Eng. & Tech. Journal, Vol.29, No10, 2011

Kinematic Coupling Analysis of Autorotation Flying Body

Dr. Muhammad. A .R. Yass*

Received on: 1/11/2009
Accepted on: 2/12/2010

Abstract

The kinematic coupling dynamic stability has been analyzed .The Laplace transformation and the coefficient matrix determinant are used to find the rolling stability characteristic equation. The effect of parameters is investigated with different value of roll rate $[p_o]$. It is found that the kinematics coupling or autorotation is critical at flying regime of low Cn_β and high Cm_q .The results can be used as real design requirements for further configuration improvements of the airplane.

تأثير البارامترات الساكنه والديناميه على ازدواج الطائرة الحركي

الخلاصة

تم في هذا البحث تحليل استقرارية الديناميكيه للرحو التلقائي وتحويل معادلات الحركه باستخدام تحويلات لابلاس من ثم حل المعادلات بعد تحويلها بطريقة المصفوفات لايجاد المعادلة المميزه لاستقراريه الدوران. ومن ثم تحديد العوامل المؤثرة على الاستقراية ودراسة العوامل المؤثرة لقيم الدوران المتغيرة $[p_o]$. فقد تبين ان ظاهرة الرحو التلقائي حرجه في انظمة الطيران لقيم Cn_β لواطئه و Cm_q العاليه.ويمكن اعتماد هذا البحث للمتطلبات الحقيقية في تصميم الاجسام الطائرة.

Introduction

The inertial coupling has been generally tamed as a potential problem in modern fighter aircraft .Even the most austere of these are equipped with stability augmentation systems that can be provide the required feedbacks to minimize excursions in rapid rolls[1].During the rolling maneuvers large angles of sideslip may occur as a result of kinematics coupling [2]. The vertical tail may produce large yawing moment that acts in the direction of roll. In such a case, it may not be possible to stop the flying body from Rolling, although the lateral control is held against the roll direction. This is Known as autorotation rolling. In this situation positive "G" would facilitate recovery [3]. As the angle of attack is increased to a positive value, kinematics coupling will be result in a moment that opposes the original direction of roll, thus alleviating the tendency for autorotation rolling [4].The divergence experienced during rolling manufacture is complex because it involves not only inertia properties, but aerodynamic as well, [4]. Coupling results when a disturbance about one aircraft axis causes a disturbance created by an elevator deflection during straight and level flight, [5]. The resulting motion is

* Electromechanical Engineering Department ,University of Technology/Baghdad

https://doi.org/10.30684/etj.29.10.13
2412-0758/University of Technology-Iraq, Baghdad, Iraq

restricted to pitching motion and no disturbance occurs in yaw or roll. An example of couple motion is the disturbance created by a rudder deflection [6]. The ensuing motion will be some combination of both yawing and rolling motion [7]. Although all lateral disturbance motion are coupled, the only motion that ever results in coupling problems large enough to threaten the structural Integrity of the aircraft is coupling as a result of rolling Motion, [8] .

T-38 jet plane was taken as case study (Figure (1), Table (1) [9].

Mathematical Analysis of Rolling Divergence

The overall equation of motion, [2].

$$\sum F_x = \frac{m}{\rho S u_o}\tilde{u} - \left[c_x(u_o,\alpha_o,\dot{q}_o) + \frac{1}{2}c_{x_o}u_o - \frac{1}{2}c_{T_u}u_o - c_T(M_o)\tilde{u}\right] + \left(2\frac{m\alpha_o}{\rho S\bar{c}} - \frac{1}{2}c_{x_q}\right)\tilde{q} + \frac{mg\cos\theta_0}{\rho s u_o^2}\tilde{\theta} + \frac{1}{2}c_{x_\alpha}\tilde{\alpha} = m(\dot{u} + qw - rv)$$

… (1)

$$\sum F_z = \frac{2m\alpha_o}{\rho s u_0}\dot{\tilde{u}} + \left[-c_{z_u} - 2c_z(u_o,\alpha_o,\dot{q}_o,\delta_{e_0})\right]\tilde{u} - \left(c_{z_q} + \frac{4m}{\rho s\bar{c}}\right)\tilde{q} + \frac{2mg}{\rho s u_o^2}\sin\theta_o\tilde{\theta} + \frac{2m}{\rho s u_o}\dot{\tilde{\alpha}} - c_{z_\alpha}\tilde{\alpha} = m(\dot{w} + pv - qu)$$

…. (2)

$$\sum M = -\left(2c_m(u_o,\alpha_o,\dot{q}_o,\delta_{e_0}) + c_{m_u}u_o\right)\tilde{u} + \frac{4I_{yy}}{\rho s u_o\bar{c}^2}\dot{\tilde{q}} - c_{m_q}\tilde{q} - c_{m_\alpha}\tilde{\alpha} = I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2)$$

Pitching moment Velocity

$$\frac{2u_o}{\bar{c}}\tilde{q} = \dot{\tilde{\theta}}$$

… (3)

$$\sum F_y = \frac{2m}{\rho s u_o}\dot{\tilde{\beta}} - c_{y_\beta}\tilde{\beta} - \left(c_{y_p} + \frac{4m\alpha_o}{\rho s b}\right)\tilde{p} - \frac{2mg}{\rho s u_o^2}\cos\theta_o\tilde{\phi} + \left(c_{y_r} + \frac{4m}{\rho s b}\right)\tilde{r} = m(\dot{v} + ru - pw)$$

