Enhancement of Guidance System using Kalman Filter

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Abstract

The operation of Modern Guidance System may be separated into two cascaded functions
1. Filtering of the noisy measurements obtained from the sensors and 2. Utilization of the estimated parameters to control the required acceleration.
Guidance system is enhanced by using 3rd order Kalman filter to estimate the separation distance and its derivatives that needed in optimal controller. Slant range and closing velocity is estimated using 2nd order Kalman filter and they also needed in estimation of time to go.

The optimal guidance system is better than the proportional navigation, biased proportional navigation, and the augmented proportional navigation system since it required simple achieved acceleration.

Fading memory filter may be used to enhance the navigation system instead of Kalman filter. It is simple in structure and need minimum time but less accuracy than the Kalman filter.

تحسين منظومات التوجيه باستخدام مرشح كالمن

الخلاصة

عمل منظومة التوجيه الحديثة يمكن أن يجزأ إلى دالتين متتاليتين: 1 . ترشيح ضوضاء القياسات المعطاة من قبل المتحسسات و 2 . استخدام العوامل المقاسة للسيطرة على التعجيل المطلوب. يمكن تحسين منظومة التوجيه من خلال استعمال مرشح كالمن Kalman الثلاثي الرتبة لحساب مسافة الفصل ومشتقاتها والتي يحتاجها المسيطر الأمثل. المدى المائل وسرعة الاقتراب يحسبان باستعمال مرشح كالمن الثنائي الرتبة واللذان مطلوبان أيضا في حساب زمن الذهاب . نظام التوجيه الأمثل أفضل من نظام الملاحة التناسبية PN ،نظام الملاحة التناسبية المتحيز BPN ، ونظام الملاحة التناسبية الموسع APN لانه يعطي تعجيل منجز بسيط. مرشح تضائل الذاكرة التدريجي FMF يمكن استخدامه لتحسين نظام الملاحة بدلا عن مرشح كالمن والذي يكون ذات تركيب بسيط مع زمن تنفيذ قليل ولكنه اقل دقة من مرشح كالمن.

1. Introduction

It is useful to divide the guidance techniques broadly into classical and modern categories, which are closely related despite differences in filtering. Classical guidance utilizes proportional navigation and classical control theory, while modern guidance system (MGS) is based on modern control theory for stochastic time varying linear systems [1].

In the analysis of flight control systems of air to air missile, it is

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usually assumed that the navigation filter and the guidance law can be designed separately based on the separation theorem [2], which also called the statistical equivalent theorem. If the guidance system is characterized by a nonlinear dynamical model, or the state variable sensors are nonlinear, then the separation theorem is generally not valid.

2.Kalman Filter (KF)

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete data linear filtering problem. Since that time, due to large part of advances in digital computing, the KF has been the subject of extensive research and application particularly in the area of autonomous or assisted navigation. The KF equations fall into two groups [3]: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for feedback, i.e. for incorporating a new measurement into the priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought as corrector equations.

(a) 3rd Order KF:

for the dynamical linear system described by

$$\dot{x} = Fx + Gu + w \qquad (1)$$

Where

x is the system state

F is the system dynamic matrix

u is the known control vector

w is the white noise process with zero mean and variance $Q = E[ww^T]$.

Where

E is an expectation

The measurements available are linearly related to the states according to

$$Z_K = Hx_K + v \qquad (2)$$

Where, v is the measurement noise with zero mean and variance R_K and H is the measurement matrix

The KF algorithms are [3, 4]

$$\hat{x}_K = \hat{x}'_K + K_K[Z_K - H\hat{x}'_K] \qquad (3)$$

$$M_K = f_K P_K f_K^T + Q_K \qquad (4)$$

$$K_K = M_K H^T [HM_K H^T + R_K]^{-1} \qquad (5)$$

$$P_K = (I - K_K H)M_K \qquad (6)$$

With

$$\hat{x}'_K = f_K \hat{x}_{K-1} + G_K u_{K-1}$$

$$G_K = \int_0^{t_s} f(t)G\,dt$$

$$Q_K = \int_0^{t_s} f(t)Qf^T(t)\,dt$$

where

P_K is the covariance matrix of state estimate before an update

M_K is the covariance matrix of state estimate after an update

K_K is the KF gain

f_K is the state transition matrix

Then

$$x_K = \begin{bmatrix} y \\ \dot{y} \\ nt \end{bmatrix} \qquad f_K = \begin{bmatrix} 1 & T_s & 0.5T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_K = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \qquad H = [1\,0\,0]$$

