Near-MDS Codes of Degree Three over the Galois Field of Order 25

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ABSTRACT. In the projective plane $PG(2,q)$ a $(n;3)$-arc may be generated by a non-singular cubic curve, which generates an almost-MDS code. In this work, 108 novel nearly-MDS codes are constructed from nonsingular cubic curves over the order 25 Galois field, and their nearly-MDS status is shown. With the exception of 26 and 26, which are incomplete, these codes are between 16 and 36 characters long. Also supplied are the covering radius and weight distributions for these codes.

Key Words: elliptic cubic curves, Arc, Weight distributions, Near-MDS-code

رموز MDS القريبة من الدرجة الثالثة فوق حقل جالوا من الرتبة 25

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خلاصة

في المستوى الإسقاطي PG(2,q) ، يمكن إنشاء قوس (n;3) بواسطة منحنى مكعب غير مفرد، والذي يولد رمز MDS تقريبًا. في هذا العمل، تم إنشاء 108 رموز جديدة تقريبًا لـ MDSمن منحنيات مكعبة غير مفردة على الترتيب 25 لحقل جالوا، ويتم عرض حالة شبه MDS الخاصة بهم. باستثناء 26 و26، وهي غير مكتملة، يتراوح طول هذه الرموز بين 16 و36 حرفًا. يتم أيضًا توفير نصف قطر التغطية وتوزيعات الوزن لهذه الرموز.

الكلمات المفتاحية: المنحنيات المكعبة الإهليلجية، القوس، توزيعات الوزن، كود MDS القريب

INRODUCTION

Let $F_q=GF(q)$ represent the order q Galois field, where q is a prime power ($q=p^m$, p is a prime number, and m is a positive integer), and let $PG(2,q)$ be the projective plane over F_q. A set of k points with r collinear points and no $(r+1)$ collinear points is known as a $(k;r)-$arc. If a $(k+1,r)$-arc does not contain a $(k;r)-$arc, it is said to be complete. For further details on the salient features of these geometric structures see [1] .

$V(n,q)$ is a vector space of dimension n over F_q , and C is a subspace of this space of size dim(C) = k. This code is also known as a $[n,k,d]$_q-code or a $[n,k,d]$ linear q- ary -code C and the shortest distance is given by $d(C)$ = d = min $w(x)|x \neq (0) \in$ C} = min $\{d(x,y)|\ x \neq y\}$.

We indicate $x=(x_1,\dots,x_n)=x_1\dots x_n$ and $y=(y_1,\dots,y_n)= y_1\cdots y_n$ where $w(x)= |\{i \mid x_i \neq 0\}|$, is the codeword's weight x. $d(x,y)=|\{i \mid x_i \neq y_i\}|$ is the Hamming distance between x and y. Generator matrix refers to a matrix G whose columns serve as the building blocks of a linear code. An $[n,k,d]$ q-code If any two columns of the generator matrix are linearly independent, C is referred to as a projective code (PG code). The term "e-error correcting code" refers to a code of distance d, where $e=\lfloor (d-1)/2 \rfloor$. The quantity A_i, where $i=0$ to n , of Hamming-weighted codewords in C. The vector $(A_0,\dots,A_n)$ represents the weight distribution of C.

For any $y \in C$, dual code C of a $[n,k,d]$_q-code C is defined as $C^{\perp}=\{x \in F_q^n \mid x \cdot y = 0\}$. for Further information on linear code's characteristics see [2].

One of the most challenging coding theory problems is determining the existence of a code with a given set of parameters (length, dimension, and Hamming distance). Also, figuring out the values of k for which a complete $(k;r)-$arc exists is one of the trickier and more challenging tasks in finite projective geometry. Since the two issues are related, the outcomes of one of them will have a beneficial impact on the other, as will be shown in the next section.

One of the main areas of study in the projective geometry $PG\ (n,q)$ is finding complete $(k;r)$-arcs with the most points. A few of them looked at this problem conceptually., as in [3]–[7], while others more researched it in the projective plane for a certain field. and $r=3$, as shown in [8], [9], and [10], $q=27$. Additionally, several writers examined codes through arcs for certain fields, as in [10], $q=17$, [11], [12], $q=19$, [13], $q=2,3,5,7$, [14], $q=11,13$, [15] , $q=27$.

