A Confined Flow over a Cylinder by the Finite Element and the Finite Difference Methods

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Abstract

The Finite element and the finite difference methods were applied to a confined flow over a cylinder. The values of velocity potential (Φ), the stream function (Ψ) and the velocity distribution are calculated by using a computer programs achieved by the researcher. The results revealed that, convergence and divergence were achieved between the equipotential lines by two methods. The finite element method has been shown to be a powerful tool. Small elements may be used in areas of rapid change and large elements may be used where variations are less severe. The boundary conditions are handled naturally by the finite element method in contrast to the finite difference method.

This study shows that the finite element method is the best technique for the solution of practical engineering problems like thermal and fluid flows (steady state or unsteady problems).

Keywords: Confined Flow, Finite element, Finite difference, velocity potential,

الجريان المحصور فوق اسطوانة بطريقتي العناصر المحددة والفروقات المتناهية

الخلاصة

استخدمت طريقتي العناصر المتناهية والفروقات المتناهية في تحليل الجريان المحصور فوق اسطوانة. ان قيم جهد السرعة ، دالة الجريان وتوزيع السرعة قد تم الحصول عليها بأستخدام برنامج على الحاسبة اعد من قبل الباحث. وان نتائج الدراسة بينت بأنه يوجد تقارب وتباعد بين خطوط تساوي الجهد بأستخدام الطريقتين المذكورتين. ان طريقة العناصر المتناهية تمثل الاداة الامثل والاكفأ في حساب قيم جهد السرعة ودالة الجريان حيث يمكن استخدام احجام مختلفة للعناصر وحسب الموقع في حقل الدراسة (تم استخدام عناصر صغيرة في المساحات او المواقع التي تكون فيها المتغيرات حادة وعناصر كبيرة في المواقع ذات المتغيرات الصغيرة) وكذلك يمكن تحديد الشروط الحدودية بصورة طبيعية عند استخدام طريقة العناصر المتناهية على العكس من ذلك عند استخدام طريقة الفروقات المتناهية. وكذلك بينت الدراسة ان طريقة العناصر المتناهية هي الطريقة المثلى لحل بعض المسائل بصورة علمية مثل مسائل الشروط الحرارية ومسائل جريان السوائل . تستخدم هذه الدراسة بصورة كبيرة في مسائل الجريان الحراري فوق اسطوانة او فوق الهدارات .

Introduction

The Finite elements method is one of the greatest advances in numerical computing of the past century. It has become an indispensable tool for simulation of a wide variety of phenomena arising in science and engineering. A tremendous asset of finite element is that they not only provided a methodology to develop numerical algorithms for simulation, but also a

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theoretical framework in which to assess the accuracy of computed solution (Dcuglas N. & Richard S. (2006)).

In engineering, physics, and applied mathematics, three main areas of application of the finite element method can indentify, these are: equilibrium problems (steady state), Eigen value problems and propagation problems. In the finite element method, approximate solutions are derived using an alternate formulation of the problem determined by variational calculus. When determining extreme using variational, a minimum is sought not for a function but for a functional. A functional is a quantity that depends upon a function of a given class rather than upon a discrete variable. Problems of variational calculus are concerned with determining extrema of functional, finding the particular function which makes a functional a minimum.

The basic steps for deriving a finite element solution to an equilibrium problem can be summarized as follows (David Roulance (2001)):-

- Sub – division of the continuum into finite element.
- Evaluation of element stiffness and load terms.
- Assembly of the element stiffness and load terms into an overall stiffness matrix and load vector,
- Solution of the resulting linear simultaneous equations for the unknown nodal variables.
- Evaluation of subsidiary element quantities.

Finite – difference methods for solving partial differential equations have been the subject of many books. The basic idea of these methods is to replace derivatives at a point by ratios of the changes in appropriate variable over small but finite intervals (Δx or Δy) (Vrushali A. Bokil & Nathan L. Gibson (2007)). This type of approximation is made at a finite number of points and reduces a continuous boundary – value problems to a set of algebraic equations. Three sets of boundary conditions may complete the specification of the problems (John Mathews (1994)):-

- Dirichlet problem. (The value of the dependent variableΦ, may be specified on all of the boundary curve (C).
- Neumann problem (the value at the normal derivative of Φ may be specified on all of the boundary curve (C).
- Mixed problem (the value of Φ may be specified on part of the boundary (C), and the normal derivative on the rest of the boundary curve (C).

