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Design of a Nonlinear Robust Controller for Vibration Control of a Vehicle Suspension System

Dr. Shibly Ahmed Al-Samarraie*, Dr. Muhsin N.Hamza**
&YasirKhudhair Abbas*
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Abstract

The suspension system is the main tool to achieve ride comfort and drive safety for a vehicle. Passive suspension systems have been designed to obtain a good compromise between these objectives, but intrinsic limitations prevent them from obtaining the best performances for both goals. In present work, a robust controller for the active suspension system has been designed to get the best performance of the suspension system. The nonlinear robust controller is designed based on adding an integrator to a two-degree-of-freedom quarter-car model. The control action will decouple the upper sprung mass subsystem from the lower (unsprung mass) subsystem after a certain small period of time. As a result, by adjusting the control law parameters, the dynamical response for the sprung mass subsystem is freely specified (the damping ratio and the natural frequency for the sprung system after decoupling).

The simulation results, which are carried out by using Matlab/Simulink, proved the effectiveness of the proposed control law. The results confirmed that the sprung mass system is decoupled from the lower unsprung system and unaffected by the change in sprung mass and the road excitation disturbance. Additionally, the time history of the sprung mass response is according to a mass spring system response with the desired damping ratio and the natural frequency.

Keywords: Quarter-car suspension, active suspension, nonlinear controller, & robust control.

تصميم مسيطر متين لاخطي للسيطرة على اهتزاز منظومة التعليق للمركبات

الخلاصــة

تعتبر منظومة التعليق في المركبات المنظومة الرئيسية التي توفر الراحة والامان اثناء القيادة للمركبة.بالنسبة لمنظومات التعليق الاعتيادية، الغير مسيطر عليها، فان التصميم يتم بحيث يكون هنالك توازن بين هذين الهدفين، اي الراحة والامان، ولكن هنالك محددات جوهرية تمنع من الحصول او تحقيق هذيين الهدفين معا لتحقيق افضل اداء. لذا فان هذا البحث يهتم بدراسة تصميم مسيطرةمتينة لاخطية للسيطرة على منظومة التعليق للمركبات للوصول الى افضل أداء لنظام التعليق.تم التصميم بالاعتماد على اضافة مسيطرة تكاملية الى نموذج التعليق ثنائي درجة الحرية. تم التصميم بحيث يكون هنالك فصل كلي، بعد برهه من الوقت، بين المنظومة الديناميكية للكتلة المعلقة (اي جسم المركبة والركاب) والكتلة الغير معلقة. وبذلك امكن اختيار الاداء الديناميكي للكتلة المعلقة بشكل حر عن طريق تعديل المعاملات الخاصة بقانون المسيطرة.

اظهرت نتائج المحكاة، باستخدام Matlab/Simulink، فعالية التصميم المقترح لقانون المسيطرة. واثبتت النتائج ان المنظومة النابضية (اي الكتلة المعلقة) قد انفصلت بفعل السيطرة على

*Control and System Engineering Department, University of Technology/Baghdad
** Machines & Equipments Engineering Department, University of Technology/ Baghdad

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

المنظومـة الغيـر نابضـية اي انهـا اصـبحت معزولـة تمامـا ولاتتـأثر الاسـتجابة بتغييـر الكتلـة النابضـية ومتغيرات إثارة الطريق (وعورة الطريق). بالاضـافة الى انه استخدام هذا النـوع مـن السيطرة المتينـة مكّن من الحصول على تصرف ديناميكي مرغوب مع قيم مرغوبة من نسبة التخميد والتردد الطبيعي.

Nomenclature

m_s	sprung mass	kg
m_u	unsprung mass	kg
k_s	suspension spring stiffnessN/m	
cs	suspension damping rate	Ns/m
k_t	tire stiffnessN/m	
c_t	tire damping rate	Ns/m
d	road excitation disturbance	m

1. Introduction

A vehicle suspension is required to perform effectively under a range of operating conditions including high levels of braking and accelerating, cornering at speed and traversing rough terrain – maneuvers which are required to be done in comfort and with safety. These requirements present the chassis engineer with some challenging problems and introduce some unavoidable design compromises [1].

However, traditional passive suspensions cannot achieve a better compromise between ride comfort and stability due to their incontrollable damping or spring characteristics. Therefore, controllable suspensions have been proposed by using controllable actuators and computer-based control devices [2-4] in recent years. Conventionally, these conflicting objectives are achieved by using a passive suspension that's damping coefficient and stiffness curves are selected carefully for a compromise solution. At the same time, it has been theoretically proved that changing the damping coefficient and stiffness according to the road disturbance can significantly improve suspension performance [5]. Nowadays, these theoretical results can be realized in practice using more capable microprocessors, sensors, and actuators that have appeared in the market [6]. To improve the performance, the damping coefficient should continuously vary according to the road disturbance, and this requires special type of dampers called continuously varying dampers (CVDs). The technique of designing the suspension system by varying damping coefficient is called semiactive suspension. The semiactive damper is capable of producing resistive force only, i.e., the damping factor can only take positive values.

Semiactive dampers and their application in suspension development are addressed in many papers [7]–[8]. There are various types of semiactive suspension control strategies such as sky-hook [7], ground-hook [8], and hybrid [9] that can be realized through semiactive dampers. There are also various types of sky-hook control strategies named ON–OFF sky-hook, continuous sky-hook, and its modified versions. A good comparison between these strategies can be found in [10]. Even though the control algorithms are simple, the actual implementation is cumbersome from the view of the available sensors, filtering the signals, estimating non-measured variables, and the control of CVDs.