To find original linear codes, this study makes use of the connection between cubic curves and the PG linear code, Near-MDS codes providing information on their weight distributions and covering radius..

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In order to establish linear coding of Near $-MDS$ code, as well as To determine the covering radius and weight distributions, it was necessary to identify the generating matrices for each of these codes. These algorithms were built on a pc with a Core i7-8565U CPU operating from Intel at 1. 99 GHz using the GAP application [16]. RAM 8 GB.

BACKGROUND AND PREVIOUS RESULTS

This section contains a discussion of the calculation techniques as well as the vocabulary and algebra used in this subject.

Definition 1 [2]. The $[n,k,d]\_q$-code's covering radius C is the smallest integer , $\bigcup_{x\in C} S(x,\omega) = F_q^n$, where the ball $S(x,\omega)$ has radius ω *and* centre x .

Definition 2[17][18]. The term "almost-MDS code" (abbreviated "$AMDS$") refers to a linear code C with $d(C) = n - k$ and a Near-MDS code, or MDS , is $d(C) = d(C^{\perp})$.

Theorem 3[18]: Suppose C is an $[n,k,d]_q$-Near $-MDS$ code, its weight distribution is $\{A_i\,|i = 0\ to\ n\}$ and the weight distribution of $C^{\perp}\ is\ \{A'_i\ |i = 0\ to\ n\}$. Then ,

$$A_{n-k+s} = \binom{n}{k-s}\sum_{j=0}^{s-1}(-1)^j\binom{n-k+s}{j}(q^{s-j}-1) + (-1)^s\binom{k}{s}A_{k-s},$$

where $s \in \{1,2,\dots,k\}$, and

$$A'_{n-k+s} = \binom{n}{k+s}\sum_{j=0}^{s-1}(-1)^j\binom{k+s}{j}(q^{s-j}-1) + (-1)^s\binom{n-k}{s}A'_k\ ,$$

$s \in \{1\ to\ n-k\}$.

In [19] and [20] theoretically , the connection between projective coding theory and the theory of a $(n;r)$-arcs is provided. Below is a summary of this relationship.

Theorem 4 . $[n,k,d]_q$- code in F_q^n is identical to an $(n;n-d)$-arc in $PG(k-1,q)$.

Therefore, there is a 1:1 correspondence between $[n,3,n-3]_q$-code C and $(n;3)$-arcs in $PG(2,q)$ for $k = 3$ and $n - d = 3$.

The primary finding upon which this study is predicated is presented in [8] and is briefly summarized in the following Table.

Suppose F stands for collection of points on F'_i (the cubic curve), where $|F|$ shows how many points are on F'_i and the size of the whole curve created by $F(F'_i)$ is shown by $M(F)$. Assume α is fundamental component of F_q.

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TABLE 1. Elliptic cubic curves parameters

