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Nonlinear Finite Element Analysis of Laterally Loaded Piles in Cohesive Sediments

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Abstract

Laterally loaded single piles are usually analyzed by methods derived directly from the classical beam on elastic foundation or *Winkler soil model*. Soil strength is characterized using modulus of sub-grade reaction.

Early solutions that employed Winkler's model assumed that the modulus of sub-grade reaction is constant with depth, or linearly varying with depth, while in fact it varies nonlinearly with depth due to the nonlinear soil response to loads. The characterization of soil using "*p-y* curves" concept; (which relates the soil reaction and pile deflection at typical points along its length, and extracted from field tests), is proper for representing nonlinearity in soil response, and gives better solutions. Another method used for the solution of the problem of laterally loaded piles is the *elastic continuum method*. In this method the soil is modeled as an elastic media, such formulation poses increased difficulty when the soil modulus varies with depth.

The Sub-grade Reaction or Winkler method of solution is used through this study. The soil is characterized by a nonlinear springs with employing the "*p-y*" model, while the pile is modeled as a two-node element.

Keywords: piles, cohesive sediments, nonlinear, finite element analysis

استخدام العناصر غير المحددة للتحليل غير الخطي لركائز محملة جانبياً في ترسبات طينية

الخلاصة

الركائز المنفردة والمحملة جانبياً يتم تحليلها بطرق غالباً ما تكون مشتقة مباشرة من الطرق التقليدية لـــ عتبة فوق اساس مرن أو ما تسمى بـــ نموذج تربة وينكلر. مقاومة التربة في هذا النموذج تمثّل باستخدام رد فعل معامل التربة التحتية.

الحلول السابقة افترضت ان معامل التربة التحتية يكون مقداره ثابت مع العمق أو يمكن أن يتغير بصورة خطية مع تغير العمق. ولكن في واقع الأمر انه يتغير و بصورة غير خطية نظراً للسلوك غير الخطي للتربة اثناء التحميل.

ان تمثيل التربة باستخدام مفهوم منحنيات *p-y* والتي ترسم العلاقة بين رد فعل التربة وانحراف الركيزة وعلى امتداد طول الركيزة والتي تستنبط من خلال الفحوصات الحقلية هي بالفعل الطريقة الأنسب لتمثيل الأستجابة الغير خطية للتربة .

الطريقة الأخرى المستخدمة هي طريقة التواصل المرن حيث يتم تمثيل التربة في هذه الطريقة باعتبارها وسط مرن. ان هذا التمثيل له نتائجه السلبية عندما يكون تغير معامل التربة التحتية كبيراً مع العمق. ان نموذج تربة وينكلر في تمثيل رد فعل التربة التحتية قد تم اعتماده في هذا البحث. وان التربة قد تم نمذجتها او تمثيلها بـــ نوابض غير خطية مع اعتماد مفهوم منحنيات *p-y* والتي افترضت بان الركيزة لها عقدتان لغرض التحليل

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Introduction

The Finite Element method is used in this study for obtaining solutions of soil-pile system under lateral loads by modelling the pile structure with beam elements and using elastic springs to represent the soil resistance, Sami (2004). Soil strength is characterized using modulus of sub-grade reaction (Georgiadis and Butterfield, 1982).

Failure Criterion: the failure criterion considered in this analysis is the yield of the maximum allowed lateral displacement at pile head, where the lateral load capacity of pile is reached when the allowable lateral displacement yields at head of pile. The allowable lateral displacement at pile head depends on some factors such as pile type, soil type, and type of structure carried by the pile, but it mainly depends on pile diameter. Some investigators specified the allowable lateral displacement as a ratio of pile diameter, and stated that it is ranging from 5% to 20% of pile diameter according to type of structure.

In the analysis carried through this study the allowable lateral displacement at head of pile is taken to be 10% of pile diameter as stated by Pyke, 1984 (ASTM STP-835, 1983) and (USACE, 1998).

Soil Representation: The soil surrounding the pile was represented by independent nonlinear springs, which is given in load-displacement (*p-y*) data sets form. There are different methods used for generating load-displacement (*p-y*) curves data in cohesive soils for different depths. Through this analysis Matlock's method is adopted for generating load-displacement (*p-y*) curves along piles embedded in soft clay soil. Reese and Welch's method is used for constructing load-displacement (*p-y*) curves data along piles embedded in stiff clay soils, Whereas Reese and Sullivan method is used for constructing load-displacement (*p-y*) curves data along piles embedded in non-homogenous cohesive soil.

Objectives of the study

The characterization of soil using "*p-y* curves" concept is adopted in this study as a means of a proper solution when compared with that of Winkler's model where the modulus of sub-grade reaction is assumed constant or linearly varying with depth, while in fact it varies nonlinearly with depth due to the nonlinear soil response to loads. It can therefore be postulated that this concept is more appropriate for representing nonlinearity in soil response, and therefore should give better solutions.

Algorithm for Analysis

The *p-y* curves described in the preceding paragraph were derived on the assumption that the lateral resistance p at any point on the pile is a function of the lateral resistance y at that point (i.e., Winkler assumption). For this assumption and the one-dimensional model of the pile soil system (shown in figure 1-a), the governing differential equation for bending in the *x-y* plane of a prismatic, linearly elastic pile is:

$$EI \cdot \frac{\partial^4 y}{\partial x^4} + P \cdot \frac{\partial^2 y}{\partial x^2} - p = 0 \quad ….(1)$$

Where, P is the Axial load on pile, y is the Lateral deflection of the pile at point x along the pile length, p is the Soil reaction per unit length, EI is the Flexural Rigidity of the pile.

