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Performance Study of a Modified Sine Wave Inverter

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Abstract

This paper presents the performance of a modified sine wave voltage source inverter with duty cycle. Evaluation of quality factors like the Total Harmonic Distortion THD of the output voltage for various values of switching-on angle α indicates that the minimum harmonic distortion occurs at $\alpha=23.218^0$, where the THD is 28.96%, about 60% of that of the square wave. The 3^{rd} harmonic is about 12.61% of the fundamental, and about third of the square wave.

The current paper, illustrates the inverter waveforms and how different types of loads behave when operating from them. Therefore, the THD of the output current depends on the load types. Results of typical load types are presented.

No heavy filters or complex timing circuits are necessary; therefore, the resulting inverter is smaller, lighter weight, and simpler, implying greater reliability and lower cost. These advantages invite a closer look at inverter-motor systems. Variations in induction motor performance may occur when the motor is driven from a modified sine wave inverter rather than a sine wave source.

Keywords: Modified Sine Wave, THD, Inverter.

دراسة أداء عاكس الفولتية ذو الموجة الجيبية المعدّلة

الخلاصة

يدرس البحث الحالي أداء عاكس الفولتية ذو الموجة الجيبية المعدّلة مع عرض النبضة. حيث تبيّن أن تقدير معاملات الأداء كمعامل التشويه الكلي للموجة الخارجة لقيم مختلفة لزاوية التشغيل α يشير إلى أن أقل تشويه للموجة يحدث عند زاوية مساوية لـــ (23.218^0) وبقيمة (28.96%), أي بمقدار (60%) من تشويه الموجة المربعة. فضلاً عن ذلك, فأن مقدار التوافقية الثالثة يكون حوالي (12.61%) من التوافقية الأساسية وبمقدار الثلث تقريباً مقارنةً بالموجة المربعة. تم التطرق إلى الأنواع الأساسية لموجات العاكس وسلوك الأحمال المختلفة التي تغذى من هذا العاكس, وبذلك فان التشويه التوافقي الكلي للتيار يعتمد على طبيعة تلك الأحمال, حيث تمّ الحصول على نتائج مختلفة لقيم مختلفة للأحمال.

يمتاز العاكس الحالي بأنه صغير الحجم وخفيف الوزن وبسيط التركيب, مما ينفي حاجته لمرشحات كبيرة أو دوائر معقدة للتوقيت الزمني, محققةً بذلك موثوقية أعلى وكلفة أقل. إن هذه المميزات تدعو للاهتمام بشكل أكبر بمنظومات المحرك المغذاة من عاكس. حيث وجد بأن التغيير الحاصل في أداء المحرك يحدث عندما يغذى المحرك من عاكس للفولتية ذو الموجة الجيبية المعدّلة أكثر منه عندما يغذى من مصدر ذو الموجة الجيبية الخالصة.

1. Introduction

With the increasing popularity of alternate power sources, such as solar and wind, the need for static inverters to convert DC energy stored in Batteries to conventional AC form has increased substantially [1].

With some equipment such as heaters and lights, the input waveform

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is not important. However with things like electric motors, and especially microwaves, the waveform is absolutely critical to achieve correct running [2].

On the market today, there are basically two different types of power inverters, Modified Sine (MS) wave and Pure Sine (PS) wave generators. These inverters differ in their outputs, providing varying levels of efficiency and distortion that can affect electronic devices in different ways.

A MS wave is similar to a square wave but instead has a “stepping” look to it that relates more in shape to a sine wave [3]. An approximation of a sine wave may be created by outputting one or more stepped square wave with the amplitudes chosen to approximate the sine [4]. This can be seen in Fig. 1, which displays how a MS wave tries to emulate the sine wave itself.

Notice that as time progresses from left to right, the three different waveforms rise at different rates. The MS waveform is easy to produce because it is just the product of switching between three values at set frequencies, thereby leaving out the more complicated circuitry needed for a PS wave. The MS wave inverter provides a cheap and easy solution to powering devices that need AC power. PS wave inverters are able to simulate precisely the AC power that is delivered by a wall outlet. Usually PS wave inverters are more expensive than MS wave generators due to the added circuitry [3]. The issue then becomes a trade-off between cost and waveform purity.

For evaluation of power quality, the Total Harmonic Distortion (*THD*) and the Distortion Factor (*DF*) are generally used. For a given voltage or current, the *THD* and *DF* are functions of the waveform [5]. In the case of a MS wave, which has constant amplitude in the switching-on period, they are only functions of the duty cycle. By optimizing the duty cycle of a MS wave Voltage Source Inverter (VSI); its *THD* can be reduced to the minimum. That is, the power quality can be improved.

Study [6] had been used the square wave VSI for line harmonics reduction in high power systems. In [7], a High Power Active Filter (HPAF) system using square wave inverters had been presented to provide supply-load harmonic isolation at dominant harmonic frequencies. In the current paper, the MS wave inverter-motor system performance will be studied.

2. Types of Inverters

There are three major inverter waveform types, Square wave, Modified Sine (MS) wave, and Pure Sine (PS) wave. Of these, only the last two are commonly seen, as the square wave is considered obsolete. One might wonder why there are so many types of inverters [2]. The primary reason is cost. Some types of devices won’t work on the cheaper MS wave of cheaper inverters and generators [3].

The first type, square wave, is not suitable for marine use because it has no voltage control. This means that the input voltage is proportional to the output [2].

This type of inverter is the simplest and least expensive on the market. However, reliability and output voltage regulation of these inverters are poor. Eventually this will damage sensitive equipment. Another shortcoming of this type of inverter is its lack of surge power which can result in an inability to power motorized equipment [8].

In order to overcome this regulation problem and maintains the output voltage over a large input voltage range, the second waveform is used. This is called the step square wave, rectangular wave, or Modified Sine (MS) wave. The big plus with this type of inverter is the cost; it is low cost and high performance, linked in with the fact that it will run 95% of general equipment. This makes it the most popular inverter choice by far (about 100 MS inverters are sold to 1 sine wave inverter) [2].

Therefore, there are basically two kinds of DC-AC inverters on the market today. One category is the “PS wave” inverter, which produces sine waves with *THD* in the range of 3%. These are typically used when there is a need for clean, near-sine-wave outputs for medical, instrument and other critical applications. For any device that requires sensitive calibration, it is advisable to use a PS wave inverter [9].

Waveforms approaching sine waves, with minimal distortion, are required in any case. Early techniques for designing these inverters incorporated significant linear technology, reducing their efficiency and contributing to their higher cost.

More recent designs used pulse-width modulation (PWM) to produce a pulsed waveform that can be filtered relatively easily to achieve a good approximation to a sine wave. However, PWM requires significant control circuitry and high-speed switching. So the PWM approach introduces significant complexities and switching losses [1].