F'_i	Coefficient of $X_0{}^3 + X_1{}^3X_2{}^3 - bX_0X_1X_2$	$\lvert\mathcal{F}\rvert - M(\mathcal{F}$	F'_i	Coefficients of $X_2{}^2X_1 + X_0{}^3 + bX_0{}^2X_1 + cX_1{}^3$	$\lvert\mathcal{F}\rvert - M(\mathcal{F}$
F'_1	α^{21}	18-36	F'_{41}	$\alpha\,,\alpha^8$	28
F'_2	α^6	27	F'_{42}	$\alpha\,,\alpha^6$	28
F'_3	0	36	F'_{43}	α^7,α	29
F'_i	$X_0X_1X_2{+}bX^3$		F'_{44}	α^7,α^{13}	29
F'_4	α^9	18-31	F'_{45}	$1\,,\alpha^{22}$	32
F'_5	α^{21}	18-31	F'_{46}	$1\,,\alpha^6$	32
F'_i	Coefficient of $X_2{}^2X_1 + X_0{}^3 - bX_1{}^3$		F'_{47}	$1\,,\alpha^{14}$	32
F'_6	α^2	21-36	F'_{48}	α^7,α^4	34
F'_7	α^4	21-36	F'_{49}	α^7,α^{14}	34
F'_i	Coefficient of $X_0X_1X_2$ $+bX^3$		F'_{50}	$\alpha^{22}\,,1$	34
F'_8	α	24	F'_{51}	$0\,,\alpha^9$	16-32
F'_9	α^7	24	F'_{52}	$\alpha^{22},0$	20-28
F'_{10}	α^{22}	24	F'_{53}	$0\,,\alpha$	31
F'_{11}`	α^5	24	F'_{54}	$0\,,\alpha^7$	31
F'_{12}	α^{14}	24	F'_{55}	$1\,,0$	32
F'_{13}	α^{11}	24	F'_{56}	$\alpha^7\,,0$	34
F'_{14}	α^{15}	27	F'_i	Coefficient of $X_0{}^3 + aX_1{}^3 + bX_2{}^3 - c\,X_0X_1X_2$	
F'_{15}	α^3	27	F'_{57}	$\alpha\,,\alpha^2,\alpha^6$	18-30
F'_{16}	α^{19}	30	F'_{58}	$\alpha\,,\alpha^2,\alpha^{22}$	18-30
F'_{17}	α^4	30	F'_{59}	$\alpha\,,\alpha^2\,,\alpha$	27
F'_{18}	α^{20}	30	F'_{60}	$\alpha\,,\alpha^2\,,\alpha^7$	27
F'_{19}	α^{23}	30	F'_{61}	$\alpha\,,\alpha^2\,,\alpha^{16}$	27
F'_{20}	α^{10}	33	F'_{62}	$\alpha\,,\alpha^2\,,\alpha^4$	27
F'_{21}	α^2	33	F'_{63}	$\alpha\,,\alpha^2\,,0$	36
F'_i	Coefficients of $X_2{}^2X_1 + X_0{}^3 + bX_0{}^2X_1 + cX_1{}^3$		F'_{64}	$\alpha\,,\alpha^2\,,\alpha^{13}$	36
F'_{22}	$1\,,\alpha^{15}$	17-32	F'_i	Coefficients of $X_0{}^2X_1{+}\,X_0{}^2X_2{+}\,a\,X_1{}^2X_2 - b(\,X_0{}^3 + a\,X_1{}^3 + a^2X_2{}^3 - c\,X_0X_1X_2\,)$	
F'_{23}	$\alpha^7\,,1$	19-31	F'_{65}	$\alpha\,,\alpha^{10},\alpha^{19}$	18-32
F'_{24}	$\alpha^7\,,\alpha^6$	19-31	F'_{66}	$\alpha^2\,,\alpha^4,\alpha^{20}$	18-32

F'_{25}	α^{22}, α^{7}	20-28	F'_{67}	$\alpha, 0$	21
F'_{26}	α^{22}, α^{19}	20-30	F'_{68}	$\alpha^{2}, 0$	21
F'_{27}	α^{22}, α^{15}	20-28	F'_{69}	$\alpha, \alpha^{7}, \alpha^{19}$	24
F'_{28}	$1, \alpha$	22-27	F'_{70}	$\alpha, \alpha^{13}, \alpha^{19}$	24
F'_{29}	$1, \alpha^{13}$	22-25	F'_{71}	$\alpha, 1, \alpha^{19}$	24
F'_{30}	$1, \alpha^{8}$	22-27	F'_{72}	$\alpha^{2}, \alpha^{13}, \alpha^{20}$	24
F'_{31}	$1, \alpha^{5}$	22-25	F'_{73}	$\alpha^{2}, \alpha^{19}, \alpha^{20}$	24
F'_{32}	α, α^{4}	23-26	F'_{74}	$\alpha^{2}, \alpha^{10}, \alpha^{20}$	24
F'_{33}	α, α^{14}	23-26	F'_{75}	$\alpha, \alpha^{19}, \alpha^{19}$	27
F'_{34}	α^{22}, α^{8}	25	F'_{76}	$\alpha^{2}, \alpha, \alpha^{20}$	27
F'_{35}	α^{22}, α^{9}	25-28	F'_{77}	$\alpha, \alpha, \alpha^{19}$	30
F'_{36}	α^{22}, α^{4}	25	F'_{78}	$\alpha, \alpha^{22}, \alpha^{19}$	30
F'_{37}	α, α	28	F'_{79}	$\alpha^{2}, 1, \alpha^{20}$	30
F'_{38}	α, α^{13}	28	F'_{80}	$\alpha^{2}, \alpha^{7}, \alpha^{20}$	30
F'_{39}	$\alpha, 1$	28	F'_{81}	$\alpha, \alpha^{4}, \alpha^{19}$	30
F'_{40}	α, α^{22}	28	F'_{82}	$\alpha^{2}, \alpha^{22}, \alpha^{20}$	30