Because the displacement y must be known before the lateral resistance p can be evaluated, numerical alternative solution of equation (1) is required. The most common approach is to represent the pile-soil system by a descretized model as illustrated in figure (2-a) where the displacement and forces are evaluated at a finite number of points (Smith, 1988). The solution proceeds as a succession of trials and corrections until forces and displacements are comparable at every node. By replacing the nonlinear *p-y* curve by equivalent linearly elastic springs during each iteration, the numerical solution can be expedited. The stiffness of these linear springs is evaluated as the secant to the *p-y* curve for the displacement calculated during the proceeding iteration; the secant to the *p-y* curve is termed the secant modulus *Es*, where:

$$E_s = -\frac{p}{y} \quad \text{….(2)}$$

Thus, $p = -E_s \cdot y$ ….(3)

By substituting this equation into equation (1), it becomes:

$$EI \cdot \frac{\partial^4 y}{\partial x^4} + P \cdot \frac{\partial^2 y}{\partial x^2} + E_s \cdot y = 0 \quad \text{..(4)}$$

It is to be noted that the ultimate lateral resistance tends to increase as depth increases. Hence, it can be concluded that the secant stiffness of the lateral resistance increases with depth below the pile head.

The second term in equation 1 represents the interaction of the axial load in the pile with the lateral displacement, which increases the bending moment in the pile (the 'beam-column' effect) (Mosher and Dawkins, 2000).

Finite Element Formulation

A laterally loaded pile problem can be handled by modelling the pile structure with beam elements and using elastic springs to represent the soil resistance. This approach in effect assumes independent Winkler type action for the springs (Sogge, 1981).

Beam Element Equation:

Consider the one-dimensional solid element (slender beam) in figure (3), the end nodes 1 and 2 are subjected to shear forces and moments which result in translations and rotations. Each node, therefore, has two degrees of freedom (Smith, 1988).

The element shown in the figure has length L, flexural rigidity EI, and carries a uniform transverse load of q per unit length. The well-known equilibrium equation for this system, for a differential length dx, is given by:

$$EI \frac{\partial^4 y}{\partial x^4} \cdot dx = q \cdot dx \quad \text{….(5)}$$

and,

$$y = \begin{Bmatrix} y_1 \\ q_1 \\ y_2 \\ q_2 \end{Bmatrix} \quad \text{…..(6)}$$

Where $q_1 = \frac{\partial y}{\partial x}$ at node 1, and so on

Integrating this equation over the length of the beam L

$$\int_0^L EI \cdot \frac{\partial^4 y}{\partial x^4} \cdot dx = \int_0^L q \cdot dx \quad \text{….(7)}$$

Evaluation of the integral gives:

$$EI\begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ & 4/L & -6/L^2 & 2/L \\ & & 12/L^3 & -6/L^2 \\ Symmetrical & & & 4/L \end{bmatrix}\begin{Bmatrix} y_1 \\ q_1 \\ y_2 \\ q_2 \end{Bmatrix} = q\begin{Bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{Bmatrix}$$

…(8)

This equation is for the case of a uniformly distributed load applied to the beam. For the case where loading is applied only at the nodes:

$$EI\begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ & 4/L & -6/L^2 & 2/L \\ & & 12/L^3 & -6/L^2 \\ Symmetrical & & & 4/L \end{bmatrix}\begin{Bmatrix} y_1 \\ q_1 \\ y_2 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

….(9)

which represent the element stiffness relationship, Hence, in matrix notation;

$$KM \cdot y = F \qquad \text{…. (10)}$$

Where KM = element stiffness matrix

F = nodal forces matrix

Beam with Axial Force:

If the beam element in the figure (3), is subjected to an additional axial force P as shown in figure (4), a simple modification to equation (4) leads to the following equation:

$$EI\frac{\partial^4 y}{\partial x^4} \pm P\frac{\partial^2 y}{\partial x^2} = q \qquad \text{..(11)}$$

Where the positive sign corresponds to a compressive axial load and the negative sign corresponds to tensile axial force. For loads applied only at the nodes, with compressive axial loads, the equation becomes:

$$EI\frac{\partial^4 y}{\partial x^4} + P\frac{\partial^2 y}{\partial x^2} = F \qquad \text{….(12)}$$

Integrating the equation over the beam length L leads to an additional matrix associated with the axial contribution:

$$\pm\frac{P}{30}\begin{bmatrix} 36/L & 3 & -36/L & 3 \\ & 4L & -3 & -L \\ & & 36/L & -3 \\ Symmetrical & & & 4L \end{bmatrix}\begin{Bmatrix} y_1 \\ q_1 \\ y_2 \\ q_2 \end{Bmatrix}$$

….(13)

if this matrix is designated by KP, the equilibrium equation becomes:

$$(KM \pm KP)\cdot y = F \qquad \text{….(14)}$$

Beam on Elastic Foundation:

In figure (5) continuous elastic support has been placed beneath the basic element. If this support has a stiffness k (force/length^3) then clearly the transverse load is related by an extra force $+ky$ leading to the differential equation:

$$EI\frac{\partial^4 y}{\partial x^4} + ky = q \qquad \text{….(15)}$$

For uniformly distributed load applied to the beam. For load applied at nodes;

$$EI\frac{\partial^4 y}{\partial x^4} + ky = F \qquad \text{…(16)}$$

Applying the Finite Element beam equation formulation, with an axial force subjected to the beam leads to the following equation (in matrix notations form) (Smith, 1988):

$$(KM \pm KP)\cdot y + ky = F \qquad [17]$$

Analysis of Problem

The ground is represented by discrete springs, which has load-displacement characteristics. These springs are simply attached to the nodes of the finite elements, as shown in figure (2-a). The load in the spring is traditionally termed '*p*' and the displacement '*y*', which may be given in *p-y* data sets, or *p-y* curves. In this analysis quite general 'curves' are allowed, by specifying any number of linear segments. These ground springs oriented horizontally and resist the horizontally applied loads. The pile distributes the load by bending and the appropriate elements are flexural with stiffness typified by equation (15). At each node a transverse displacement and rotation are permitted. In order to allow for the effect for an axial force on the bending stiffness, the element stiffness matrices in the program are actually given by equation (13) (Smith, 1988). The method of solution used in the program is by controlling the displacement, which is more efficient than load controlling, where in the latter, convergence to within the specified tolerance will be increasingly difficult to achieve as the ultimate load is approached. In addition, the solution of the problem of laterally loaded piles by controlling the displacement is more compliant with the design criterion, which specifies the allowed lateral displacement for the laterally loaded piled foundation according to the super-structure conditions.

Results

The results obtained from carrying out a parametric study gave a description for the behavior of the laterally loaded piles under working loads.

It is shown that there is a critical relative length $(L/D)_c$, which characterizes the behavior of pile, where piles having relative lengths less than the critical behave as short piles, while piles having relative length more than the critical, behave as long piles. Table (1) gives the values of critical relative length for free headed piles embedded in soft clay and stiff clay soils. Piles that behave as long piles have a fixity point along its depth, where no lateral displacement occurs below this point and bending moment approaches zero at this point. The depth of this point is specified by the critical relative length $(L/D)_c$. The lateral load capacity and bending moment values along pile For piles behaving as short piles, increases with increase in length of pile, while increase in length of pile that behave as long piles does not make any change to the values of lateral load capacity and bending moment along pile.

Figure (6) shows the relation of lateral load versus relative length for free headed piles embedded in cohesive soil of different un-drained cohesion values

Figure (7) shows critical relative length versus un-drained cohesion.

Location of point of maximum bending moment along pile depends on stiffness of pile and soil, and displacement at pile head. Table 2 show the depth of maximum moment in terms of pile diameter for free headed piles embedded in soft clay and stiff clay.

It is concluded that Increase in axial load applied to the pile decreases the lateral load capacity of pile, and has no worth effect on bending moment values along pile and its depth, as shown in figure (8).

Also it is concluded that Increase in moment applied to the head of pile, decreases the lateral load capacity and decreases the depth of maximum moment along pile, as shown in figure (9).

It is also shown in this study that Restriction of pile head against rotation increases the lateral load capacity of pile, and decreases the maximum positive moment along pile depth, while the head of pile exposes to negative moment. Restriction of pile head also increases the value of the critical relative length $(L/D)_c$, as shown in figure (10) and (11).

For piles embedded in layered cohesive soil, the top layer seems to exert an overwhelming influence on the lateral load capacity of pile. The effective thickness of top layer is expressed as a ratio of total thickness of soil sediment around pile, this ratio is about 0.25 for free headed long piles loaded laterally to 10% of its diameter, as shown in figures (12) and (13), while for short piles the ratio of effective top layer thickness is less than this value, and equals to 0.375 for piles with relative length of 16, as shown in figures (14) and (15).

Concerning *p-y* curves data used for soil representation in the analysis of laterally loaded piles embedded in soft clay soil, *Matlock* method (Matlock, 1970) when compared to *Reese and Sullivan* method (Reese and Sullivan, 1980) overestimates the soil resistance. Thus using *Reese and Sullivan* method gives more conservative results than those given using *Matlock* method. On the other hand concerning *p-y* curves data for representing soil in the analysis of laterally loaded piles embedded in stiff clay, using *Reese and Welch* method (*Reese and Welch, 1975)* rather than *Reese and Sullivan* method gives more conservative results, as shown in figures (16) and (17).

Conclusions

1- The results showed that the lateral load capacity of a pile depends mainly on the stiffness of soil-pile system, and in the case of piles embedded in layered cohesive soil, it is shown that only a surface layer of limited thickness (25% of the total length of pile) is controlling the behavior and the capacity for lateral load of piles.
2- Increasing the axial load applied to the pile decreases the lateral load capacity by 10% and has no effect on the bending moment values along pile length.
3- By increasing the moment applied to the pile head, the lateral load capacity will decrease by a ratio of 25%, i.e. an increase of 100 KN-m will result in a decrease of 25 KN of the lateral load capacity.
4- Regarding *p-y* curves data used for soil representation in the analysis of laterally loaded piles embedded in soft clay soil, *Matlock* method as compared to *Reese and Sullivan* method overestimates the soil resistance by 23%. Thus using *Reese and Sullivan* method gives more conservative results than those given using *Matlock's* method.
5- On the other hand concerning *p-y* curves data for representing soil in the analysis of laterally loaded piles embedded in stiff clay, using *Reese and Welch* method rather than *Reese and Sullivan* method gives more conservative results, i.e. The latter gives maximum value of 30% more compared to former method.