The construction of finite difference approximations to derivatives are classified by (forward – differences, backward – differences and central – differences) approximations.

Construction of Finite element and Finite difference approximations to steady flow problems.

The two dimensional potential flow, (inviscid incompressible flow problems can be formulated in terms of velocity potential function (Φ) or stream function (Ψ). In terms of velocity potential, the governing equation is the Laplace's equation (elliptic equation):

$$\frac{\partial 2\Phi}{\partial x^2} + \frac{\partial 2\Phi}{\partial y^2} = 0 \qquad \ldots\ldots.(1)$$

$$\frac{\partial 2\Psi}{\partial x^2} + \frac{\partial 2\Psi}{\partial y^2} = 0 \qquad \ldots\ldots\ldots(2)$$

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where F = velocity potential and Y = stream function, the velocity components are given by:-

$$u = \frac{\partial \Phi}{\partial x} \;,\; v = \frac{\partial \Phi}{\partial y} \qquad \ldots\ldots(3)$$

And the flow velocities can be determined as:-

$$= \frac{\partial \Psi}{\partial y} \;,\; v = -\frac{\partial \Psi}{\partial x} \qquad \ldots\ldots\ldots(4)$$

Finite *Element method:*

The basic steps of the finite element method can be stated by using the interpolation function of a three nodes triangle element. The triangle element is one of the popular elements which can be employed to approximate irregular surface; also it can be used to create a large number of straight sided elements along the curved boundaries in order to achieve a reasonable geometric representation. (Steven C. Chapra (1988)).

$$\Phi(x,y) = [N(x,y)]\Phi \rightarrow e \quad \ldots(5)$$

Where:-

[N(x,y)]=[Ni(x,y)….Nj(x,y).. Nk(x,y)]

$$\left.\begin{aligned} N_i(x,y) &= \frac{1}{2A}(a_i + b_i x + c_i\, y) \\ N_j(x,y) &= \frac{1}{2A}(a_j + b_j x + c_j\, y) \\ N_k(x,y) &= \frac{1}{2A}(a_k + b_k x + c_k\, y) \end{aligned}\right\}$$

$\rightarrow$ *is the shape fuction* ...(6)

And

$$\Phi \rightarrow e = \begin{Bmatrix} \Phi_i \\ \Phi_j \\ \Phi_k \end{Bmatrix} = \textit{vector or nodal unkown of element } e\,.$$

$$A = \frac{1}{2}\,(x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j) \qquad \ldots\ldots\ldots(7)$$

[illegible]

…………………..…. (8)

And by using Galerkin's procedure which is given by:-

$$\int_A R(x,y) N_i(x,y)\,dx\,dy = 0 \;\ldots\; (9)$$

where:- $R(x,y) = \frac{\partial 2\Phi}{\partial x^2} + \frac{\partial 2\Phi}{\partial y^2}$

The solution of the above equation (using Green's theorem) is:-

$$\int_A \left(\frac{\partial \Phi}{\partial x}\frac{\partial N_i}{\partial x} + \frac{\partial \Phi}{\partial y}\frac{\partial N_i}{\partial y}\right)dx\,dy - \int_S \left(\frac{\partial \Phi}{\partial x} L_x + \frac{\partial \Phi}{\partial y} L_y\right) N_i\, ds = 0 \qquad \ldots\ldots 10)$$

where L is the direction cosine.

The finite element method reduces the above equations to the equilibrium equations of the form:

$$\left.\begin{aligned} [K^{(e)}]\Phi^{\rightarrow(e)} &= \vec{Q}^{(e)} \\ [K^{(e)}]\Psi^{\rightarrow(e)} &= \vec{Q}^{(e)} \end{aligned}\right\} \qquad \ldots\ldots (11)$$

The typical terms in the matrix $[K^{(e)}]$ are of the form (Tottenham H. (1970)) :

$$[K^{(e)}] = \iint [R^T][S][R]\,dx\,dy \qquad \ldots\ldots \quad (12)$$

where

$$[R] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} \end{bmatrix} \qquad \ldots(13)$$

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus by differentiation the shape functions in equation (6) with respect

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to x and y, the characteristics matrix of each element becomes:

$$[K^{(e)}] = \frac{1}{4A^e}\begin{bmatrix} b_i^2 + c_i^2 & b_i b_{j+} c_i c_j & b_i b_{k+} c_i c_k \\ b_i b_{j+} c_i c_j & b_j^2 + c_j^2 & b_j b_{k+} c_j c_k \\ b_i b_{k+} c_i c_k & b_j b_{k+} c_j c_k & b_k^2 + c_k^2 \end{bmatrix} \quad \ldots\ldots (14)$$

The vector Q of nodal inflow/ outflows can be expressed by:

$$Q = \int_{c_2^e} V_o N_i dC_2 \quad \ldots\ldots\ldots\ldots (15)$$

Where: V_o is the velocity normal to the boundary surface, and

C_2^e is the boundary of the element.