…. (4)

$$\sum \ell = -c_{\ell_\beta}\tilde{\beta} - \frac{4I_{xx}}{\rho s u_o b^2}\dot{\tilde{p}} - c_{\ell_p}\tilde{p} - \frac{4I_{xz}}{\rho s u_o b^2}\dot{\tilde{r}} - c_{\ell_r}\tilde{r} = I_{xx}\dot{p} - (I_{yy} - I_{zz})qr - I_{xz}(\dot{r} - qp)$$

… (5)

$$\sum N = -c_{n_\beta}\tilde{\beta} - \frac{4I_{xz}}{\rho s u_o b^2}\dot{\tilde{p}} - c_{n_p}\tilde{p} + \frac{4I_{zz}}{\rho s u_o b^2}\dot{\tilde{r}} - c_{n_r}\tilde{r} = I_{zz}\dot{r} - (I_{xx} - I_{yy})pq - I_{xz}(p' - qr)$$

… (6)

Rolling Velocity

$$\frac{2u_o}{b}\tilde{p} = \dot{\tilde{\phi}} - \sin\theta_o\dot{\tilde{\psi}}$$

Yawing Velocity

$$\frac{2u_o}{b}\tilde{r} = \cos\theta_o\dot{\tilde{\psi}}$$

The approach for solving the autorotation rolling equations was derived based on some necessary assumption to fit into the present analysis of autorotation rolling [1].

I. Velocity remains constant during the roll maneuver

$\dot{u} = 0, u = u_o.$

2 .The rate roll rate is constant;

$\dot{p} = 0, so\ that\ p = po.$

3. v, w, q, r are small therefore their products are negligible.

4. Engine gyroscopic effect is negligible.

5 .Rudder and elevator are fixed in their initial trim position.

6. Aerodynamic coefficients are negligible with the exception of $c_{m_\alpha}, c_{m_q}, c_{n_\beta}$ and c_{n_r}.

7. Small angle assumption on α and β.

When these assumptions are applied to the six equations of Motion the following results are obtained

$$\sum F_x = 0 \quad \ldots (7)$$

$$\sum F_y = mu_o(\dot{\beta} + r - p_o\alpha) = 0 \quad \ldots (8)$$

$$(\dot{\beta} + r - p_o\alpha) = 0 \quad \textit{from assumption}$$

$$\sum F_z = mu_o(\dot{\alpha} + p_o\beta - q) = 0 \quad \ldots (9)$$

(*Both lift and side* Force will average zero throughout a roll)

$$\sum \ell = -(\dot{r} + q\,p_o)\,I_{xz} = 0 \quad \ldots (10)$$

This is a reasonable condition because one considers the motion to be pricipally a steady state rollbecause such a situation the aileron moment and damping in roll exactly oppose one another.

$$\sum \mathcal{M} = \dot{q}I_{yy} + p_o r(I_{xx} - I_{zz}) + p_o^2 I_{xz} = \frac{1}{2}\rho u_o^2 s\bar{c}\left[c_{m_\alpha}\alpha + c_{m_q} q\frac{\bar{c}}{2u_o}\right] \quad \ldots (11)$$

$$\sum \mathrm{N} = \dot{r}I_{zz} + p_o q\left(I_{yy} - I_{xx}\right) = \frac{1}{2}\rho u_o s b\left[c_{n_\beta}\beta + c_{n_r} r\frac{b}{u_o}\right] \quad \ldots (12)$$

Rewriting the equations in neater form

$$\dot{\alpha} + p_o\beta - q = 0 \quad \ldots (13)$$

$$\dot{\beta} + r - p_o\alpha = 0 \quad \ldots (14)$$

$$\frac{1}{2}\rho u_0^2 s\bar{c}c_{m_\alpha}\alpha - \dot{q}I_{yy} - p_o r(I_{xx} - I_{zz}) + \frac{\rho u_o s\bar{c}^2}{4}c_{m_q} q = p_o^2 I_{xz} \quad \ldots (15)$$

$$\frac{1}{2}\rho u_0^2 s b c_{n_\beta}\beta - \dot{r}I_{zz} - p_o q\left(I_{yy} - I_{xx}\right) + \frac{\rho u_0 s b^2}{4}c_{n_r} r = 0 \quad \ldots (16)$$

Note that there are four equations in four unknowns $(\alpha, \beta, q, and\ r)$. Particular solution to these equations exists because the pitching moment equation is not homogenous. However, the investigation of the particular solution holds only for design interest. On the other hand the homogenous solution represents motion which is indicative of stable or unstable coupling. Accordingly, the equations are Laplace transformed and coefficient matrix determinant becomes.

$$\begin{vmatrix} s & p_o & -1 & 0 \\ -p_o & s & 0 & 1 \\ \frac{1}{2}\frac{\rho u_o^2 s\bar{c}}{I_{yy}}c_{m_\alpha} & 0 & s - \frac{\rho u_o^2 s\bar{c}c_{m_q}}{4I_{yy}} & p_o\frac{(I_{xx} - I_{zz})}{I_{yy}} \\ 0 & -\frac{1}{2}\frac{\rho u_o^2 s b c_{n_\beta}}{I_{zz}} & p_o\frac{(I_{yy} - I_{xx})}{I_{zz}} & s - \frac{\rho u_o s b^2 c_{n_r}}{4I_{zz}} \end{vmatrix}$$