The good side of a PS wave is it will run all equipment as well as the mains; however, the bad side is its cost [2].

The second category consists of relatively inexpensive units, producing MS wave outputs. They are basically square waves with some dead spots between positive and negative half cycles. Switching techniques rather than linear circuits are used in the power stage, because switching techniques are more efficient and thus less expensive.

These inverters unlike conventional PWM inverters require no high frequency switching, as the switching takes place at line frequency [1].

These types of inverters are much cheaper than PS wave inverters and therefore are attractive alternatives [3] and the most common, general-use inverters available [9].

The typical MS wave inverter has a waveform as shown in Fig. 1. It is evident that, if the waveform is to be considered a sine wave or a MS wave, it is a sine wave with significant distortion [1].

3. Problem Statement

In the market of power inverters, there are many choices. They range from the very expensive to the very inexpensive, with varying degrees of quality, efficiency, and power output capability along the way. High quality combined with high efficiency exists, though it is often at a high monetary cost [3].

The original inverters produced a square waveform output that was cheaper and easier to produce. These inverters are very limited in the equipment they will operate and can be used for resistive electrical equipment [10]. Computers, sound equipment such as stereos, televisions, induction motors, transformer loads and even light bulbs are not recommended to run on square wave inverter, because its output waveform has a high harmonic content [8].

The newest inverters, output PS waveforms (see Fig. 1). These inverters will operate any load within their power rating range [10].

However, to produce a PS wave from DC is a complex task with some problem areas:

1) It is very difficult to produce a PS waveform electronically.

2) Reduced conversion efficiency generally 10% less efficient than MS wave.

3) Increased circuit complexity. Microprocessor technology is needed to produce the PS wave for maximum reliability.

4) The high frequency techniques used to produce the PS wave can have a bad side effect: RFI (Radio Frequency Interference).

5) Do not have the motor starting capability of MS inverters.

6) The price of PS wave inverter is greater than MS wave inverter.

The high end PS wave inverters tend to incorporate very expensive, high power capable digital components.

For applications that are predictable as to what loads will be used, the MS wave has served without the above disadvantages [10,11].

The MS wave units can be very efficient, as there is not much processing being performed on the output waveform. Many of the very cheap devices output a square wave, perhaps a slightly MS wave, with the proper *rms* voltage, and close to the right frequency [3].

The MS wave inverters do maintain correct *rms* voltage for most input voltages, but will only have the correct peak voltage for one input voltage. They may also have high surge power capabilities, and indeed are the correct selection for running some types of motors and incandescent lighting [8].

4. Applications of the MS Wave Inverter

The power inverters come in all shapes and sizes, from low power functions such as powering a car radio to that of backing up a building in case of power outage. They can come in many different varieties, differing in price, power, efficiency and purpose [3].

Most loads will run without trouble from a MS wave. It is suitable for a variety of applications such as induction motors (*i.e.* refrigerators, drill presses); universal motors (*i.e.* hand tools, vacuum cleaners) and general electrical equipment including most small motors (except for some variable speed motors); resistive loads (*i.e.* heaters, toasters) as well as microwaves and computers [12].

For your kids to watch TV in the car, or to power lights at the summer cabin, MS wave output should be quite satisfactory.

Some appliances which are likely to require PS wave include digital clocks, battery chargers, light dimmers, variable speed motors, and audio / visual equipment [9].

Inductive loads may run with a little more noise and get warmer. Inductive loads are found in voltage transformers and motors like those often found in refrigerators, freezers and washing machines. Induction motors also need a comparatively high surge current to start up. For a MS wave inverter to handle an inductive load well, it needs to have a good surge capacity [12].

Affordable and readily available from the automotive aisle of many department stores, MS wave inverters offer an inexpensive alternative for most mobile-power applications [13].

They are the simplest and most economical for UPS design [14].

Some appliances will run noticeably less well on square and MS wave AC than on PS wave. Those affected include:

- Some of the latest sewing machines
- Some programmable timers
- Microwave ovens (which operate more slowly)
- Some battery chargers
- Some cordless appliances
- Some dimmer switches
- Some digital clocks
- Some variable speed devices such as fans
- Some hi-fi and other sound equipment
- Some TVs and video equipment
- Some Fax's and Laser Printers
- Iron ballasted fluorescent lights [1].

The main drawback with MS wave inverters is that some appliances with timing devices, light dimmers, and some battery chargers, as well as variable speed devices may not work well, or indeed, may not work at all [9].

5. Analysis of Current Technology

It is well known that any periodic waveform such as that mentioned previously can be represented by a Fourier series, an infinite sequence of sines and cosines, at the fundamental frequency of the waveform and its harmonics.

The actual percent distortion is not usually quoted in the specifications for inverters other than the PS wave versions, so it is instructive to compute the distortion products to get a feel for the relative distortion involved with the different approaches. For purposes of comparison, let us look first at a conventional square wave as shown in Fig. (1).

The coefficients of the Fourier series are computed with a pair of integrals that produce the coefficients of the sine and cosine terms in the series.

For a signal *f(ωt)* with a zero DC component, the integrals are:

$$a_n = \left(\frac{1}{p}\right)\int_0^{2p} f(wt)\cos(nwt)dwt \quad n>0$$

$$b_n = \left(\frac{1}{p}\right)\int_0^{2p} f(wt)\sin(nwt)dwt \quad n>0$$

, where a_n and b_n are the coefficients of the cosine and sine terms at the n^{th} harmonic, respectively, in the series. The Fourier series is then:

$$f(\omega t) = a_1\cos(\omega t) + a_2\cos(2\omega t) + a_3\cos(3\omega t) + \ldots + b_1\sin(\omega t) + b_2\sin(2\omega t) + b_3\sin(3\omega t) + \ldots$$

Both the square wave and the MS wave have half- and quarter-wave symmetries, therefore, the Fourier series only contains the odd harmonics and the sine terms. Thus, the integral used to compute the coefficients for the conventional square wave becomes:

$$b_n = \left(\frac{4}{p}\right)\int_0^{\frac{p}{2}} f(wt)\sin(nwt)dwt$$

$$= \frac{4}{np} \qquad (n = 1,3,5,\ldots) \qquad (1)$$

The series is then: $(4/\pi)\sin(\omega t) + (4/3\pi)\sin(3\omega t) + (4/5\pi)\sin(5\omega t) + \ldots$

6. *THD* and *DF* of a MS Wave

The quality of the inverter output waveform can be expressed by using the Fourier analysis data to calculate the *THD* and *DF* [5].

Fig. 2 illustrates a typical MS waveform of amplitude V_{dc} and period 2π. This wave represented by *f(ωt)* can be either a voltage or a current.

