Numerical Simulation for Laminar Natural Convection with in a Vertical Heated Channel

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Abstract

A numerical investigation has been performed to estimate the induced flow rate for a laminar natural convection flow of air in a vertical channel with isothermal walls conditions (one hot and another is cold). The two-dimensional governing equations have been solved using finite volume method. The coupling between the continuity and momentum equations is solved by using the SIMPLE algorithm. After the validity of the present code by comparing results with these of previous study for the similar conditions, solutions have been obtained for Prandtl number of 0.7, aspect ratio of (4 to 20) and wall temperature difference of (10 to 30). The effects of the changes in these parameters on the induced flow rate, Grashof number and flow patterns within the channel have been predicted. A mathematical form of flow rate correlation is presented for these cases.

Keywords: Natural convection, vertical channel, laminar flow, numerical analysis.

التمثيل العددي للحمل الطبيعي الطباقي خلال مجرى عمودي مسخن

الخلاصة

تم تقديم دراسة عددية لتخمين معدل تدفق الهواء للجريان الطباقي خلال مجرى عمودي بتاثير الحمل الحر حيث فرض ان الجدارين عند درجة حرارة ثابتة احدهما ساخن والاخر بارد. تم حل المعادلات الثنائية البعد التي تمثل الجريان باستخدام طريقة الحجوم المحددة . الترابط بين معادلة الاستمرارية ومعادلات الزخم تم حسمه باستخدام (SIMPLE algorithm). بعد اجراء التحقق من وثوقية البرنامج المعد في هذه الدراسة بواسطة مقارنة النتائج العددية الحالية مع نتائج لباحث سابق لنفس الظروف، بعد التحقق من وثوقية البرنامج الحالي استخدم البرنامج لدراسة الجريان عند عدد (Pr) يساوي (0.7) وعند (aspect ratio) متغير يتراوح بين (4-20) وفرق درجة الحرارة للجداريين تتراوح بين (10-30). تم دراسة تاثير تلك المتغيرات على معدل وشكل الجريان وعلى عدد (Gr) للحالات المدروسة وتم التوصل الى صيغة رياضية تمثل العلاقة بين معدل الجريان وتلك المتغيرات المتغيرات

Nomenclature

A	aspect ratio (L/S), area for control volume face (m)
a	coefficient for the discretization equation
c_p	specific heat (J/kg.K^o)
D	diffusion term (kg/m.s)
F	convection term (kg/m.s)
g	gravitational acceleration (m/s^2)
Gr	Grashof number, $\left(\dfrac{g\beta\Delta T S^3}{\nu^2}\right)$
K	thermal conductivity (W/m K)
L	channel height (m)
P	pressure (N/m^2)
Q	induced flow rate (m^3/s)
S	channel width (m), source term for discretization equation.

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Γ_Φ diffusion coefficient used in discretization equation.
μ dynamic viscosity (kg/m s)
ν kinematic viscosity (m^2/s)
ρ density of the fluid (kg/m^3)
θ non-dimensional temperature (T/T_H).
θ_1 non-dimensional temperature $(T-T_\infty/T_H-T_\infty)$.
δ_x , δ_y grid space (m)

Subscripts

E,W,N,S neighboring grids; east, west, north, south.
e,w,n,s control volume faces.

T temperature (K)
ΔT wall temperature difference.
u velocity in x direction (m/s)
V non-dimensional axial velocity component (v/v_{in})
v velocity in y direction (m/s)
X non-dimensional channel width (x/S)
x coordinate (m)
y coordinate (m)

Greek symbols

β volumetric coefficient of thermal expansion (K^{-1})
Φ dependent variable used in discretization equation.

Introduction

Natural convection in smooth vertical channels occurs usually when the two plates forming the channel are heated. As one (or both) of the two walls is maintained at a temperature different from the ambient, flow occurs due to buoyancy effects. Depending on the wall temperature, the flow can be laminar or turbulent. The interaction of the flows close to both walls tends to increase the level of velocity gradient. Accordingly, the Rayleigh number (Ra) at which flows become turbulent ($Ra>10^{10}$) in this geometry is different from that for flows over vertical flat plates.

Natural convection in smooth vertical channels received the attention and interest of several researchers. This has been associated with increasing efforts to enhance the cooling of electronic components. The problem is also relevant to air conditioning systems, solar heating systems and nuclear fuel elements. Understanding the flow pattern in these equipment may significantly improve their design and, consequently, the performance of their operation.

Early Laminar natural convection heat transfer in smooth parallel-walled vertical channels has been investigated theoretically and experimentally. A detailed study of the thermal characteristics of cooling by natural convection in smooth parallel-walled vertical channels using air as the fluid was documented by Etenbss [1]. Numerical studies, using the finite difference method for the development of free convection in air between heated vertical parallel plates were performed by Bodia and Osterle [2]. Aung and Worku [3] presented a numerical study for the effects of buoyancy on the hydrodynamic and thermal parameters in the laminar, vertically upward flow of a gas in a parallel plate channel.