Finite difference method:

The Differential equations could be solved by two general methods, either by direct method or iterative method. Starting with Laplace's equation (Ray C. Wylie (1982)):

$$\Phi_{i+1,j} + \Phi_{i-1,j} + \Phi_{i,j+1} + \Phi_{i,j-1} - 4\Phi_{i,j} = 0 \quad \ldots\ldots (16)$$

The above equation is rewritten in the following form that suitable for iteration:

$$\Phi_{i,j} = \Phi_{i,j} + r_{i,j} \quad \ldots. (17)$$

where $r_{i,j} = (\Phi_{i+1,j} + \Phi_{i-1,j} + \Phi_{i,j+1} + \Phi_{i,j-1} - 4\Phi_{i,j})/4$ …… (18)

For $2 \le i \le n-1$, and $2 \le j \le m-1$

Starting values for all interior grid points must be supplied. Successive iterations sweep the interior of the grid with the equation (17) until the residual term $r_{i,j}$ is reduced to zero. The speed of convergence for reducing all the residuals { $r_{i,j}$ } to zero is increased by using the method called "successive over relaxation (SOR) ". The SOR method uses the iteration formula:

$$\Phi_{i,j} = \Phi_{i,j} + \omega\, r_{i,} \quad \ldots.. (19)$$

where

$$\omega = 4\Big/\left(2 + \sqrt{4 - \left(\cos\left(\frac{\pi}{n-1}\right) + \cos\left(\frac{\pi}{m-1}\right)\right)^2}\right) \quad \ldots\ldots.. (20)$$

The parameter ω lies in the range $1 \le \omega \le 2$, which depends on the value of n and m and it will be calculated in section (3-2). Equation (19) is swept across the grid until $|r_{i,j}| < \varepsilon$.

The application of finite element and finite difference methods.

The objective of this study is to find the values of the velocity potential (Φ), the stream function (Ψ) and the velocity distribution by the two methods to the steady state flow over a cylinder. All the results are computed by using computer programs.

***Finite elements*:**

Figure (1) shows the field of the study of a confined flow over a cylinder. The application of finite element method can be stated as follows:

- Discretization: divide the region into (42) of triangular finite elements. There will be (33) quantities namely $\Phi_1, \Phi_2, \Phi_3, \ldots\ldots\ldots, \Phi_{33}$ to be determined in the problem. The numbers of the elements and their nodes are shown in Figure (1).
- Interpolation model: For two dimensional flows, the simplest alternative is a linear polynomial

$$F(x,y) = a_1 + a_2 x + a_3 y \quad \ldots. (21)$$

This function must pass through (x_1,y_1), (x_2,y_2) and (x_3,y_3).

- Evaluation of the shape function constants and the area of each element:

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The area of each element $A^{(e)}$ and the constants (a, b and c) as defined by equations (7) and (8) respectively are calculated. The constants a_i ,a_j and a_k are not required in the computations because by differentiating equation (6) with respect to x and y and equaling the result to zero (or in another words $\frac{\partial N}{\partial x} = 0$ & $\frac{\partial N}{\partial y} = 0$) leads to make the values of the constants (a_i ,a_j and a_k) equal to zero.

Boundary conditions

As a boundary conditions, the velocity function will assumed to be equal to zero(*F = 0*) along the line through the nodes 5, 10, 14, 25. The velocity (u) normal to the boundary surface through the nodes (1, 6, and 19) and the velocity (v) parallel to the boundary surface are assumed to be equal to (2m/sec.) and (0) respectively.

For the stream function Ψ(x,y), (Ψ= 0) will be assumed along the line through the nodes (19, 20, 21, 22, 24, 33, 32, 31, 30, 29, 28, 27, 26, 25) and Ψ = 20.0 along the line through the nodes (1, 2, 3, 4, 5). The stream function for node (6) is equal to 10.0 (half away between the nodes 1 and 19).