The determinant must be expanded to solve for the characteristic equation

$$AS^4 + BS^3 + CS^2 + DS + E = 0$$

The equation must be tested for stability in several methods such as Routh Discriminant [6] which conditioned for stability

$$BCD - B^2E - AD^2 \geq 0$$

The stability derivatives formation which is given below, [2] was helpful in this analysis for determining

$$c_{m_\alpha} = c_{L_\alpha}(\bar{x} - \bar{x}_n) \quad \ldots (17)$$

$$c_{m_q} = -2\zeta\eta_t\bar{v}\left(\frac{\partial c_L(\alpha_o, M_o)}{\partial\alpha}\right)_t \frac{\ell_t}{\bar{c}} \quad \ldots (18)$$

$$c_{n_r} = -2\left(\frac{\partial c_L(\alpha_o, \sigma_o)}{\partial\alpha}\right)_F \bar{V}_F\frac{\ell_F}{b} \quad \ldots (19)$$

$$c_{n_\beta} = c_{L_{\alpha_F}}\left(1 - \frac{d\sigma}{d\beta}\right)_F \bar{V}_F + c_{n_{\beta_{fus}}}$$

… (20)

Results and Discussion

All the parameters exits in autorotation characteristic equation are selected as effective parameters, which may be tested with different roll rate $[p_o]$ .Wing mean chord line $[\bar{c}]$ and wing span $[b]$ hvae negative effect toward autorotation stability because any increment in these parameters decrease the directional stability (C_{n_β}), Fig (2) and Fig (3).

Autorotation stability is much better at low altitude due to lift increase, Fig (4).Any change in moment of inertia in x, y plane (I_{xx}, I_{yy}) has limited effect on kinematics coupling dynamic stability (Fig (5), Fig (6)), but any change in moment of inertia in z-plane has great effect on roll coupling (Fig (7)).Stability Derivatives have different behavior because each derivative is depending on the way of its generation. c_{m_α} represents the longitudinal stability so it has limited positive effect toward autorotation stability, Fig (8), c_{m_q} presents the damping in pitch .It has great negative effect toward autorotation stability, Fig (9), c_{n_r} represents damping in yaw .It has limited negative effect toward autorotation stability, **but it should be maintain below zero ($cn_r < 0$) to keep the B coefficient of characteristic equation greater than zero in order to avoid autorotation** Fig (10) and c_{n_β} represents the directional stability and it has great positive effect toward autorotation stability **and should be kept larger than zero, since this parameter determine the C coefficient which it should be positive to avoid the autorotation**, Fig (11).

Conclusions

1. **Vertical Tail stability design more important than wing-body and horizontal tail the for kinematic coupling**.
2. Autorotation stability was found much better at low altitude
3. Decrease weight distribution in Z-plane and increase weight distribution in Y-plane one of best solution of kinematic coupling of autorotation.
4. The most serve cases naturally should be expected in the flight regime of low c_{n_β} and high c_{m_q}.
5. It can be notice that $p_o = 20\ deg / sec$ was quite reasonable for optimum stability.

References

[1]Bandu.Pamadi",Performance,stability,dynamic,and control of airplane",Amazon 2004.
[2]Aircraft from a Steady Sideslip Flight Conditions ", Airforce Base Ohio , Dec. 1974.
[3]Wells , W.R, and Quam, D.L " Aircraft Lateral Parameter Estimation From Flight Data with Unsteady Aerodynamic Modeling ", Journal of Aircraft, volume 19, No. 3, March 1982.
[4]Stalford, H. J., "High – Alfa Aerodynamics model Identification of T – 2C Aircraft using the EBM Method ", Journal of Aircraft, vol. 18. Number 10, October 2000.
[5]Pallet, E. H. J., "Automatic Flight Control ", 3rd ed. BSP Professional Books, 1997.
[6]Calico, Jr. R, and Fuler S.G, "Stability of Steady Sideslip Equilibrium for High Alpha

"Journal of Aircraft, vol. 20, No. 4, April 1983.
[7]Perkins, C.D., "Airplane Performance Stability and Control ", Wiley 1976.
[8]Moul, M.T. and Paulson, J., "Dynamic Lateral Behavior of High Performance Aircraft ", NACA RM L58E16, Aug 1958.
[9]USAF Stability and Control Datcum, Flight Control Division, Airforce Flight Dynamics Laboratories, Wright Patterson, Airforce Base, OH 2000.

List of symbols

Symbols	Definition	Units
b	Wing Span	ft
$\bar{c}$	Wing Chord Line	ft
S	Wing Area	ft^2
G	Ground Force	$slug.ft^2/\sec^2$
m	mass	$slug$
q	Dynamic Pressure	$slug/ft.\sec^2$
u	Air Sped	ft/sec
u_o	Initial Air Sped	ft/sec
p_o	Roll Rate	deg/sec
I_{xx}	$X-Axis$ moment of Inertia	$slug.ft^2$
I_{yy}	$Y-Axis$ moment of Inertia	$slug.ft^2$
I_{zz}	$Z-Axis$ moment of Inertia	$slug.ft^2$
ρ	Air Density	$slug/ft^3$
c_{m_q}	Damping in pitch	$1/radian$
c_{m_α}	Static longitudinal stability	$1/rad$
c_{n_r}	Damping in yaw	$1/rad$
c_{n_β}	Directional Stability	$1/rad$
α	angle of attack	deg
α_o	Initial angle of attack	
β	angle of attack	deg
r	angle of attack	deg
$\dot{\alpha}$	Rate of angle of attack	deg/sec
$\dot{\beta}$	Rate of angle of attack	deg/sec
$\dot{r}$	Rate of angle of attack	deg/sec
$\dot{q}$	Rate of Dynamic Pressure	$slug/ft.\sec^3$
C_X	Coefficient of $X-force$	$--------$
C_T	Coefficient of thrust force	$--------$
C_{T_u}	Coefficient of Thrust in $X-Axis$	$--------$