$$G_K = \begin{bmatrix} -0.5T_s^2 \\ -T_s \\ 0 \end{bmatrix}$$

(b) 2nd order KF: the slant range R_s which measured by radar and the closing velocity [5], $V_c = -\dot{R}_s$, that measured by the Doppler radar may contains errors affected the system accuracy. Here we proposed using of 2nd order KF to estimate the range and its rate. The KF equations are [3]:

$$\begin{bmatrix} \hat{R} \\ \hat{V}_c \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R \\ V_c \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}\left\{Z - [1\,0]\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R \\ V_c \end{bmatrix}\right\}$$

$$M = fPf^T = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} M_{11}/(M_{11} + s_R^2) \\ M_{12}/(M_{11} + s_R^2) \end{bmatrix}$$

$$P = (I - KH)M = \begin{bmatrix} 1-k_1 & 0 \\ -k_2 & 1 \end{bmatrix}\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}$$

3.Optimal Guidance Law (OPG)

The particular problems is chosen as a minimization of the integral square system acceleration that perpendicular to the line of sight (LOS), and

$$J = \frac{1}{2}\int_{t_0}^{t_f} n_c^2\,dt \qquad (7)$$

Where , n_c is the normal acceleration. The relative motion is given by equation (1) with the system dynamics represented by a single time lag τ. And

$$x = \begin{bmatrix} y \\ \dot{y} \\ n_c \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{t} \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{t} \end{bmatrix}$$

The solution for P is obtained from Riccati equation [6]

$$\dot{P} + F^T P = 0; \quad P(t_f) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/t \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

The solution of the above equation is

$$P(t) = \begin{bmatrix} 1 \\ t_{go} \\ t^2(e^{-T} + T - 1) \end{bmatrix}$$

Where

$T = \dfrac{t_f - t}{t} = \dfrac{t_{go}}{t}$, t_{go} is time to go.

Then the normal acceleration is given by:

$$n_c = \frac{N'}{t_{go}^2}[y + \dot{y}t_{go} + t^2(e^{-T} + T - 1)] \quad (8)$$

Where N' is the navigation constant which depends upon the system capabilities. At large value of time to go (t_{go}), N' is asymptotically equal 3 [1]. OPG needs range, t_{go} , and system acceleration measurements. The t_{go} is the ratio of slant range to closing velocity. If t_{go} estimation is lacking or inaccurate, OPG system performance degrades.

4. Proportional Navigation (PN)

It is a method of guidance in which the required acceleration is proportional to the LOS rate. The target movement and the required movement cause the LOS to rotate through a small angle λ. Then the PN law is given by [7]

$$n_c = N' V_c \dot{\lambda} \quad (9)$$

In practice the navigation ratio is held constant with acceptable values ($3 \langle N' \langle 6$). The LOS angle can be expressed as

$$\lambda = \frac{y}{R} = \frac{y}{V_c(t_f - t)} \quad (10)$$

We can find the LOS rate by taking the derivative of the preceding expression

$$\dot{\lambda} = \frac{y + \dot{y}\, t_{go}}{V_c t_{go}^2} \quad (11)$$

Thus we can express the PN law as

$$n_c = \frac{N'}{t_{go}^2}[y + \dot{y}t_{go}] \quad (12)$$

Modification may be occurs when maneuvering is taking into account by using the augmented proportional navigation APN, or the biased proportional navigation BPN.

In the APN, the zero effort miss distance must be augmented by an additional term as a function of target acceleration. Then the APN law can be given by:

$$n_c = \frac{N'}{t_{go}^2}[y + \dot{y}t_{go} + 0.5 n_t t_{go}^2] \quad (13)$$

In BPN it is suggested that a bias may be added to the PN algorithm to takes the form [8]

$$n_c = N' V_c (\dot{\lambda} + \dot{\lambda}_b) \qquad (14)$$

Where $\dot{\lambda}_b$ is a constant or time varying bias added to the angular velocity of the LOS.