To calculate the *THD* and *DF*, the analytical expression of the MS wave shown in Fig. 2 is expanded into the

Fourier series. Because of the symmetry of the waveform, the Fourier series only contains the odd harmonics and the sine terms and can be expressed as:

$$f(wt) = \sum_{n=1,3,5,...}^{\infty} b_n \sin(nwt) \quad ....(2)$$

, where:

$$b_n = \frac{4V_{dc}}{np}\cos(nwt) \quad (n = 1,3,5,...) \quad (3)$$

is the amplitude of the n^{th} odd harmonic of angular frequency $n\omega$, and ω is the angular frequency of the fundamental component. A Fourier analysis of the MS waveform is presented in the Appendix.

The *THD* and *DF* of an arbitrary non-sinusoidal waveform are defined as [5]:

$$THD = \frac{\sqrt{\sum_{n>1}^{\infty} b_{n,rms}^2}}{b_{1,rms}} = \frac{\sqrt{b_{rms}^2 - b_{1,rms}^2}}{b_{1,rms}} \quad (4)$$

$$DF = \frac{b_{1,rms}}{b_{rms}} \quad ... (5)$$

, where

$$b_{n,rms} = b_n / \sqrt{2} \quad (n = 1,3,5,...) \quad ..(6)$$

is the *rms* value of the n^{th} harmonic, and

$$b_{rms} = \sqrt{\frac{1}{2p}\int_0^{2p} f^2(wt)dwt} \quad ...(7)$$

is the *rms* value of the waveform. The *THD* and *DF* are respectively, related by:

$$THD = \sqrt{\frac{1}{DF^2} - 1} \quad ...(8a)$$

$$DF = \sqrt{\frac{1}{1 + THD^2}} \quad ... (8b)$$

For the MS wave *f(ωt)* shown in Fig. 2, the *rms* value can be determined by Eq. 7 as following:

$$b_{rms} = V_{dc}\sqrt{\frac{1}{2p}\left[\int_a^{p-a} dwt - \int_{p+a}^{2p-a} dwt\right]}$$

$$= V_{dc}\sqrt{\frac{p - 2a}{p}} \quad (9)$$

Substituting Eqs. 3, 6, and 9 into Eqs. 4 and 5, respectively:

$$THD = \sqrt{\frac{p(p - 2a)}{8\cos^2 a} - 1} \quad .. (10)$$

$$DF = \sqrt{\frac{4\cos a}{2p(p - 2a)}} \quad .. (11)$$

, where α is the switching-on angle.

7. Minimization of *THD*

A. Minimizing *THD* of the Inverter Output Voltage

When a MS wave VSI is used as a power supply, the load requires the output voltage contain as less harmonics as possible.

For a given load, the *THD* of the MS wave output voltage THD_{υ} should be minimized in order to meet as much as possible the requirement of the load, while keeping the *rms* value of the output voltage V_{rms} constant. In this situation, it is a common practice to choose $\alpha = 30^{o}$ to eliminate the 3^{rd} harmonic. The following analysis, however, shows that this is not quite right for the minimization of THD_{υ}.

To minimize the THD_{υ}, let: $dTHD_{\upsilon}/d\alpha = 0$, where THD_{υ} is expressed in a general form in Eq. 10, and obtain:

$$\cot a = p - 2a \quad ...(12)$$

By Eq. 9, we obtain the *rms* value of the output voltage of a MS wave VSI:

$$V_{rms} = V_{dc}\sqrt{\frac{p - 2a}{p}} \quad .. (13)$$

It shows that when α varies, in order to keep V_{rms} constant, the magnitude of the MS wave V_{dc} should be adjusted accordingly by:

$$\left(\frac{V_{dc}}{V_{rms}}\right)^2=\frac{p}{p-2a} \qquad ..(14)$$

The magnitudes of harmonic components of the output voltage can be obtained by Eq. 3 as:

$$V_n=\frac{4V_{dc}}{np}\cos(na) \quad (n=1,3,5,...) ..(15)$$

B. Minimizing *THD* of the Output Load Current

The quality of output waveform that is needed from an inverter depends on the characteristics of the load. Therefore, the total harmonic distortion of the output current THD_i is determined by both the output voltage waveform and the type of load [5]. Three typical load types are considered in the analysis of THD_i below.

(1) Resistive Load

When a resistive load is connected to the output terminals of a MS wave VSI, the output current can be determined by the Ohm's law as:

$$i(t)=u(t)/R \qquad (16)$$

, where R is the resistance of the load. Since the output current is proportional to the load resistance, the waveforms of the output current and voltage are same. Therefore, THD_i for a resistive load is same as that of the output voltage, *i.e.*

$$THD_i=THD_u \qquad \ldots (17)$$

(2) Inductive Load

When an inductive load which consists of an inductor L in series with a resistor R is connected to a MS wave VSI, the output current waveform will not be a square wave since the load has different impedance to different voltage harmonics. The *rms* value of each current harmonic component can be determined by:

$$I_{n,rms}=\frac{V_{n,rms}}{Z_n}=\frac{V_n}{\sqrt{2}\;Z_n} \qquad \ldots(18)$$

, where $V_{n,rms}$ is the *rms* value of the n^{th} voltage harmonic, and

$$Z_n=\sqrt{R^2+(nwL)^2} \quad (n=1,3,5,...) \ (19)$$

is the load impedance to the n^{th} voltage harmonic. Substituting Eqs. 15 and 19 into Eq. 18, obtain:

$$I_{n,rms}=\frac{4V_{dc}\cos na}{\sqrt{2}\,np\,R\sqrt{1+\left(n\frac{wL}{R}\right)^2}} \qquad ..(20)$$

By the definition of *THD* in Eq. 4, the THD_i for an inductive load supplied by a MS wave VSI can be written as:

$$THD_i=\sqrt{\frac{\sum_{n=3,5,...}^{\infty} I^2_{n,rms}}{I^2_{1,rms}}}$$

$$=\sqrt{\sum_{n=3,5,...}^{\infty}\frac{1+\left(\frac{X_L}{R}\right)^2}{1+\left(\frac{nX_L}{R}\right)^2}\left(\frac{\cos na}{n\cos a}\right)^2} \ . \ (21)$$

It is shown that the THD_i is a function of α and the ratio between the inductive reactance to the fundamental voltage component and the resistance of the load; X_L/R, where, $X_L = \omega L$. For a given load, the THD_i can be minimized by choosing a proper α. It should be noted that the load power factor P_f can be found as: $P_f = \cos(\theta)$; $q=\tan^{-1}(X_L/R)$ is the phase angle between the inverter output voltage and the load current.