Computation of the element characteristics vectors:

Equation (15) could be used to yield the element characteristic vector Q which is used in obtaining an approximation to the solution of equation (11). In order to compute the stream functionΨ, the vector Q is equal to zero for all elements, because of no velocity parallel to the boundary but for the computation of the velocity function Φ , the vector Q will be non zero only for the elements 1 and 16. The velocity normal to the edge *(ij)* for the element 1 and 16 is equal to (2m/sec.) and the length of the edge (ij) is equals to 5 units for both elements. The global nodes corresponding to the local nodes(ij) in element 1 are (6 and 1) and in element 16 are (19 and 6), so to establish the integral values associated to the global nodes, equation (15) could be used to evaluate the vector Q for nodes 6,1 and 19 only.

$$\bar{Q}^{(1)} = \frac{V_0(y_2 - y_1)}{2} \begin{Bmatrix} N_i \\ N_j \\ N_k \end{Bmatrix} \begin{matrix} 6 \\ 1 \\ 7 \end{matrix} = \frac{2(5)}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \begin{matrix} 6 \\ 1 \\ 7 \end{matrix} = \begin{Bmatrix} 5 \\ 5 \\ 0 \end{Bmatrix} \begin{matrix} 6 \\ 1 \\ 7 \end{matrix}$$

$$\bar{Q}^{(16)} = \frac{2(5)}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \begin{matrix} 16 \\ 6 \\ 15 \end{matrix} = \begin{Bmatrix} 5 \\ 5 \\ 0 \end{Bmatrix} \begin{matrix} 19 \\ 6 \\ 15 \end{matrix}$$

These values are places in element characteristic vector location (1,1) , (6,1) and (19,1) . The vector Q has one column and 33row. The vector values of Q are zero. Combination of the two integrals yields the final values of the vectors Q:

[illegible]

[illegible]

Element assembly

For each element, there is an equation which is described by equation (12). The total number of equations for the mesh is 33. Assembly of the element characteristic matrices will yield the total system matrix (33x33). Table (1) shows the program's output.

Solution of the equilibrium equation

After assembly, including specification of boundary condition,

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equation (11) was solved rapidly by using the Gaussian elimination method with back substitution. The values of *F* and *Y* are listed as follow:-

$$[\Phi^{(e)}] = \begin{Bmatrix} 32.3415 \\ 23.2148 \\ 15.8299 \\ 7.3125 \\ 0.0000 \\ 32.5255 \\ 23.2768 \\ 14.8033 \\ 7.5811 \\ 0.0000 \\ 23.4441 \\ 13.6821 \\ 6.4101 \\ 0.0000 \\ 23.8085 \\ 17.1226 \\ 13.0648 \\ 10.8335 \\ 32.7454 \\ 23.9428 \\ 17.6900 \\ 14.0666 \\ 13.9050 \\ 13.2615 \\ 0.0000 \\ 4.2011 \\ 7.6039 \\ 10.1016 \\ 12.0889 \\ 13.003 \\ 13.1363 \\ 13.1488 \\ 13.0369 \end{Bmatrix} \qquad [\Psi^{(e)}] = \begin{Bmatrix} 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 10 \\ 14.6968 \\ 13.8415 \\ 12.3136 \\ 13.1010 \\ 9.5351 \\ 8.0694 \\ 4.9722 \\ 5.2757 \\ 4.6774 \\ 3.8936 \\ 2.0418 \\ 3.4093 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.7191 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

A flow net consisting of contours of the velocity potential *F* and the stream function *Y* is shown in Figure (2).

The velocity of the flow for arbitrary points through the region was determined by using the following equation (Kidger, D.j.(1994)) :

$$\left.\begin{aligned} q_x &= \frac{-(\Phi_{i+1,j} - \Phi_{i-1,j})}{(x_{i+1,j} - x_{i-1,j})} \\ q_y &= -\frac{(\Phi_{i,j+1} - \Phi_{i,j-1})}{(y_{i,j+1} - y_{i,j-1})} \end{aligned}\right\} \quad \ldots(21)$$

The resultant velocity is:

$q_n = \sqrt{q_x^2 + q_y^2}$

$\theta = \tan^{-1}(q_y/q_x)$ … for $q_x > 0$

$\theta = \tan^{-1}(q_y/q_x) + 180$ for $q_x < 0$

If $q_x = 0$, θ is (90°) or (270°) depending on the value of q_y if it is positive or negative respectively.