Laminar natural convection heat transfer in smooth parallel-walled vertical channels has been investigated experimentally and numerically [4-5]. An experimental investigation of natural convection of air in a vertical channel [4] and vertical finite-length channel in free space [6] were conducted. Correlations were obtained for the average Nusselt number in terms of Rayleigh number and channel geometry. Numerical investigation on the transport mechanism of laminar natural convection motion of a Trombe-wall channel and turbulent combined convection between two vertical parallel plates that were uniformly healed were performed by Inagaki and Komori [7]. The results showed significant deviation between the obtained Nusselt number and previous predictions for the case of semi-infinite plates due to the effects of vertical thermal diffusion and free space stratification. The heat transfer characteristics of natural convection flow in a rectangular cross-sectional vertical finned channel were experimentally investigated by Daloglu and Ayhan [5]. The results show that the Nusselt number for finned channels is less than that for smooth channels for values of modified Rayleigh number (defined as Rayleigh number multiplied by the ratio of the channel width over the channel length) ranging from 20 to 90 and an aspect ratio (length-to-width) of 66.

In the beginning of the last decade Habib et al. [8] presented the results of velocity measurements of natural convection in symmetrically and asymmetrically heated vertical channels. In the first case of symmetrical flow, the two channel plates were both kept at a constant temperature higher than the ambient. In the second case of asymmetrical flow, one plate was kept at a temperature higher than the ambient temperature while the other plate was kept at a temperature lower than the ambient temperature. The velocity measurements were performed using a Laser Doppler Anemometer. Anwar [9] investigated the problem of buoyancy driven turbulent natural convection flow in a vertical channel numerically. The investigation is limited to vertical channels of uniform cross-section (parallel-plate channels) but with different modes of heating. Singh and Paul [10] had studied the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls when convection between the vertical parallel walls is set up by a change in the temperature of the walls as compared to the fluid temperature. A non-dimensional parameter is used in order to characterize the vertical wall temperature with respect to the fluid temperature.

During the last three years, a number of studies involving numerical and experimental measurements of heat transfer in laminar free convection flows between two vertical plates were reported. Tanda [11] investigated the effect of repeated horizontal protrusions on the free-convection heat transfer in a vertical, asymmetrically heated, channel experimentally. The protrusions have a square section and are made of a low-thermal-conductivity material. Experiments were conducted by varying the number of the protrusions over the heated surface (whose height was held fixed) and the aspect ratio of the channel. The convective fluid was air and the wall-to-ambient air temperature difference was set equal to 45 K. Ospir et al.[12] represented an experimental and numerical dynamical study of the flow in an asymmetrically heated vertical plane channel. The experiments are carried out in water for modified Rayleigh numbers in a range corresponding to the boundary layer flow regime. Mokni [13] expressed a numerical investigation of mixed convection in a vertical heated channel. This flow results from the mixing of the up-going fluid along walls of the channel with the one issued from a flat nozzle located in its entry section. The fluid dynamic and heat-transfer characteristics of vented vertical channels were investigated for constant heat-flux boundary conditions. Cakar [14] studied the steady-state natural convection from vertically placed rectangular fins numerically by means of a commercial CFD program called ICEPAK. The effects of geometric parameters of fin arrays on the performance of heat dissipation from fin arrays are examined. A combined convection process between two parallel vertical infinite walls, containing an incompressible viscous fluid layer and a fluid saturated porous layer had been presented analytically by Srivastava and Singh [15]. Recently Qing et al. [16] presented the experimental research on the steady laminar natural convection heat transfer of air in three vertical thin rectangular channels with different gap clearance. The much higher ratio of width to gap clearance (60–24) and the ratio of length to gap clearance (800–320) make the rectangular channels similar with the coolant flow passage in plate type fuel reactors.

Now, the focal point in this study is to predict a mathematical correlation for induced flow rate in terms of Grashof number, walls temperature differences and channel aspect ration. A numerical investigation of two-dimensional flow of air in a vertical channel with isothermal walls is presented. A computer program in FORTRAN-90 is written to solve a set of the partial differential equations that govern the flow field. The governing equations have been solved using finite volume method. Solutions have been obtained for Prandtl number of 0.7, aspect ratio of (4-20) and wall temperature difference of (10-30). The effects of the changes in these parameters on the induced flow rate and Grashof number for flow patterns within the channel have been studied and correlated.