5. Fading Memory Filter (FMF)

Discarding old data can be accomplished by weighting them according to when they occurred. This means that the covariance of the measurement noise must somehow be increased for past measurements. This filter is recursive and weights new measurements more heavily than older measurements. The FMF estimates essentially the old estimate plus the residual, difference between the current measurement and previous estimate. The filter gains are constant and are function of only one parameter, β. This parameter is associated with the memory length of the filter and is constant between zero and unity [9]. The FMF has a set of equations that nearly identical to KF sets, and the only difference is the appearance of the age-weighting factor υ in the equation of error covariance matrix after update [9];

$$M_K = u f_K P_K f_K^T + Q_K \qquad (15)$$

The gain of the 3rd order FMF is given by

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 1 - b^2 \\ 1.5(1+b)(1-b)^2 / t_s \\ (1-b)^3 / t_s^2 \end{bmatrix} \qquad (16)$$

6. Results

The proposed navigation system with OPG law is shown in Fig.(1). The 2nd order KF is used to estimate the slant range and the closing velocity while the 3rd order KF is used to estimate the relative separation distance and its derivatives. Optimal control law used these parameters to guide the missile navigation system with acceptable acceleration. The autopilot describes the behavior of the flying object. The limitation in flying acceleration is taken into account (10 g). All navigation laws are able to guide the missile to hit the target but the advantage of OPG law is clear since it gives a minimum required acceleration than the other types of guidance laws. This fact is shown in Fig. (2). Navigation constant is equal 3 but in OPG it is variable as shown in Fig (3). The characteristic of the 3rd order KF is shown in

Fig. (4). the measured and estimated relative separation distance are shown in Fig. (5) and Fig.(6) respectively. It is clear that the proposed filter is succeeded in error minimization and therefore in estimation of the parameter and its derivatives, see Fig.(7), from the noisy data. The estimated LOS angle is shown in Fig.(8).

The characteristics of the 2nd order KF are shown in Fig. (9).

This filter estimates the slant range and its rate as shown in Fig.(10) and Fig.(11) respectively. The initial slant range is taken as 12200 m. The steady state value of the estimated closing velocity is 1220 m/s, therefore the time to go is equal 10 second. The 3rd order KF may be replaced by 3rd order FMF that succeeded in estimation of the relative separation distance, velocity, and acceleration. The gain constant β is taken as 0.95 to estimate the achieved acceleration as shown in Fig.(12) .

The FMF is easy implementation and required less calculation than the KF but the later is more accurate. The sampling time is taken as 0.01.

Measurement of relative separation distance is linear operation. One can measure the LOS angle which is nonlinear operation. In this case the