Another type of suspension system is active suspension. Active suspension supports a vehicle and isolates its passengers from road disturbances for ride quality and vehicle handling using force-generating components under feedback control. Not with standing its complexity, high cost, and power requirements, active suspension has been used by the luxury car manufacturers such as BMW, Mercedes-Benz, and Volvo [11]. Development of an active-suspension system should be accompanied by the methodologies to control it. Considering costly commercial vehicles with active suspension, Allen constructed a quarter-car test bed to develop the control strategies [12].

Many researchers developed active-suspension control techniques. These research results can be categorized according to the applied control theories. When it comes to the linear-quadratic, LQ, control, Peng, *et al.* presented the virtual input signal determined by the LQ optimal theory for active-suspension control [13]. Tang and Zhang applied linear-quadratic-Gaussian (LQG) control, neural networks, and genetic algorithms in an active suspension and presented simulation results [14]. Sam, *et al.* applied LQ control to simulate an active-suspension system [15].

As for the robust control, Lauwerys, *et al.* developed a linear robust controller based on the synthesis for the active suspension of a quarter car [16].Wang; *et al.* presented the algorithm to reduce the order of the controller in the application of active suspension [17]. They were able to reduce the controller's order by nearly one third while the performance was only slightly degraded. Concha and Cipriano developed a novel controller combined with the fuzzy and LQR controllers [18]. Gobbi, *et al.* proposed a new control method based on a stochastic optimization theory assuming that the road irregularity is a Gaussian random process and modeled an exponential power spectral density [19].

Savaresi, *et al.* developed a novel control strategy, called Acceleration-Driven-Damper (ADD) in semi-active suspensions. They minimized the vertical sprung mass acceleration by applying an optimal control algorithm [20]. Then Savaresi and Spelta had ADD compared to sky-hook (SH) damping [21]. Recently, they proposed an innovative algorithm that satisfies quasi-optimal performance based on an SH-ADD control algorithm [22]-[23]. Abbas, *et al.* applied sliding-mode control for nonlinear full-vehicle active suspension [24]. They considered not only the dynamics of the nonlinear full-vehicle active-suspension system but also the dynamics of the four actuators. Many neural-network controllers were also applied to active suspension. Jin, *et al.* developed a novel neural control strategy for an active suspension system [25]. By combining the integrated error approach with the traditional neural control, they were able to develop a simple-structure neural controller with small computational requirements, beneficial to real-time control. Kou and Fang established active suspension with an electro-hydrostatic actuator (EHA) and implemented a fuzzy controller [26]. They could attenuate the suspension deflection by 26.76% compared with passive suspension. Alleyne and Hedrick developed a nonlinear adaptive controller for active suspension with an electro-hydraulic actuator [27]. They analyzed a standard parameter adaptation scheme based on the Lyapunov analysis and presented a modified adaptation scheme for active suspension.

In this paper a nonlinear robust controller is designed based on integrator addition. The added integral will help in designing a controller that will decouple the sprung mass subsystem from the total vehicle dynamical system where the remainder (the unsprung mass subsystem) affected by the road disturbance. Such a design will enable the suspension system of a vehicle to isolates its passengers from road disturbances for ride quality and vehicle handling. In addition the controller will force the sprung system to behave like a first degree of freedom system with the desired damping ratio and natural frequency.

2. Nonlinear Controller Design

In the present work a two-degree-of-freedom quarter-car model, depicted in Figure (1), will be analyzed. In this model, the sprung and unsprung masses are denoted, respectively, by m_s and m_u. This suspension system has a linear spring stiffness k_s and a linear damper with a damping rate c_s. The tire is modeled by a linear spring of stiffness k_t and a linear damper with a damping rate c_t. And it is assumed in the present analysis that the tire, always, follow the road profile.

The mathematical model for a two –degree–of–freedom quarter-car mode (refer to Figure (1)) is described by the following set of differential equations:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m_s\dot{x}_2 &= -k_s x_1 - c_s x_2 + k_s x_3 + c_s x_4 + u \\ \dot{x}_3 &= x_4 \\ m_u\dot{x}_4 &= k_s x_1 + c_s x_2 - (k_s + k_t)x_3 - (c_s + c_t)x_4 - u + D(t) \end{aligned} \quad \textbf{(1)}$$

Where:

$$D(t) = k_t d(t) + c_t \dot{d}(t)$$
$$d(t) = d_o[1 - \cos(\omega_r t)]$$

With d_o as the peak amplitude, and ω_r is a constant frequency in the disturbance model which depends on the vehicle velocity and on the width of the disturbance on the road. Also the states (x_1, x_2) and $(x_3, x_4)$are the position and velocity for the sprung and unsprung masses respectively.

As a first step in the present design is the following input transformation, with integrator addition is imposed:

$$u = x_5$$
$$\dot{x}_5 = v \quad \textbf{(2)}$$

This is also named as a dynamic feedback. Accordingly, by augmenting the transformation equation (2) with system model in Equation (1), we get

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m_s\dot{x}_2 &= -k_s x_1 - c_s x_2 + k_s x_3 + c_s x_4 + x_5 \end{aligned} \quad \textbf{(3-}\boldsymbol{a}\textbf{)}$$

$$\begin{aligned} \dot{x}_3 &= x_4 \\ m_u\dot{x}_4 &= k_s x_1 + c_s x_2 - (k_s + k_t)x_3 - (c_s + c_t)x_4 - x_5 + D(t) \\ \dot{x}_5 &= v \end{aligned} \quad \textbf{(3-}\boldsymbol{b}\textbf{)}$$

Now let x_5 be determined such that the upper subsystem in equation (3-*a*) has a desired damping coefficient c_d and a desired spring stiffness k_d. Thus, we have

$$-k_d x_1 - c_d x_2 = -k_s x_1 - c_s x_2 + k_s x_3 + c_s x_4 + x_5$$
$$\Rightarrow x_5 = -(k_d - k_s)x_1 - (c_d - c_s)x_2 - k_s x_3 - c_s x_4 \quad \textbf{(4)}$$

