An Attitude Navigation System Based on the GPS

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Abstract

In this paper, the use of multi-GPS receiver to estimate the parameters of attitude (orientation) of a platform is developed. The GPS receiver has two measurements; pseudorange and carrier phase. The latter is highly accurate (sub centimeter-level).Therefore, it is used to give precise attitude parameters. But the carrier phase has one problem; an initial integer ambiguity must be resolved first. Without resolution of this integer, the carrier phase is meaningless. Therefore, the attitude determination technique based on the carrier phase observable of the GPS involves two steps; integer ambiguity resolution and attitude estimation. Here, two methods are used for attitude estimation; first, Single-point method that is based on the least square approach is developed using the quaternion representation. Second, Eigenproblem algorithm that is used to minimize a quartic quaternion-based cost function. In order to resolve the integer ambiguity, an attitude-independent algorithm is developed. This algorithm first incorporates an instantaneous integer search to significantly reduce the search space using a geometric inequality. Then, a batch-type loss function is used to check the remaining integers in order to determine the optimal integer. The results show that the Single-point method is more accurate (with RMS 0.137, 0.079 and 0.197 degree in yaw, pitch and roll respectively), and it convergences exponentially to the correct solution. The Eigenproblem may diverge when the initial quaternion is far.

نظام الملاحة الاتجاهي باستخدام منظومة تحديد الموقع العالمي (GPS)

الخلاصة

في هذا البحث ، تم استخدام عِدة مستلمات الـــ (GPS) لتطوير نظام تحديد الاتجاه. ان مستلمة الـــ (GPS) تستخدم مقياسين، المدى الكاذب (pseudorange) وناقل الطور (carrier phase) .ان الاخير يملك دقة عالية لذا يُستخدم لإعطاء اتجاه دقيق .ولكن توجد مشــكلة فــي قياسات الـــ carrier phase وهي وجودعدد صحيح غامض يجب ان يحل اولاً. بدون معرفــة هذا العدد فان الـــ carrier phase يبقى بلا معنى. لذا فان تقنية تحديد الاتجاه المستندة على الـــ carrier phase تَتضمّن خطوتين ، حل غموض العدد الصحيح ومن ثم احتساب الاتجاه. هنا تم استخدام طريقتين لحساب الاتجاه , الاولى كانت النقطة الوحيدةSingle point التي تعتمد علـــى التربيعات الصغرى(Least square) والتي طُوّرت لتستخدم الـــ quaternion. امـــا الطريقــة الثانية فهي Eigenproblem والتي تقوم بايجاد الـــ quaternion المثالي مـــن دالـــة الكلفــة الرباعية. ولحل غموض العددالصحيح ،تم تطوير خوارزمية لا تعتمد على الاتجاه. تتضمن هذه الخوارزمية اولاً عملية بحث آنية ضمن الاعداد الصحيحة باستخدام متباينة هندسية لتقليل مجال البحث بشكل ملحوظ . بعد ذلك يتم استعمال دالة خسارة (loss function) لفحـــص الاعـــداد الصحيحة المرشحة من عملية البحث لكي يتم تحديد العدد الصحيح المثالي. اظهرت النتائج ان طريقة Single-point اكثر دقة (الجذر التربيعي لمتوسط الخطأ يســـاوي 0.137, 0.079 و

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0.197 درجة بالنسبة الى زوايا pitch,yaw, roll على التوالي) وتقترب تصاعديا من الحل الصحيح. في حين ان طريقة Eigenproblem ربما تبتعد عن الحل الصحيح عندما تكون نقطة البداية بعيدة.

Keywords: Global Positioning System (GPS), attitude determination, carrier phase double difference, integer ambiguity, least-square method, Eigenproblem method.

1-Introduction

With the advance of satellite navigation technology, the Global Positioning System (GPS) has become widely used as an accurate sensor in many navigation and location systems [1]. Although GPS was originally designed as a positioning and timing system, its usage in attitude determination for ship, aircraft and spacecraft has been heavily discussed in the past [2,3].With multiple closely-spaced GPS antennas, the GPS attitude system can precisely estimate the 3D attitude parameters of mobile platforms without error drift over time [1].