The resulting solution is shown graphically in Figure (3). In Figure (2), it can be seen that the flow lines and the equipotential lines intersect at right angles and the contours are very little ragged, however the results are acceptable. Small elements are used in areas of rapid change (near the cylinder) and large elements are used where variations are less severe. Also Figure (2) shows that the net flux through a region of influence near the boundary is correct since that $\frac{\partial \Phi}{\partial n} = 0$, where n is the normal to the boundary. The boundary fixed potential of (on the left side of the field of the study at x=0.0 (Dirichlet condition). Conditions at the region (elements (37, 38, 39, 40, 41 and 42)) are not defined as it is a stagnation point (where the fluid is at rest).

In Figure(3) and by using equation (21), the maximum velocity was (4 m/sec.) and occurs on the top of the cylinder which is equal to ($2V_o$). Hence the maximum velocity is in good agreement with the velocity components at p(r,θ), where *a*=*r* and *q*

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$=p/2$ which are given by equation (22) (William F.Hughes(1967)):

$$\left.\begin{aligned} u &= -V_o\left(\frac{a^2}{r^2}\cos 2\theta - 1\right) = 2V_o \\ v &= -V\frac{a^2}{r^2}\sin 2\theta = 0 \end{aligned}\right\} \quad \ldots (22)$$

Finite Difference

The field of the study of a confined flow over a cylinder is divided into 24 squares with sides Δx = 2.5 and Δy =2.5 (see Figure (4)). The initial value of the interior grid points was set at $\Phi_{i,j}$ = 15.0 for each i = 2, 3, 4 and j = 2, 3, 4, 5, 6. Then SOR method, (equation (19)) was used with the parameter ω = 1.2365 (the parameter ω was calculated by substitute n=5 and m =7 in equation (20)), after (25) iterations, the residual was uniformly reduced ($|r_{i,j}| \le 0.0000001$) .

A computer program is used to solve this problem; the resulting approximations are given as follow:

27.6566	22.9272	17.7351	12.5659	6.8108
27.8093	23.1171	18.4471	13.9479	8.9572
27.9335	23.2047	18.9104	15.9104	12.0901

The same values of Φ at the boundary surface which were used for finite element method are used to compute the approximate solution to Laplace's equation.

In the finite differences method, a uniform grid spacing Δx= Δy was used. This leads to the simplest equations that can be obtained. The generation of these equations on a computer is relatively easy.

It can be seen that convergence was achieved between the equipotential lines by Comparing the results obtained by the finite difference and the finite element method at Φ = 28 and Φ = 32 (see Figure (5)). So the Dirichlet condition is satisfied.

In addition, the divergence are increasing rapidly at Φ =8 to Φ = 24. Indicating that for complicated problems involving steady flow, the finite element method has been shown to be a powerful tool. If n specifies a unit normal velocity. The direction at any point p on the boundary, there must be no relative normal velocity. The finite difference method shows that the equipotential lines make angle (α ≠90) with the cylinder boundary ($\frac{\partial \Phi}{\partial n} = 0$,)

Conclusions

1) In the finite element method, the results are acceptable. The flow lines and equipotential lines intersect at right angles and the contours are very little ragged due to using small elements in areas of rapid change near the cylinder and large elements where variations are less severe.
2) The results show that there is no difference between the velocity potential Φ which was evaluated by the finite difference and the finite element method at x= 0 or in another word the value of Φ is a constant along x=0. And hence, the Dirichlet condition is satisfied.
3) Condition at the region near the boundary (y= 0) are not defined as it is a stagnation point (where the fluid is at rest).
4) The accuracy of the finite element method has been tested by the results of the velocity distribution. The velocity of flow at the top of the cylinder surface is $2V_o$ which has been checked by two equations,(equations 21 and 22)

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5) Comparing the finite element method with the finite difference method show that there is a divergence between the equipotential lines, indicating that for complicated problems involving either steady or unsteady flow, the finite element method has been shown to be a powerful tool.
6) The finite difference method shows that the equipotential lines make angle not equal to 90^{o} with the cylinder boundary so there will be a flow through the fixed boundary.