Problem Formulations

Consider a two dimensional vertical channel and Cartesian coordinates *x* and *y*, as shown in Fig.(1) Channel height (L) is chosen much larger than its width (S) to ensure the laminar flow and fully developed flow conditions at the channel exit. As a result of the nature of flow within the channel a two dimensional analysis is employed Both ends of the channel are open to the ambient with temperature T_0. Vertical walls of the channel are kept at uniform temperature of (hot) T_H and (cold) T_C ,which is larger than the ambient temperature. The effects of compressibility and viscous dissipations are neglected due to low speed flows associated with laminar flow condition. Under the Boussinesq approximation, the governing equations for two dimensional, steady, and laminar flow are the following:

Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

x-Momentum:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad (2)$$

y-Momentum:

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\rho g\beta(T-T_o) \quad (3)$$

Energy:

$$\left(u\frac{\partial(\rho c_p T)}{\partial x}+v\frac{\partial(\rho c_p T)}{\partial y}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \quad (4)$$

Where β is volumetric thermal expansion coefficient, μ is the dynamic viscosity, c_p is the specific heat at constant pressure and k is thermal conductivity.

Boundary Conditions

Fig 1. Shows the geometry under consideration in the present investigation with axis system. Both walls are maintained at uniform temperature as T_H for left hand side wall, while the other one is T_C. The fluid is assumed to enter into the channel from lower at ambient temperature T_o and ambient pressure. At the upper boundary, the stream wise variation of the velocity and temperature are set to be smooth exit (gradient zero). The gravity acceleration (g) acts vertically downwards. No slip boundary condition is applied for velocity components at vertical walls.

Boundary conditions can be summarized by the following equations:

$$0\le x\le S \text{ and } y=0$$
$$\frac{\partial u}{\partial y}=0 \quad ,\frac{\partial v}{\partial y}=0 \quad ,\frac{\partial T}{\partial y}=0$$
$$0\le x\le S \text{ and } y=L$$
$$\frac{\partial u}{\partial y}=0 \quad ,\frac{\partial v}{\partial y}=0 \quad ,\frac{\partial T}{\partial y}=0 \quad (5)$$
$$0\le y\le L \text{ and } x=0$$
$$u=0 \quad ,v=0 \quad ,T=T_H$$
$$0\le y\le L \text{ and } x=S$$
$$u=0 \quad ,v=0 \quad ,T=T_C$$

Numerical Solution

The methodology used to solve Eqs. (1-4) and the associated boundary conditions (5) is based on the finite volume technique with staggered grid arrangement. The discretization procedure gives an algebraic equation which are solved using a Semi-Implicit Procedure (SIP) solver. The pressure and velocities are linked by the SIMPLE algorithm. The SIMPLE algorithm is a general algorithm of elliptic fluid flow and heat transfer problems. It is based on the finite-difference scheme developed by Patankar and Spalding [17], which was formulated to solve the conservation laws written in the following general form:

$$\frac{\partial(\rho u\phi)}{\partial x}+\frac{\partial(\rho v\phi)}{\partial y}=\frac{\partial}{\partial x}\left[\Gamma_\phi\left(\frac{\partial\phi}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\Gamma_\phi\left(\frac{\partial\phi}{\partial y}\right)\right]+S_\phi \quad (6)$$

where ϕ is called the dependent variable while Γ_ϕ and S_ϕ are the diffusion coefficient and the source term, respectively.

Integrating equation (6) over the typical CV around node P, will deduce the following equation:

$$a_P \phi_P = \sum_{nb} a_{nb} \phi_{nb} + S_\phi \qquad (7)$$

Where :

$$a_P = \sum_{nb} a_{nb} ,$$

$$a_E = \max[-F_e, (D_e - 0.5F_e), 0]$$

$$a_W = \max[F_w, (D_w + 0.5F_w), 0] \qquad (8)$$

$$a_N = \max[-F_n, (D_n - 0.5F_n), 0]$$

$$a_S = \max[F_s, (Ds + 0.5F_s), 0]$$

where (F and D) represent the strength of the convection and diffusion term for the control volume respectively, which are equal to:

$$F_e = (\rho u)_e A_e , F_w = (\rho u)_w A_w$$

$$F_n = (\rho v)_n A_n , F_s = (\rho v)_s A_s$$

$$D_e = \frac{\Gamma_e}{\delta x_{EP}} A_e , D_w = \frac{\Gamma_w}{\delta x_{PW}} A_w$$

$$D_n = \frac{\Gamma_n}{\delta y_{NP}} A_n, \; D_s = \frac{\Gamma_s}{\delta y_{PS}} As (9)$$

The coefficients (a_n) are derived using a hybrid scheme of Patankar [18] .

Where P_r is Prandtl number. The solution domain is extended in the x and y-directions as shown in Fig. 1. The final computations are carried out for a grid containing 40*40 nodal points. Fig (2) presents a flow chart for the present code.

