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Employing a proposed method to estimate the parameters of a beta regression model in the presence of multicollinearity

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Abstract :
The multiple linear regression analysis shows and explains the effects between the explanatory variables and the dependent variable, as its function is to represent the data to understand the nature of the relationship between the explanatory variables and the response variable among the problems that the regression suffers from is that the data does not follow the normal distribution and the emergence of different types of regression models such as the beta regression model, and to know the estimates of the parameters of the multi-linear model, several methods are used for estimation, as this stage is considered one of the important and necessary stages in the analysis of the model, and because some of these methods are traditional and do not give accurate results because the random error does not have a normal distribution, so other methods with good results and suitable for the error distribution are used, so the beta regression model will be studied in which the random errors follow the beta distribution and where the data is smaller than one and it is confined within the period (0,1), the beta regression model will be addressed in this research when there is a problem of multicollinearity, and a comparison will be made between Liu's method and the proposed method used to estimate the model's parameters, as the comparison is done by simulation in two methods.

Keywords: beta regression model, multicollinearity, Liu's method, proposed method, simulation.

1- Introduction:

Regression analysis is one of the most important statistical topics widely used in the field of statistics. Its significance has been highlighted in various fields, including medicine, economics, social sciences, and many aspects of life. One of the major challenges faced by regression models is the multicollinearity problem among explanatory variables [3] [30]. Additionally, regression models sometimes encounter non-normal data distributions [38] [49], leading to the emergence of various regression models such as the Poisson regression model, logistic regression model [8] [10], and beta regression model [37] [18]. These models have provided significant support for statistical theories in numerous applications in life sciences and have offered solutions to mitigate these issues [40] [31]. To estimate the parameters of multiple linear regression models[9][28], various estimation methods are employed[43]. Some of these methods may not yield accurate results [11][42], especially when the random error does not follow a normal distribution[33][34]. In such cases, alternative methods that produce good and suitable results for error distribution are sought [4][29] . The beta distribution is one of the most important continuous probability distributions defined within the range (0,1), with parameters greater than zero. It is derived from the beta function [14]. The beta distribution is adaptable and essential, as it is used for modeling continuous random variables with values between (0,1) [13] Numerous books and previous research studies have addressed the effects of multicollinearity in various forms of multiple linear regression models and methods for detecting and addressing this problem [1][23]. Additionally, there are specific estimation techniques for beta regression. In 1961, researchers (Stein & James) introduced a new method for estimating regression parameters based on different distributions, such as spherical distributions, among others. They theoretically discussed the characteristics of a good estimator that can be extracted from models suffering from linear relationships among explanatory variables. In 2004, researchers (Ferrari & Cribari-Neto) proposed a multiple linear regression model when its random errors follow a beta distribution, with parameters representing the mean and dispersion parameters. They explained that beta regression is useful when the data is continuous and confined to the range (0,1). The Maximum Likelihood Estimation (MLE) method for beta regression was theoretically elaborated. In 2020, researchers (Kabria and others) suggested using the Liu method to estimate the parameters of the beta regression model. They also theoretically explained the Maximum Likelihood Estimation (MLE) method for the beta regression model and estimated the parameters using the Liu method [17] [6]. They derived formulas to calculate the shrinkage parameter for the Liu method [45] [35], in addition to proposing other formulas for calculating the shrinkage parameter for the Liu method, In 2022 the researchers (Pirmohammadi and Bidram) suggested using the Liu method to compare between beta regression model and kumaraswamy regression model and using MSE [36].

2- Important of the research:

The significance of this research lies in the estimation of beta regression parameters when dealing with the multicollineaity problem using the Liu method and utilizing both original and proposed shrinkage parameters. furthermore, the research introduces an effective approach for parameter estimation in beta regression model. the proposed method has demonstrated lower mean squared error values compared to the previously mentioned method.

3- Research Hypotheses:

The research assumes that the original Liu method uses original shrinkage parameters to estimate the parameters of the beta regression model, the research assumes that there are proposed shrinkage parameters used to improve the performance of the Liu method and estimate the model parameters more effectively, the hypothesizes the expansion of the specific formula of the Liu method to incorporate the proposed parameters and enhance the method's performance. there is a proposed employee method for parameter estimation that enhances the accuracy of the model's parameters. Additionally, use Monte-Carlo Method to perform the analysis and estimate the parameters, by generating random data to test the two proposed methods using a comparison criterion mean squared error (MSE) evaluate the performance of the two proposed methods.