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Table (1): Total System Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	[illegible]	[illegible]	0	0	0	[illegible]	[illegible]	0	0	0	0						
2	[illegible]	[illegible]	[illegible]	0	0	0	[illegible]	0	0	0							
3	0	[illegible]	[illegible]	[illegible]	0	0	[illegible]	[illegible]	[illegible]	0	0						
4	0	0	[illegible]	[illegible]	[illegible]	0	0	0	[illegible]	0	0	0					
5	0	0	0	[illegible]	[illegible]	0	0	0	[illegible]	[illegible]	0	0	0				
6	[illegible]	0	0	0	0	[illegible]	[illegible]	0	0	0	[illegible]	0	0	0	[illegible]	0	0
7	[illegible]	[illegible]	[illegible]	0	0	[illegible]	[illegible]	[illegible]	0	0	[illegible]	[illegible]	0	0			
8	0	0	[illegible]	0	0	0	[illegible]	[illegible]	[illegible]	0	0	[illegible]	0	0			
9	0	0	[illegible]	[illegible]	[illegible]	0	0	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	0			
10	0	0	0	0	[illegible]	0	0	0	[illegible]	[illegible]	0	0	[illegible]	[illegible]			
11	0	0	0	0	0	[illegible]	[illegible]	0	0	0	[illegible]	[illegible]	0	0	[illegible]		
12	0	0	0	0	0	0	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]
13	0	0	0	0	0	0	0	0	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	0	0	0
14										[illegible]	0	0	[illegible]	[illegible]	0	0	0
15						[illegible]	0	0	0	0	[illegible]	[illegible]	0		[illegible]	[illegible]	0
16											0	[illegible]	0		[illegible]	[illegible]	[illegible]
17												[illegible]	0		0	[illegible]	[illegible]
18												[illegible]	[illegible]		0	0	[illegible]
19															[illegible]	0	0
20															[illegible]	0	0
21																[illegible]	0
22																0	0
23																[illegible]	[illegible]
24																	
25														[illegible]			
26													[illegible]				
27													[illegible]	0			0
28																	[illegible]
29																	[illegible]
30																	[illegible]
31																	
32																	
33																	

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Continue to Table (1)

	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
1																
2																
3																
4																
5																
6	0	-.312														
7																
8																
9																
10																
11																
12	[illegible]															
13	-.833	0							-.862	-.840						
14	0	0						-.674	-.381							
15	0	-.278	-1.6													
16	0	0	0	-1.2		-.167										
17	1.33	0	0	0		1.48					.406	.548	.812			
18	4.415	0	0	0		0				-.238	-1.01					
19		.876	.778	0		0										
20		.778	[illegible]	.867		0										
21			.867	1.974	.167	[illegible]										
22				.167	1.917	[illegible]	[illegible]									
23				-.25	-.3	2.129	-.107						-.704	.089		
24					[illegible]	[illegible]	[illegible]							[illegible]	[illegible]	[illegible]
25								1.281	-.623							
26								-.623	2.327	-.359						
27	-.286								-.559	2.409	-.868					
28	-1.001									-.868	2.284	-.873				
29											.876	[illegible]	.876			
30						-.756						-.875	2.809	-1.22		
31						[illegible]	[illegible]						[illegible]	[illegible]	[illegible]	
32							[illegible]							[illegible]	[illegible]	[illegible]
33							[illegible]								[illegible]	[illegible]