(3) Capacitive Load

When a MS wave VSI is supplying a capacitive load which consists of a capacitor C in series with a resistor R, the output current is also not a square wave because of the different impedance to different voltage harmonics. Applying the same procedure used for inductive loads, we can minimize the THD_i for different capacitive loads as follows:

$$THD_i = \sqrt{\sum_{n=3,5,...}^{\infty} \frac{1+\left(\frac{X_C}{R}\right)^2}{1+\left(\frac{X_C}{nR}\right)^2}\left(\frac{\cos na}{n\cos a}\right)^2} \quad (22)$$

, where: $X_C = 1/\omega C$ and

$$Z_n = \sqrt{R^2 + \left(\frac{1}{nwC}\right)^2} \quad ...(23)$$

We also have that; $P_f = \cos(\theta) = \cos\left(\tan^{-1}(X_C / R)\right)$.

8. Simulation Results

The solutions for Eq. 12 are α = 0.4052, 1.5708, and 2.7363, when $0 \le \alpha \le \pi/2$. Among them, only α=0.4052 or 23.218° is a reasonable solution. Numerical evaluation of the coefficients for the MS wave indicates that, if the MS wave is to be considered, a sine wave with distortion, the *THD* is in the range of 28.96% (at α=23.218°) about 60% of that of the square wave (see Table 1). The 3rd harmonic of the output voltage, the hardest to filter out, is about 12.61% of the fundamental with the MS wave while, it is 33.33% (one-third) for that of the square wave. Table 2 shows the percentage of the fundamental for each harmonic component of the square wave and MS wave VSI output waveform for various values of α.

When α=30°, the 3rd harmonic component can be eliminated, but the other harmonic components are not minimized. The choose of α should take into account the *THD* rather than an individual harmonic component.

Table 1 compares the THD_υ's, DF_υ's, and the fundamental, 3rd, 5th, ... 15th harmonic components of the output voltage of the square wave and MS wave VSI for each value of α; α =30^0, 18^0...90^0/n...6^0 used to eliminate the corresponding harmonics 3rd, 5th...15th, respectively.

It can be seen that, the THD_υ at α=30^0, can be reduced by 36% of that of the square wave. That is, the quality of the output voltage can be improved.

From the Fourier analysis in the Appendix, it can be seen that, the fundamental *rms* voltage V_{1rms} for the optimum MS wave (α=23.218^0) in terms of a PS wave is equal to 1.1701 while, it is 1.1027 when using α=30^0.

Figs. 3 and 4 show the output voltage and motor current spectra, respectively, of the square wave and MS wave VSI with α=23.218^0, 30^0, and 18^0. Fig. 5(a) shows the α solution with modulation index m; $m = (4V_{dc} / \pi) \cos(\alpha)$. he MS wave inverter is loaded by a capacitor-run induction motor with the data listed in the Appendix.The inverter power quality (THD_υ and DF_υ) and motor performance (current; I, input power; P_{in}, efficiency; η, power factor; P_f, motor additional losses; P_{add} and additional torque pulsations; T_{add}, due to harmonics content) are evaluated for each value of the α range ($0 \le \alpha \le \pi/2$) or m range ($0 \le m \le 1.2732$) to choose the optimum one as shown in Figs. 5(b-f). These figures show that the minimum THD_υ; $THD_{\upsilon min}$ and maximum DF_υ; $DF_{\upsilon max}$ occur at α=23.218^0, while they have variable values of α to get maximum performance (*i.e.* η_{max}= 65.7 % at α=28.67^0, and so on, as shown in Table 3). From the table, we conclude that, the MS wave inverter-motor system has a slightly 2.5% lower efficiency and 2.8% lower power factor than a sine wave motor system. Fig. 6 shows the inverter-motor system performance with α; (α=30^0, 18^0... 90^0/n ... 0^0) specified for eliminating harmonics;

(3^{rd}, 5^{th}... n^{th}), respectively. It can be seen that *THD* decreases, while *DF* increases with α increase. The THD_{vmin} and DF_{vmax} happen at α=18^0, while the maximum motor performance happened at α=30^0 (see Table 1), where α here is only specified for eliminating harmonics.

When the output terminals of a MS wave VSI is loaded by a variable inductive or capacitive load by varying the ratios; X_L/R or X_C/R for limited ranges, respectively, or their corresponding load power factors, the minimum THD_i (THD_{imin}) and the corresponding values of α are found as shown in Fig. 7.

Table 4 tabulates the THD_{imin}'s and the corresponding values of α for inductive loads with different ratios of X_L/R, ranging from 0 (pure resistive load) to 10^6 (almost pure inductive load). The table shows that α increases when the load inductance increases, but it is always smaller than 27.99^o. The table also lists the THD_{imin}'s and the corresponding values of α for capacitive loads with different ratios of X_C/R, ranging from 0 to 10^6. It is shown that α decreases as the ratio X_C/R increases. When the ratio X_C/R increases, the THD_{imin} increases too and reaches very high values. The table shows that, a smaller THD_{imin} can be reached as the load becomes more inductive, while the THD_{imin} that can be reached by adjusting α is larger as the load becomes more capacitive.

This is reasonable because a *R-L* circuit is in fact a low-pass filter, which blocks the higher order current harmonics, whereas a *R-C* circuit is a high pass filter, which shows higher impedance to the fundamental current component. Figs. 8(a and b) shows the 3-D plot of the THD_i for each value of α and ratios; (a) X_L/R for inductive load, and (b) X_C/R for capacitive load.

The bond lines in the figures point to the minimum values of THD_i's.

The proposed motor, have P_f = 0.9726 or X_L/R=0.239. From Tables 3 and 4, the corresponding optimum α=26.306^0 and THD_{imin}=14.981. Fig. 9 shows the inverter output voltage and motor current spectra at $THD_{i\,min}$.

9. Conclusions

The relationship between the *THD* of MS wave VSI's and the duty cycle has been analyzed. The analysis shows that the *THD* of the output voltage is at its minimum when the switching-on angle is 23.218^o. Also it has been shown that the voltage *THD* at α=30^0, can be reduced by 36% of that of the square wave. That is, the quality of the output voltage can be improved.

On the one hand, it has been found that the *THD* of the output current depends on the load types, when the output terminals of a MS wave VSI is loaded by a variable inductive or capacitive load. It has been shown that α increases when the load inductance increases, but it is always smaller than 27.99^o, while it decreases with the increase of load capacitance. Also, the results show that a smaller current *THD* can be reached as the load becomes more inductive, while it is larger as the load becomes more capacitive. Even though the MS wave inverter-motor system has a slightly 2.5% lower efficiency and 2.8% lower power factor than a sine wave system, the MS system is much smaller and simpler, more efficient and reliable with fewer parts and therefore, less expensive. If a MS inverter is used, the MS system actually produces a higher fundamental voltage than the sine. Thus, in terms of overall system design, the MS system is adequate for virtually all applications.