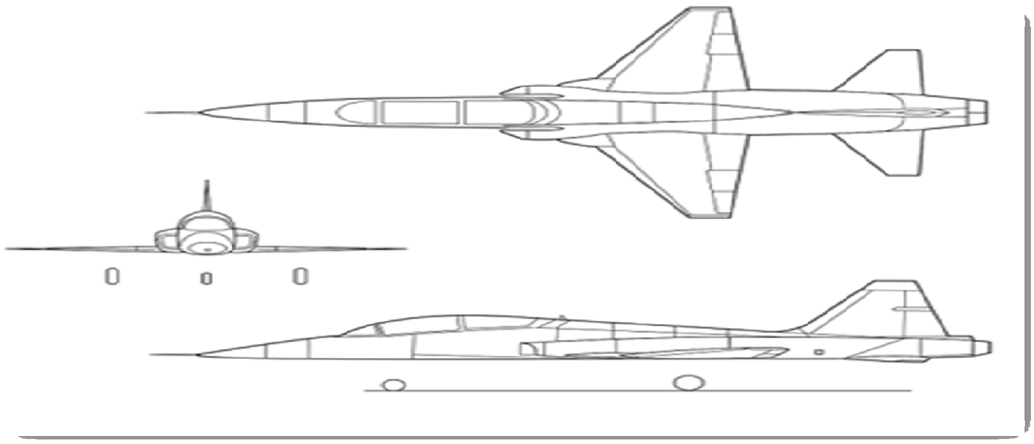


Figure (1):-Views of Supersonic Aircraft T-38 Taylon (Case Study)

Aerodynamic Data						
Wing Span (ft)	**Wing Area (ft^2)**	**Wing Mean Chord (ft)**	**Aspect Ratio**	**Wing Sweep Angle**	**Taper Ratio**	**Airfoil Section**
25 .3	170	7.73	3.75	24(deg)	0.2	NACA 65A004.8

Stability Derivatives			
Cm_{α}	Cm_{q}	Cn_{r}	Cn_{β}
-0.16/rad	-8.4/rad	-0.54/rad	+0.28/rad

Other Data								
I_{xx} (Sl-ft2)	I_{yy} (Sl-ft2)	I_{zz} (Sl-ft2)	I_{xz} (Sl-ft2)	**Max Speed (M)**	**Weight(lbf)& mass(slug)**	**Density M=0.8 Alt=20000 ft (Sl/ft^3)**	**Density M=0.8 Alt=20000 ft(ft/sec)**	**Engine Type**
1479	28166	29047	-80	1.63	9000 280	0.001267	831	J85-GE_5 Turbojet