Then by using x_5 above, the upper subsystem (Equation (3-*a*)) becomes

$$\dot{x}_1 = x_2$$
$$m_s\dot{x}_2 = -k_d x_1 - c_d x_2$$

or

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(k_d/m_s)x_1 - (c_d/m_s)x_2 = -2\zeta\omega_n x_2 - \omega_n^2 \quad \textbf{(5)}$$

In Equation (5), we can select the desired dynamical response according to our choice for$\zeta$and ω_n. Accordinglyk_d and c_d are determined as

$$k_d = m_s\omega_n^2 \& c_d = 2m_s\zeta\omega_n \quad \textbf{(6)}$$

The upper subsystem will behave, if Equality (4) satisfied, as a second order system with a desired natural frequency ω_n and damping ratio $\zeta$and unaffected by the external disturbance (the road excitation $d(t)$). Consequently, the main controller job (the virtual controller term) is to force x_5 to be equal to the right hand side of Equation (4). This can be formulated by introducing the following functional:

$$\vartheta = x_5 + (k_d - k_s)x_1 + (c_d - c_s)x_2 + k_s x_3 + c_s x_4 \qquad \textbf{(7)}$$

The goal now is to regulate ϑ to zero. Hence, when $\vartheta \to 0$ the equality condition in (4) is satisfied which means that the upper subsystem is **decoupled** from the remainder system and behaved as in Equation (5) irrespective to the disturbance coming from the road.

To evaluate v, we consider ϑ as an output which it has a relative degree equal to one with respect to the input v. So, by differentiate ϑ with time we get

$$\dot{\vartheta} = (k_d - k_s)\dot{x}_1 + (c_d - c_s)\dot{x}_2 + k_s\dot{x}_3 + c_s\dot{x}_4 + \dot{x}_5$$
$$= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + v + gD(t) \qquad \textbf{(8)}$$

where

$a_1 = -(k_s(c_d - c_s)/m_s) + (c_s\, k_s/m_u)$,
$a_2 = (k_d - k_s) - (c_s(c_d - c_s)/m_s) + (c_s^2/m_u)$,
$a_3 = (k_s(c_d - c_s)/m_s) - (c_s(k_s + k_t)/m_u)$,
$a_4 = ((c_d - c_s)c_s/m_s) + k_s - (c_s(c_s + c_t)/m_u)$,
$a_5 = ((c_d - c_s)/m_s) - (c_s/m_u)$,
$g = (c_s/m_u)$

To get the control law for$v$, that will regulate ϑ in the presence of the disturbance $D(t)$, the following Lyapunov function is candidate:

$$V = \frac{1}{2}\vartheta^2 \qquad \textbf{(9)}$$

By differentiating V, we get

$$\frac{dV}{dt} = \vartheta\dot{\vartheta} = \vartheta * \{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + v + gD(t)\} \qquad (10)$$

Now, let

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + v = -\mu * \tan^{-1}(\alpha\vartheta)\,, \alpha > 0 \qquad (11)$$

So $\frac{dV}{dt}$ becomes,

$$\frac{dV}{dt} = \vartheta * \{-\mu * \tan^{-1}(\alpha\vartheta) + gD(t)\}$$
$$= -\mu * \vartheta * \tan^{-1}(\alpha\vartheta) + \vartheta * gD(t)$$
$$\le -\mu * \vartheta * \tan^{-1}(\alpha\vartheta) + |\vartheta| * g * |D(t)|$$
$$\le -\mu * \vartheta * \tan^{-1}(\alpha\vartheta) + |\vartheta| * g * \delta$$
$$= -\mu * |\vartheta| * \tan^{-1}(\alpha|\vartheta|) + |\vartheta| * g * \delta$$
$$= -|\vartheta| * \{\mu * \tan^{-1}(\alpha|\vartheta|) - g * \delta\} \qquad (12)$$

Where $|D(t)| \le \delta$.If it is required for the steady state error of ϑ not to exceedρ, then μ can be evaluated from the following inequality:

$$\mu > (|g| * \delta/\tan^{-1}(\alpha\rho)) \qquad (13)$$

For which $\frac{dV}{dt} < 0\,, \forall|\vartheta| > \rho$. In this inequality, α and ρ are the design parameters and their selection will specify the steady state error of ϑ. The controller term v now is equal to

$$v = -a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4 - a_5x_5 - \mu * \tan^{-1}(\alpha\vartheta) \qquad (14)$$

Finally the nonlinear controller u with dynamic feedback is taken in the following form:

$$\left.\begin{array}{c} u = x_5 \\ \dot{x}_5 = -a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4 - a_5x_5 - \mu\tan^{-1}(\alpha\vartheta) \end{array}\right\} \qquad (15)$$

In the following section the simulations result are presented which proves the effectiveness of the proposed nonlinear controller in Equation (15).

3. Analysis of Simulation Results

To verify the control design concept, simulations were performed using Matlab/Simulink based simulation model (refer to Figure (2)). The quarter-car model parameters are found in Table (1). All simulations are carried out under road excitation disturbance (Equation (1-b)) shown in Figure (3) with the maximum excitation amplitude applied being 0.2 m. The nonlinear controller parameters are taken as in Table (2), where the values of desired damping coefficient c_d and a desired spring stiffness

$k_d$have been calculated using (Equations (6)) by choosing the desired damping ratio ζ and natural frequency ω_n to be 1 and 20 rad/sec, respectively.