Validity Verification

The code which developed in the present study has been validated by comparing the results obtained with those of Aung and Worku [3]. The non-dimensional outlet velocity (V) and temperature θ_1 profiles are compared for an aspect ratio (A) of (4) and G_r/Re = 250, and there is a good agreement as shown in Fig.3 .

Results and Discussions

In the present study the effect of some parameters on the behavior of natural convection flow within a channel is studied, thus the discussion of results will be summarized as follow.

Aspect ratio influence

The effect of aspect ratio on the behavior of flow within the channel is presented in three sets of figures. Figs. (4a to 4e) show the non-dimensional temperature distribution (θ) within the channel for different aspect ratio. Figs. (5a to 5e) represent velocity vector within the channel at the same conditions for the first set of figures. While the third set of figures (6a to 6e) illustrate the non-dimensional axial velocity component at channel exit. From these figures the uniform temperature distribution is seen for a larger aspect ratio, while there is a some deformation occurs when the aspect ration is decreased, and there is a strong acceleration of the fluid near the heated wall whereas a deceleration near the cold wall. The negative velocities indicating the recirculation zone. The recirculation zone is created by cold fluid which tends to descend with greater density than the hot fluid.

Wall temperature difference influence

As discussed in the aspect ratio influence there is a three set of figures illustrate the effect of wall temperature difference on the behavior of flow within the channel, Figs. (4f to 4h), (5f to 5h) and (6f to 6h). From these figures, it will be noted that the changes in wall temperature difference have a major influence on the flow structure as well as the change of aspect ratio. The negative values of (V) which represent the reverse flow are decreasing with decreasing the temperature difference of the channel walls and it will be vanished when the walls temperature are equal as shown in Fig. (6h). The velocity magnitude decreasing with decreasing the walls temperature difference as a result of the decreasing in the Grashof number.

Finally, Figs. 7,8 and 9 illustrate that the induced flow increased with increasing these factors with aid of Grashof number. With aid of these figures the induced flow rate can be correlated in terms of these factors. The correlation can be presented by the equation of forms:

$$Q = a\, Gr^{b}\ S^{c}\ \Delta T^{d} \ldots \quad (8)$$

Employing the least-squares techniques the overall correlation equation is given by:

$$Q = 0.0016\, Gr^{0.1198} S^{0.1892} \Delta T^{0.4214} \quad (9)$$

From the above equation it can be noted that the wall temperature difference has the major effect on the induced flow rate as shown in Figs. (7,8 and 9).

Conclusions

A numerical study of two-dimensional laminar convection flow of air in a vertical channel with isothermal walls is presented. The governing equations have been solved using finite volume method with staggered grid arrangement. Solutions have been obtained for Prandtl number of 0.7, aspect ratio of (4-20) and wall temperature difference of (10-30). The effects of the changes in these parameters on the induced flow rate and Grashof number for flow patterns within the channel have been studied. A correlation of flow rate in terms of these parameters is presented.

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Table 1. Dependent variables ϕ, corresponding Diffusion coefficient and Source term

Equation	ϕ	Γ_ϕ	S_ϕ
Continuity	1	0	0
x-momentum	u	μ	$-\frac{\partial p}{\partial x}$
y-momentum	v	μ	$-\frac{\partial p}{\partial y} + \rho g \beta (T - T_o)$
Energy	T	$\mu / \Pr$	0

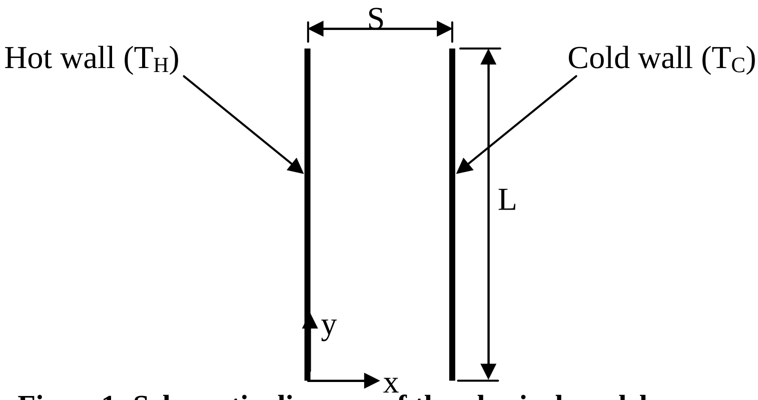


Figure1: Schematic diagram of the physical model.