4- Research Problem:

The problem addressed by this research is the application of linear regression analysis to data that contains non-normally distributed random error [41] [48], such as a Beta Distribution., linear regression faces the issue of multicollinearity among explanatory variables. This problem makes the use of traditional methods like Least Squares and Maximum Likelihood Estimation inadequate for obtaining accurate estimates.

To address this problem, the research employs the Liu method and proposes an employee method for estimating the model parameters [47] [5]. The goal is to obtain accurate estimates of the model parameters in the presence of non-normally distributed random error and the multicollinearity problem. This problem represents a statistical challenge that requires the development and utilization of specialized methods for analysis and estimation.

5- Estimation Methods:

5.1. Liu Method:

The Liu method addresses the inflation that occurs in the variance of estimated model parameters. In 1993, Liu developed this method to deal with linear regression models suffering from multicollinearity [32][7].

The parameter vector estimation using the Liu method is carried out as follows [24][15]:

$$\hat{\beta}(d) = (X'X + I)^{-1}(X'X + dI)\hat{\beta}_{ML} \qquad \ldots\ldots(1)$$

And based on the constraint introduced by Liu [16][39]:

$$E'E = \left(d\hat{\beta}_{ML} - \hat{\beta}_{LBR}\right)'\left(d\hat{\beta}_{ML} - \hat{\beta}_{LBR}\right) \qquad \ldots\ldots(2)$$

By taking the weighted sum of squares of errors for the beta regression model after incorporating the constraint and differentiating, we obtain the following[27]:

$$E'WE = \left(Y - X\hat{\beta}_{LBR}\right)'\,\widehat{W}\left(Y - X\hat{\beta}_{LBR}\right) + \left(d\hat{\beta}_{ML} - \hat{\beta}_{LBR}\right)'\left(d\hat{\beta}_{ML} - \hat{\beta}_{LBR}\right)$$
$$= Y'\widehat{W}Y - 2\,\hat{\beta}_{LBR}X'\widehat{W}Y + \hat{\beta}'_{LBR}X'\widehat{W}X\hat{\beta}_{LBR} + d'\hat{\beta}'_{ML}\hat{\beta}_{ML}d - 2d\hat{\beta}_{ML}\hat{\beta}_{LBR} + \hat{\beta}'_{LBR}\hat{\beta}_{LBR} \qquad \ldots\ldots(3)$$

By differentiating Equation (3) with respect to the parameter vector $\hat{\beta}_{LBR}$, we obtain:

$$\frac{\partial E'W\,E}{\partial\hat{\beta}_{LBR}} = -2X'\widehat{W}Y + 2X'\widehat{W}X\hat{\beta}_{LBR} - 2d\hat{\beta}_{ML} + 2\hat{\beta}_{LBR} \qquad \ldots\ldots(4)$$

So:

$$Y = X\hat{\beta}_{ML}$$

When setting the derivative equal to zero, the weighted estimate of Liu is as follows:

$$\hat{\beta}_{LBR} = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + dI\right)\hat{\beta}_{ML} \qquad \ldots\ldots(5)$$

additionally, the bias vector for the Liu method in beta regression is [20][2]:

$$\mathrm{Bias}\left(\hat{\beta}_{LBR}\right) = \left(X'\widehat{W}X + I\right)^{-1}(d - I)\hat{\beta}_{ML} \qquad \ldots\ldots(6)$$

The estimate of Liu can be expressed in the following formula [24]:

$$\hat{\beta}_{LBR} = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + dI\right)\hat{\beta}_{ML} = \delta\hat{\beta}_{ML} \qquad \ldots\ldots(7)$$

So:

$$\delta = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + dI\right) \qquad \ldots\ldots(8)$$

Furthermore, the variance and covariance matrix for the Liu estimators is as follows:

$$\mathrm{Var-Cov}\left(\hat{\beta}_{LBR}\right) = \delta\,\mathrm{Var-Cov}\left(\hat{\beta}_{ML}\right)\delta' \qquad \ldots\ldots(9)$$