Figures (4) and (5) show the responses of sprung mass displacement x_1 and sprung mass velocity $x_2$respectively for both controlled and uncontrolled cases for comparison. It is obvious that the comfort of the passengers is considerably enhanced for the sprung mass displacement in a control mode (Figure (1-a)) if compared with the uncontrolled mode (damper mode) as in figure (1-b). This indicates that the proposed controller (Equation (15)) is very efficient in a way so that it is able to decouple or isolate the upper system (sprung mass) from the lower system making upper system unaffected with the applied road excitation.

The simulation results of unsprung mass displacement x_3, and unsprung mass velocity x_4 are illustrated in Figure (6) and (7), respectively for both controlled mode (a) and uncontrolled mode (b). The concept of isolation or decoupling between the upper system and lower system is obviously noticed through the behavior of the unsprung mass x_3 for controlled mode (Figure (6-a), where it is resemble the profile of the road excitation disturbance, which indicates that the proposed nonlinear controller force the lower system to absorb the impact and influence of the road excitation (Figure (3)) thus the isolation of the upper system from any road disturbance vibrations is performed. While in Figure (6-b) when there is no control applied, the unsprung mass system stills fluctuate after the impact of the road excitation. The simulation results of both sprung mass and unsprung mass prove the robustness nature of the proposed controller as it is able to attenuate the effect of the road excitation disturbance in an efficient way.

Figure (8) shows plots of r.m.s acceleration against time for passive and active suspension. It is well known, according to ISO 2631 that human body has an ability to withstand vibration or discomfort for a certain period of time for each frequency value at a certain value of r.m.s acceleration. ISO 2631 standard distinguishes between vibrations with a frequency in the range between 0.5 Hz and 80 Hz that may cause a reduction of comfort, fatigue, and health problems, and vibrations with a frequency in the range between 0.1 Hz and 0.5 Hz that may cause motion sickness [1].Standards refer to the acceleration due to vibration and suggest weighting functions of the frequency to compute the root mean square values of the acceleration. Such functions depend both on the point of the body where the acceleration is applied and the direction along which it acts.

According to ISO 2631 it is clear that the frequency range in which humans are more affected by vibration lies between 4 and 8 Hz [1]. And since, in present According to ISO 2631 it is clear that the frequency range in which humans are design, under the control action, that the displacements of the sprung mass are rapidly died out this will give the passengers more comfort ride.

Figure (9) shows the difference between the displacement of the unsprung mass with that of the road profile, and it is clearly seems from this figure that the tire practically follows the road profile and our assumption for this is accurate.

Figure (10) shows the simulation result for the actuator force (controller action). The maximum actuation force needed to perform the isolation of the sprung mass upper system is about (16 KN) (the negative behavior indicates the reversal direction of the action with respect to the road excitation disturbance).

To demonstrate how the decoupling concept is performed through the proposed nonlinear controller (Equation (15)). The error functional ϑ (Equation (7)) is simulated and shown in (Figure (11)). It can be seen that the objective goal of the designed controller to regulate ϑ to zero is achieved in about 1 sec. Hence, when $\vartheta \to 0$ the equality condition in equation (4) is satisfied which means that the upper subsystem is decoupled from the remainder system and behaved as in Equation (5) (the desired system with specific chosen damping ratio ζ and natural frequency ω_n) irrespective to the disturbance excitation coming from the road.

To testify the robustness of the proposed nonlinear controller, the simulation test is repeated by doubling the sprung mass value from 2500 kg to 5000 kg, as a perturbation to the system parameters, this is may be caused by the changing in sprung mass due to the changing of passengers mass or goods loaded to the vehicle. The simulation results are shown in Figures (12) – (19). Although the sprung mass is doubled, the time history of the simulation results are similar to the time history of simulation results obtained in the first test, this is justified due to the robustness of proposed nonlinear controller as it is able to handle the model uncertainties and external disturbances.

4. Conclusions

In present paper a nonlinear robust control design techniques on a quarter-car suspension system has been applied to obtain a fully controlled suspension system, or so called active suspension system, resulted in a controller that is able to significantly increase comfort since the road excitation affect on the passenger mass will rabidly die out. As a comparison, the comfort in this case will be superior when compared with the comfort obtained with a passive suspension system where the road excitation causes the oscillation to the passenger mass. The design of robust controller, proposed in this paper, is based on adding integrator to the control channel. The proposed controller has been approved mathematically and via simulation to decouple the suspension system to the upper sprung system and the lower unsprung subsystem with a desired upper system dynamical behavior. This ability makes the active suspension system robust to the road disturbance. Also the upper, decoupled, subsystem is still stable and has a proper dynamical behavior in spite of the change in sprung mass (robustness to the variation in sprung mass).

The simulations result using Matlab/Simulink, demonstrate the ability of the proposed controller to decouple the upper subsystem from the suspension system and makes it robust with respect to the road disturbance ($0.2\,m$ peak amplitude) and to the variation in the sprung mass (the change from $2500\ kg$ to $5000\ kg$).

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Table (1): Parameters Values for Quarter-Car Suspension model.

Parameter	Values	Units
m_s	2500	kg
m_u	320	kg
k_s	80000	N/m
k_t	500000	N/m
c_s	350	$N\,s/m$
c_t	15020	$N\,s/m$
d_o	0.1	m
ω_r	6.7230	rad/sec

Table (2): Parameters Values for Nonlinear Controller.

Parameter	Values	Units
k_d	10^6	N/m
c_d	10^5	$N\,s/m$
α	1	-
μ	13500	-

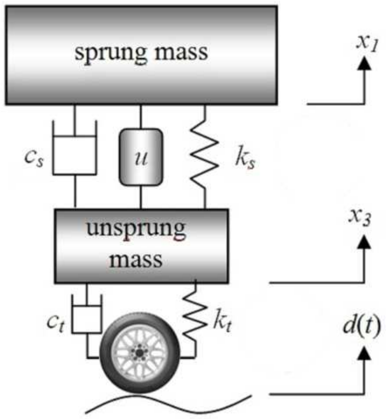

Figure (1): Quarter-Car Suspension Model for The Vehicle.