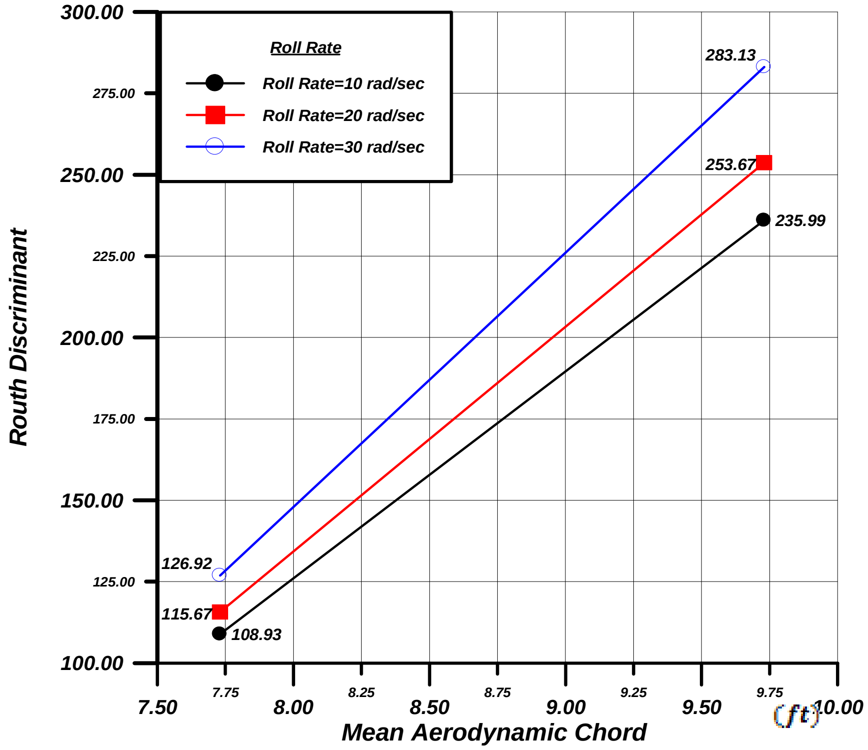


Figure (2) Effect of Wing Mean Chord on Aircraft Autorotation

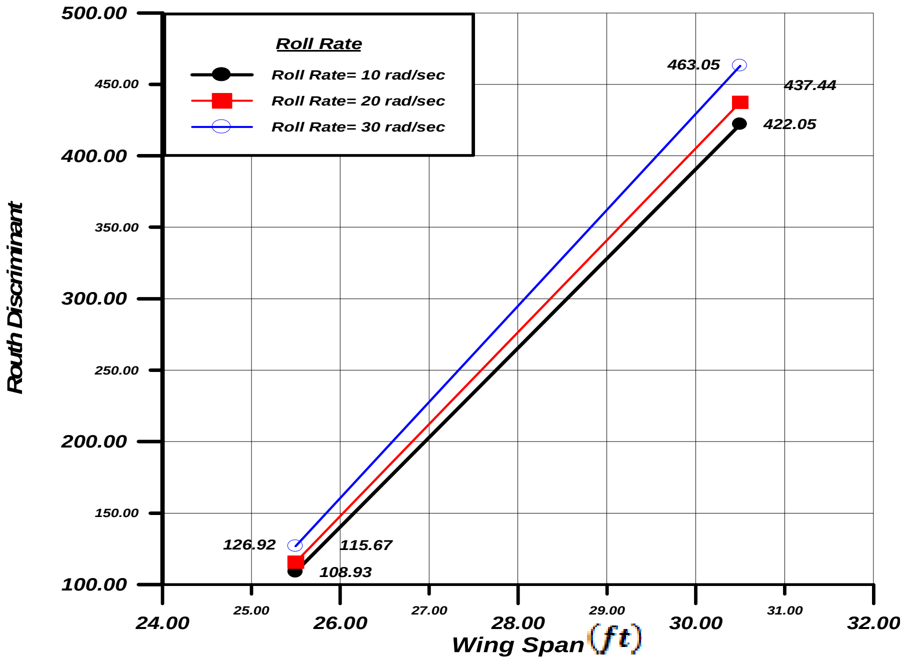


Figure (3) Effect of Wing Span on Aircraft Autorotation

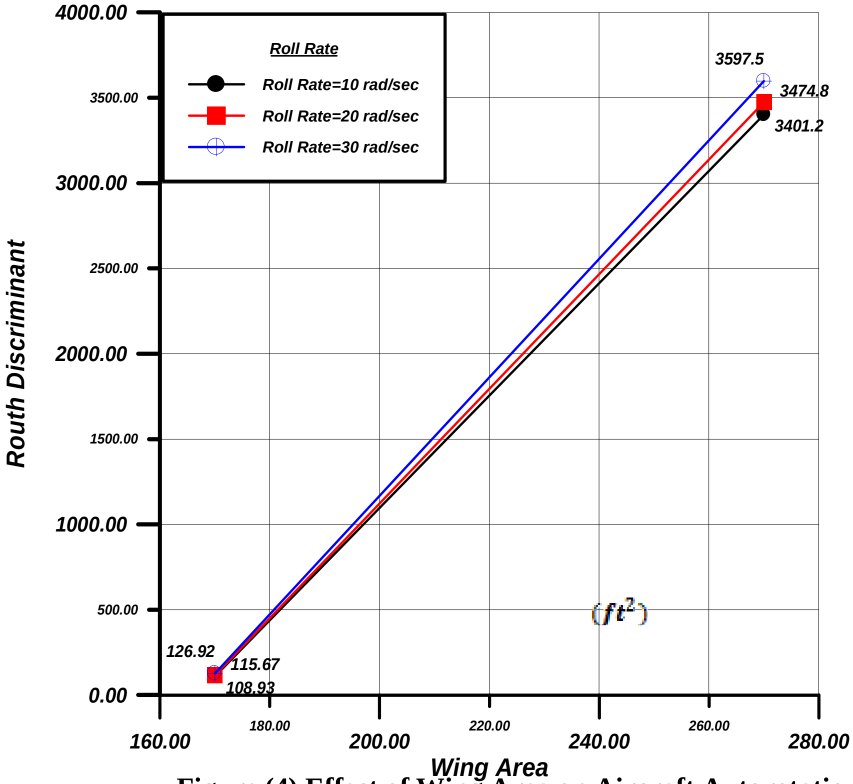


Figure (4) Effect of Wing Area on Aircraft Autorotation