The Mean Squared Error (MSE) for the parameters of the beta regression model using the Liu method is as follows [39][25]:

$$\mathrm{MSE}\left(\hat{\beta}_{LBR}\right) = E\left[(\hat{\beta}_{LBR} - \beta)'(\hat{\beta}_{LBR} - \beta)\right]$$
$$= \frac{1}{\varphi}\sum_{j=1}^{p}\frac{(\lambda_j + d)^2}{\lambda_j(\lambda_j + 1)^2} + (d - 1)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{\left(\lambda_j + 1\right)^2} \qquad \ldots\ldots(10)$$

And the value of d is as follows [2]:

$$d_L = \frac{\sum_{j=1}^{p}\left[\frac{\left(\widehat{\alpha}_j^2 - \frac{1}{\varphi}\right)}{(\lambda_j + 1)^2}\right]}{\sum_{j=1}^{p}\left[\frac{\left(\frac{1}{\varphi\lambda_j} + \widehat{\alpha}_j^2\right)}{(\lambda_j + 1)^2}\right]} \quad \dots\dots (11)$$

As for the proposed function for the value of d, it is:

$$d_{PL} = \text{Median}\left(\sqrt{\frac{\alpha\, j^2}{\lambda j}}\right) \quad \dots\dots (12)$$

2.5 Proposed Method:

The researcher has proposed another method for estimating the parameters of the beta regression model [26][19]in the presence of multicollinearity by extending Equation (5) as follows [3][22]:

$$\widehat{\beta}_{PTPBR} = \left(X'\widehat{W}X + I\right)^{-1}\left(X'\widehat{W}X + kdI\right)\widehat{\beta}_{ML} \quad \dots\dots (13)$$

So [12][50]:

$k > 0 \;\; ; \;\; 0 < d < 1$

So, the shrinkage parameters (k, d) are as follows:

The shrinkage parameter k is determined by the following formula:

$$k_{TPBR} = \frac{1}{P}\sum_{j=1}^{p}\left(\frac{\lambda_j}{\varphi(\lambda_j\widehat{\alpha}_j^2(1 - d_{TPBR}) - d_{TPBR/\varphi})}\right) \quad \dots\dots (14)$$

And the shrinkage parameter d is determined by the following formula:

$$d_{TPBR} = \frac{1}{2}\min\left(\frac{\lambda_j\widehat{\alpha}_j^2}{\frac{1}{\varphi} + \lambda_j\widehat{\alpha}_j^2}\right)_{j=1}^{p} \quad \dots\dots (15)$$

The proposed functions for the shrinkage parameters are as follows:

The proposed formula for the shrinkage parameter k is given by:

$$k_R = \frac{1}{\varphi\sum_{j=1}^{p}\widehat{\alpha}_j^2} \quad \dots\dots (16)$$

And the shrinkage parameter d is determined by the following formula:

$$d_M = \frac{\sum_{j=1}^{p}\left[\frac{\left(\frac{1}{\varphi} - k_M \hat{\alpha}_j^2\right)}{\left(\lambda_j + k_M\right)^2}\right]}{\sum_{j=1}^{p}\left[\frac{\left(\frac{1}{\varphi} + \lambda_j \hat{\alpha}_j^2\right)}{\lambda_j\left(\lambda_j + k_M\right)^2}\right]} \quad \ldots\ldots (17)$$

6- Simulation:

The concept of simulation is defined as a mathematical method used to address problems related to the studied phenomenon through the use of computers. It can also be defined as a method for representing reality without actually realizing it practically [44][29]. The Monte Carlo Method is one of the most important and widely used simulation methods because it is employed in solving complex problems for which practical experiments are challenging[46]. Additionally, it can generate random data for various probability distributions [21][1]. The comparison process relies on the mean squared errors in generating random data using MATLAB. The random error variable (y_i) in the beta regression model is generated according to a beta distribution with two parameters, (μ_i) and (φ).