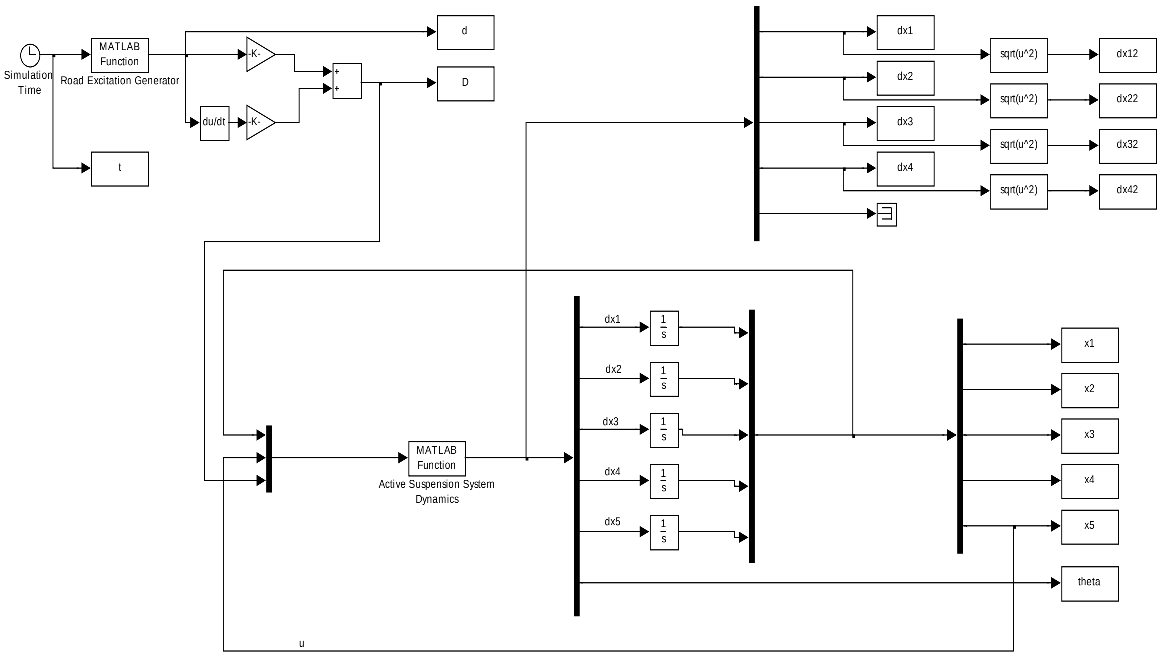

Figure (2): Matlab/Simulink Quarter-Car Simulator with The Proposed Nonlinear Controller.

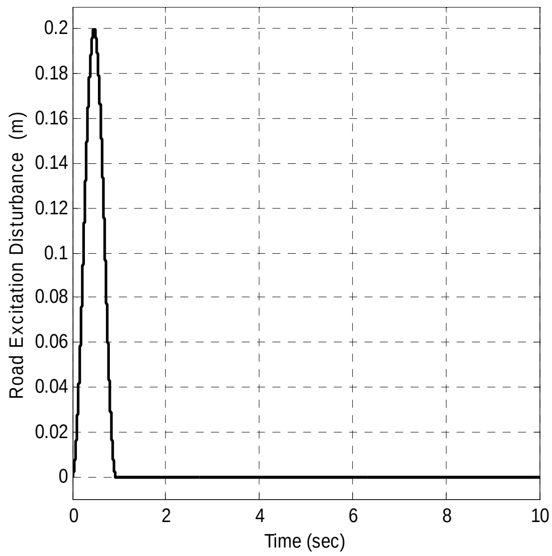

Figure (3): Road Excitation Disturbance $d(t)$ (m).

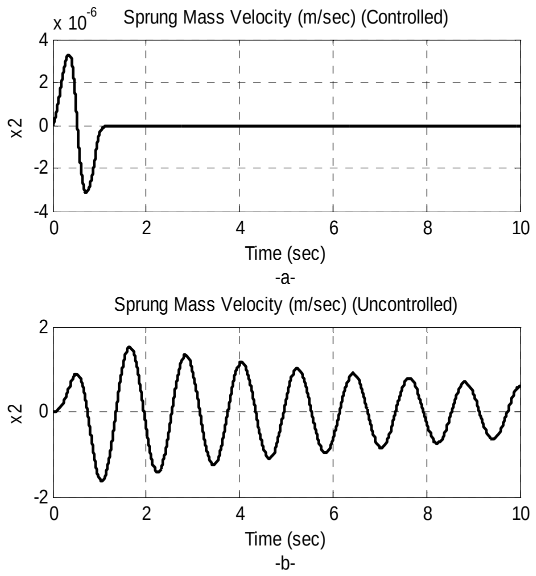

Figure (5): Sprung Mass (2500 kg) Velocity. (a) Under Nonlinear Control, (b) No Control Applied.