Although code measurements can be used in attitude determination [4], the carrier phase measurements are preferable in estimating attitude parameters because of their high accuracy. The use of carrier phase measurements results in the ambiguity problem of carrier phase integer cycles. This is because the GPS receiver can only measure a fraction of a cycle very accurately and the number of cycles after tracking the signal, while the initial number of complete cycles remains unknown, or ambiguous. Therefore, the attitude determination techniques based on GPS carrier phase measurements involve two steps, ambiguity resolution and attitude estimation. [1,2,5]

2-GPS Observables

Two main kinds of measurements are simultaneously available from the GPS signals. The pseudorange measurement is estimated from the coarse/acquisition (C/A) random noise code while the carrier phase measurement is derived from the carrier waves of L1(1575.42MHz) or L2(1227.60MHz) for more details see Ref. [6,7].

2.1 Pseudorange Measurement

The basic observation equation of the pseudorange measurement can be expressed as: [8]

$$p_{\alpha}^{j} = \rho_{\alpha}^{j} + d\rho^{j} + d_{ion} + d_{trop} + c(dT_{\alpha} - dt^{j}) + \varepsilon(p_{mult}) + \varepsilon(p_{nois}) \quad .....(1)$$

where

p_{α}^{j}	is the pseudorange measurement from satellite *j* to receiver α (*m*) ,
ρ_{α}^{j}	is the geometric range from satellite j to receiver α (*m*),
$d\rho^{j}$	is the orbital error of satellite j (*m*),
d_{ion}	is the ionospheric delay (*m*),
d_{trop}	is the tropospheric delay (*m*),
dt^{j}	is the clock error of satellite j (*sec*),
dT_{α}	is the clock error of receiver α (*sec*),
$\varepsilon(p_{mult})$	denotes the pseudorange multipath errors (*m*),
$\varepsilon(p_{nois})$	represents the measurement error due to the receiver noise (*m*)and
c	is the speed of light (*m/sec*) .

The geometric range equation expressed in terms of Earth-fixed coordinates is:

$$\rho^j_\alpha = |r^j - r_\alpha| = [(x^j - x_\alpha)^2 + (y^j - y_\alpha)^2 + (z^j - z_\alpha)^2]^{1/2} \quad \ldots(2)$$

r^j is the satellite position vector referenced to the Earth-fixed frame computed using the broadcast ephemeris at epoch t_k (*m*),and

r_α is the position vector for antenna α referenced to the Earth-fixed frame at epoch t_k (*m*).

2.2 Carrier-phase Measurement

The equation of the carrier phase measurement can be written as [8]:

$$\lambda\Phi^j_\alpha = \rho^j_\alpha + d\rho^j + c(dT_\alpha - dt^j) + \lambda N^j_\alpha - d_{ion} + d_{trop} + \varepsilon(\Phi_{mult}) + \varepsilon(\Phi_{nois}) + \varepsilon(\Phi_{ant}) \quad (3)$$

where

Φ^j_α is the carrier phase measurement made from receiver α to satellite j at t_k (*cycles*).

N^j_α is the carrier phase ambiguity,

$\varepsilon(\Phi_{mult})$ is the error in the carrier phase measurement due to multipath (*m*),

$\varepsilon(\Phi_{nois})$ is the error in the carrier phase measurement due to receiver noise (*m*),

$\varepsilon(\Phi_{ant})$ is the antenna phase center variation (*m*), and

λ is the carrier wavelength (*m*) .

while the remaining terms in Eq.(3) are the same as those defined in Eq.(1).

3- Attitude Determination

Attitude determination with GPS and multiple antennas is based on the interferometric model that is shown in Figure (1). The signal wavefront can be considered planar when it arrives to the antennas because the distance between GPS antennas is much smaller as compared to the distance to GPS satellites. A signal transmitted from a distant GPS satellite arrives at the closest antenna slightly before reaching the other. By measuring the difference in carrier phase between the two antennas, it is possible to determine the relative distance **Δρ** between the antennas in the direction of the GPS satellite [1,9]. In this case we can define the relative distance as the projection of the baseline vector **b** onto the sightline **s** vector as [10]:

$$\Delta\rho = |b| \cos(\alpha) = b^T s \quad \ldots(4)$$

The components of the sightline vector are known in the reference frame ECEF and the baseline vector is given in the body frame, therefore Eq.(4) becomes as:

$$\Delta\rho = b^T A\, s \quad \ldots(5)$$

where the $A \in \Re^{3\times3}$ is the rotation matrix from the ECEF frame to body frame which represents the attitude parameters .