Algorithm B :

Cic= points on the projective cubic curves F_i as a group

1- for g in Cic do

PIO=[] A collection of points within the code C, denoted as g, is observed.

M= A matrix is constructed such that each column represents a point within the set denoted as "g".

KL=[]; The collection of Hamming distances between codewords.

2- for s in [1..q^n] Let n be the number of rows in M.

y= build from M at point

m= amount of y coordinates that are not zero;

Add(KL,m);

2- od;

$d(C)$= The code's shortest Hamming distance derived from the KL divergence is calculated.

W= Theorem 3's weight distribution for the code.

PO A subroutine is executed to evaluate positive integers in order to calculate covering radius. Due to the excessive time required for this process, it is separated into multiple partitions.)

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The subsequent theorem presented in this section provides a concise summary of the research findings pertaining to Near-MDS codes derived from elliptic cubic curves in the projective plane $PG(2,25)$.

Because it takes up a lot of space, the generating matrix for the linear coding is not printed; just the results are.

.**Theorem 5.** The omission of the generating-matrix for the linear-coding is attributed to its extensive size, with only the outcomes being presented instead.

There exist a total of 82 $PG\ Near-MDS$ codes of dimension three over the field F_{25}, with a length ranging from 16 to 36, that are derived from cubic curves..

The weight distributions and covering radius are supplied.

Proof.

Suppose F'_i indicates the cubic forms, $C_{F'_i}$ is the linear code constructed from F'_i, and $C^+_{F'_i}$ is the extension linear code from $C_{F'_i}$.

Following PG codes may be determined from Theorem 4 and Table 1 using procedure B.

For any cubic curve in $PG(2,25)$ there is PG Near-MDS $[n,k\ ,d]$ −code C_{F_i} $,i=1,2,\dots,82$, their weight distribution wd , error correcting code e and covering radius cr of these codes it has been organized in the following table:

F_i: PG Near-MDS $[n,k\ ,d]$ −code C_{F_i}	Weight distribution wd	Error correcting code e	covering radius cr
F_1 : [18,3,15] -code C_{F_1}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1152, 216, 7344, 6912)	7	$13 \le \omega \le 15$
$F_1{}^+$: [36,3,33] -code $C_{F_1^+}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4392, 1944, 5400, 3888)	16	$28 \le \omega \le 33$
F_2: [27,3,24] -code C_{F_2}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2664, 432, 7992, 4536)	11	$20 \le \omega \le 24$
F_3 : [36,3,33] -code C_{F_3}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	7	$13 \le \omega \le 15$

