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Development of Algorithms to Represent Intermediate Layers for Machining Sculptured Surfaces

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Abstract

The objective aim of this research is to develop an algorithm for design and manufacture sculptured surfaces that are common in a wide variety of products such as dies, automobile, and aircrafts components. In the design stage Bezier technique has been used to represent the desire surface. In rough-machining stage, the number of intermediate layers depend on the geometry of the desired surface and on the maximum allowable depth of cut, an algorithm has been proposed to represent these layers, whereas another efficient algorithm has been proposed to represent the semi-finished layer depends on the tangents and normal vectors along all the points of the desired surface to create the offset surface. The desired surface generated points using Bezier technique are used as cutter location points for the finish machining tool path. Flat end mill (φ 12mm) has been used for the intermediate stages machining (roughing), while (φ 12mm) ball end mill have been used for both semi-finish and finish machining. The developed algorithm have been tested by several designed sculptured surfaces, its proved good flexibility and efficiently in all of its stages, the results have been implemented for machining one of these surfaces [fifteen intermediate layers, semi-finished layer, and finished] using 3-axis vertical CNC machine. The proposed rough machining algorithm reduces the machining time as compared with contour tool path by 15% for case study two and reduces the NC file size 52% for case study three.

Keywords: Sculptured surfaces, Offset surfaces, Bezier techniques, Intermediate layers, Rough machining, Semi finishing and finishing.

تطوير خوارزمية لتمثيل الطبقات البينية في تشغيل السطوح المتعرجة

الخلاصة

يهدف هذا البحث الى ابتكار وتطوير خوارزمية لتصميم وتصنيع السطوح المتعرجة والشائعة الاستخدام في منتجات متنوعة مثل القوالب واجزاء السيارات والطائرات وغيرها, تم استخدام تقنية بيزر لتوليد وتمثيل السطوح لمتعرجة لجميع مراحل التصميم. أن عدد الطبقات البينية لمرحلة التشغيل الاولي (الخشن) يعتمد بشكل كبير على هندسة السطح وعلى اعظم عمق قطع مسموح به, ومن خلال هذا البحث تم اقتراح خوارزمية لتوليد وتمثيل الطبقات البينية للسطوح المتعرجة , وكذلك تم اقتراح خوارزمية كفوءة اخرى لتمثيل السطح شبه المكتمل اعتمادا على المتجهات المماسية والمتجه العمودي لكل نقاط السطح لتوليد السطح الموازي , اما نقاط السطح التي تم توليدها بطريقة بيزر لوصف السطح النهائي فقد تم اعتمادها كنقاط مواقع للعدة لتوليد مسار العدة للتشغيل النهائي.

تم استخدام عدة تفريز اصبعية ذات نهاية مستوية وبقطر 12 ملم لتشغيل الطبقات البينية وعدة تفريز اصبعية ذات نهاية كروية و بقطر 12 ملم لتشغيل السطح الشبه نهائي والسطح النهائي. تم اختبار الخوارزمية المقترحة من خلال تصميم عدد من السطوح المتعرجة واثبتت مرونة وكفاءة جيدة في جميع المراحل, واستثمرت نتائج التصميم لتشغيل احد هذه السطوح (15 طبقة بينية , سطح شبه نهائية و السطح النهائي) باستخدام ماكنة تفيز عمودية ذات تحكم رقمي . أن الخوارزمية المقترحة لوصف و تشغيل السطوح المتعرجة اعطت نتائج ممتازة عند مقارنتها مع بعض الخوارزميات الجاهزة حيث تم تقليل الزمن اللازم للتشغيل بـ (15%) للحالة الثانية وبنفس الوقت فأن الحجم الكلي لبرنامج التشغيل تم تقليله بحدود (52%) للحالة الثالثة.

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Introduction

Many products are designed with sculptured surfaces to enhance their aesthetic appeal. It is an important factor in customer's satisfaction, especially in the automotive and consumer-electronics industries. Other products have sculptured shapes to meet functional requirements, while these aesthetic and functional surfaces are created by using CAGD (computer-aided geometric design) techniques, it is the role of SSM (sculptured surface machining) to realize them in physical form.

A sculptured surface is the surface that can only be represented as the image of a sufficiently regular mapping of a set of points in a domain into a 3D space[1]. These surfaces cannot be totally described by the analytical representation it is usually described by a series of “patches” in the same way that patchwork quilt is put together [2].

A sculptured surface can be represented by a set of curves that connect the design points of the surface. Two main approaches are commonly used for obtaining the curved surfaces: the first approach exploits the parametric curves representation, while the second one uses contouring planes (frequently geometrically equally-spaced parallel planes) to intersect the surface for obtaining a curved surface [1].

Sculptured surfaces are generally produced in two stages: roughing and finishing. Roughing cuts are used to remove most of the material from a work piece while leaving the part slightly oversized. Finish machining of a sculptured surface removes as much as possible of the remaining material from the roughed out work piece and attempts to machine the part to its final dimensions. The resulting surface is left with a large number of scallops, as shown in figure 1 [3].