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Table (1) Values of critical relative length $(L/D)_c$, for piles in soft clay, and stiff clay soil, with different lateral displacement at pile head

Soil type	Displacement at pile head (as ratio of pile diameter)				
	5%	10%	20%	30%	40%
Soft clay (c_u=20 kPa)	6D	6D	8D	8D	8D
Stiff clay (c_u =110 kPa)	4D	4D	6D	-	-

Table (2) Values for depth of maximum moment along pile, for piles in soft clay, and stiff clay soil, with different lateral displacement at pile head

Soil type	Displacement at pile head (as ratio of pile diameter)				
	5%	10%	20%	30%	40%
Soft clay (c_u =20 kPa)	22	24	26	28	30
Stiff clay (c_u =110 kPa)	13	14	16	-	-

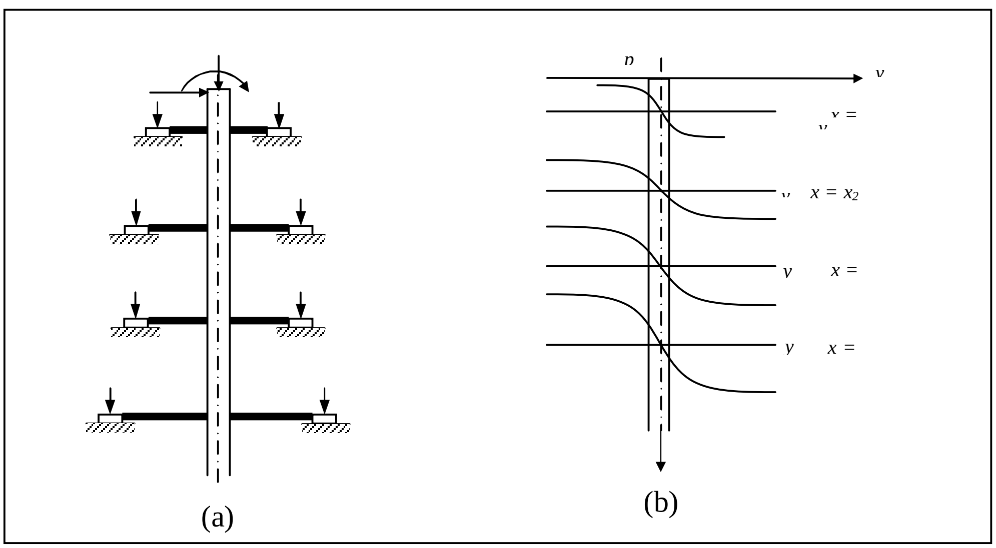


Figure (1) (a) Model of pile under lateral loading; (b) Shape of curves at various

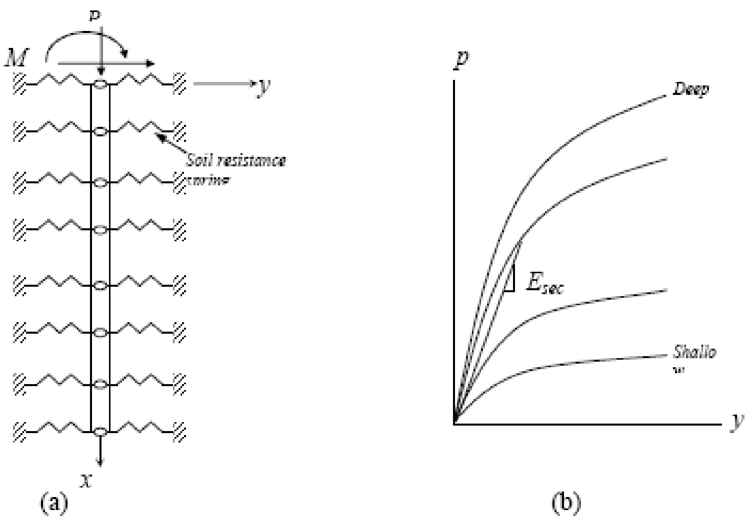


Figure (2) (a) Descretized model used for laterally loaded pile; (b) Nonlinear p-y curves used for representing soil resistance

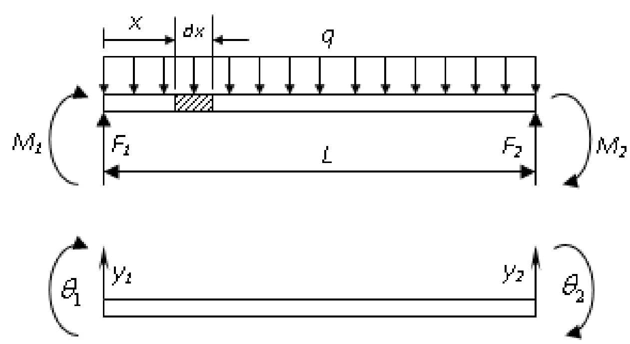
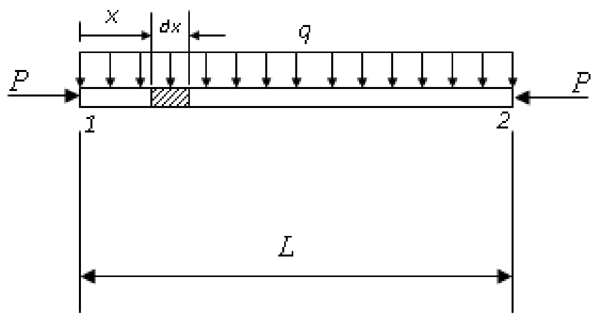