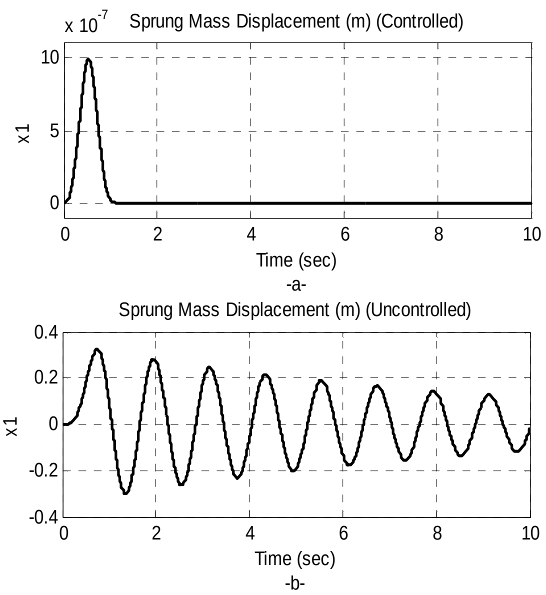

Figure (4): Sprung Mass (2500 kg) Displacement. (a) Under Nonlinear Control, (b) No Control Applied.

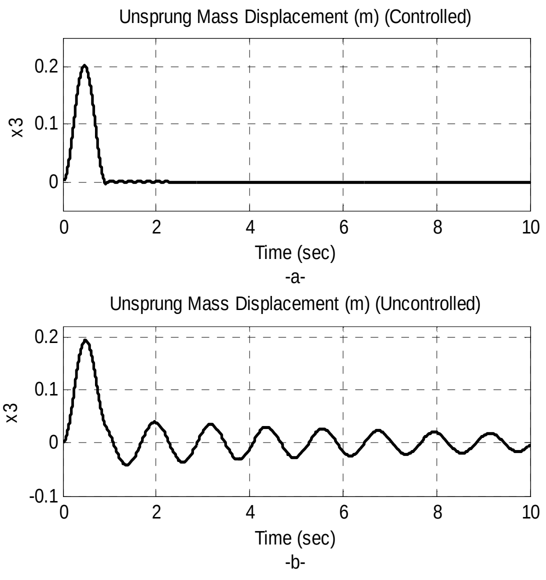

Figure (6): Unsprung Mass (320 kg) Displacement. (a) Under Nonlinear Control, (b) No Control Applied.

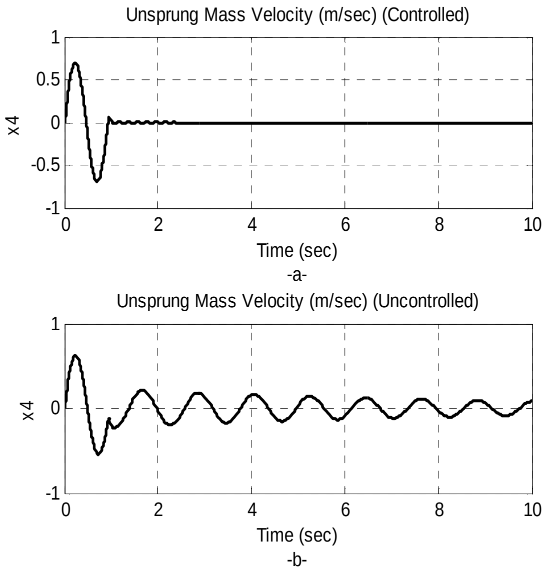

Figure (7): Unsprung Mass (320 kg) Velocity. (a) Under Nonlinear Control, (b) No Control Applied.

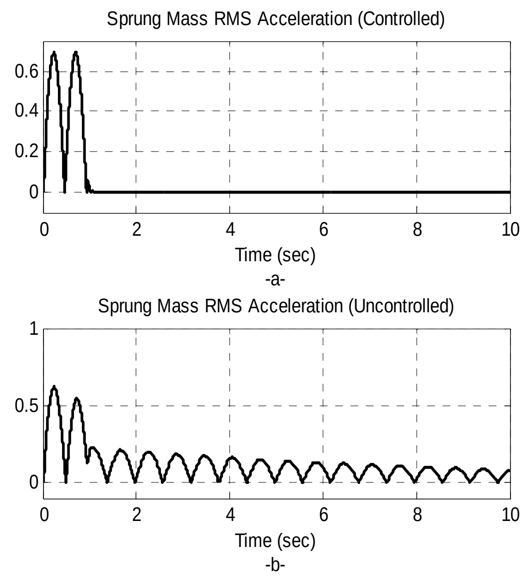

Figure (8): Sprung Mass (2500 kg) RMS Acceleration. (a) Under Nonlinear Control, (b) No Control Applied.

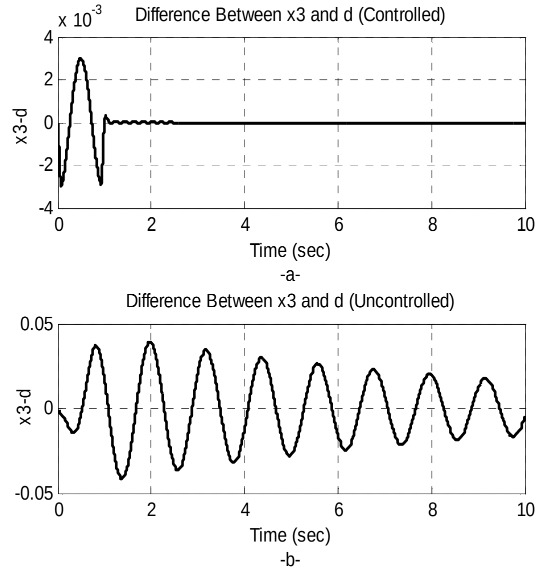

Figure (9): The Difference Between Unsprung Mass (320 Kg) Displacement and Road Excitation Disturbance. (a) Under Nonlinear Control, (b) No Control Applied.