	0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4824, 648, 6696, 3456)		
F_4 : [18,3,15] -code C_{F_4}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1104, 360, 7200, 6960)	7	$13 \le \omega \le 15$
$F_4{}^+$: [31,3,28] -code $C_{F_4^+}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 3384, 4800, 4848)	13	$24 \le \omega \le 28$
F_5 : [18,3,15] -code C_{F_5}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1104, 360, 7200, 6960)	7	$13 \le \omega \le 15$
$F_5{}^+$: [31,3,28] -code $C_{F_5{}^+}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 3384, 4800, 4848)	13	$24 \le \omega \le 28$
F_6: [21,3,18] -code C_{F_6}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1536, 432, 7632, 6024)	8	$15 \le \omega \le 18$
$F_6{}^+$: [36,3,33] -code $C_{F_6{}^+}$:	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3960, 3240, 4104, 4320)	16	$28 \le \omega \le 33$
F_7 : [21,3,18] -code C_{F_7}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1536, 432, 7632, 6024)	8	$15 \le \omega \le 18$
$F_7{}^+$: [36,3,33] -code $C_{F_7{}^+}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3960, 3240, 4104, 4320)	16	$28 \le \omega \le 33$
F_8: [24,3,21] -code C_{F_8}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504, 7848, 5232)	10	$18 \le \omega \le 21$
F_9: [24,3,21] -code C_{F_9}	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504,7848, 5232)	10	$18 \le \omega \le 21$
F_{10}: [24,3,21] -code $C_{F_{10}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504,7848, 5232)	10	$18 \le \omega \le 21$
$F_{10}{}^+$: [28,3,25] -code $C_{F_{10}{}^+}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2472, 1656, 6744, 4752)	12	$21 \le \omega \le 25$

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F_{11}: [24,3,21] -code $C_{F_{11}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504, 7848, 5232)	10	$18 \leq \omega \leq 21$
F_{12}: [24,3,21] -code $C_{F_{12}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504, 7848, 5232)	10	$18 \leq \omega \leq 21$
${F_{12}}^{+}$: [28,3,25] -code $C_{{F_{12}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2472, 1656, 6744, 4752)	12	$21 \leq \omega \leq 25$
F_{13}: [24,3,21] -code $C_{F_{13}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2040, 504, 7848, 5232)	10	$18 \leq \omega \leq 21$
F_{14}: [27,3,24] -code $C_{F_{14}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2664, 432, 7992, 4536)	11	$20 \leq \omega \leq 24$
F_{15}: [27,3,24] -code $C_{F_{15}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2616, 576, 7848, 4584)	11	$20 \leq \omega \leq 24$
F_{16}: [30,3,27] -code $C_{F_{16}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3264, 648, 7632, 4080)	13	$23 \leq \omega \leq 27$
F_{17}: [30,3,27] -code $C_{F_{17}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3264, 648, 7632, 4080)	13	$23 \leq \omega \leq 27$
F_{18}: [30,3,27] -code $C_{F_{18}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3264, 648, 7632, 4080)	13	$23 \leq \omega \leq 27$
F_{19}: [30,3,27] -code $C_{F_{19}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3264, 648, 7632, 4080)	13	$23 \leq \omega \leq 27$
F_{20}: [33,3,30] -code $C_{F_{20}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3984, 720, 7200, 3720)	14	$26 \leq \omega \leq 30$
F_{21}: [33,3,30] -code $C_{F_{21}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3984, 720,	14	$26 \leq \omega \leq 30$

	7200, 3720)		
F_{22}: [17,3,14] -code $C_{F_{22}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 960, 384, 6960, 7320)	6	$12 \leq \omega \leq 14$
${F_{22}}^{+}$: [32,3,29] -code $C_{{F_{22}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2952, 3048, 5016, 4608),	14	$25 \leq \omega \leq 29$
F_{23} : [19,3,16] -code $C_{F_{23}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1224, 432, 7320, 6648)	7	$13 \leq \omega \leq 16$
${F_{23}}^{+}$: [31,3,28] -code $C_{{F_{23}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2832, 2664, 5520, 4608)	13	$24 \leq \omega \leq 28$
F_{24}: [19,3,16] -code $C_{F_{24}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1224, 432, 7320, 6648)	7	$13 \leq \omega \leq 16$
${F_{24}}^{+}$: [31,3,28] -code $C_{{F_{24}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2952, 2304, 5880, 4488)	13	$24 \leq \omega \leq 28$
F_{25}: [20,3,17] -code $C_{F_{25}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1368, 456, 7464, 6336)	8	$14 \leq \omega \leq 17$
$F_{25^{+}}$: [28,3,25] -code $C_{F_{25^{+}}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2232, 2376, 6024, 4992)	12	$21 \leq \omega \leq 25$
F_{26}: [20,3,17] -code $C_{F_{26}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1368, 456, 7464, 6336)	8	$14 \leq \omega \leq 17$
${F_{26}}^{+}$: [30,3,27] -code $C_{{F_{26}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2448, 3096, 5184, 4896)	13	$23 \leq \omega \leq 27$
F_{27}: [20,3,17] -code $C_{F_{27}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1368, 456, 7464, 6336)	8	$14 \leq \omega \leq 17$
${F_{27}}^{+}$: [28,3,25] -code $C_{{F_{27}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2232, 2376, 6024, 4992)	12	$21 \leq \omega \leq 25$
F_{28}: [22,3,19] -code $C_{F_{28}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1680,	9	$16 \leq \omega \leq 19$