If Eq.(5) is substituted in the carrier phase single difference equation will become as :

$$\lambda\Delta\Phi = b^T As + \lambda\Delta N + c\Delta dT \quad \ldots(6)$$

In fact, the single difference technique is used to eliminate the satellite clock error and the residual ionospheric and the tropospheric refraction effects are negligible. In order to eliminate the receiver clock error, the carrier phase double difference technique will be used as follows:

for satellite i^{th} and receivers α and β , the single difference is :

$$\lambda\Delta\Phi^i_{\alpha\beta} = b_{\alpha\beta}As^i + \lambda\Delta N^i_{\alpha\beta} + c\Delta dT_{\alpha\beta} \,.(7)$$

and for satellite j^{th} and receiver α , β the single difference is :

$$\lambda\Delta\Phi^j_{\alpha\beta} = b_{\alpha\beta}As^j + \lambda\Delta N^j_{\alpha\beta} + c\Delta dT_{\alpha\beta} \,.(8)$$

then the carrier phase double difference measurement is :

$$\lambda\nabla\Delta\Phi^{ij}_{\alpha\beta} = \lambda\Delta\Phi^{i}_{\alpha\beta} - \lambda\Delta\Phi^{j}_{\alpha\beta}$$
$$= (b^{T}_{\alpha\beta}As^{i} + \lambda\Delta N^{i}_{\alpha\beta} + c\Delta dT_{\alpha\beta})$$
$$- (b^{j}_{\alpha\beta}As^{j} + \lambda\Delta N^{j}_{\alpha\beta} + c\Delta dT_{\alpha\beta}) \quad \ldots(9)$$

or

$$\lambda\nabla\Delta\Phi^{ij}_{\alpha\beta} = b^{T}_{\alpha\beta}As^{ij} + \lambda\nabla\Delta N^{ij}_{\alpha\beta} + \varepsilon \quad ..(10)$$

where

$$s^{ij} = s^{i} - s^{j} \quad ..(11)$$

If Eq.(11) is multiplied by $1/\lambda$, it can be rewritten as :

$$\nabla\Delta\Phi^{ij}_{\alpha\beta} = b^{T}_{\alpha\beta}As^{ij} + \nabla\Delta N^{ij}_{\alpha\beta} + \varepsilon \quad (12)$$

In this case the $b_{\alpha\beta}$ will be in wavelengths (cycles) units. The integer ambiguity double difference $\nabla\Delta N^{ij}_{\alpha\beta}$ must be determined at startup before any attitude determination can take place, and will be constant as well as there is no cycle slip (will be described later) [2,3]. In order to estimate the three attitude parameters (roll, pitch and yaw), the minimum number of antennas must be three (two baselines) [2,9].

An optimal attitude solution for a given set of range measurements, $\nabla\Delta\Phi^{ij}_{\alpha\beta}$, taken at a single epoch for k^{th} baseline and difference sight of lines s^{ij} (difference between a master s^{i} and a slave s^{j}) is obtained by minimizing the quadratic cost function [2]:

$$J(A) = \sum_{k}^{m}\sum_{j}^{n} w^{ij}_{k}(\nabla\Delta\Phi^{ij}_{k} - b^{T}_{k}As^{ij})^{2} \quad \ldots (13)$$

where m is the number of baselines, n is the number of satellites and w^{ij}_{k} is a weight assigned to each range measurement. The value of the weights w^{ij}_{k} should be an indication of the uncertainty in the measurements. If the measurement noise ε is zero-mean Gaussian white noise with covariance $(\sigma^{ij}_{k})^{2}$, then w^{ij}_{k} is given by $(\sigma^{ij}_{k})^{-2}$.