In other words:

$y_i \sim Beta\ (\mu_i, \varphi)$

So:

$\varphi = \{1, 2\}$

As for (μ_i), its value is determined by the following formula:

$$\mu_t = \frac{e^{x_t'\beta}}{1 + e^{x_t'\beta}} \quad \ldots\ldots (18)$$

The explanatory variables $x_i = (x_{i1}, \ldots, x_{ip})'$are subjected to interrelationships to create the problem of multicollinearity through the following formula:

$$x_{ij} = (1 - \rho^2)^{0.5} Z_{ij} + \rho Z_{ij} \quad \ldots\ldots (19)$$

Where: i= 1 ,2 , … , n & j= 1 , 2 , ... , p

ρ: Represents the simple correlation coefficient between each pair of explanatory variables.

Z_{ij} : Represents random numbers that are independent and generated in two ways as follows:

1- The first method: Z_{ij} is generated when the standard normal distribution (Pesado Standard Normal Distribution) is used.

2- The second method$Z_{ij}$is generated based on real data as follows:

$$Z_{ij} = L(X) + (U(X) - L(X))\text{rand}\ (n, P) \quad \ldots\ldots (20)$$

Where:

L(X): Represents the minimum value of the explanatory variable in the real experimental data.

U(X): Represents the maximum value of the explanatory variable in the real experimental data.

n: Represents the sample size.

P: Represents the number of explanatory variables.

The selected sample sizes were (20, 30, 50, 60, 100, 150), with correlation coefficient values of (0.90, 0.95, 0.99), and assuming a number of explanatory variables (2, 5). The experiment was repeated 1000 times to ensure robustness and reliability of the results.

6.1- Results of Simulation

Table1: Comparison between the estimation methods with the original and proposed shrinkage parameters for the number of variables (2, 5) based on the Mean Square Error criterion for the first simulation method.

P	n	ρ	ϕ	Original Shrinkage Parameters		Proposed Shrinkage Parameters	
				Liu Method	The Proposed Method	Liu Method	The Proposed Method
2	20	0.90	1	**0.332458925472224**	**0.283446344443627**	**0.283446342875468**	**0.283432029996273**
			2	**0.148730622352262**	**0.108217867734875**	**0.108217855388534**	**0.108186401884304**
		0.95	1	**0.334023395414060**	**0.280119849279413**	**0.280119846928161**	**0.280101687189807**
			2	**0.147809249905319**	**0.104274943295425**	**0.104274923992683**	**0.104234973497546**
		0.99	1	**0.339345083927692**	**0.274250966763917**	**0.274250961739766**	**0.274222203051999**
			2	**0.147845160291178**	**0.0974528038206978**	**0.0974527559204756**	**0.0973892878948825**
	30	0.90	1	**0.294668629650899**	**0.263921951305882**	**0.263921951151812**	**0.263918464503835**
			2	**0.126358631997553**	**0.101423625769447**	**0.101423624658593**	**0.101415837327966**
		0.95	1	**0.294497824629690**	**0.260747565839922**	**0.260747565610912**	**0.260743141701201**
			2	**0.124378626911089**	**0.0976715802379758**	**0.0139180158860250**	**0.0139154933363830**
		0.99	1	**0.295729385799876**	**0.255146051963372**	**0.255146051472415**	**0.255139044882173**
			2	**0.121875070251927**	**0.0911778347327004**	**0.0911778306642575**	**0.0911621171583017**
	50	0.90	1	**0.276812338982277**	**0.258940125368407**	**0.258940125359924**	**0.258939481455286**
			2	**0.113848630210115**	**0.0996648052386794**	**0.0996648051805156**	**0.0996633370220148**
		0.95	1	**0.275408749970499**	**0.255812111085303**	**0.255812111072790**	**0.255811294072111**
			2	**0.111142507978657**	**0.0959805481395204**	**0.0139459176474980**	**0.0139454693447124**