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	504, 7680, 5760)		
${F_{28}}^{+}$: [27,3,24] -code $C_{{F_{28}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2208, 1800, 6624, 4992)	11	$20 \le \omega \le 24$.
F_{29}: [22,3,19] -code $C_{F_{29}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1680, 504, 7680, 5760)	9	$16 \le \omega \le 19$
${F_{29}}^{+}$: [25,3,22] -code $C_{{F_{29}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1896, 1512, 6888, 5328)	10	$19 \le \omega \le 22$
F_{30}: [22,3,19] -code $C_{F_{30}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1680, 504, 7680, 5760)	9	$16 \le \omega \le 19$
${F_{30}}^{+}$: [27,3,24] -code $C_{{F_{30}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2208, 1800, 6624, 4992)	10	$20 \le \omega \le 24$
F_{31}: [22,3,19] -code $C_{F_{31}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1680, 504, 7680, 5760)	9	$16 \le \omega \le 19$
${F_{31}}^{+}$: [25,3,22] -code $C_{{F_{31}}^{+}}$	1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1896, 1512, 6888, 5328)	10	$19 \le \omega \le 22$
F_{32}: [23,3,20] -code $C_{F_{32}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1848, 528, 7752, 5496)	9	$17 \le \omega \le 20$
${F_{32}}^{+}$: [26,3,23] -code $C_{{F_{32}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2112, 1464, 6960, 5088)	11	$20 \le \omega \le 23$
F_{33}: [23,3,20] -code $C_{F_{33}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1848, 528, 7752, 5496)	9	$17 \le \omega \le 20$
${F_{33}}^{+}$: [26,3,23] -code $C_{{F_{33}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2112, 1464, 6960, 5088)	11	$20 \le \omega \le 23$.
F_{34}: [25,3,22] -code $C_{F_{34}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2208, 576, 7824, 5016)	10	$19 \le \omega \le 22$
F_{35}: [25,3,22] -code $C_{F_{35}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2208, 576, 7824, 5016)	10	$19 \le \omega \le 22$
${F_{35}}^{+}$: [28,3,25] -code $C_{{F_{35}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	10	$19 \le \omega \le 22$

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	2208, 576, 7824, 5016)		
F_{36}: [25,3,22] -code $C_{F_{36}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2208, 576, 7824, 5016)	10	$19 \leq \omega \leq 22$
F_{37}: [28,3,25] -code $C_{F_{37}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{38}: [28,3,25] -code $C_{F_{38}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{39}: [28,3,25] -code $C_{F_{39}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{40}: [28,3,25] -code $C_{F_{40}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{41}: [28,3,25] -code $C_{F_{41}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{42}: [28,3,25] -code $C_{F_{42}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2808, 648, 7752, 4416)	12	$21 \leq \omega \leq 25$
F_{43}: [29,3,26] -code $C_{F_{43}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3024, 672, 7680, 4248)	12	$22 \leq \omega \leq 26$
F_{44}: [29,3,26] -code $C_{F_{44}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 96, 2688, 1104, 7440, 4296)	12	$25 \leq \omega \leq$ 26.
F_{45}: [32,3,29] -code $C_{F_{45}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3720, 744, 7320, 3840)	14	$25 \leq \omega \leq 29$
F_{46}: [32,3,29] -code $C_{F_{46}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3720, 744, 7320, 3840)	14	$25 \leq \omega \leq 29$
F_{47}: [32,3,29] -code $C_{F_{47}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3720, 744, 7320, 3840)	14	$25 \leq \omega \leq 29$