When a sculptured part is machined from prismatic stock a large amount of rough machining, up to 90 percent of the total machining, is required. This is due to the significant shape difference between the sculptured part and the stock. Machining time reduction in rough machining can considerably improve the productivity of sculptured part machining and subsequently lower production costs [4].

Related Previous Work

A considerable amount of research has been carried out to describe sculptured surfaces mathematically. A progression of various methods starting from Hermit patches, Bezier surfaces to B-spline surfaces have proposed to solve this problem.

Weiss V. et al. (2002) provided practical solutions to overcome the surface fitting problems. They presented an advanced surface fitting algorithm, by means of which point clouds with irregular topology can be approximated with high accuracy. In this work they used Bezier surface and least square algorithm [5]. Hayong Shin, and Su K. Cho (2002) Proposed a 3D curve offset method, named directional offset. Since the normal vector of a 3D curve at a point is not unique, a 3D curve offset definition is about how to select the offset direction vector on the normal plane of the curve. In directional offset, the offset direction vector on the normal plane is chosen to be perpendicular to the user-specified projection direction vector. Each point on the original curve is then moved along the offset direction by a given offset distance [6]. Renner G.

et al. (2004) presented a method to explicitly compute the curves in three-dimensions; practical algorithmic issues are discussed concerning the efficiency of the implementation. Good approximations are important because of the quite high degree of exact curves on surfaces [7]. Debananda Misra, et al. (2004) provided a systematic approach to tool path generation of sculptured surfaces using 3-axis CNC machines. Tool paths have been generated for finishing operation only. Tool path planning has been done on the offset surface. Iso-planar zig-zag tool paths, due to their robustness and simplicity [8]. Hsi-Yung Feng and Zhengji Teng (2005) presented a method of generating iso-planar piecewise linear NC tool paths for three-axis surface machining using ball-end milling directly from discrete measured data points [9]. Young-Keun Choi and A. Banerjee (2007) focused on developing algorithms that generate tool paths for free-form surfaces based on the accuracy of a desired manufactured part[10]. Sotiris L. Omirou and Andreas C. Nearchou (2007) proposed a machining strategy for milling a particular set of surfaces, obtained by the technique of cross-sectional design. The surfaces considered are formed by sliding a Bezier curve (profile curve) along another Bezier curve (trajectory curve)[11]. Tao Ye and Cai-Hua Xiong (2008) presented a systematic method for the determination of optimal geometric machining parameters in multi-axis machining. Machining accuracy is considered to be determined by the design parameters of the cutter, the positioning of the cutter, and the orientation of the cutter [12].

Most of the previous researchers produced the offset surface either by offsetting the control points then generated the sculptured surface for the new control points or by finding the partial derivative of the sculptured surface in both direction, both of these methods are approximated , whereas an exact offset surface data can be generated using the proposed algorithm presented in this paper, on other hand, Iso-parametric have a wide use in the sculptured surfaces tool path generation because it is easy to find the CL-points, while the zig-zag pattern have a wide use in sculptured surface tool path generation, it have less machining time, therefore it have been used as a tool path strategy in the present work.

Surface Representation by Using Bezier TechniquesBezier curves

The Bezier curve representation is one that utilizes most frequently in computer graphics and geometric modeling. The curve is defined geometrically which means that the parameters have geometric meaning which are points in three dimensional space. Bezier curves can be developed through dividing approach whose basic operation is the generation of midpoints on the curve. There are many ways to represent a polynomial curve. A better formulation comes with the Bernstein basis functions as the building block for Bezier curves. The standard procedure is to evaluate Bezier curves for t $\in$ [0, 1] [13].

Given the control points P0.P1, P2, P3, the cubic Bezier curve can be defined as:

$$P(t) = \sum_{i=0}^{3} P_i B_{i,3}(t) \quad .. (1)$$

Where

$$B_{0,3}(t) = (1-t)^3$$
$$B_{1,3}(t) = 3t(1-t)^2$$
$$B_{2,3}(t) = 3t^2(1-t)$$
$$B_{3,3}(t) = t^3$$

are the Bernstein polynomials of degree three.

Mathematically a parametric Bezier curve of nth-degree is defined by:

$$P(t) = \sum_{i=0}^{n} P_i B_{i,n}(t) \qquad (2)$$

Given the control points P0, P1, P2, P3 the cubic Bezier curve can be defined in matrix form as:

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} \qquad (3)$$

Bezier patches

The definition of a surface is "a surface is the locus of a curve that is moving through space and thereby changing its shape"[3]. A Bezier patch is a special type of surface patch, defined by a given doubly indexed set of control points (Pij), forming a control net to define the individual curves. Suppose the double indexing as an integer points in a rectangular grid, each point in the grid is associated to a control point, and each connecting line is used to shape the surface. The surface can be viewed as a mapping of the rectangular grid into space.The definition of a Bezier patch is as a tensor product. The doubly indexed set of control points are viewed as an [(m+1) × (n+1) × 3] matrix, that is a three dimensional matrix or a tensor. To generate a surface from the control net, each column or row of the control net can generate a Bezier curve. To extend Bezier techniques for curves to a surface form a parametric surface is the result of a map of the real plane into 3-space. This plane, or domain, is defined by a (s, t)-coordinate system.