Figure (3) Slender beam element



Figure(4) Beam with axial force

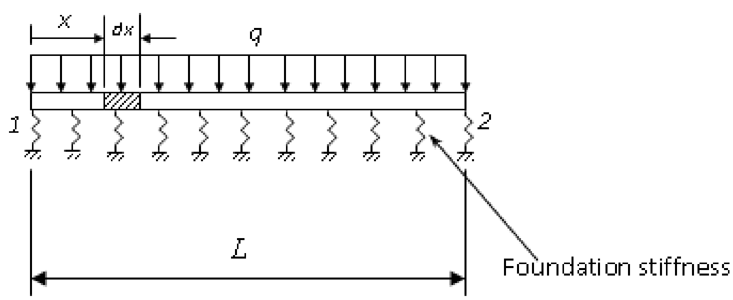


Figure (5) Beam on elastic foundation

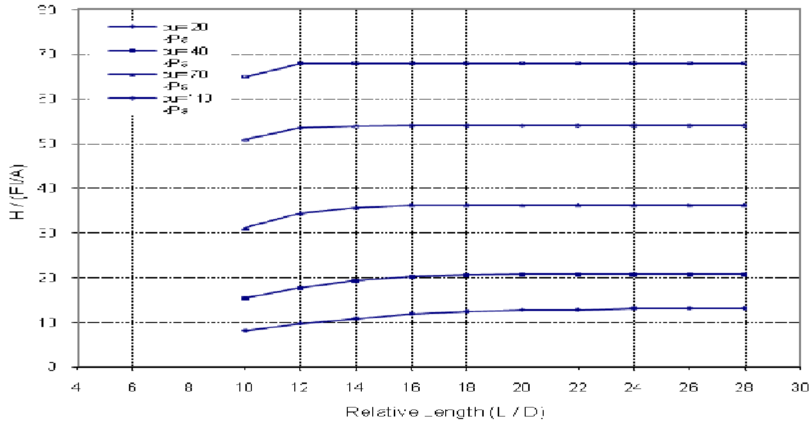


***Figure 6**: Lateral Load versus Relative Length for free headed pile embedded in cohesive soil of different undrained cohesion values*