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Table (1) the Quality Factors and Performance of the Output of the MS Wave Inverter-motor System

ptimum Performance	$THD_{v\,min}$ (%)	$DF_{v\,max}$ (%)	$THD_{i\,min}$ (%)	I_{min} (A)	$P_{in\,min}$ (W)	$P_{add\,min}$ (W)	$T_{add\,min}$ (N.m)	η_{max} (%)	$P_{f\,max}$
	28.96	96.05	14.9815	1.2854	266.899	3.8834	0.0042	65.7	0.9454
α (Degree)	23.218	23.218	26.306	28.26	28.91	29.14	29.10	28.69	26.21
m	1.1701	1.1701	1.1414	1.1215	1.1146	1.1121	1.1125	1.1169	1.1423

Table (2) the Harmonics Percentage of Fundamental for the MS Wave with Different Values of α

Wave Type		Square Wave	Modified Sine Wave (MSW)								
α (Deg)		0	23.218	26.306	30	18	12.8571	10	8.1818	6.9231	6
Eliminated Harmonic		---	---	---	3rd	5th	7th	9th	11th	13th	15th
		Percentage of Fundamental (%)									
Harmonic Order (n)	1	100	100	100	100	100	100	100	100	100	100
	3	33.3333	12.6058	7.14912	0	20.6045	26.7300	29.3165	30.6356	31.3924	31.8724
	5	19.9969	9.5804	14.7889	19.9964	0	8.9020	13.0553	15.2741	16.5823	17.4129
	7	14.2868	14.8278	15.8928	14.2831	8.8281	0	4.9605	7.7997	9.5411	10.6768
	9	11.1137	10.5717	6.79867	0	11.1074	4.9464	0	3.1659	5.1978	6.5703
	11	9.0873	2.4870	3.36429	9.0868	9.0924	7.2505	3.1581	0	2.1915	3.7195
	13	7.6893	4.4184	8.15665	7.6902	4.7568	7.6935	5.0243	2.1900	0	1.6110
	15	6.5269	7.1020	6.12406	0	0	6.6704	5.8617	3.6420	1.6060	0

Table (3) the Switching Angles at the Maximum Performance of the MS Wave Inverter-motor System

Inductive Load			*Capacitive Load*		
X_L / R	*Optimum Switching Angle α (Degree)*	*$THD_{i\,min}$*	*X_C / R*	*Optimum Switching Angle α (Degree)*	*$THD_{i\,min}$*
0	23.218	28.960	0	23.218	28.960
0.1	24.614	21.7403	0.1	23.213	29.0929
0.239	26.306	14.9815	0.239	23.182	29.7406
0.3	26.743	13.0601	0.3	23.171	30.1832
0.5	27.441	9.3031	0.5	23.071	32.2378
1	27.839	6.2022	1	22.660	40.2919
2	27.95	4.9739	2	21.193	61.0178
4	27.98	4.6025	4	21.00	100.7875
5	27.984	4.5555	5	16.0430	115.9274
10	27.989	4.4920	10	11.187	178.8617
20	27.99	4.4759	20	7.228	265.1669
50	27.99	4.4714	50	3.807	431.6210
10^2	27.99	4.4708	10^2	2.259	615.2050
10^3	27.99	4.4706	10^3	0.2430	1847.5
10^4	27.99	4.4706	10^4	0.026	3073.8
10^6	27.99	4.4706	10^6	0.026	3126.9

Table (4) the Minimum THD_i and Corresponding α for Loads with Different Values of X_L / R or X_C / R

nverter Wave Type	*Square Wave*	*Modified Sine Wave (MSW)*								
		Optimum ($THD_{\upsilon\ min}$)	*Optimum* ($THDi_{min}$)	*Eliminated Harmonic Order*						
				3rd	*5th*	*7th*	*9th*	*11th*	*13th*	*15th*
α (Deg)	0	23.218	26.306	30	18	12.8571	10	8.1818	6.9231	6
V_{dc}/V_{rms}	1	1.1609	1.1887	1.2247	1.1180	1.0801	1.0607	1.0488	1.0408	1.0351
$h_1 = m$	1.2732	1.1701	1.1414	1.1027	1.2109	1.2413	1.2539	1.2603	1.2640	1.2663
h_3	0.4244	0.1475	0.0816	0	0.2495	0.3318	0.3676	0.3861	0.3968	0.4036
h_5	0.2546	- 0.1121	- 0.1688	- 0.221	0	0.1105	0.1637	0.1925	0.2096	0.2205
h_7	0.1819	- 0.1735	- 0.1814	- 0.158	- 0.107	0	0.0622	0.0983	0.1206	0.1352
h_9	0.1415	- 0.1237	- 0.0776	0	- 0.135	- 0.061	0	0.0399	0.0657	0.0832
h_{11}	0.1157	- 0.0291	0.0384	0.1002	- 0.110	- 0.090	- 0.04	0	0.0277	0.0471
h_{13}	0.0979	0.0517	0.0931	0.0848	- 0.058	0.0955	- 0.063	- 0.028	0	0.0204
h_{15}	0.0849	0.0831	0.0699	0	0	- 0.083	- 0.074	- 0.046	- 0.020	0
THD (%)	48.34	28.96	29.1441	31.089	30.19	33.55	36.15	38.04	39.45	40.52
DF (%)	90.03	96.05	95.94	95.49	95.73	94.81	94.04	93.46	93.02	92.68
I (A)	1.7342	1.3255	1.2913	1.2903	1.4193	1.5397	1.6049	1.6425	1.666	1.6815
P_{in} (W)	346.886	274.966	268.572	267.207	291.348	312.404	323.957	330.659	334.836	337.598
P_{add} (W)	60.476	10.134	5.3103	4.023	22.053	36.975	45.001	49.6	52.44	54.306
T_{add}(N.m)	0.0685	0.0112	0.0058	0.0044	0.0249	0.0420	0.0511	0.0564	0.0596	0.0617
η (%)	53.41	64.27	65.43	65.61	61.43	58.13	56.465	55.541	54.980	54.614
P_f	0.9092	0.9429	0.9454	0.9413	0.933	0.9223	0.9175	0.9150	0.9136	0.9126