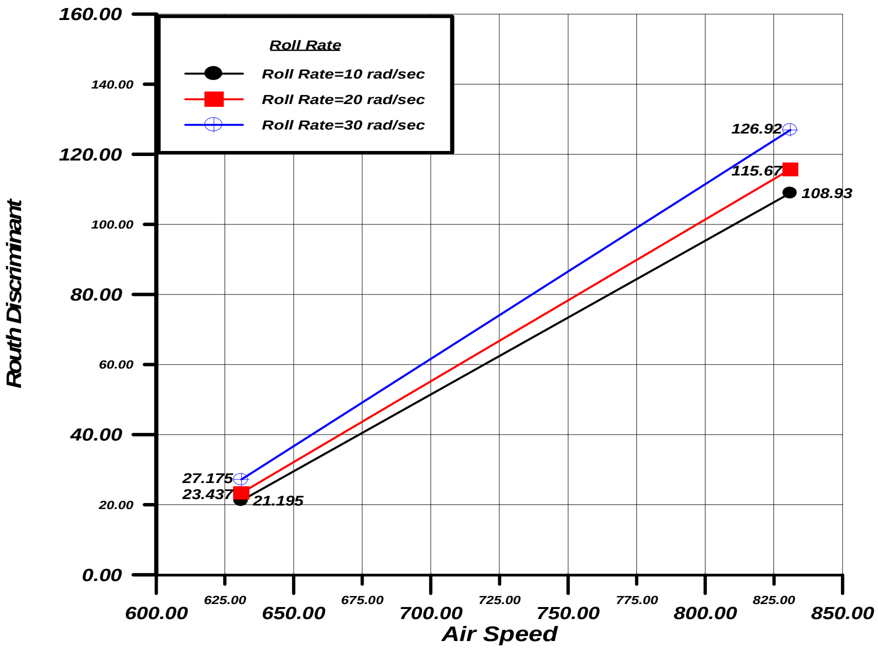


Figure (5) Effect of Wing Span on Aircraft Autorotation

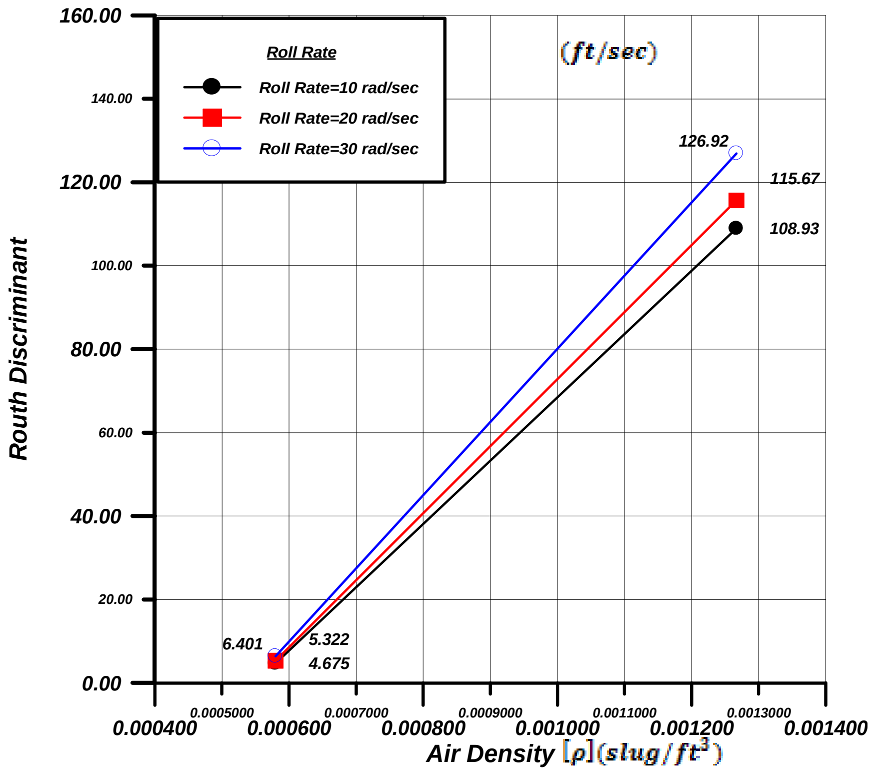


Figure (6) Effect of Air Density on Aircraft Autorotation

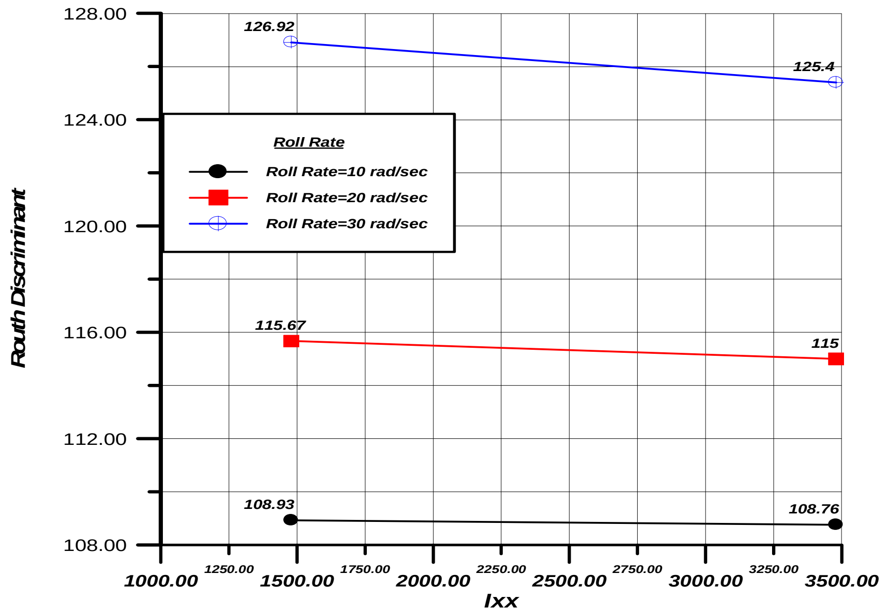


Figure (7) Effect of X-Axis moment of inertia on Aircraft Autorotation $(slug.ft^2)$