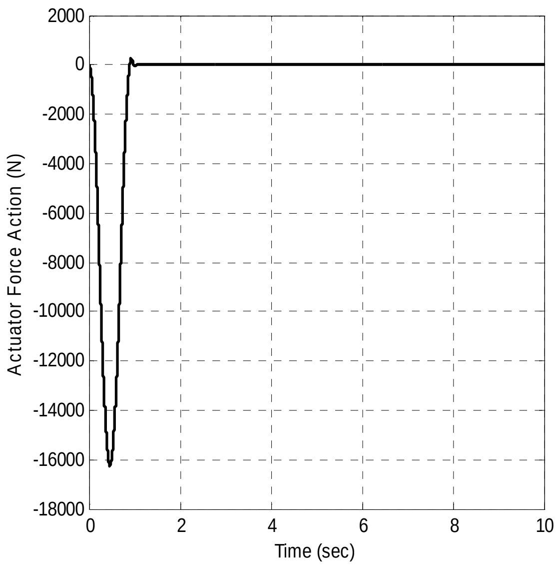

Figure (10): Actuator Force Control Action for Sprung Mass (2500 kg).

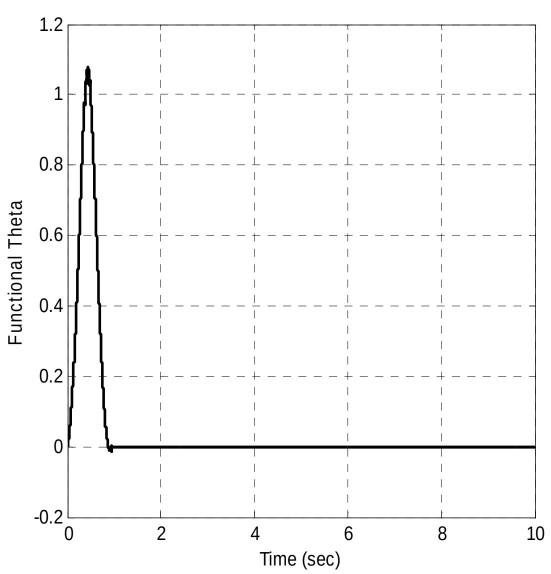

Figure (11): Functional ϑ (Equation (7)) for Sprung Mass (2500 kg).

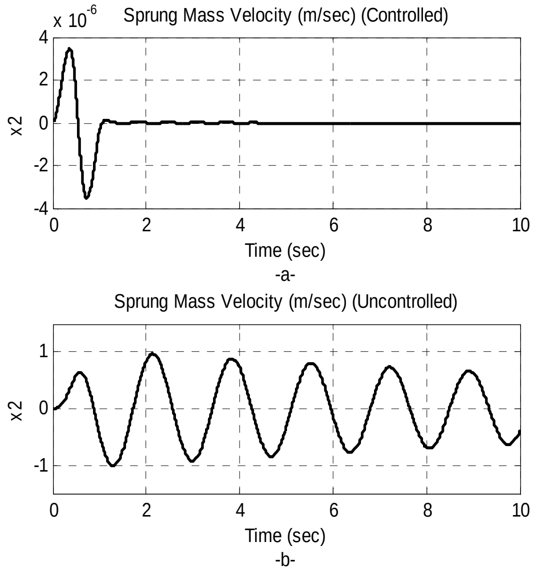

Figure (13): Sprung Mass (5000 kg) Velocity. (a) Under Nonlinear Control, (b) No Control Applied.

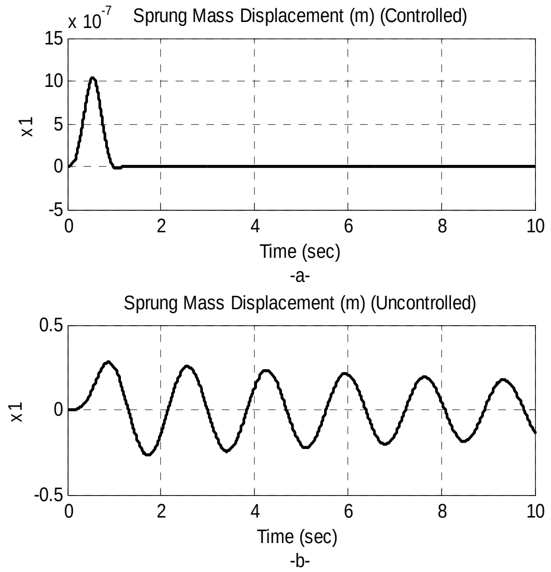

Figure (12): Sprung Mass (5000 kg) Displacement. (a) Under Nonlinear Control, (b) No Control Applied.

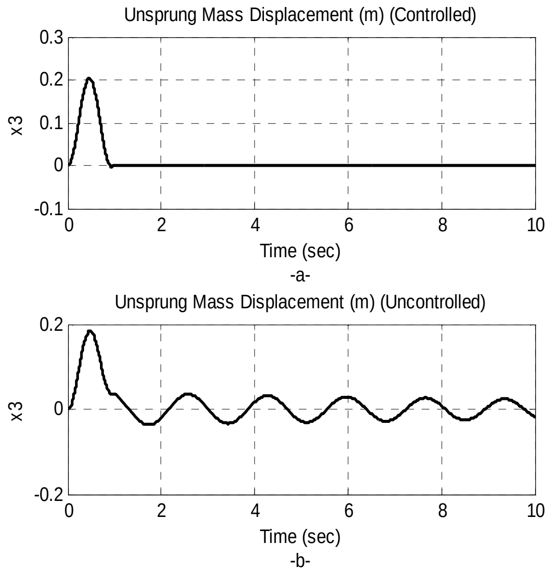

Figure (14): Unsprung Mass (320 kg) Displacement. (a) Under Nonlinear Control, (b) No Control Applied.