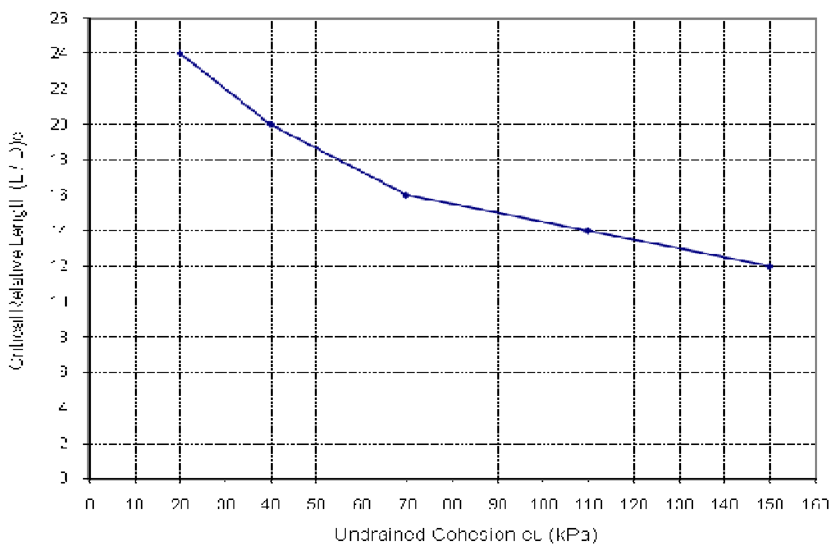


***Figure 7**: Critical Relative Length versus Undrained cohesion*

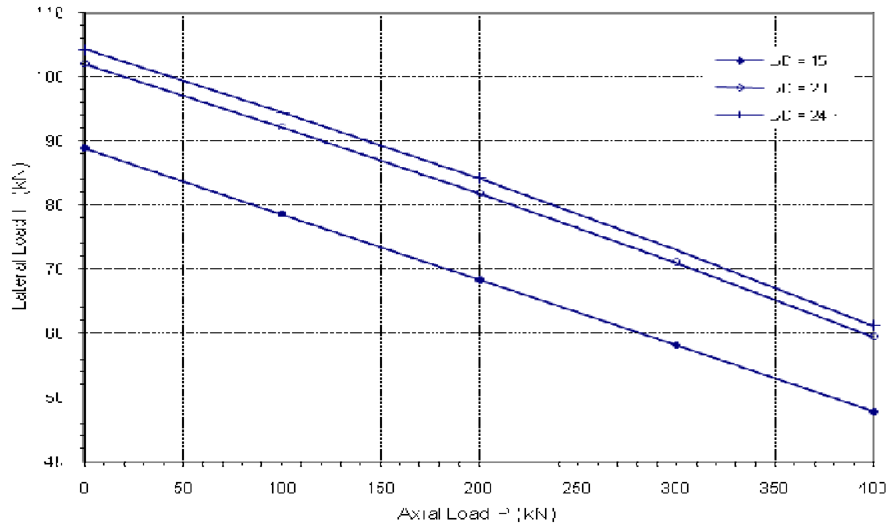


Figure 8 *: Lateral Load, versus Axial Load Diagrams for different length to diameter ratios (based on pile diameter = 0.5 m)*

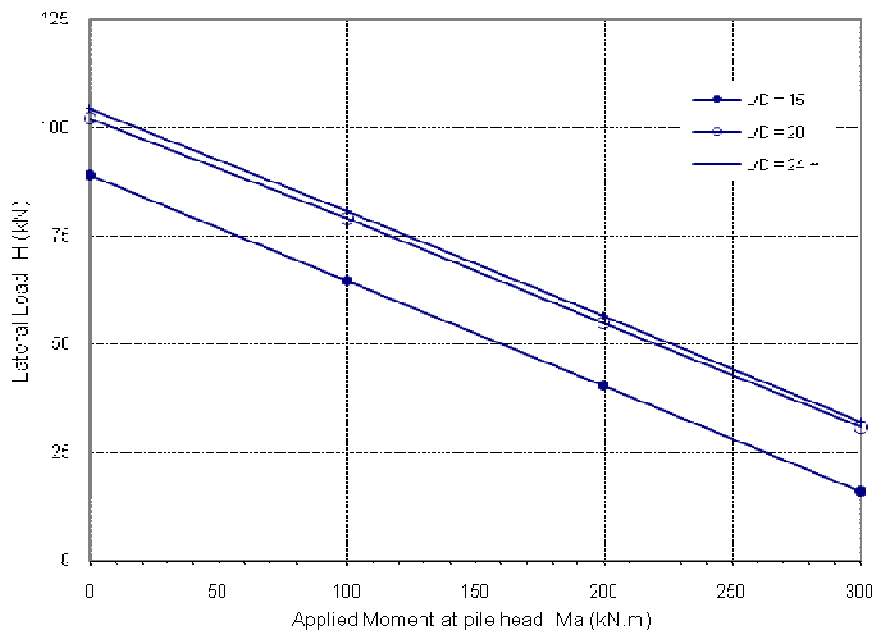


Figure 9 *: Lateral Load versus Applied Moment at pile head diagram for different length to diameter ratios (based on pile Diameter = 0.5 m)*

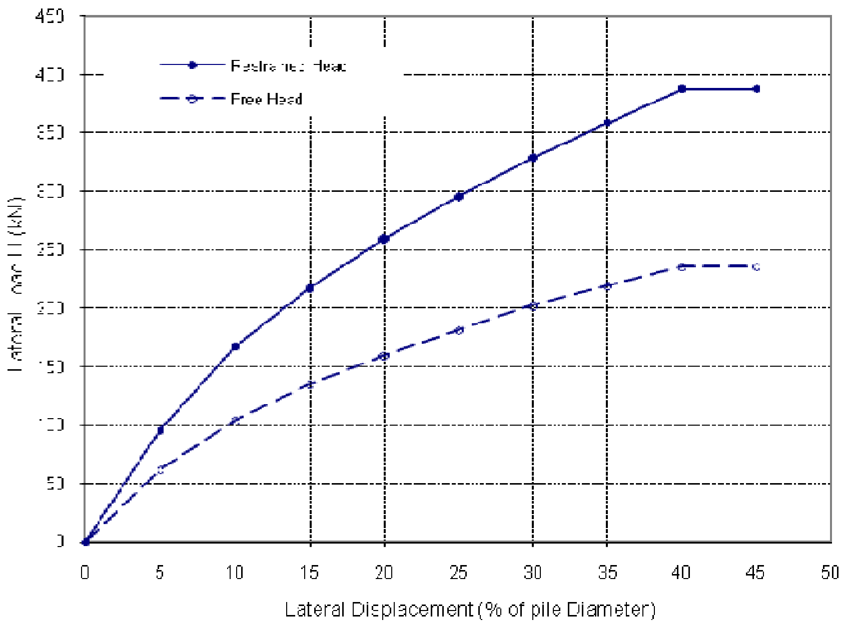


Figure 10: *Lateral Load versus Lateral Displacement at pile head Diagram for 15 m length, 0.5 m diameter piles, with Restrained head and Free head, embedded in soft clay soil*