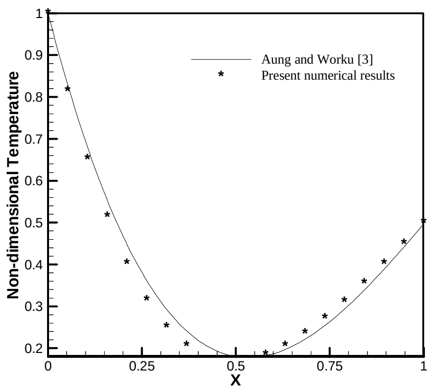
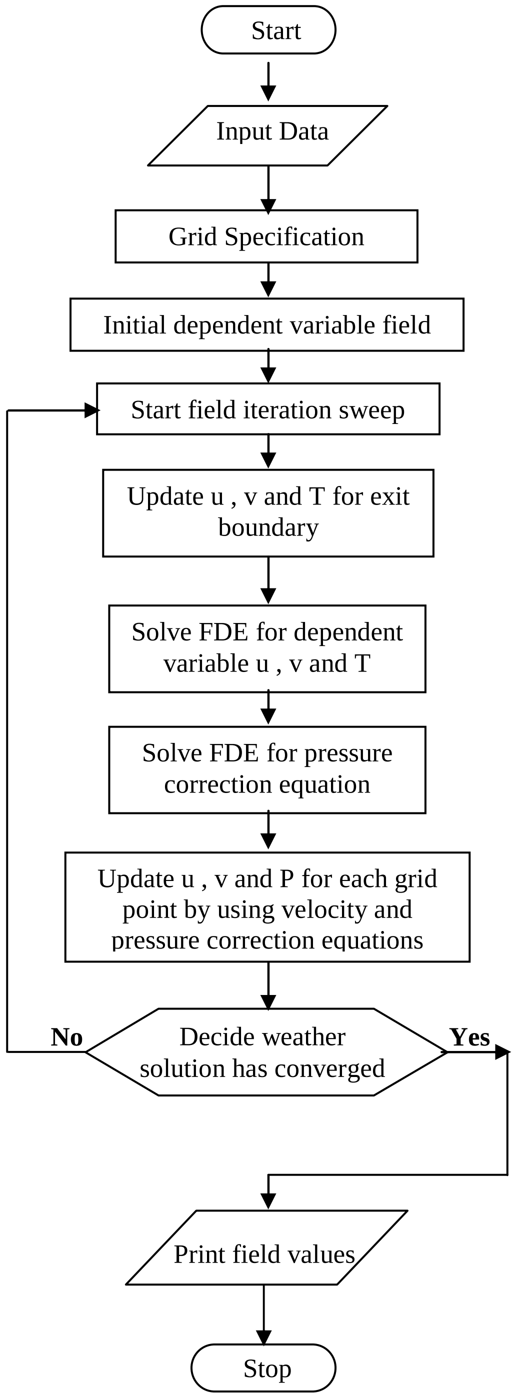
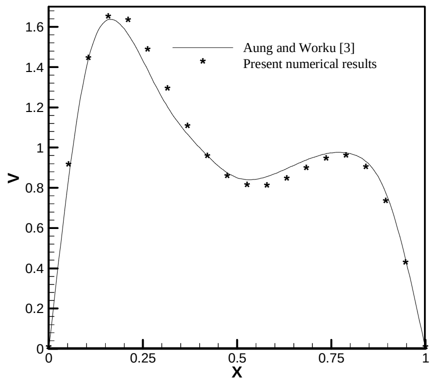


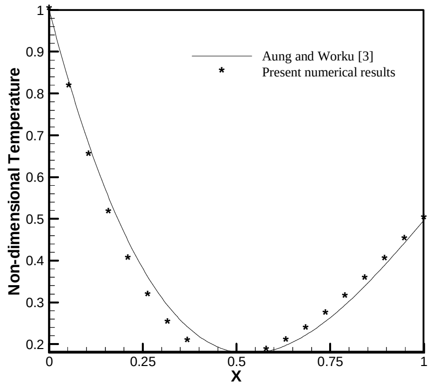
Figure2: Flow chart for present code.



Figure(2): Flow chart for present code.



(3a)



(3b)

Figure 3: Comparison of the present numerical results with those of Aung and Worku [3].