The size of the rectangular grid determines the type of Bezier patch. Given the grid is (m+1) × (n+1) the surface is a polynomial function of degree (m × n), meaning the surface is sum of mth degree polynomial in the variable (t) time's nth degree polynomial in the variable (s) .

To formalize this concept, in order to arrive at a mathematical description of a Bezier surface, the first assumption that the moving curve is a Bezier curve of constant degree m. At any time, the moving curve is then determined by a set of control points. Each original control point moves through space on a curve. The next assumption is that this curve is also a Bezier curve, and that the curve on which the control points move are all of the same degree. This can be formalized as follows:

Let the initial curve be a Bezier curve of degree m:

$$P(t) = \sum_{i=0}^{m} P_i B_{i,m}(t)$$

Let each [P_i] traverse a Bezier curve of degree n:

$$P(s) = \sum_{j=0}^{n} P_{i,j} B_{j,n}(s)$$

Combine these two equations and obtain the point $P_{m,n}(t,s)$ on the surface as:

$$P(t,s) = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{i,j} B_i^m(t) B_j^n(s) \qquad (4)$$

A cubic Bezier patch can be written in a matrix form:

$$P(t,s)=\sum_{j=0}^{3}\sum_{i=0}^{3}P_{i,j}B_i^3(t)B_j^3(s)$$

$$P(t,s)=\sum_{j=0}^{3}\left[\sum_{i=0}^{3}P_{i,j}B_i^3(t)\right]B_j^3(s)$$

$$=\sum_{j=0}^{3}[t^3\quad t^2\quad t\quad 1]\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 3 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}P_{0,j}\\ P_{1,j}\\ P_{2,j}\\ P_{3,j}\end{bmatrix}B_j^3(s)...3-12$$

(5)

And so the cubic Bezier patch is frequently written in compact form as:

$$P(t,s)=T.M.G.M^T.S^T \quad .... (6)$$

Where:

$$T=[t^3\quad t^2\quad t\quad 1]$$

$$M=\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 3 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

$$G=\begin{bmatrix}P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3}\\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3}\\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3}\\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3}\end{bmatrix}$$

$$S=[s^3\quad s^2\quad s\quad 1]$$

The previous mathematical formulation of Bezier techniques have been invested, implemented and integrated with MATLAB package (V7) to generate the interior data and represent the desired bi-cubic sculptured surfaces depending on initial control points as illustrated in next sections.

Methodology

The methodology of this work is to perform and develop several algorithms for design and manufacture sculptured surfaces, the proposed algorithms contain the following five main stages :

- Representation of Sculptured Surfaces by using Bezier method
- Intermediate Layers Generation and Representation (Roughing)
- Representation of Offset Layer (Finishing)
- DXF file generation
- NC file creation (for all machining stage)

SculpturedSurface Representation

In this section, Bezier techniques have been used to generate and represent the sculptured surfaces. Bezier surfaces can be generated according to equation (6), a proposed program have been linked with MATLAB package (V.7) to represent sculptured surfaces by using Bezier techniques, the input to this program are the surface control points and the surface increment value in both (s) and (t) directions while the output is the interior data of the desired surface , the figures (1,2,3 and 4) illustrate four different sculptured surfaces which are the output of this stage, with their original control points which are the input of this stage as examples to test and evaluate the proposed algorithm in this paper.

Intermediate Layers Generation

In this section a morphing algorithm has been proposed to calculate and represent the intermediate layers for rough machining stage. Morphing can generally be defined as the process of smooth and continuous transformation of one shape into another shape .

The property of the shape of intermediate layers between the designed surface and the upper surface of the block (reference planer surface) is changed parametrically with the change of the layer height, therefore it is easy to mention that the first layer seems to be similar to the block upper surface (planer), while the last layer seems to be similar to the original surface. To make this algorithm more efficient, the maximum depth of cut was used as an input to the algorithm. This means that the minimum number of intermediate layers depend on the maximum depth of cut. This can minimize the rough stage cutting time through the minimizing of the intermediate layers number. The morphing proposed algorithm applied only to the surface(z) coordinate while the (x and y) coordinates will be the same because the block as assumed to be a solid rectangular.

The algorithm consist the following steps:

Step one: Input the data of the desired surface

Step two: Calculate the difference between maximum Z-value and minimum Z-value

$$D_Z = (Z_{\max} + 5^*) - Z_{\min}$$

Step three: Subtract the offset distance for semi-finish (D_{off1}) and finish (D_{off2}) layers from Dz

$$D_{layers} = D_Z - (D_{off1} + D_{off2})$$

Step four: Choose the maximum allowable depth of cut depending on the cutting condition and tool work piece material (D_{cut})

Step five: Determine the number of roughing layers (N_{layers})

$$N_{layers} = (D_{layers} / D_{cut})$$

Step six: Calculate and represent each of intermediate layer

This algorithm has been invested to represent the intermediate layers of three desired sculptured surfaces and determine the interior data of each layer for further manipulation, the visual output of this algorithm is represented in figures (5), (6), and (7) .