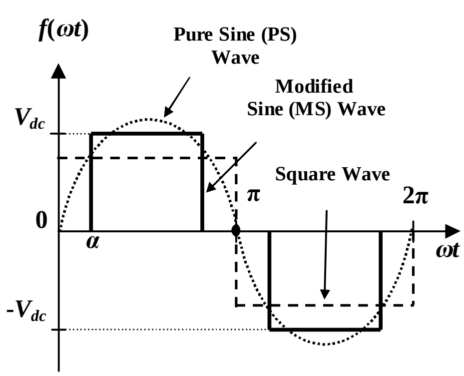

Figure (1) Different T ypes of Inverter Waveforms

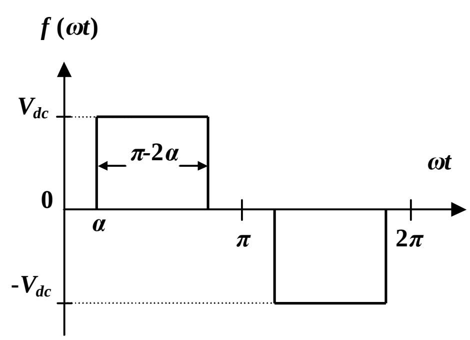

Figure (2) Typical Modified Sine (MS) Wave

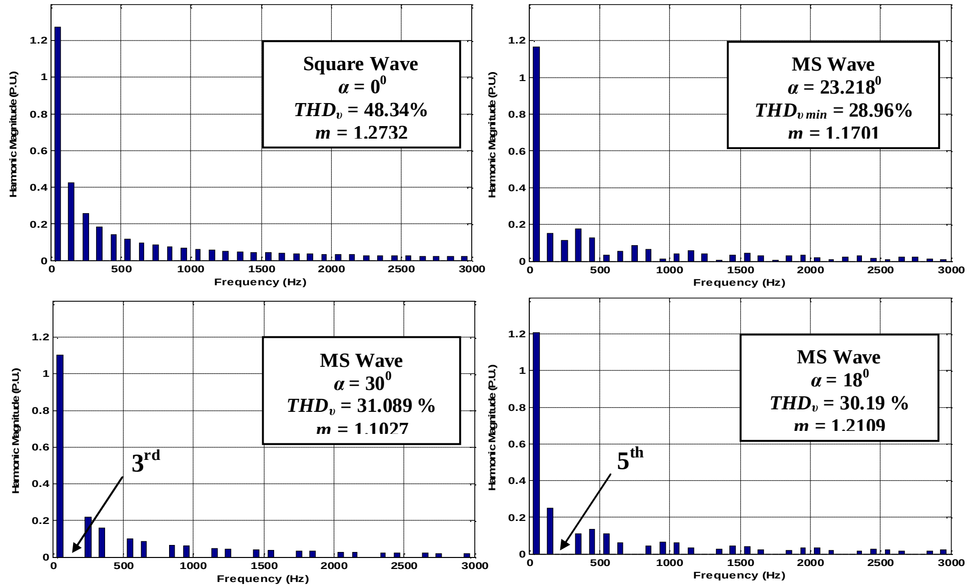

Figure (3) the Output Voltage Spectra of the MS Wave VSI with Different Values of Switching Angles α

Fig. 4: the Current Spectra of a Motor Fed by MS Wave VSI with Different Value

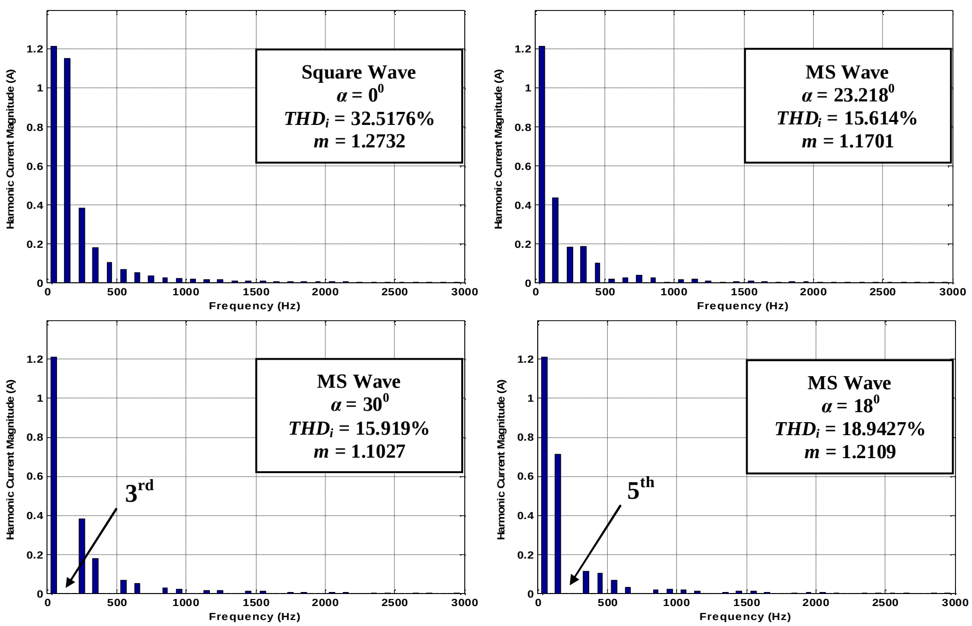

Figure (4) the Current Spectra of a Motor Fed by MS Wave VSI with Different Value s of Switching Angles α