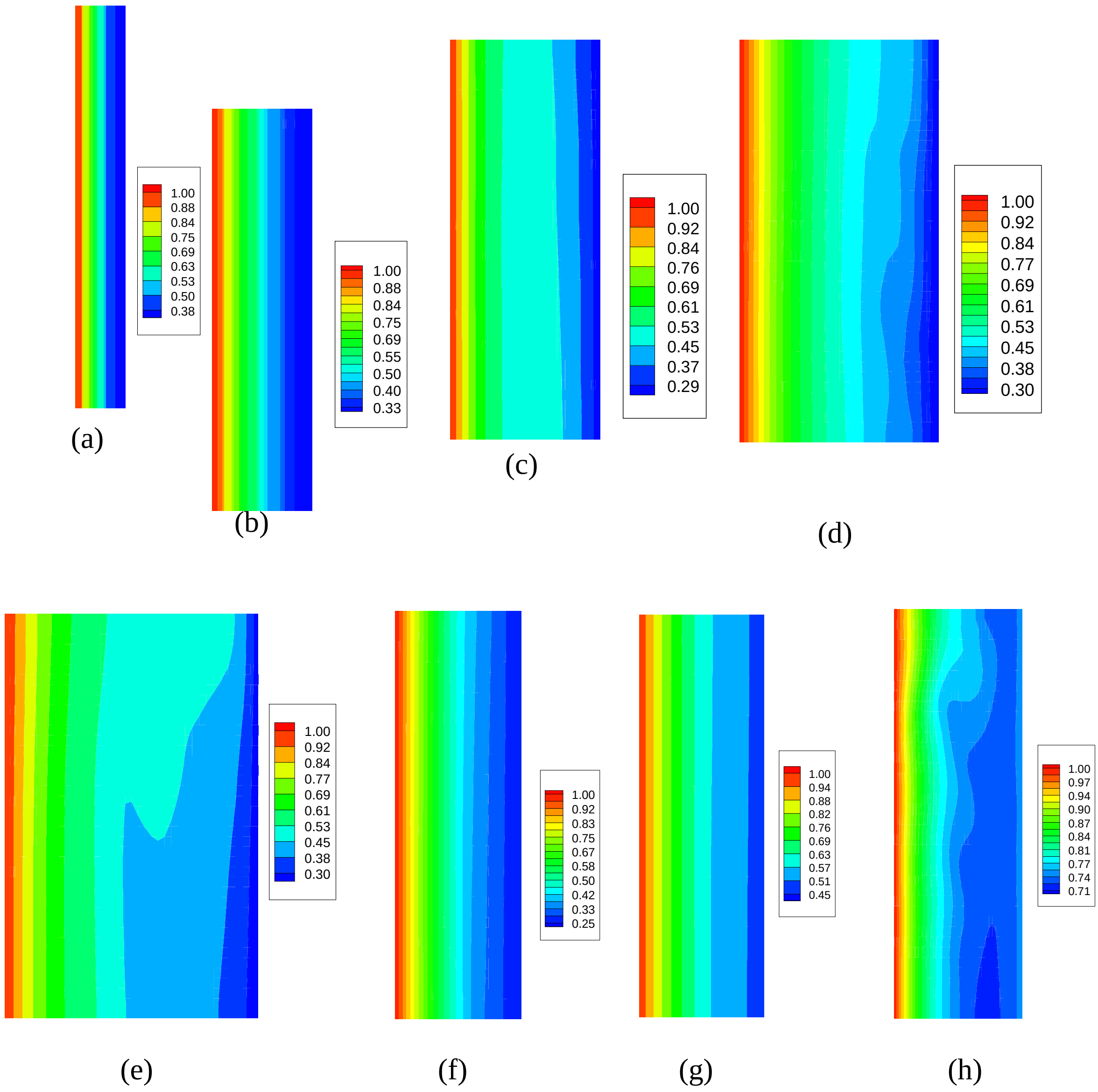


Figure 4. Non-dimensional temperature contour (θ) at $\Delta T = 30$ for (a) A=20, (b) A=10, (c) A=6.6, (d) A=5, (e) A=4, and $\Delta T = 30, 20 \; and \; 10$ for A= 8 in f, g and h