Offset Surface Generation (for semi-finish)

Surface offset algorithm has been used for calculating and representing the semi-finish surface data, this surface can be generated by offsetting the finished surface (the design surface) along the surface normal by the amount of the ball end mill radius.

Approximation techniques are always used to describe the offset surface [8] such as generate the Bezier offset surface by offsetting the control points of the surface or by derivation the surface equation (in approximate way) and calculating the surface normal for the surface points, therefore there is no exact mathematical representation for the offset surface. An offset algorithm has been proposed in this paper to generate the surface offset 'numerical offset algorithm', by using the surface point data to calculate the normal vector through calculating the tangent vectors in s and t direction for each surface point numerically using the mathematical vectors equation. To calculate the normal vectors on the surface points, the tangent vectors for both s and t directions must calculated as below

First , the t tangent vector computed as:

$$\left.\begin{aligned}
\frac{\partial}{\partial t}P(t,s) &= \frac{\partial}{\partial t}(T.M.G.M^T.S^T) \\
\frac{\partial}{\partial t}P(t,s) &= \frac{\partial}{\partial t}(T).M.G.M^T.S^T \\
\frac{\partial}{\partial t}P(t,s) &= [3t^2 \quad 2t \quad 1 \quad 0].M.G.M^T.S^T
\end{aligned}\right\} \quad \dots(7)$$

Next, the tangent vector in s direction is computed as :

$$\left.\begin{aligned}\frac{\partial}{\partial s}P(t,s) &= \frac{\partial}{\partial s}(T.M.G.M^{T}.S^{T})\\ \frac{\partial}{\partial s}P(t,s) &= T.M.G.M^{T}.\frac{\partial}{\partial s}(S^{T})\\ \frac{\partial}{\partial s}P(t,s) &= T.M.G.M^{T}.[3s^{2} \quad 2s \quad 1 \quad 0]^{T}\end{aligned}\right\} \quad ..(8)$$

Normal vectors for the surface patch are computed by the cross product of these tangents.

For example computing the normal vector at the point P[0.5,0.5] can be as follows:

$$n(t,s) == \sum_{i=0}^{3}\sum_{j=0}^{3}P_{i,j}\frac{\partial B_i^3}{\partial t}B_j^3(s) \times \sum_{i=0}^{3}\sum_{j=0}^{3}P_{i,j}\frac{\partial B_j^3}{\partial s}B_i^3(t)$$

$$n(0.5,0.5) = \sum_{i=0}^{3}\sum_{j=0}^{3}P_{i,j}\frac{\partial B_i^3(0.5)}{\partial t}B_j^3(s) \times \sum_{i=0}^{3}\sum_{j=0}^{3}P_{i,j}\frac{\partial B_j^3(0.5)}{\partial s}B_i^3(t)$$

The offset layer of the desired sculptured surfaces are illustrated in figures (8), (9), and (10).

DXF File Generation

It is useful to use MATLAB to write a surface modeling program due to MATLAB facilities, but the problem is with the limitation of drawing exporting (interchange), the output of the Bezier surface representation program which been described in the previous section is either interior surface data or image extension files such as (.jpg, .bmp, .tif, . . . etc), the image files are not useful to evaluate the surface or for surface post processing, it can be used only for visible evaluation . The surface data can be export to another program such as (Microsoft Office Excel) then to read by another CAD or CAM program, this surface interchange interpolation or approximation is too complicated and always subjected to errors.

To avoid this problem, a MATLAB program has been proposed in this paper to generate the surface DXF file. DXF file carry all the surface data and properties required to representing and machining these surfaces. The most important section is the ENTITIES section which contains the drawing entities, therefore only the ENTITIES section has been considered when the program was written while the other section was neglected.

NC File Generation

An algorithm has been proposed to generate the machining surface NC file which can be transmitted to CNC milling machine to perform surface machining, linear interpolation was used to interpolate surface points, zig-zag tool path pattern was used for surface machining. The generated NC file can be used for machining the 3D surface with either (2 1/2) or 3 axis machine. Figures 11,12 and 13 illustrates the output of the proposed NC files generation algorithm to generate zig-zag tool path for semi finish stages of the three previous sculptured surfaces

Implementation

The proposed algorithms was developed and implemented for several surfaces for which cutter contact CC and cutter location CL points were generated and one of these surfaces "concave-convex" has been machined by using a vertical CNC milling machine.

Results and Conclusions :

This section summarized the results of the work done within the framework of the proposed algorithm that described in this paper, where three dimensional sculptured surfaces were designed using Bezier technique. The effectiveness of the proposed strategy for both design and manufacture was demonstrated by actual machining using vertical CNC milling machine.

Results

1. The proposed rough machining algorithm reduces the machining time as compared with contour tool path by 15% for case study two, while it is the same for case study three. This is due to the wide distribution of the surface points along the z-axis values for the case study three, while the case study two z value surface is in small range.
2. The proposed rough machining algorithm reduces the NC file size as compared with contour tool path, where it is reduced by small percentage for case studies one and two but by 52% for case study three. This is due to the geometry of case study three, where the contour-map tool path depends on the intersection of the surface with the slices surface along the z axis.
3. The surface parameters (s& t) can be inputted in term of the required scallop height and surface's accuracy to generate the forward and side steps, the proposed algorithm keep change the surface parameters and check the required conditions (scallop height and surface's accuracy) on the entire surface until it reach to the required conditions, but this lead to increase the processing time of calculating the suitable surface parameters.
4. The number of intermediate layers in rough machining proposed algorithm depends on the minimum surface z value, the block surface height, and the maximum depth of cut .