$(slug.ft^2)$

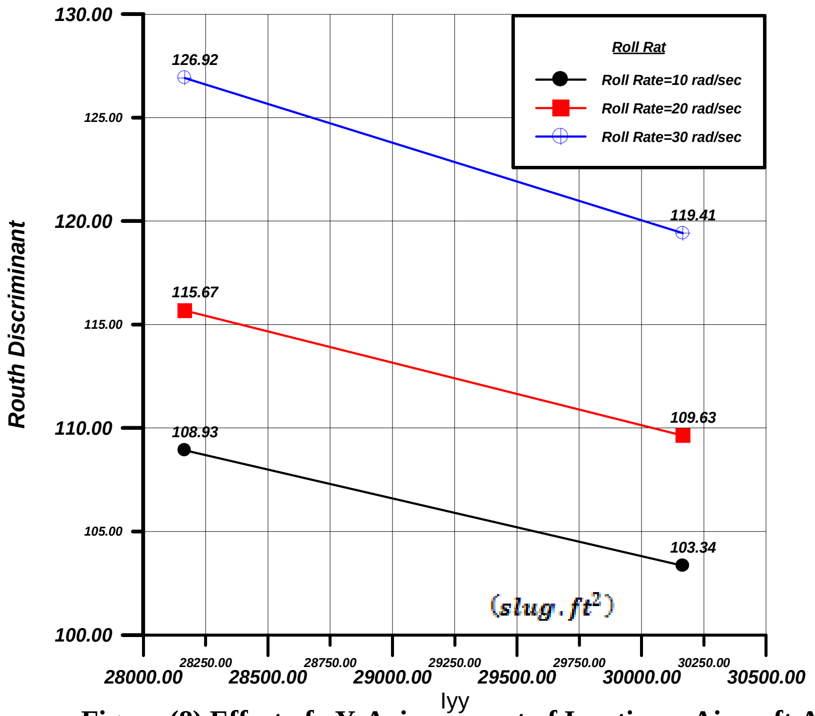


Figure (8) Effect of Y-Axis moment of Inertia on Aircraft Autorotation

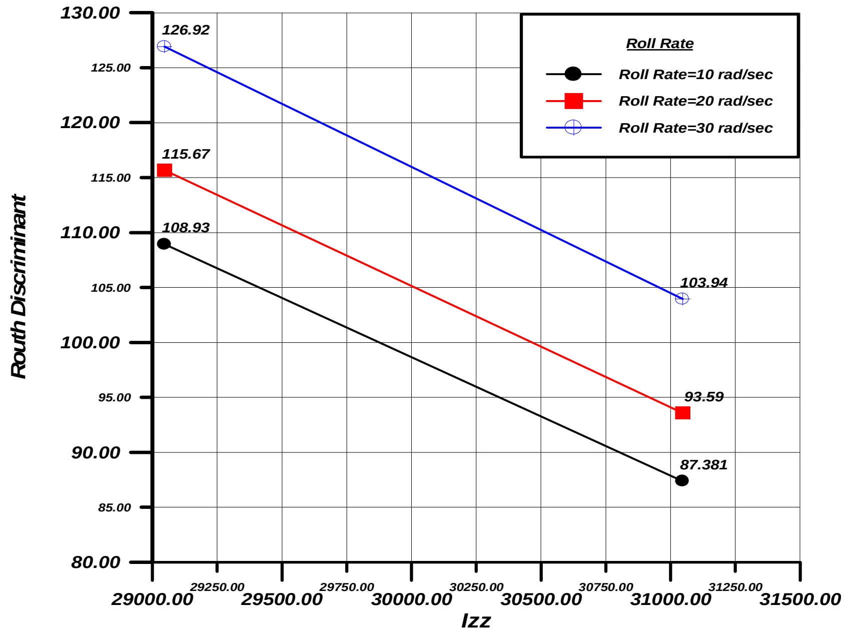


Figure (9) Effect of Z-Axis moment of Inertia on Aircraft Autorotation $(slug.ft^2)$