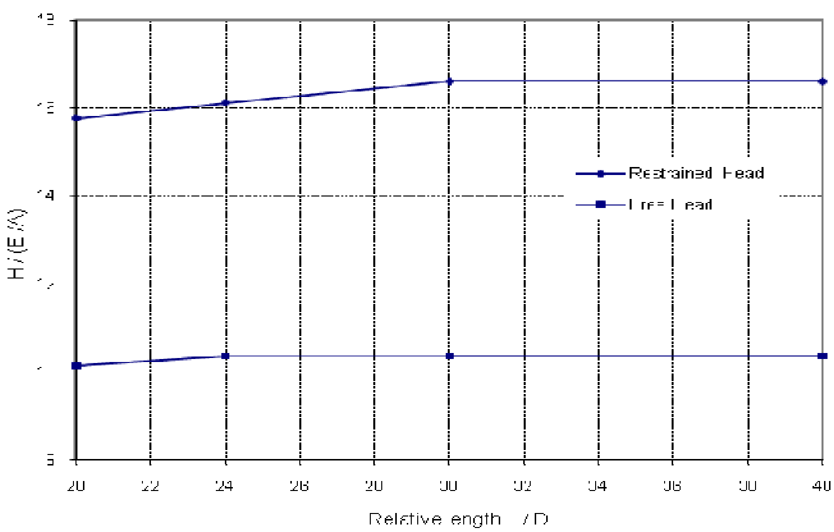


Figure 11: *Relative lateral load capacity of pile H / (EI/A) versus Relative Length (L / D) diagram*

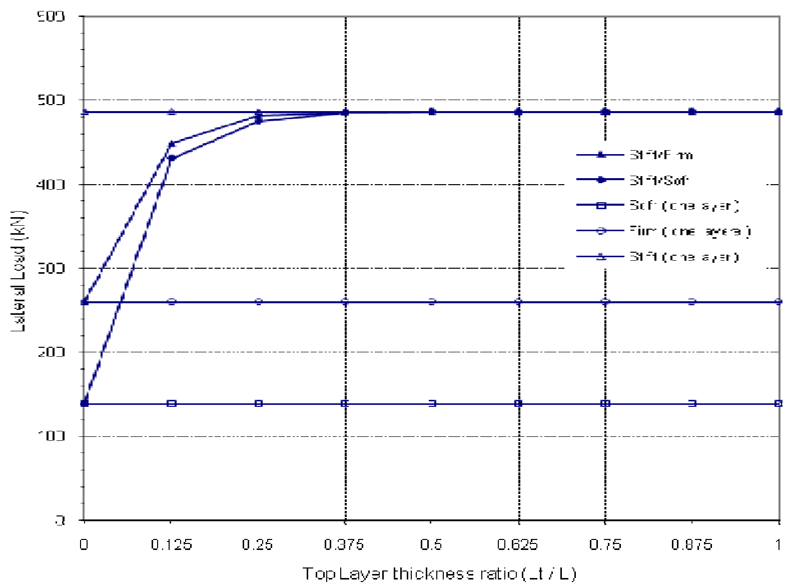


***Figure 12**: Lateral Load versus top layer thickness ratio (stiff layer overlying a weaker layer), for piles with (L/D)=32*

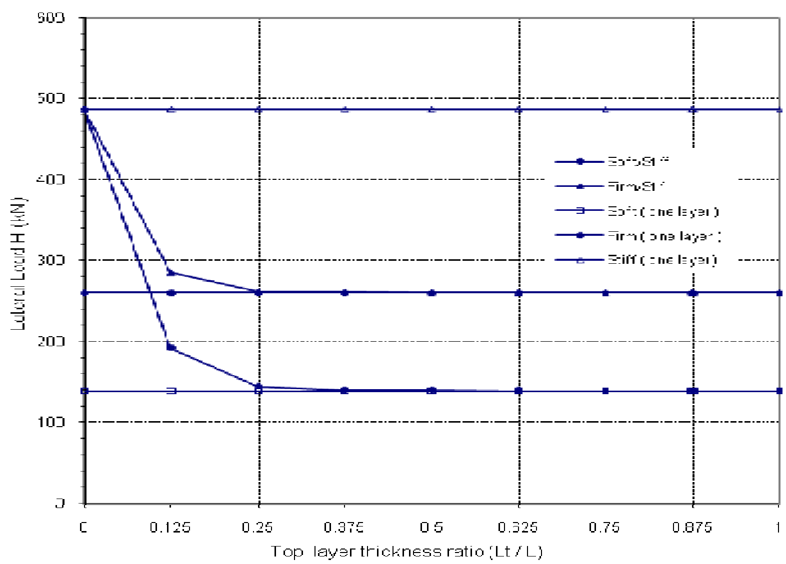


***Figure 13**: Lateral Load versus top layer thickness ratio (stiff layer underlying a weaker layer), for piles with (L/D)=32*