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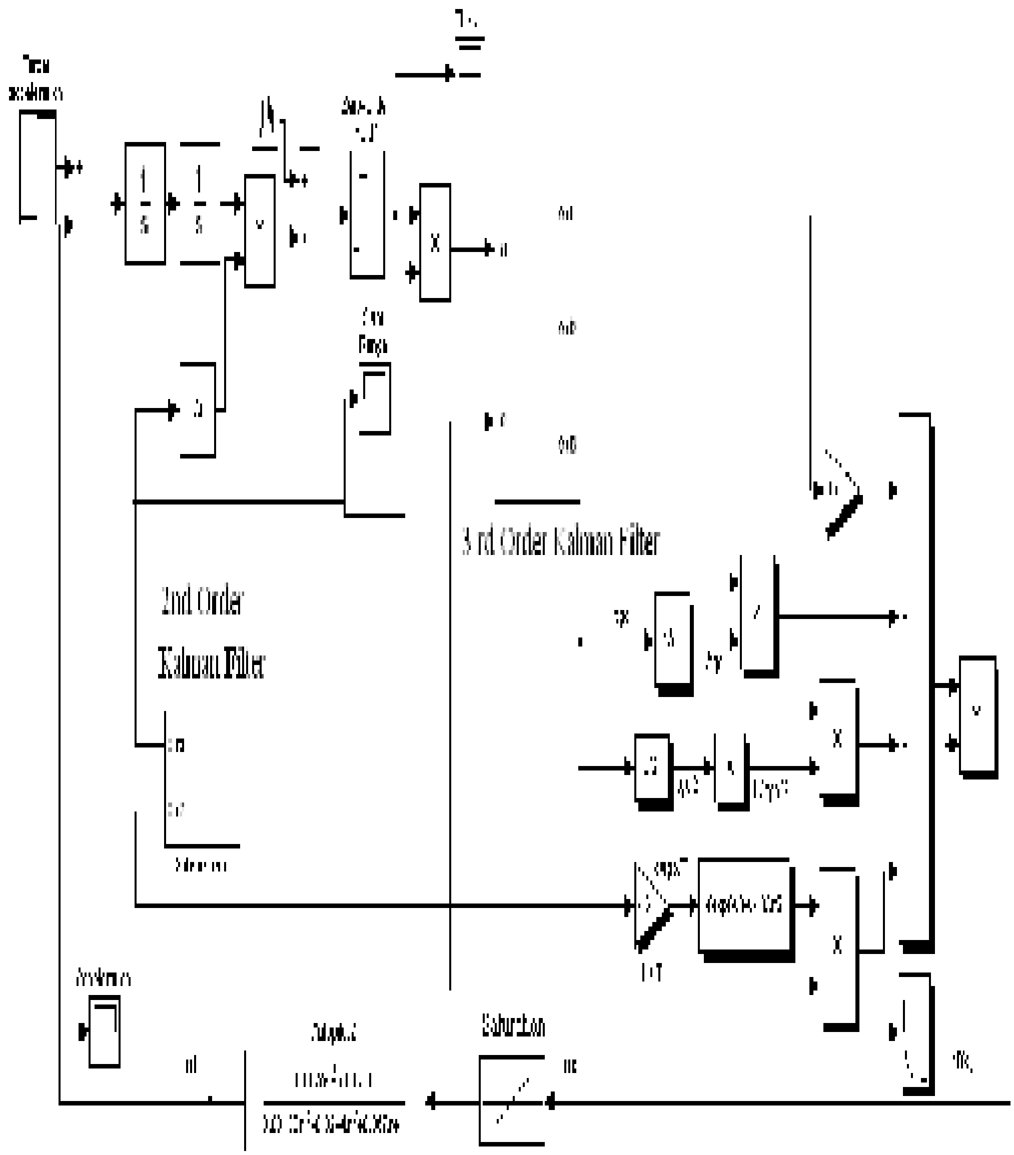
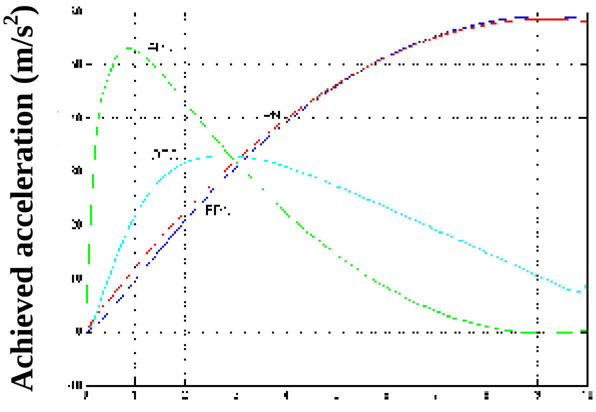


Figure (1) Simulink model of optimal navigation system



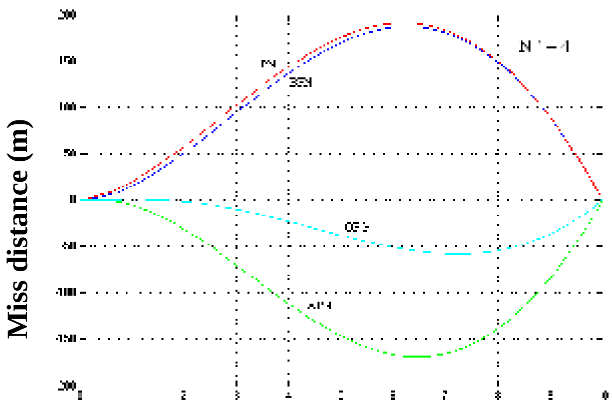


Figure (2) Miss distance of Navigation Systems.

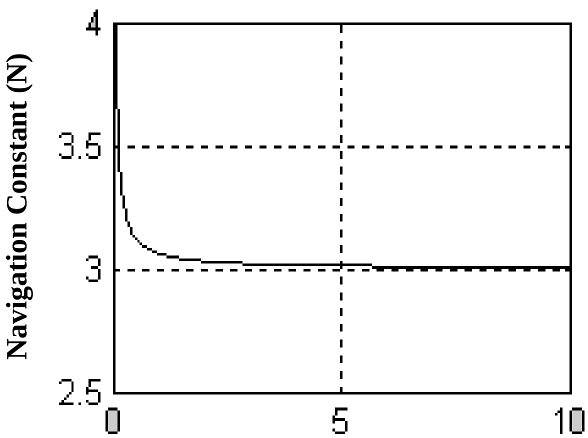
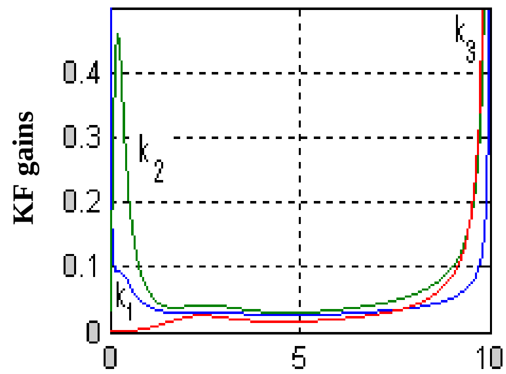


Figure (3) OPG Navigation Constant.