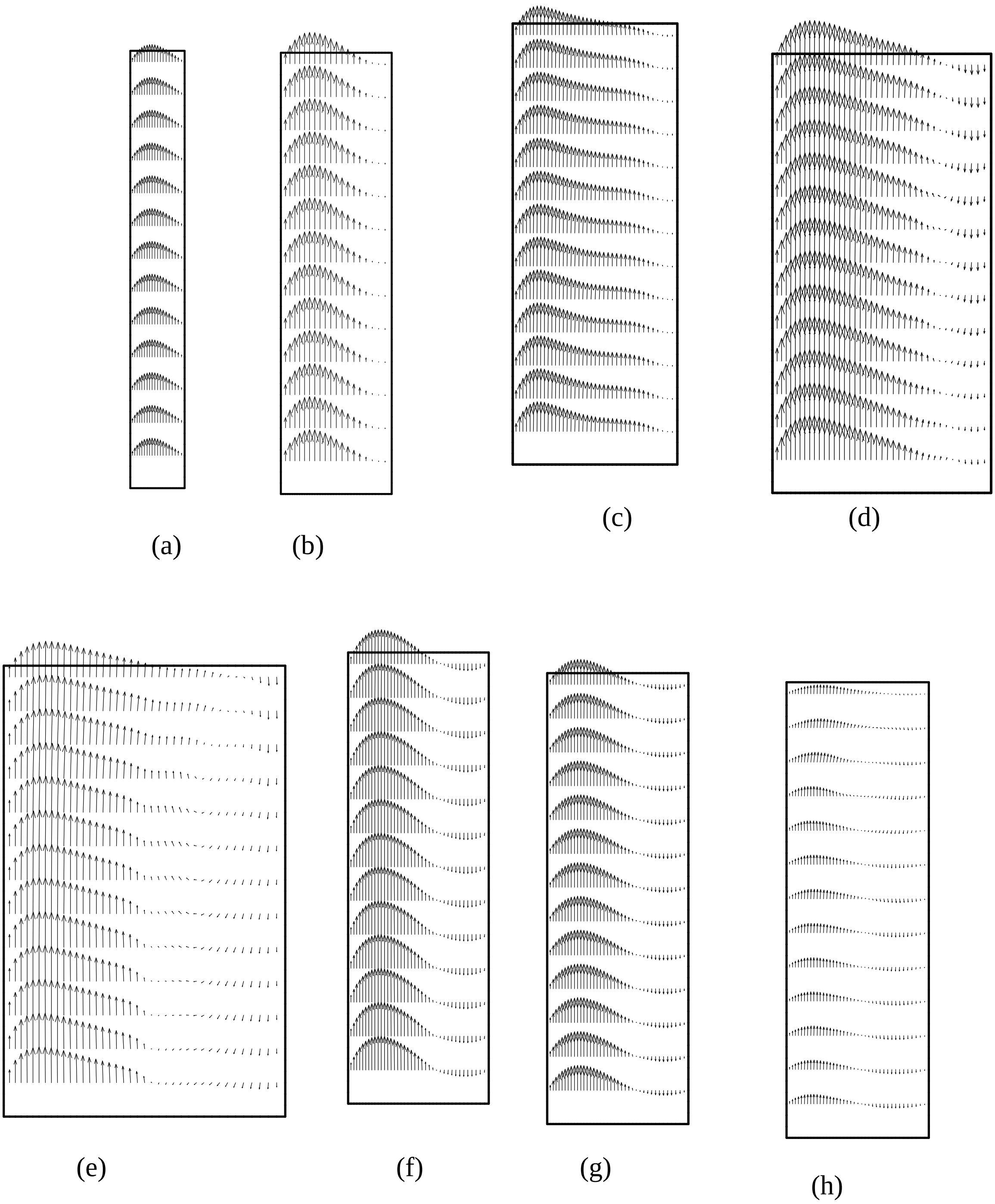


Figure 5. Velocity vector within the channel at $\Delta T = 30$ for (a) A=20, (b) A=10, (c) A=6.6, (d) A=5, (e) A=4, and $\Delta T = 30, 20$ *and* 10 for A= 8 in f, g and h respectively.