Conclusions

1. The NC file can be generated either to the 2-axis machining or to the 3-axis machining, and it have been tested and its works probably for both.
2. The DXF file can represent the sculptured surfaces in mesh representation. It worked probably after testing by transferring it to the other software.
3. A tool path planning algorithm generates the optimum cutter location data with determination of optimum step length and path interval.
4. The methodology of this work indicates that the proposed algorithm to control the machining and to generate tool path presents valid alternative methods of multi axis machining.
5. Surfaces offset algorithm has been tested with the surfaces, curves, and lines, and it gives good results.

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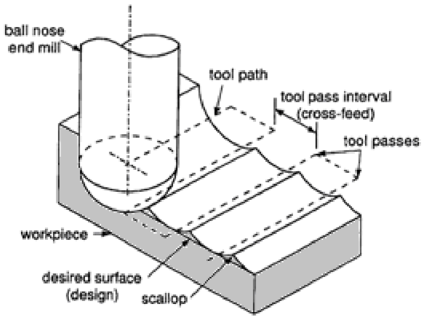

Figure1. Scallops left after machining [3].

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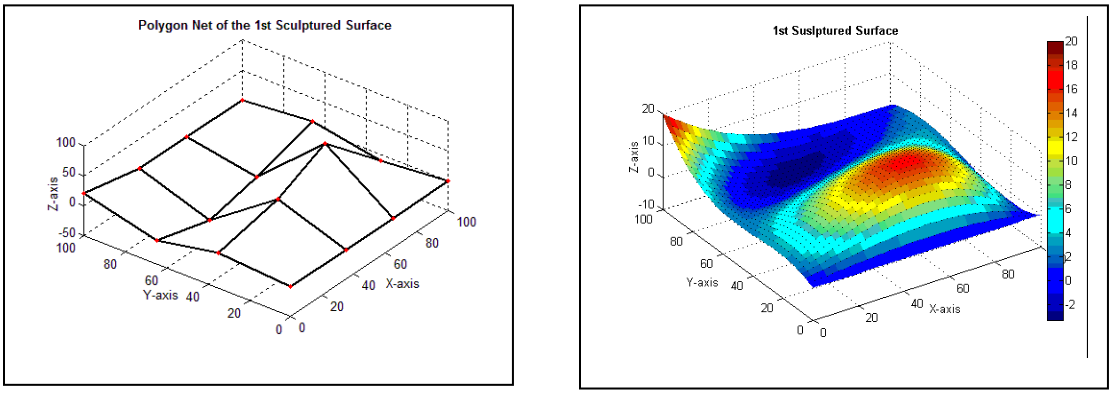