3.1 Single-point Algorithm

The Single-point algorithm uses nonlinear least squares technique to find the correction to a priori estimate of the attitude. Firstly, Eq.(12) must be linearized as:

$$\nabla\Delta\Phi^{ij}_{k}(\hat{x}) \approx \nabla\Delta\Phi^{ij}_{k}(x_{c}) + H^{ij}_{k}\delta x \quad \ldots(14)$$

where

$\delta x = \delta q_{13}$ (the first three components of quaternion), $x_c = q_c$ (nominal state) and the Jacobian matrix H^{ij}_{k} is given by [11] :

$$H^{ij}_{k} = \frac{\partial}{\partial q}[b^{T}_{k}A(q)s^{ij}]\Big|_{q_c} = -2(A(q_c)s^{ij})^{T}[b^{\times}_{k}] \quad .(15)$$

where

$$[b^{\times}] = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} \quad \ldots.(16)$$

Then, the measurement residual can be written as:

$$e = \nabla\Delta\Phi^{ij}_{k} - \nabla\Delta\Phi^{ij}_{k}(\hat{x})$$
$$\approx \nabla\Delta\Phi^{ij}_{k} - \nabla\Delta\Phi^{ij}_{k}(x_c) - H^{ij}_{k}\delta x$$
$$= \delta y^{ij}_{k} - H^{ij}_{k}\delta x \quad \ldots.(17)$$

For all the available measurements, the overall Jacobian matrix is:

$$H = \begin{bmatrix} . \\ . \\ -2(A(q_c)s^{ij})^{T}[b^{\times}_{k}] \\ . \\ . \end{bmatrix}_{\text{for all k,ij}} \quad \ldots(18)$$

and the overall measurements residual is:

$$e_r = dy - Hdx \quad \ldots(19)$$

where

$$\delta y = \begin{bmatrix} . \\ . \\ \nabla\Delta\Phi_k^{ij} - \nabla\Delta\Phi_k^{ij}(x_c) \\ . \\ . \end{bmatrix}_{\text{for all k,ij}} \quad \ldots(20)$$

Then we can estimate the optimal correction of the first three components of quaternion (δq_{13}) by using:

$$\delta x = (H^T W H)^{-1} H^T W \delta y \quad ..(21)$$

Then the full correction quaternion is obtained by forcing the fourth component to 1 and then normalizing to ensure a length of 1 as:

$$\delta q = \frac{1}{\sqrt{\delta q_1^2 \quad \delta q_2^2 \quad \delta q_3^2 \quad 1}} \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ 1 \end{bmatrix} \quad ..(22)$$

This is done to keep the small angle assumption made in the linearization of Eq.(12). If the fourth component is calculated directly from the three other ($dq_4 = \sqrt{1 - dq_1^2 - dq_2^2 - dq_3^2}$) then for large values of $\delta q_1 \ldots \delta q_3$, the fourth component becomes imaginary and the algorithm fails. Therefore, by use of Eq.(22) the robustness is increased. Finally, the a-priori quaternion estimate (old) can be updated using the standard quaternion multiplication as: [7]

$$q_{new} = dq \otimes q_{old}$$

$$= \begin{bmatrix} \delta q_4 & -\delta q_3 & \delta q_2 & \delta q_1 \\ \delta q_3 & \delta q_4 & -\delta q_1 & \delta q_2 \\ \delta q_2 & \delta q_1 & \delta q_4 & \delta q_3 \\ -\delta q_1 & -\delta q_2 & -\delta q_3 & \delta q_4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_{old} \quad \ldots(23)$$

3.2 Using Quartic Cost Function

If the attitude matrix **A** is a known function of quaternion **q**, the J(A) can be replaced by J(q), where [12] :

$$J(q) = \sum_k^m \sum_j^n w_k^{ij} (\nabla\Delta\Phi_k^{ij} - b_k^T A(q) s^{ij})^2 \quad ..(24)$$