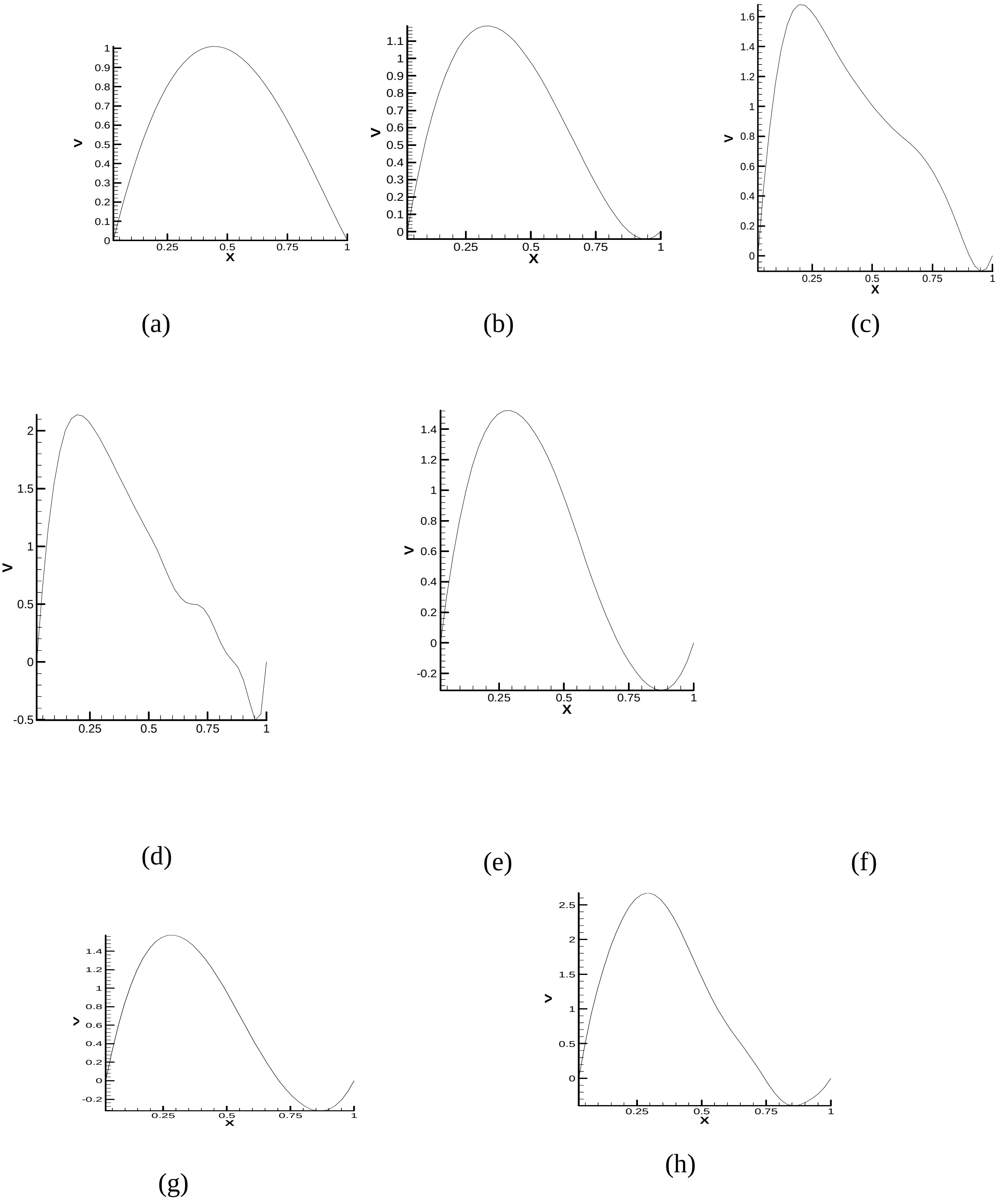


Figure 6. Non-dimensional axial velocity component at exit (V) at $\Delta T = 30$ for (a) A=20, (b) A=10, (c) A=6.6, (d) A=5, (e) A=4, and $\Delta T = 30$,20 and 10 for f, g and h respectively.

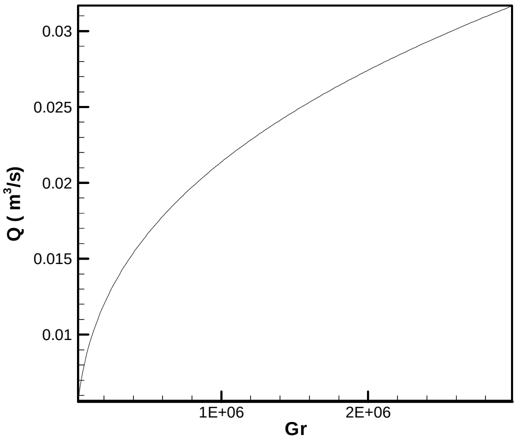


Figure7: The relation between induced flow rate with Grashof number.

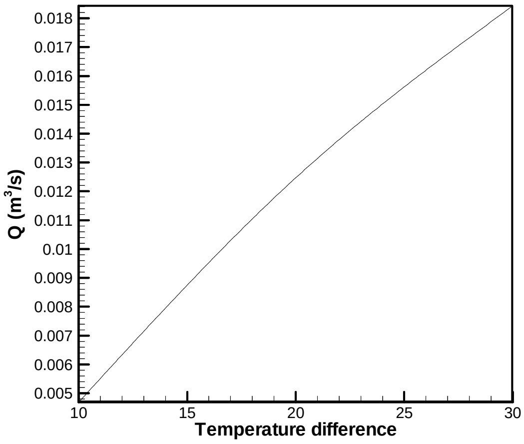


Figure8: The relation between induced flow rate with the temperature difference of the walls.

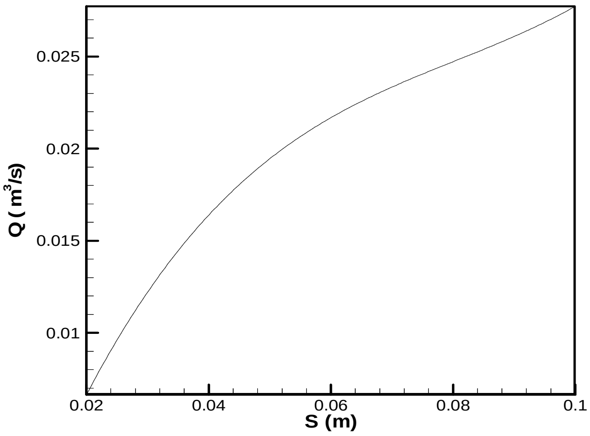


Figure 9: The relation between induced flow rate with the width of the channel.