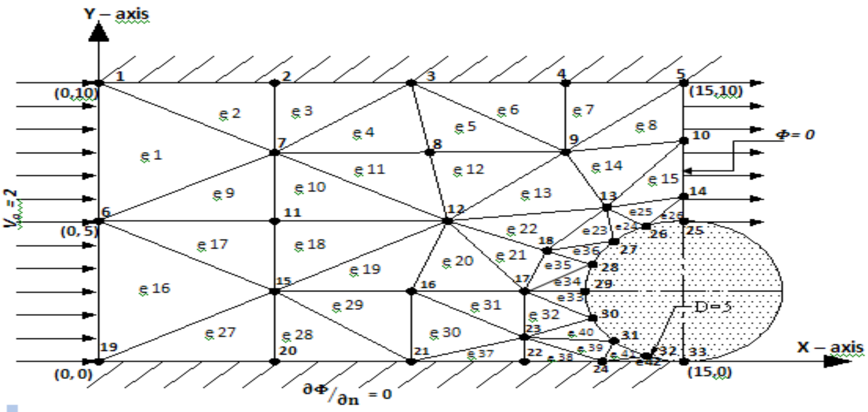

Figure (1): Finite Element mesh for A Confined Flow over a cylinder

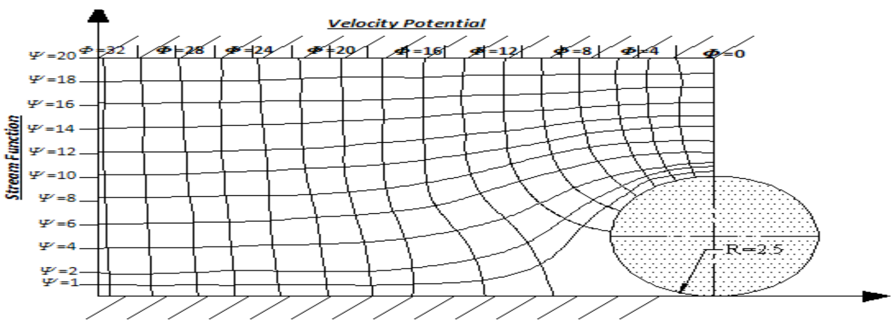

Figure (2): A Flow net Consisting of Contours of the Potential Φ and the stream Function Ψ

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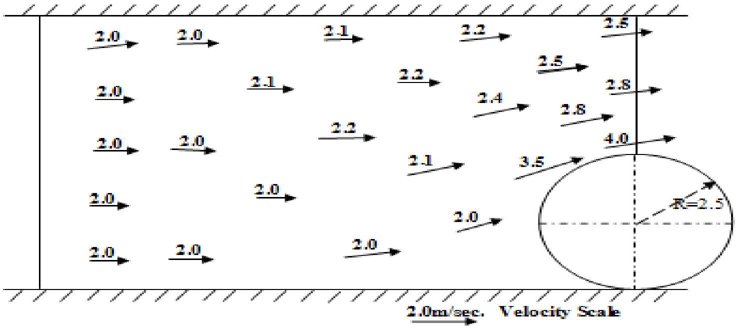

Figure (3): Velocity Distribution over a Cylinder

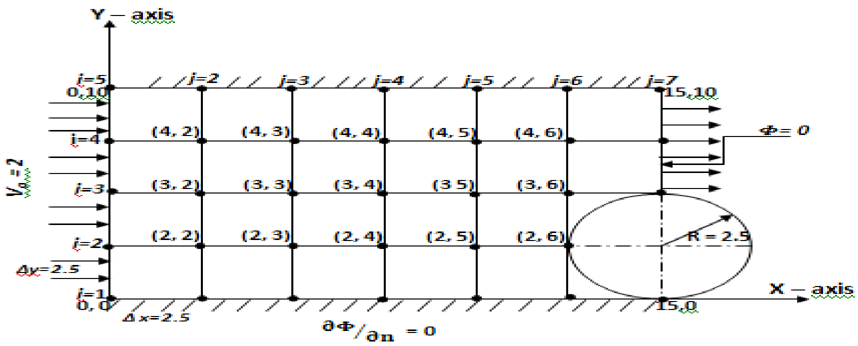

Figure (4): The grid of 24 squares for finite difference method

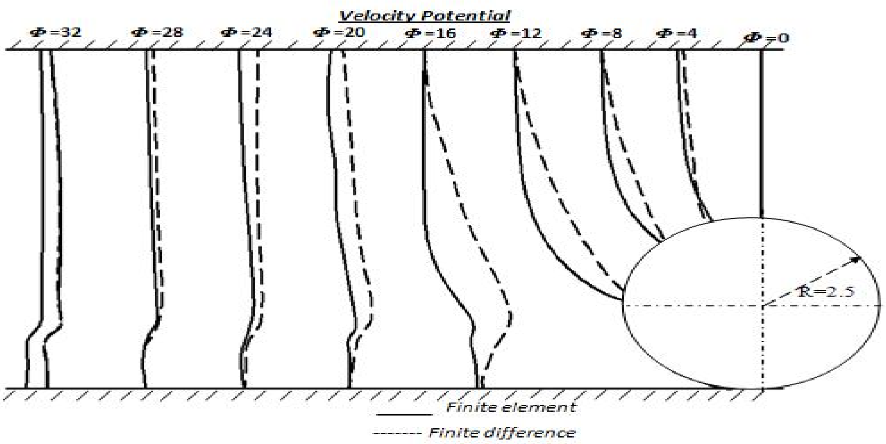

Figure (5): Comparison between the Finite element and the Finite difference results

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