		0.99	1	**0.273792524341921**	**0.250290537629153**	**0.250290537602630**	**0.250289243662351**
			2	**0.106947222777822**	**0.0895996964165088**	**0.0895996962121370**	**0.0895967367666585**
	60	0.90	1	**0.267657881811717**	**0.252935167660355**	**0.252935167657076**	**0.252934792232806**
			2	**0.109286895736244**	**0.0976484476415595**	**0.0976484476193803**	**0.0976475902113131**
		0.95	1	**0.265987659435113**	**0.249850253288365**	**0.249850253283539**	**0.249849777074854**
			2	**0.106449512993921**	**0.0940171516070056**	**0.0940171515730111**	**0.0940160633003151**
		0.99	1	**0.263742440503926**	**0.244404751437116**	**0.244404751426907**	**0.244403997658279**
			2	**0.101930762945247**	**0.0877282335198951**	**0.0877282334422956**	**0.0877265071454265**
	100	0.90	1	**0.260929856223121**	**0.252168012338842**	**0.252168012338627**	**0.252167932485273**
			2	**0.103790809586828**	**0.0969555633414096**	**0.0969555633399801**	**0.0969553780929112**
		0.95	1	**0.258704063206700**	**0.249104705778472**	**0.249104705778155**	**0.249104604507524**
			2	**0.100655884908050**	**0.0933613429249114**	**0.0933613429227368**	**0.0933611078525614**
		0.99	1	**0.255186286866964**	**0.243696425803441**	**0.243696425802775**	**0.243696265567336**
			2	**0.0954488198862582**	**0.0871345931192361**	**0.0163033497390579**	**0.0163032394253467**
	150	0.90	1	**0.254337083488041**	**0.248563130596479**	**0.248563130596453**	**0.248563106882843**
			2	**0.100392448998483**	**0.0959033760324419**	**0.0959033760322717**	**0.0959033205884214**
		0.95	1	**0.251843753798151**	**0.245520054203631**	**0.245520054203593**	**0.245520024131927**
			2	**0.0971232879564150**	**0.0923354486892223**	**0.0923354486889631**	**0.0923353783361447**
		0.99	1	**0.247710241349461**	**0.240147072266889**	**0.240147072266810**	**0.240147024692717**
			2	**0.0916028354911237**	**0.0861532874938601**	**0.0861532874932816**	**0.0861531759702754**
5	20	0.90	1	**0.441172679484375**	**0.303177278804670**	**0.303177258307814**	**0.302820732485195**

			2	0.244795706892470	0.119620649120096	0.119620485663466	0.118770956229578
		0.95	1	0.452629612023244	0.299351152731410	0.299351122329262	0.298853454248347
			2	0.251452473246764	0.115045146315321	0.115044888272907	0.113901291204030
		0.99	1	0.481801311626514	0.292607172097181	0.292607106222557	0.291759925605732
			2	0.269687554814697	0.107142743910377	0.107142113570237	0.105152850191314
	30	0.90	1	0.357752472943688	0.279426469196989	0.279426467566513	0.278851047495082
			2	0.176412108179004	0.109127539470571	0.109127528329431	0.109000091470281
		0.95	1	0.362458115490634	0.275956939634395	0.275956937218640	0.275708311816442
			2	0.177779529215666	0.105000387304085	0.105000369938843	0.104835918254557
		0.99	1	0.375275979738363	0.269838389160450	0.269838384010244	0.269690815459483
			2	0.183319279275096	0.0978655746747896	0.0978655336508724	0.0975947016658641
	50	0.90	1	0.295954827124558	0.254284531347325	0.254284531258692	0.254274671965453
			2	0.133942968620578	0.0995292526479306	0.0995292520981774	0.0995073078157302
		0.95	1	0.296899284359586	0.251076314060390	0.251076313929534	0.251063745143376
			2	0.132696088609299	0.0957268645176980	0.0957268636719092	0.0956987493798468
		0.99	1	0.300744538339033	0.245417158464408	0.245417158187364	0.245397043007345
			2	0.131927752077610	0.0891511105014805	0.0891511085547064	0.0891055872315785
	60	0.90	1	0.282470620683526	0.248600448906686	0.248600448874488	0.248595046651389
			2	0.125334291808271	0.0975989773152428	0.0975989771190013	0.0975871197210548
		0.95	1	0.282658488792873	0.245446269909733	0.245446269862244	0.245439396314185
			2	0.123614513525194	0.0938630630826673	0.0938630627817659	0.0938479115981679