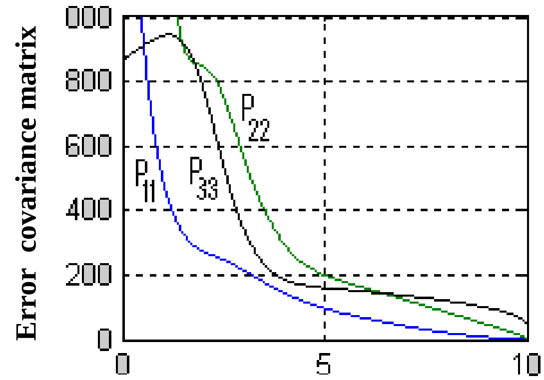


Figure (4) Ch/s of 3rd order KF.

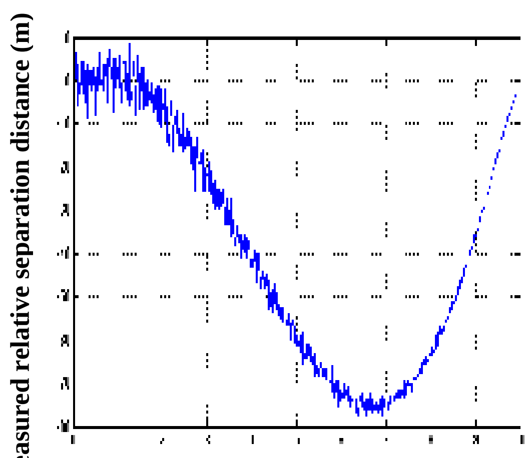


Figure (5) Measured Relative Separation Distance.

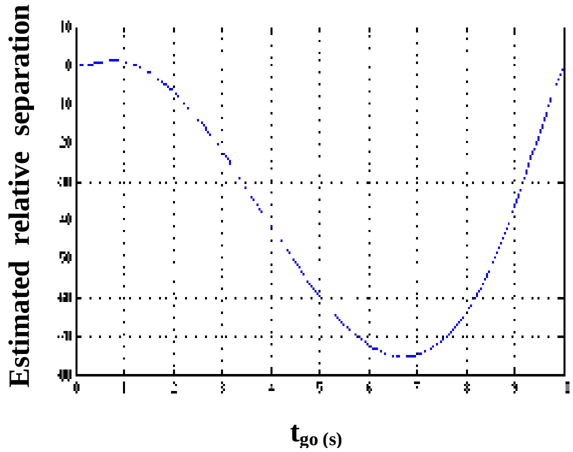
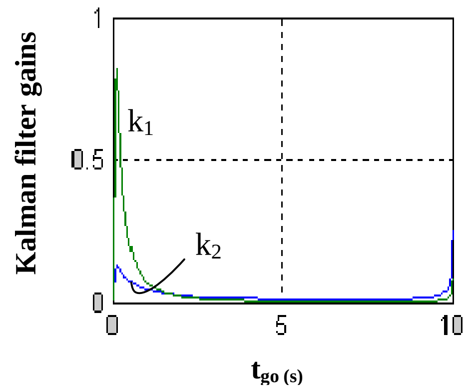


Figure (6) Estimated Relative Separation Distance.



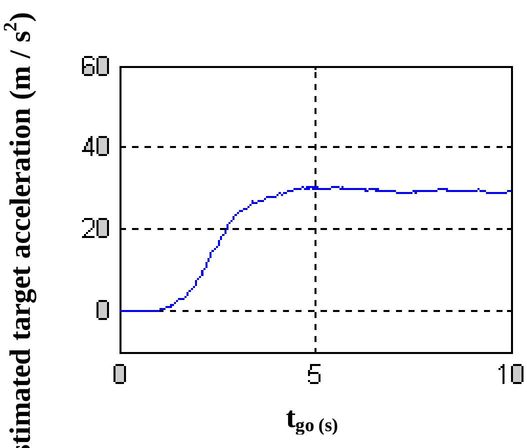


Figure (7) Estimated target Acceleration.

