	0.0546	0.2337
Quadratic	100	0.0514	0.0006	0.0242
Cubic	100	0.0000	0.0000	0.0000
Linear	225	0.2891	0.0488	0.2208
Quadratic	225	0.0512	0.0007	0.0257
Cubic	225	0.0000	0.0000	0.0000

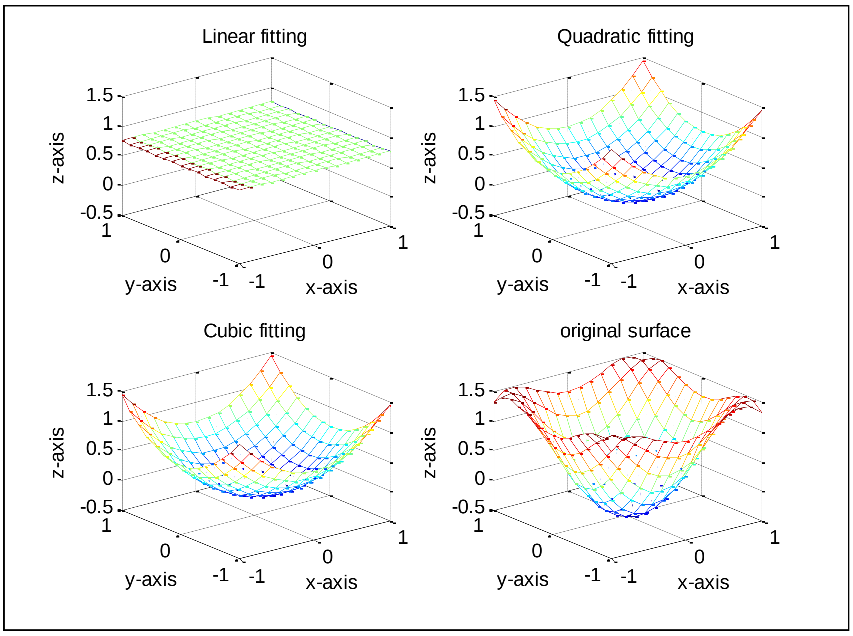

Figure (1) Fitted surface of the sine function (15x15 points)

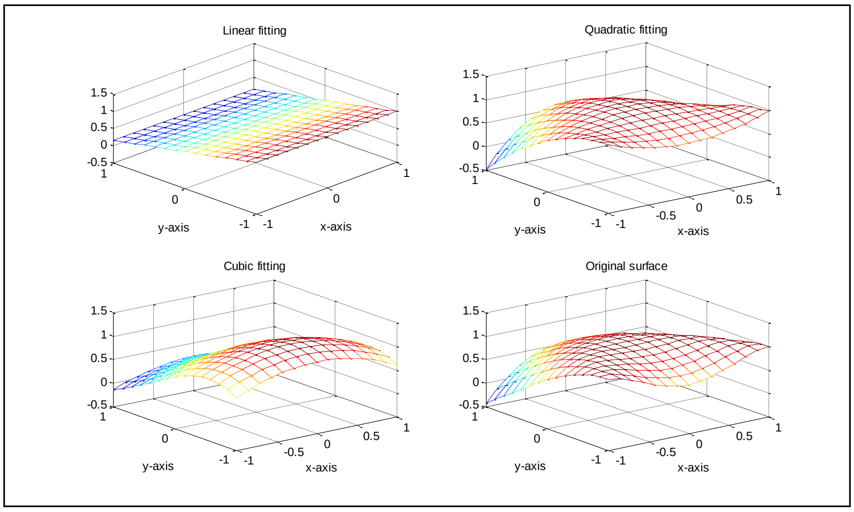

Figure (2). Fitted surface of the cosine function (15x15 points)

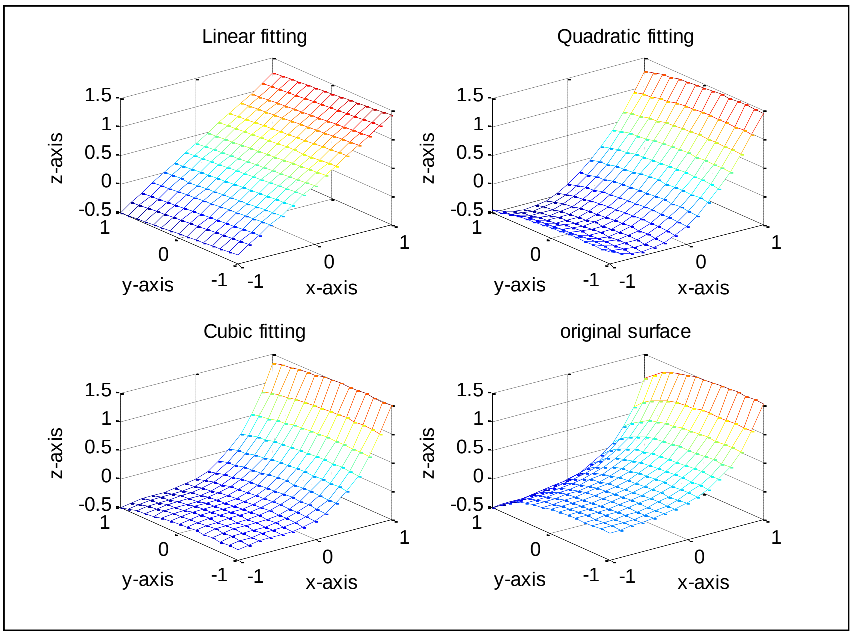

Figure (3) Fitted surface of the exponential function (15x15 points)

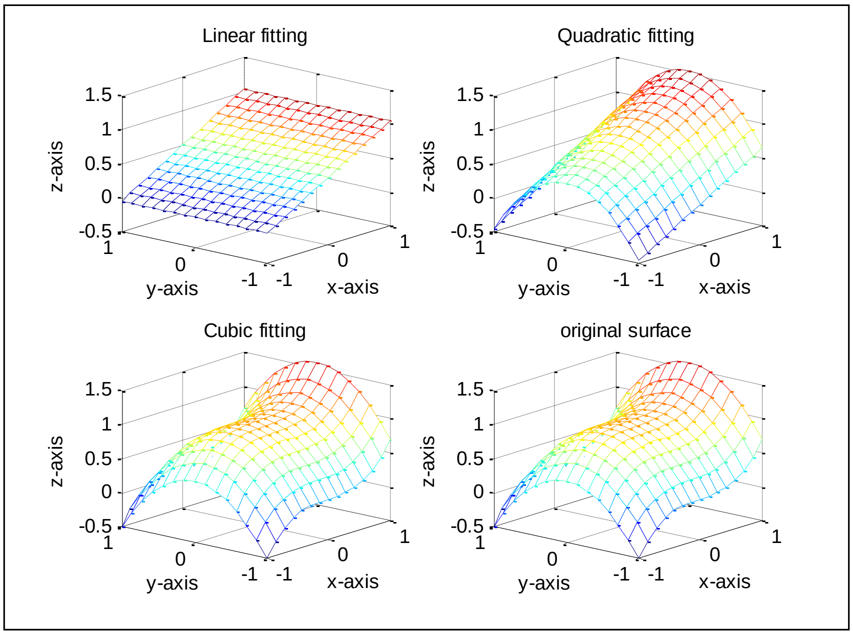

Figure (4) Fitted surface of the cubic function (15x15 points)