Figure 2. (4×4) Polygon net and bi-cubic Bezier surface No.1

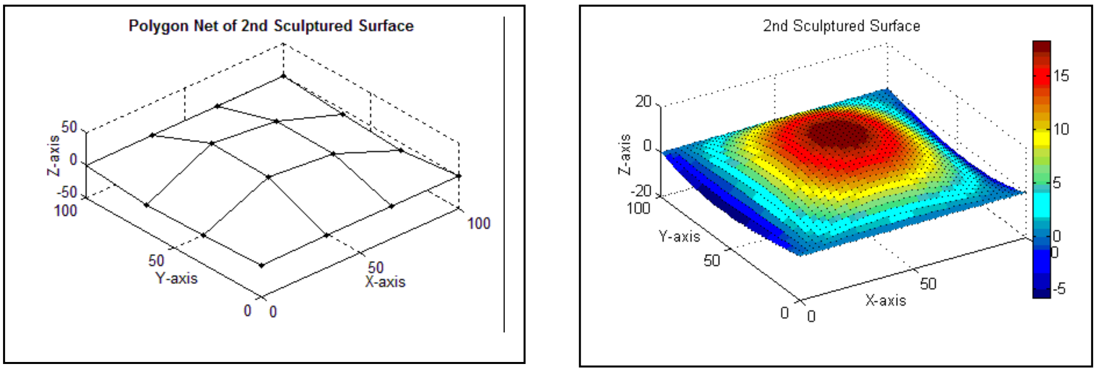

Figure 3. (4×4) Polygon net and cubic Bezier surface No.2

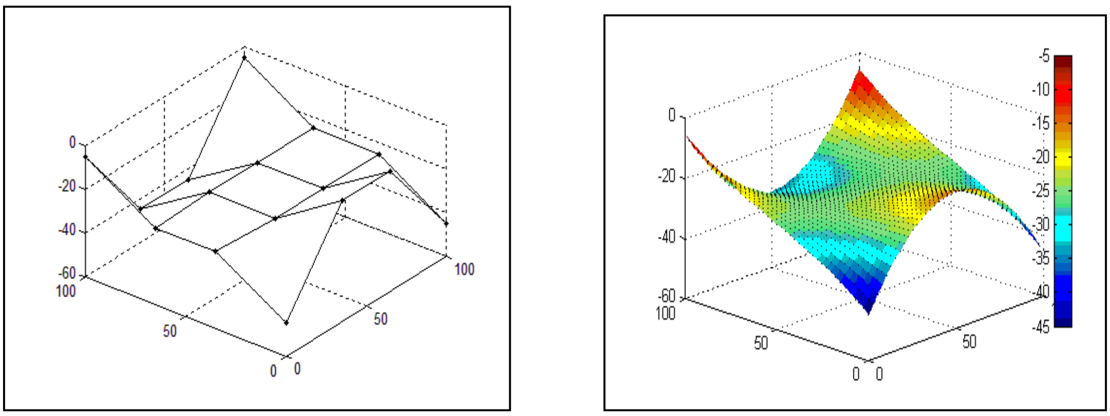

Figure 4. (4×4) Polygon net and bi-cubic Bezier surface No.3

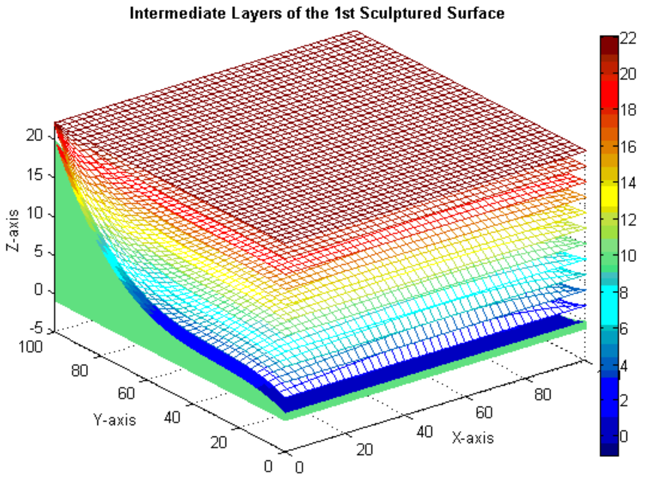

Figure 5. Intermediate layers of the 1st sculptured surface

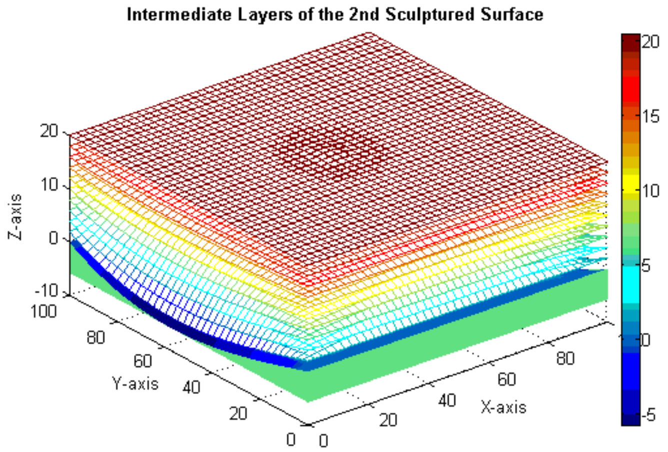