F_{48}: [34,3,31] -code $C_{F_{48}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4224, 792, 6960, 3648)	15	$27 \leq \omega \leq 31$
F_{49}: [34,3,31] -code $C_{F_{49}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4224, 792, 6960, 3648)	15	$27 \leq \omega \leq 31$
F_{50}: [35,3,32] -code $C_{F_{50}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4488, 816, 6744, 3576)	15	$28 \leq \omega \leq 32.$
F_{51}: [16,3,13] -code $C_{F_{51}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 840, 360, 6744, 7680)	6	$11 \leq \omega \leq 13$
${F_{51}}^{+}$: [32,3,29] -code $C_{{F_{51}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2856, 3336, 4728, 4704)	14	$25 \leq \omega \leq 29$
F_{52}: [20,3,17] -code $C_{F_{52}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1368, 456, 7464, 6336)	8	$14 \leq \omega \leq 17$
${F_{52}}^{+}$: [28,3,25] -code $C_{{F_{52}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2136, 2664, 5736, 5088)	12	$21 \leq \omega \leq 25$
F_{53}: [31,3,28] -code $C_{F_{53}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3480, 720, 7464, 3960)	13	$24 \leq \omega \leq 28$
F_{54}: [31,3,28] -code $C_{F_{54}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3480, 720, 7464, 3960)	13	$24 \leq \omega \leq 28$
F_{55}: [32,3,29] -code $C_{F_{55}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3720, 744, 7320, 3840)	14	$25 \leq \omega \leq 29$
F_{56}: [34,3,31] -code $C_{F_{56}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4224, 792, 6960, 3648)	15	$27 \leq \omega \leq 31$

F_{57}: [18,3,15] -code $C_{F_{57}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1080, 432, 7128, 6984)	7	$13 \leq \omega \leq 15$
${F_{57}}^{+}$: [30,3,27] -code $C_{{F_{57}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2520, 2880, 5400, 4824)	13	$23 \leq \omega \leq 27$
F_{58}: [18,3,15] -code $C_{F_{58}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1080, 432, 7128, 6984)	7	$13 \leq \omega \leq 15$
${F_{58}}^{+}$: [30,3,27] -code $C_{{F_{58}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2520, 2880, 5400, 4824)	13	$23 \leq \omega \leq 27$
F_{59}: [18,3,15] -code $C_{F_{59}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1080, 432, 7128, 6984)	7	$13 \leq \omega \leq 15$
${F_{59}}^{+}$: [32,3,29] -code $C_{{F_{59}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2904, 3192, 4872, 4656)	14	$25 \leq \omega \leq 29$
F_{60}: [18,3,15] -code $C_{F_{60}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1080, 432, 7128, 6984)	7	$13 \leq \omega \leq 15$
${F_{60}}^{+}$: [32,3,29] -code $C_{{F_{60}}^{+}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 96, 456, 1680, 3168, 6288, 3936)	14	$25 \leq \omega \leq 29$
F_{61}: [21,3,18] -code $C_{F_{61}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1512, 504, 7560, 6048)	8	$15 \leq \omega \leq 18$
F_{62}: [21,3,18] -code $C_{F_{62}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1512, 504, 7560, 6048)	8	$15 \leq \omega \leq 18$
F_{63}: [24,3,21] -code $C_{F_{63}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2016, 576, 7776, 5256)	10	$18 \leq \omega \leq 21$
F_{64}: [24,3,21] -code $C_{F_{64}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2016, 576, 7776, 5256)	10	$18 \leq \omega \leq 21$
F_{65}: [24,3.21] -code $C_{F_{65}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	10	$18 \leq \omega \leq 21$