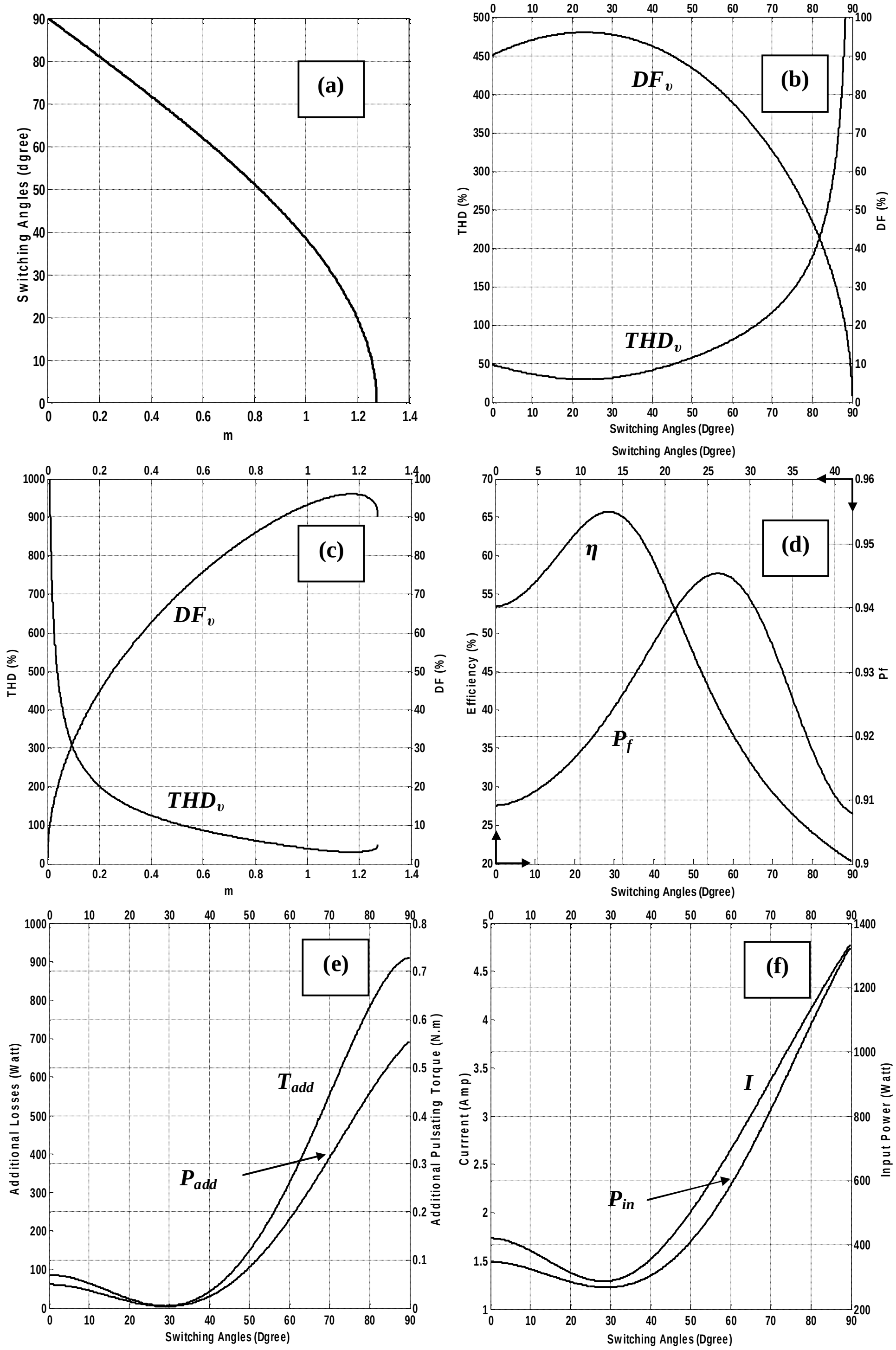

Figure (5) (a) the Solution of α for MS Wave VSI and (b-f) the Performance of the Inverter-motor System over the whole Range of Switching Angles α; $[0 \leq \alpha \leq 90^0]$

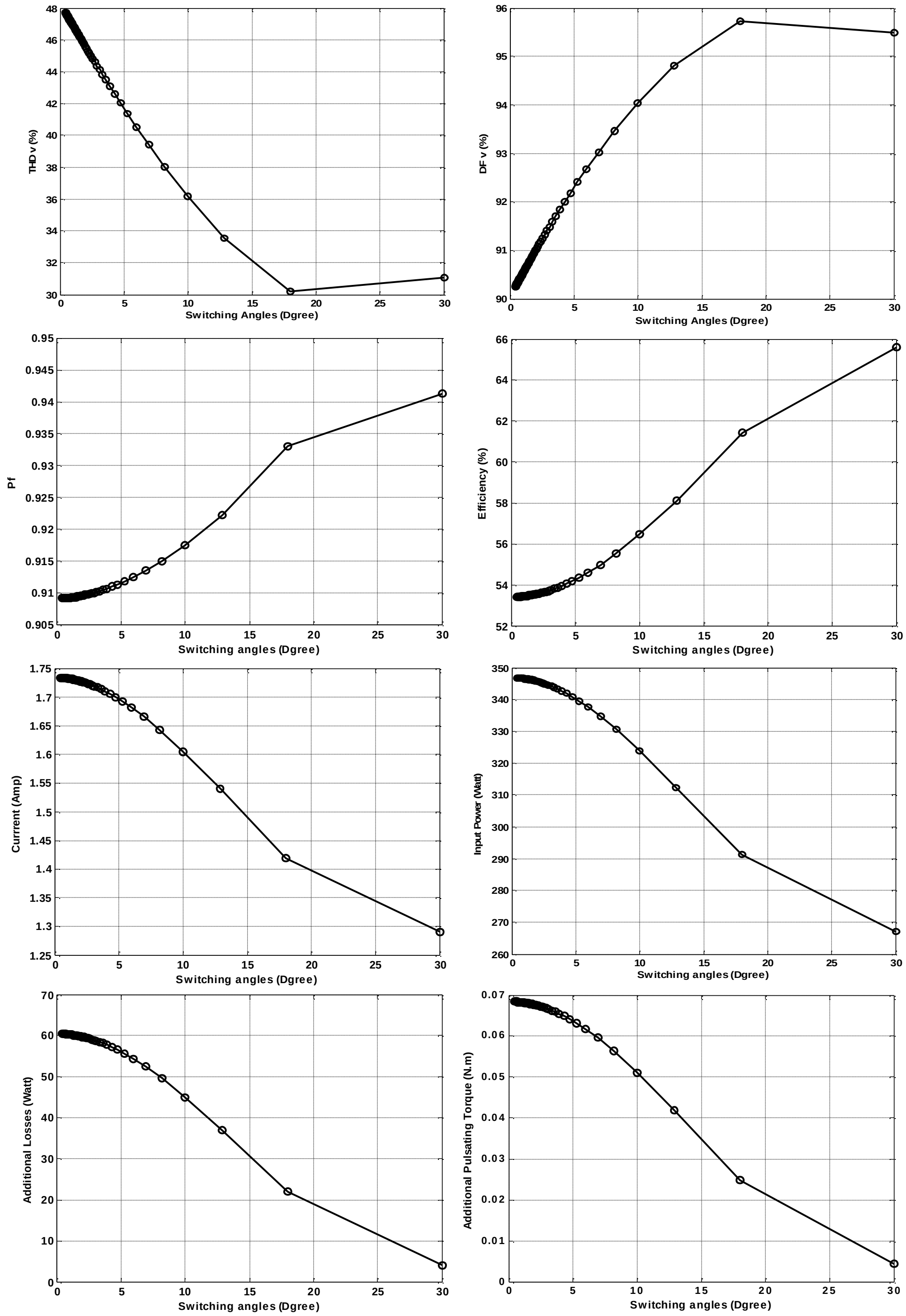

Figure (6) the Inverter-motor System Performance with the Switching Angles α Specified for Eliminating Harmonics; [$\alpha = 90^0/n$, $0 \le \alpha \le 30^0$]