P	n	ρ	ϕ				
		0.99	1	**0.284721053735982**	**0.239881728319621**	**0.239881728219265**	**0.239870782406545**
			2	**0.121714348587699**	**0.0874005732708022**	**0.0874005725831931**	**0.0873761933377301**
	100	0.90	1	**0.258510193465854**	**0.239329442319361**	**0.239329442317352**	**0.239328372399815**
			2	**0.109580412021631**	**0.0941514594595895**	**0.0941514594476544**	**0.0941491185508561**
		0.95	1	**0.257305008793048**	**0.236271589815823**	**0.236271589812863**	**0.236270229704927**
			2	**0.107031298861072**	**0.0905357250600202**	**0.0905357250418277**	**0.0905327405377785**
		0.99	1	**0.256111798755755**	**0.230875916161530**	**0.230875916155297**	**0.230873754162920**
			2	**0.121714348587699**	**0.0842790007866748**	**0.0842790007456414**	**0.0842742220738357**
	150	0.90	1	**0.247233873416865**	**0.234756898166703**	**0.234756898166474**	**0.234756593167488**
			2	**0.102089476229095**	**0.0921822256864877**	**0.0921822256851250**	**0.0921815578434422**
		0.95	1	**0.245424699898455**	**0.231754092564085**	**0.231754092563746**	**0.231753705024409**
			2	**0.0992156580369994**	**0.0886405927087558**	**0.0886405927066843**	**0.0886397422790556**
		0.99	1	**0.242824977663306**	**0.226454803491789**	**0.226454803491078**	**0.226454188057490**
			2	**0.121714348587699**	**0.0825106651964999**	**0.0825106651918562**	**0.0825093072706840**

Table2: Comparison between the Estimation Methods with Original and Proposed Shrinkage Parameters for Five Explanatory Variables Using Mean Squared Error Criterion for the Second Simulation Method

P	n	ρ	ϕ	Original Shrinkage Parameters		Proposed Shrinkage Parameters	
				Liu Method	The Proposed Method	Liu Method	The Proposed Method
5	100	0.90	1	**0.00386706530008319**	**0.00386702784569174**	**0.00386702784569173**	**0.00386685690377991**
			2	**0.00333042452304966**	**0.00333040431918202**	**0.00333040431918201**	**0.00333040775964425**
		0.95	1	**0.00378754492718652**	**0.00378750859876497**	**0.00378750859876496**	**0.00378751948704917**
			2	**0.00324212826248407**	**0.00324210445827372**	**0.00324210445827369**	**0.00324211109415606**

		0.99	1	**0.00364914991272251**	**0.00364911632376885**	**0.00364911632376883**	**0.00364909823252707**
			2	**0.00309353684194253**	**0.00309350194546274**	**0.00309350194546269**	**0.00309349413452687**

7- Discussion of Results:

The simulation experiment results revealed the following:

1- The proposed method showed better results than the Liu method, with smaller mean squared error values when using both original and proposed shrinkage parameters. This was observed when considering explanatory variables equal to (2, 5) and two levels of dispersion parameters (1, 2) for both simulation methods.

2- When comparing the proposed method with original shrinkage parameters to the proposed method with proposed shrinkage parameters, it was evident that the proposed method with the proposed shrinkage parameters performed better. The mean squared error values were slightly lower, especially when considering (2, 5) explanatory variables and two levels of dispersion parameters for the first simulation method.

8- Conclusions:

1- The simulation results, along with comparisons between the estimation methods, indicate that the proposed method is superior to the Liu method when considering both original and proposed shrinkage parameters. This conclusion holds for both the first and second simulation scenarios.

2- The proposed method performs exceptionally well when there are either two or five explanatory variables, especially in the first simulation scenario.

3- In the second simulation scenario, the proposed method with proposed shrinkage parameters outperforms the Liu method when there are five explanatory variables.

4- When considering the first simulation scenario, the proposed method with proposed shrinkage parameters is better than the proposed method with original shrinkage parameters when there are either two or five explanatory variables.

5- These findings highlight the effectiveness of the proposed method, particularly when utilizing the proposed shrinkage parameters, in improving parameter estimation in beta regression models, especially in scenarios with a small number of explanatory variables and varying levels of dispersion.

9- Recommendations:

1- Based on the simulation results, it is recommended to use the proposed method for estimating parameters in beta regression models with both original and proposed shrinkage parameters as an alternative to the Liu method with both original and proposed shrinkage parameters.

2- Explore and develop enhanced techniques for the proposed employee method to address various regression issues, such as heteroscedasticity.

3- Suggest additional shrinkage parameters for both the proposed method and the Liu method, considering different scenarios and applications in beta regression modeling

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