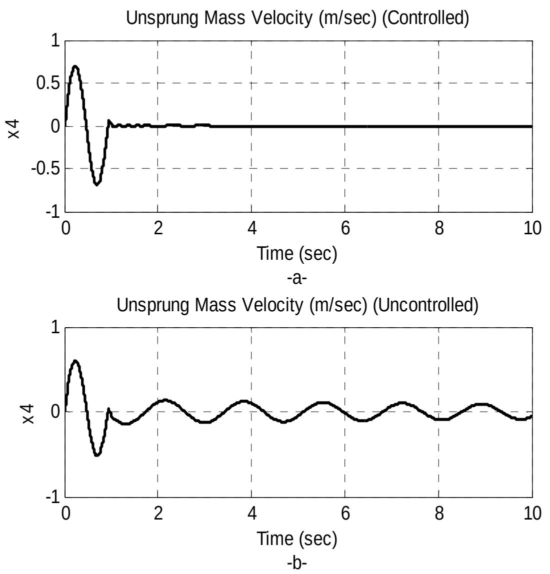

Figure (15): Unsprung Mass (320 kg) Velocity. (a) Under Nonlinear Control, (b) No Control Applied.

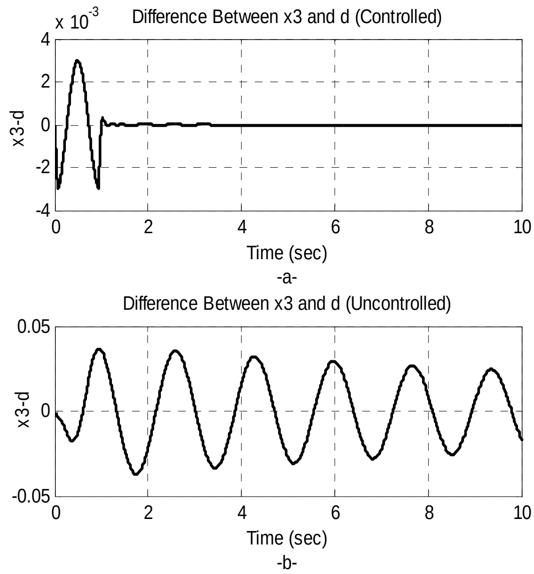

Figure (17): The Difference Between Unsprung Mass (320 Kg) Displacement and Road Excitation Disturbance. (a) Under Nonlinear Control, (b) No Control Applied.

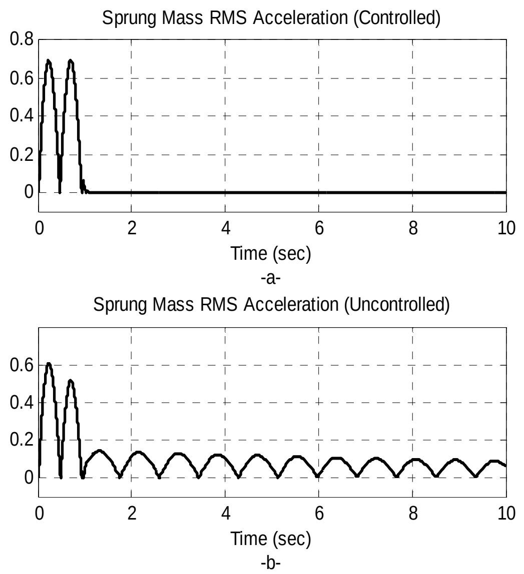

Figure (16): Sprung Mass (5000 kg) RMS Acceleration. (a) Under Nonlinear Control, (b) No Control Applied.

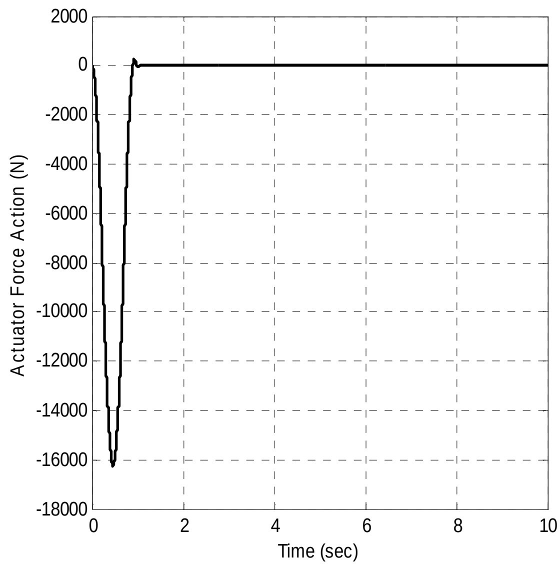

Figure (18): Actuator Force Control Action for Sprung Mass (5000 kg).

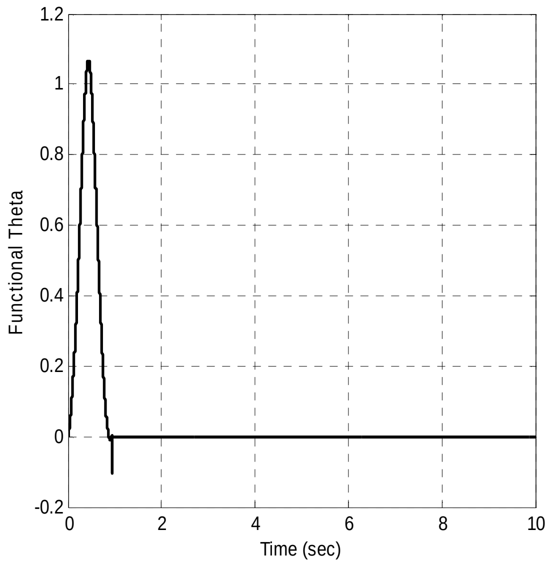

Figure (19): Functional ϑ (Equation (7)) for Sprung Mass (5000 kg).