Figure 6. Intermediate layers of the 1st sculptured surface

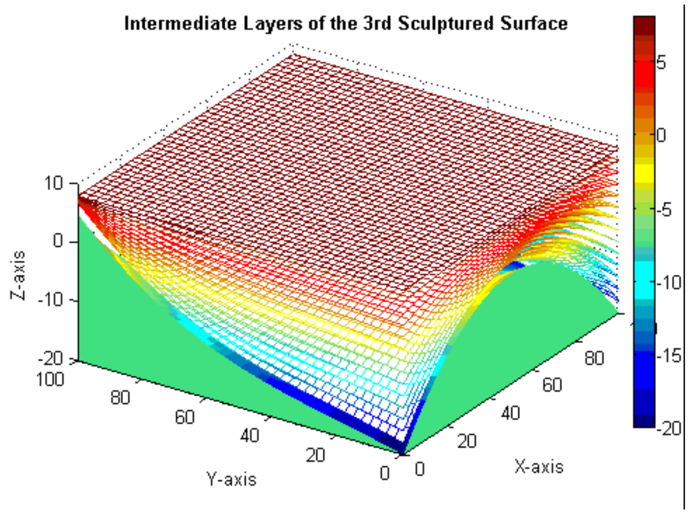

Figure 7. Intermediate layers of the 1st sculptured surface

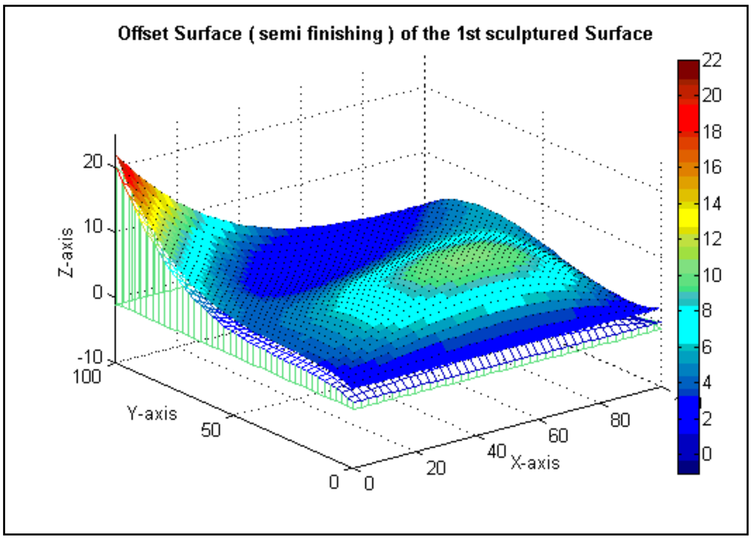

Figure 8. Offset layers of the 1st sculptured surface

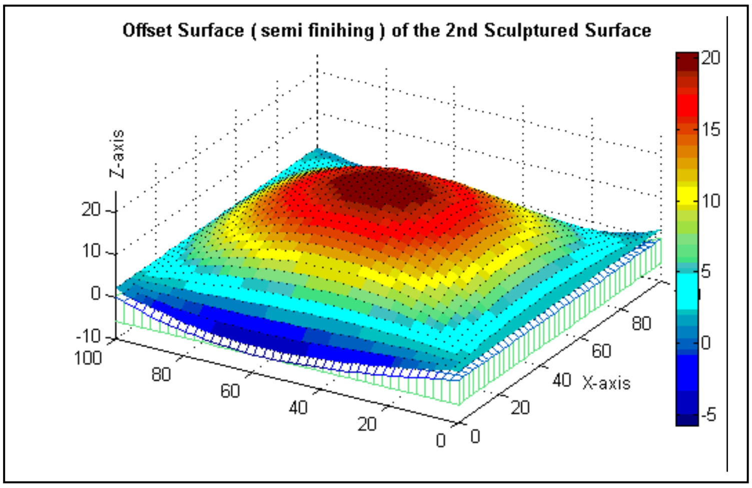

Figure 9. Offset layers of the 2nd sculptured surface

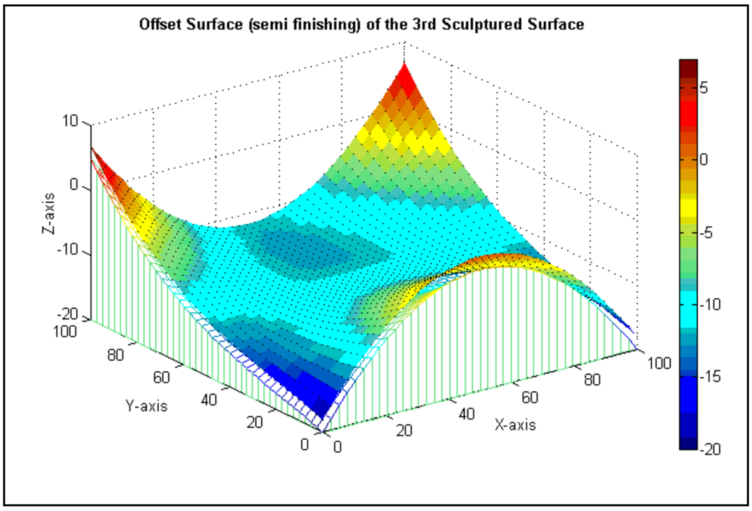