	2016, 576, 7776, 5256)		
F_{66}: [24,3,21-code $C_{F_{56}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2016, 576, 7776, 5256)	10	$18 \leq \omega \leq 21$
F_{67}: [24,3,21] -code $C_{F_{67}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2016, 576, 7776, 5256)	10	$18 \leq \omega \leq 21$
F_{68}: [24,3,21] -code $C_{F_{68}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2016, 576, 7776, 5256)	10	$18 \leq \omega \leq 21$
F_{69}: [27,3,24] -code $C_{F_{69}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$20 \leq \omega \leq 24$
F_{70}: [27,3,24] -code $C_{F_{70}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$12 \leq \omega \leq 24$
F_{71}: [27,3,24] -code $C_{F_{71}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$20 \leq \omega \leq 24$
F_{72}: [27,3,24] -code $C_{F_{72}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$20 \leq \omega \leq 24$
F_{73}: [27,3,24]-code $C_{F_{73}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$20 \leq \omega \leq 24$
F_{74}: [27,3,24]-code $C_{F_{74}}$	wd (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2592, 648, 7776, 4608)	11	$20 \leq \omega \leq 24$
F_{75}: [30,3,27]-code $C_{F_{75}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3240, 720, 7560, 4104)	13	$23 \leq \omega \leq 27$
F_{76}: [30,3,27]-code $C_{F_{76}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3240, 720, 7560, 4104)	13	$23 \leq \omega \leq 27.$
F_{77}: [30,3,27]-code $C_{F_{77}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3240, 720, 7560, 4104)	13	$23 \leq \omega \leq 27$
F_{78}: [30,3,27]-code $C_{F_{62}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	13	$23 \leq \omega \leq 27$

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	0, 0, 0, 0, 3240, 720, 7560, 4104)		
F_{79}: [33,3,30]-code $C_{F_{79}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3960, 792, 7128, 3744)	14	$26 \leq \omega \leq 30$
F_{80}: [33,3,30]-code $C_{F_{80}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3960, 792, 7128, 3744)	14	$26 \leq \omega \leq 30.$
F_{81}: [36,3,33]-code $C_{F_{81}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4752, 864, 6480, 3528)	16	$28 \leq \omega \leq 30$
F_{82}: [36,3,33]-code $C_{F_{82}}$	(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4752, 864, 6480, 3528)	16	$28 \leq \omega \leq 30$

CONCLUSION

There exist 108 non-metric distance scaling ($NMDS$) codes that exhibit projective distinctiveness over the finite field F_{25}, with a dimension of three. The length of these codes spans all values between 16 and 36, excluding the value 26. Among the set of Near $- MDS$ codes, there exist 26 codes that are not capable of being extended. These codes possess a length that corresponds to one of the following values:$21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35$,36.

COMPARISON BETWEEN THE PARAMETERS OF THE LINEAR CODES IN $PG(2,25)$ AND THE PARAMETERS OF THE LINEAR CODES IN $PG(2,27)$

From [15] and above results , we conclude the following:

$PG(2\ ,25)$ | ***$PG(2,27)$***

(i) The cell $\tau : \rho$ refers to the number of points the projective linear code F_{c_i} ($F_{c_i}{}^{+}$ for extended codes) τ and corresponding number of such distinct projective code ρ :

$PG(2,25)$	$PG(2,27)$
F_{c_i} ; 16:1 17:1 18: 7 19:2 20:4 21:4 22:4 23:2 24:12 25:3 27:9 28:6 30:8 31:2 32:4	F_{c_i} ; 18: 2 19:1 20:4 21:6 23:3 24:12 26:6 27:9 28:2 29:3 30:12 32:6 33:6 35:3 36:8 37:1 38:1

33:4 34:3 35:1 36:3 $F_{c_i^+}$: 25:2 26:3 27:2 28:2 30:3 32:3 36:3	$F_{c_i^+}$: 25:3 26:1 27:8 28:3 29:4 30:6 32:4 33:2 34:1

(ii) The cell m : n refers to the value of e (error correcting code) for the projective linear code F_{c_i} m and corresponding number of such distinct projective code n :

6:2 7:10 8:8 9:6 10:19 11:12 12:13 13:17 14:12 15:4 16:5	7:3 8:10 9:3 10:15 11:21 12:12 13:18 14:18 15:4 16:9 17:1
(iii) Minimum value of the covering radius cr for projective linear code F_{c_i} is 11 and Maximum value is 33	Minimum value of the covering radius cr for projective linear code F_{c_i} is 13 and Maximum value is 35

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