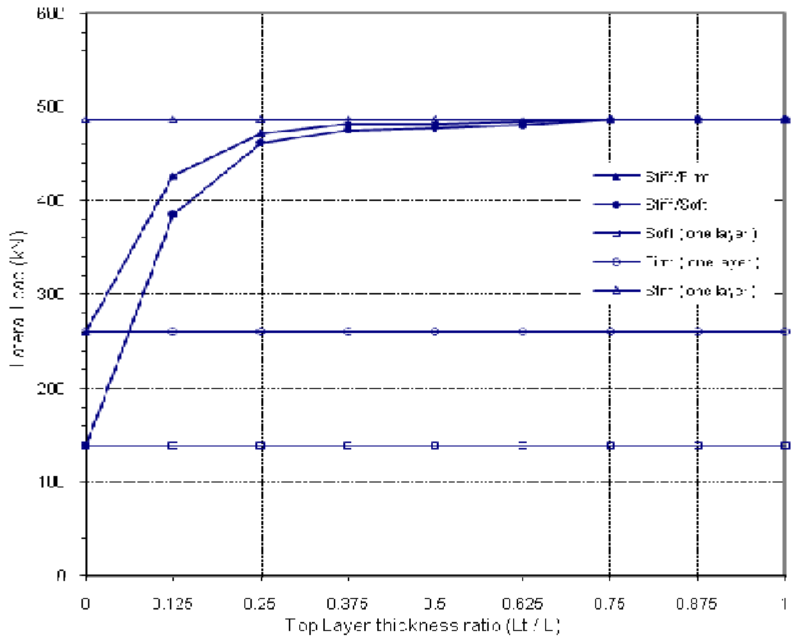


***Figure 14**: Lateral Load versus top layer thickness ratio (stiff layer overlying a weaker layer), for piles with (L/D)=16*

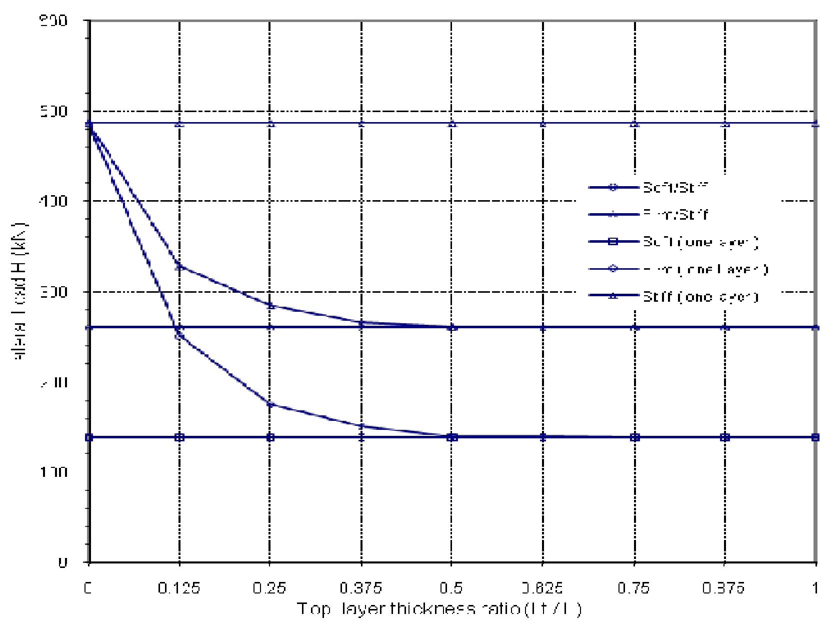


***Figure 15**: Lateral Load versus top layer thickness ratio (stiff layer underlying a weaker layer), for piles with (L/D)=16*

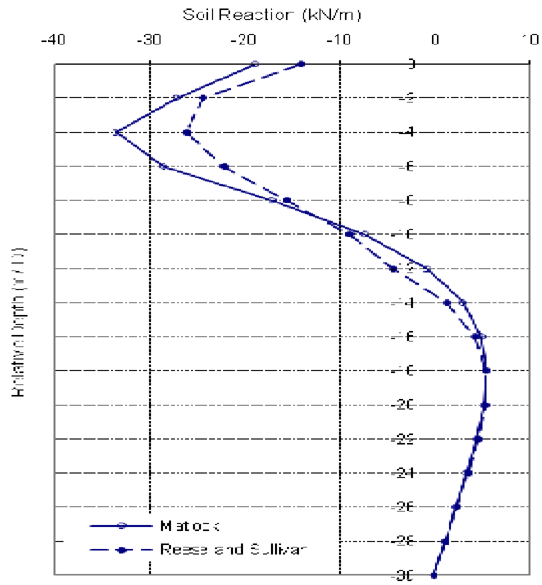


Figure 16: *Soil Reaction diagram, for15 m length, 0.5 m diameter piles embedded in soft clay soil, which has load-displacement data being determined using* ***Matlock*** *method and* ***Reese and Sullivan*** *method*

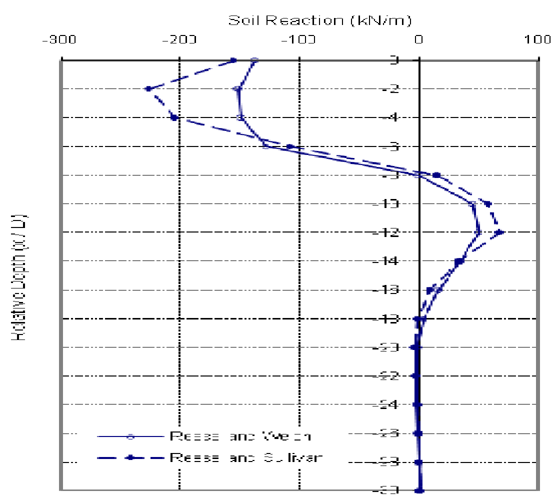


Figure 17: *Soil Reaction diagram, for15 m length, 0.5 m diameter piles embedded in stiff clay soil, which has load-displacement data being determined using* ***Reese and Welch*** *method and* ***Reese and Sullivan*** *method*