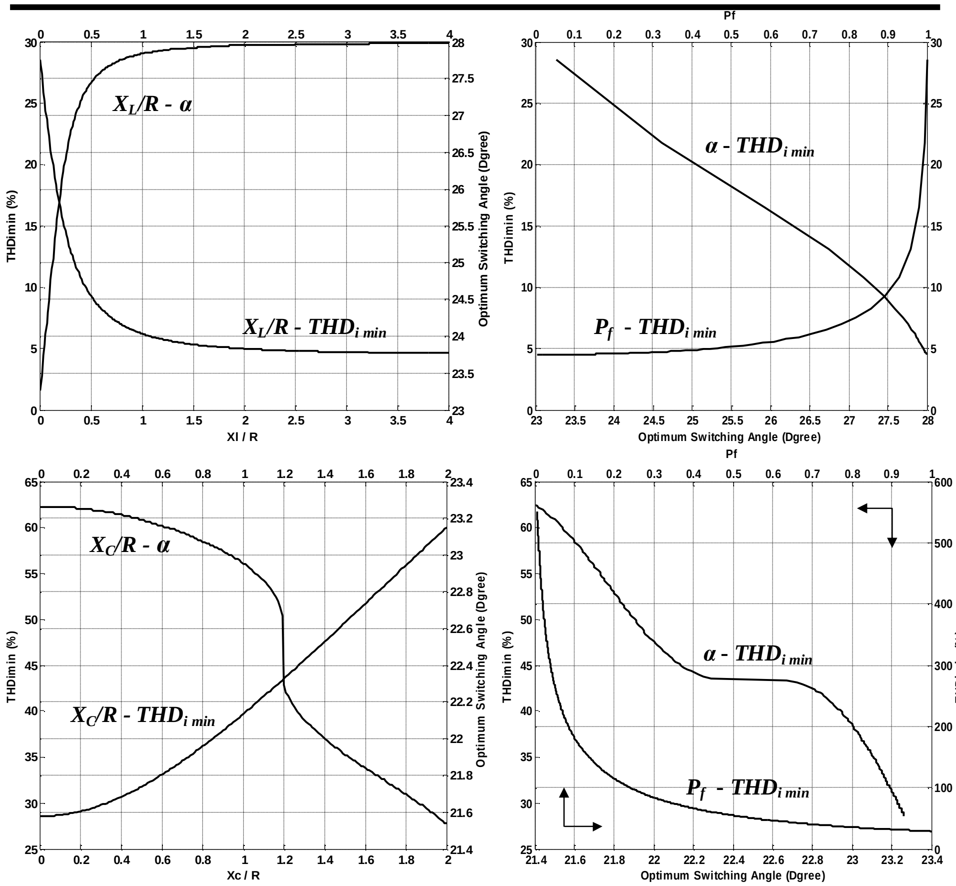

Figure (7) the Relationship between $THD_{i\ min}$, α, P_f, and (a) X_L / R of Inductive Load, or (b) X_C / R of Capacitive Load Fed by MS Wave VSI

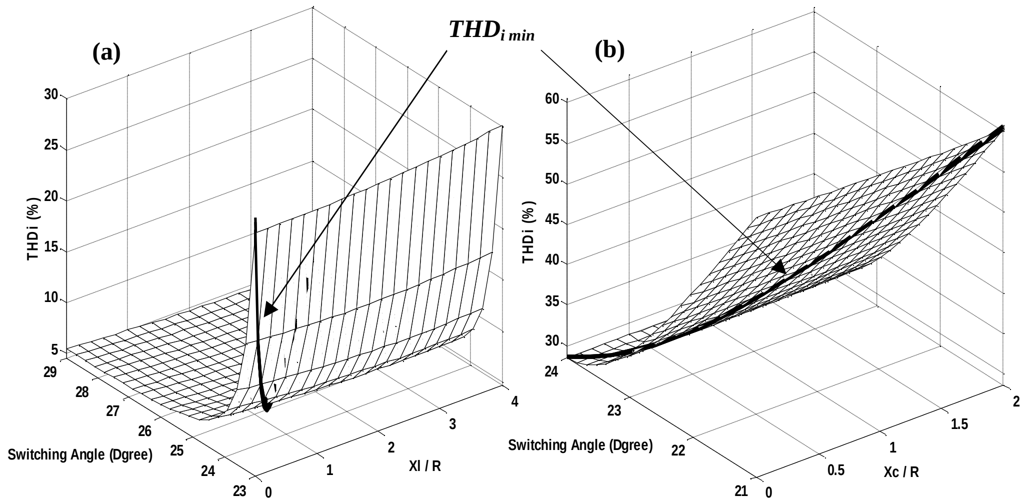

Figure (8) the THD_i as a Function of α and Ratio (a) X_L / R for Inductive Load, or (b) X_C / R for Capacitive Load

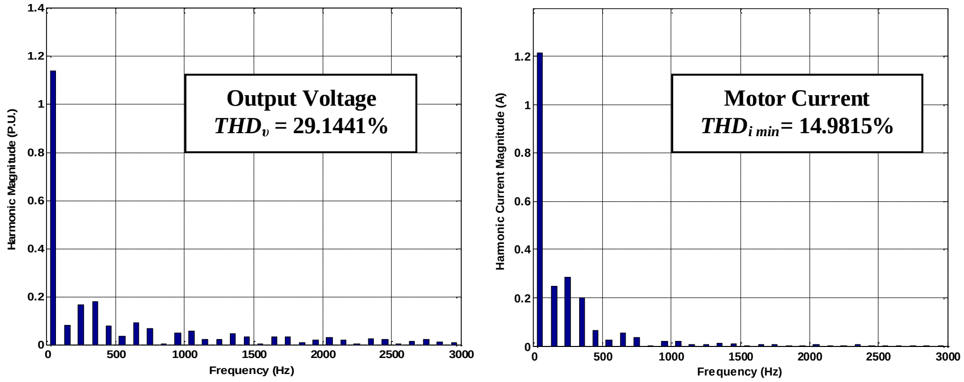

Figure (9) the Output Voltage Spectra of MS Wave VSI and Current Spectra of the Proposed Motor Fed from at the Minimum THD_i; [α = 26.306^0, m = 1.1414, and X_L / R = 0.239]

Appendix

A. Fourier Analysis of the MS Wave:

Note that a_n = 0, (due to half symmetry):

$$b_n = 2\left[\frac{1}{p}\int_a^{p-a} V_{dc}\sin(nwt)\,dwt\right] = \frac{2V_{dc}}{np}\left[-\cos na\Big|_a^{p-a}\right] = \frac{2V_{dc}}{np}\left[\cos(na) - \cos n(p-a)\right]$$

Expanding: cos n (π-α) = cos ($n\pi$ - $n\alpha$) = cos $n\pi$ cos $n\alpha$ + sin $n\pi$ sin $n\alpha$

$$b_n = \frac{2V_{dc}}{np}\left[\cos(na) - \cos np\,\cos na\right] = \frac{2V_{dc}}{np}\cos(na)\left[1-\cos np\right]$$

B. Harmonic Control

b_n = 0, n is even
b_n = ($4V_{dc}/n\pi$) cos ($n\alpha$), n is odd

In particular, amplitude of the fundamental is: b_1= ($4V_{dc}/\pi$) cos (α) = m (modulation index). Note that, the fundamental, b_1, is controlled by varying α. Harmonic can also be controlled by adjusting α.

C. Harmonic Elimination: For example if α=30^0, then b_3=0, the 3rd harmonic is eliminated from the waveform. In general, harmonic n will be eliminated if: α=90/n, where n = 1, 3, 5...

D. Motor Data: 1-phase, Capacitor-rum induction motor; 175W, 220V, 1.22A, 1275Rpm, η = 67.38%, and P_f = 0.9726.