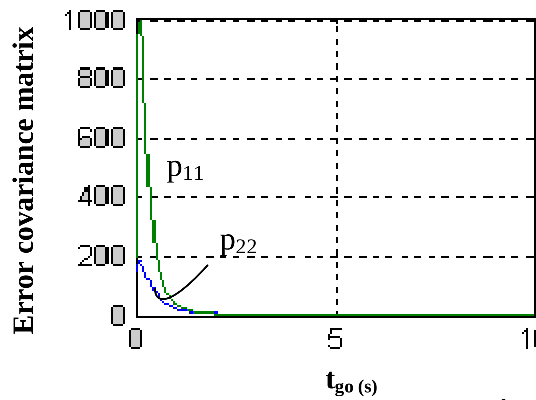


Figure (9) characteristics of 2nd order kalman filter.

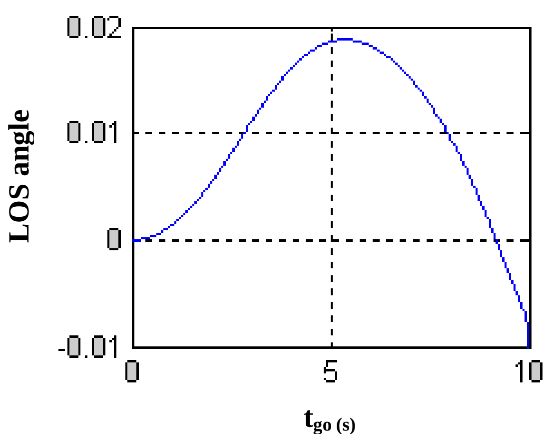


Figure (8) Estimated LOS angle.

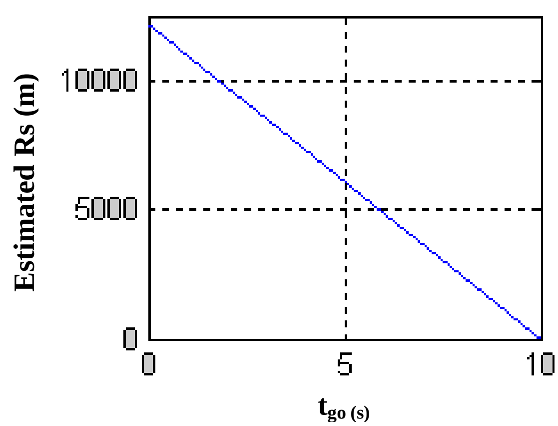


Figure (10) Estimated Slant Range.

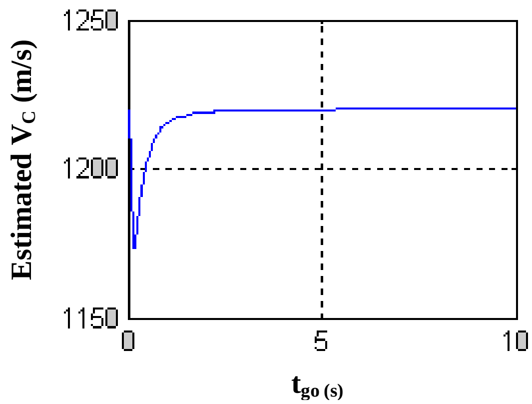


Figure (11) Estimated Closing Velocity.

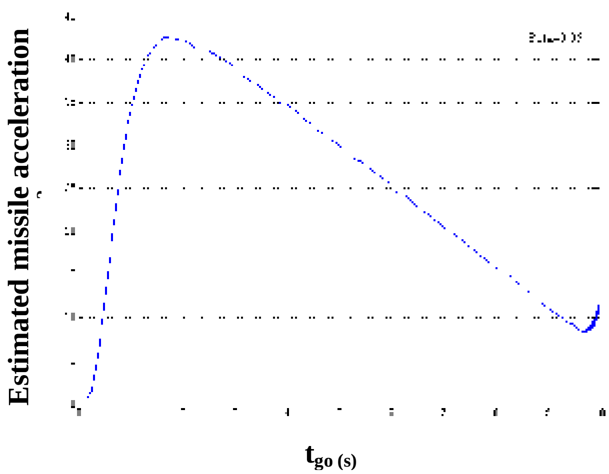


Figure (12) FMF Estimation of Achieved acceleration.