Figure 10. Offset layers of the 3rd sculptured surface

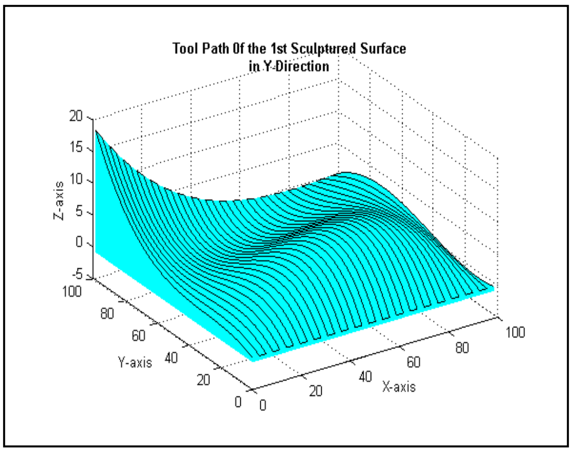

Figure 11. Tool path of the 1st sculptured surface

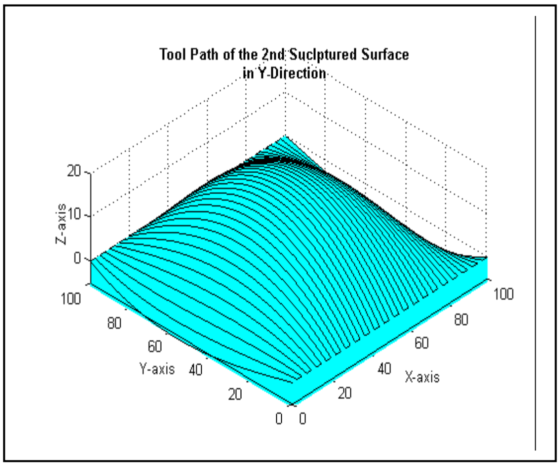

Figure 12.Tool path of the 2nd sculptured surface

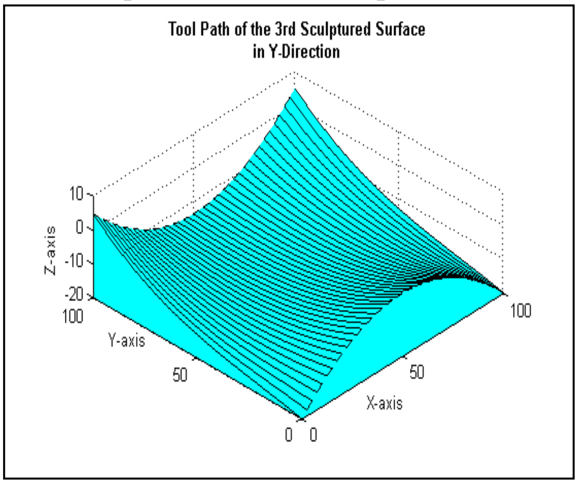

Figure 13. Tool path of the 3rd sculptured surface

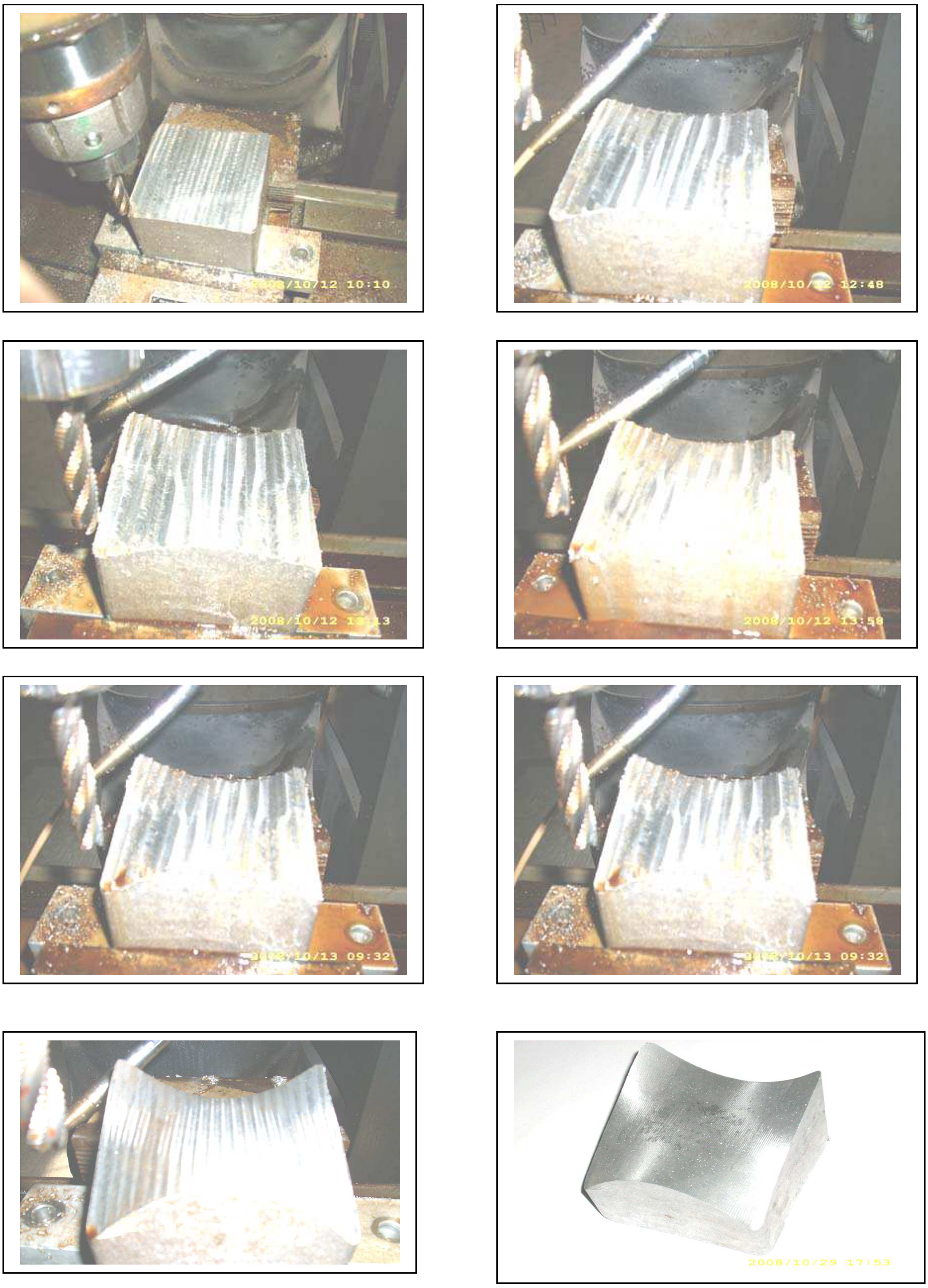

Figure 13.Various machining stages of the 3rd sculptured surface