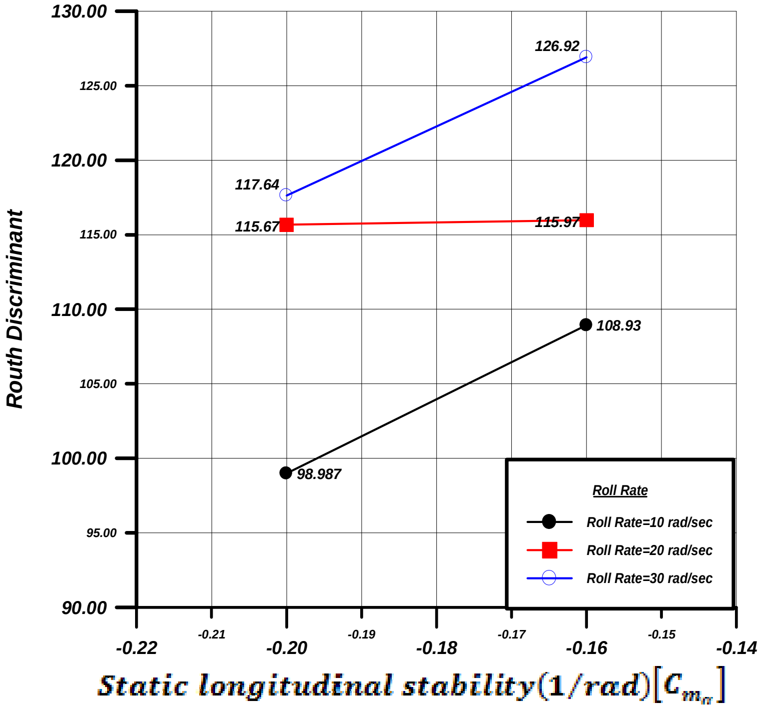


Figure (10) Effect of Static Longitudinal Stability on Autorotation

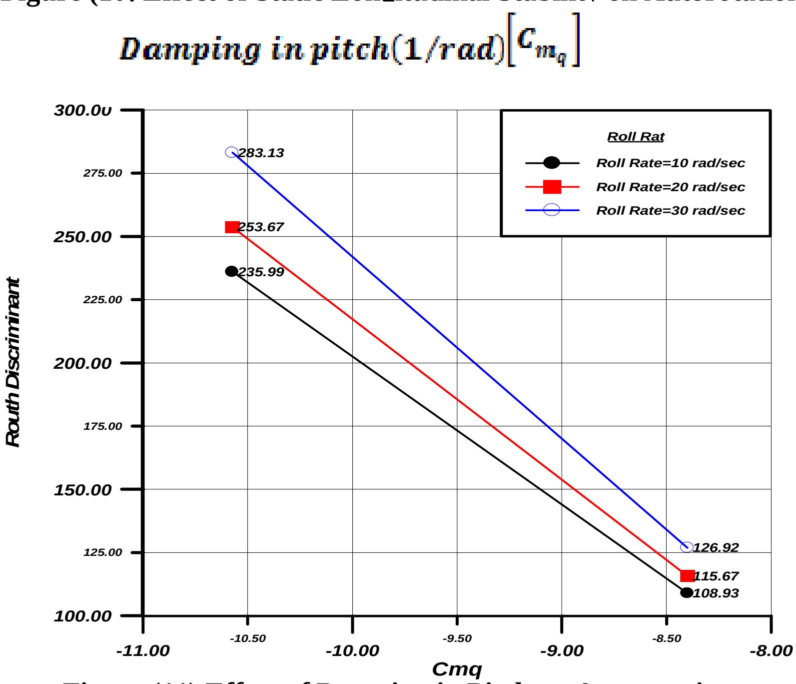
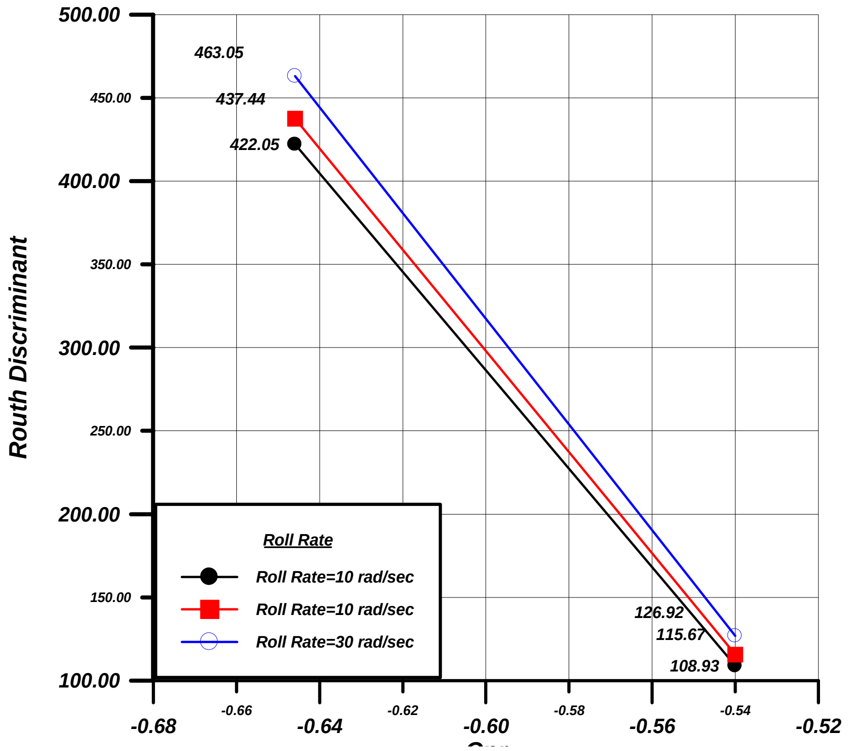
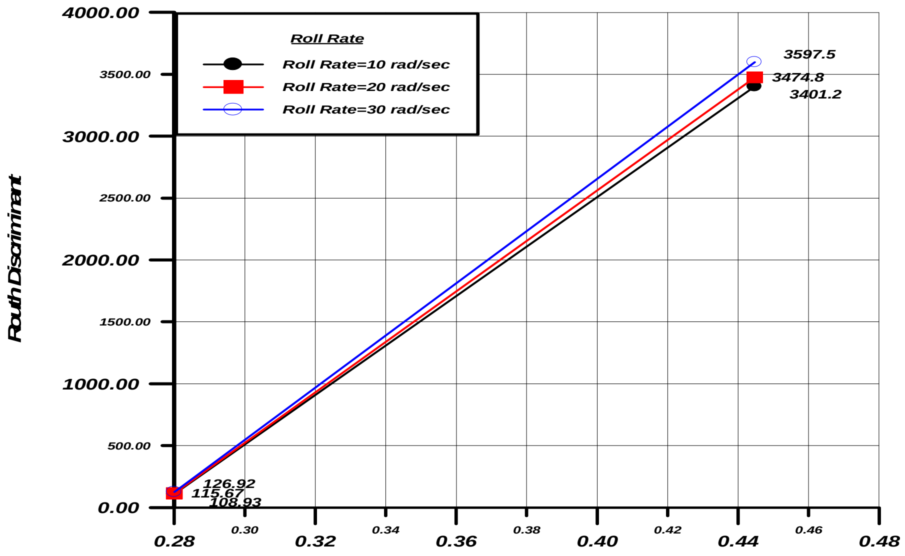


Figure (11) Effect of Damping in Pitch on Autorotation



$Damping\ in\ Yaw(1/rad)[C_{n_r}]$

Figure (12) Effect of Damping in Yaw on Autorotation



$Derictional\ stability(1/rad)[C_{n_\beta}]$

Figure (13) Effect of Dedicational stability on Autorotation