In order to facilitate the search for the quaternion q* that minimizes J(q) , the latter is now converted into an explicit function of q . To meet this end, define:

$$L_k^{ij} = \begin{bmatrix} E_k^{ij} - \mu_k^{ij} I_3 & p_k^{ij} \\ p_k^{ijT} & \mu_k^{ij} \end{bmatrix} \quad ..(25)$$

where

$$C_k^{ij} = s^{ij} b_k^T \quad ..(26)$$

$$E_k^{ij} = C_k^{ij} + C_k^{ijT} \quad ..(27)$$

$$p_k^{ij} = b_k \times s^{ij} \quad ..(28)$$

$$\mu_k^{ij} = b_k^T s^{ij} \quad ..(29)$$

and I_3 is identity matrix 3×3

It can be shown that: [13]

$$b_k^T A(q) s^{ij} = q^T L_k^{ij} q \quad ..(30)$$

Substitution of Eq.(30) into Eq.(24) yields:

$$J(q) = \sum_k^m \sum_j^n w_k^{ij} (\nabla\Delta\Phi_k^{ij} - q^T L_k^{ij} q)^2 \quad (31)$$

Because $\nabla\Delta\Phi_k^{ij}$ is a scalar and $q^T q=1$, assuming:

$$B_k^{ij} = \nabla\Delta\Phi_k^{ij} I_4 \quad ..(32)$$

$$\nabla\Delta\Phi_k^{ij} = q^T B_k^{ij} q \quad ..(33)$$

where $B_k^{ij} \in \Re^{4\times4}$

Therefore,

$$\nabla\Delta\Phi_k^{ij} - q^T L_k^{ij} q = q^T B_k^{ij} q - q^T L_k^{ij} q$$

$$= q^T (B_k^{ij} - L_k^{ij}) q \quad ..(34)$$

Using the definition:

$$M_k^{ij} = (w_k^{ij})^{\frac{1}{2}} (B_k^{ij} - L_k^{ij}) \quad ..(35)$$

where M_k^{ij} is a 4×4 symmetric matrix.

Then Eq.(24) can be written as :

$$J(q) = \sum_k^m \sum_j^n (q^T M_k^{ij} q)^2 \quad ..(36)$$

or

$$J(q) = q^T \left[\sum_k^m \sum_j^n M_k^{ij} q q^T M_k^{ij} \right] q \quad ..(37)$$

Obviously, the problem of finding the matrix A^* that minimizes J(A) defined in Eq.(13), has been transformed into finding q^* that minimizes J(q) of either Eq.(36) or Eq.(37) which are explicit of q . Unfortunately, J(q) is quartic in q,therefore an iterative algorithm can be used to minimize the quartic cost function J(q) .

3.2.1 Minimization of Quartic Cost Function

Define C(q) as follows:

$$C(q)=\sum_{k}^{m}\sum_{j}^{n}M_k^{ij}qq^TM_k^{ij} \quad .....(38)$$

Then, the quartic cost function of Eq.(37) can be written as:

$$J(q)=q^TC(q)q \quad ......(39)$$

We wish to minimize **J** with respect to **q** where the latter has to satisfy the normality constraint :

$$q^Tq=1 \quad ....(40)$$

To accomplish this, define the following Lagrange function:

$$L(q,\lambda) = J(q) + \lambda h(q) \quad ..(41)$$

where

$$h(q) = 2(1-q^Tq) \quad .... (42)$$

The necessary condition for $\mathbf{q}^*$ to be a local minimum is as follows [14]:

$$L_q=\frac{\partial L}{\partial q}=0$$

(43a)

$$L_\lambda=\frac{\partial L}{\partial \lambda}=0 \quad .....(43b)$$

When performing the differentiation and after some elaborate manipulations one obtains the following equations:

$$L_q=4C(q)q-4q\lambda=0 \Leftrightarrow C(q)q\lambda q \quad ..(44)$$

or

$$\lambda=q^TC(q)q \quad ..(45)$$

and

$$L_\lambda = 2(1-q^Tq) = 0 \Leftrightarrow q^Tq = 1 \quad ..(46)$$

where $q\in\Re^4$, $\lambda\in\Re$

Observe that:

$$\lambda = q^T C(q)\, q = J(q) \geq 0 \quad ..(47)$$

and, therefore :

$$\min_q J(q)=\lambda_{min}\geq 0 \quad ...... (48)$$

This observation is used later to initialize the minimization algorithms near the minimum point of the quartic cost function by selecting $\lambda_0 = 0$. Also observe that C(q) should be nonnegative definite to guarantee $\lambda \geq 0$.

3.2.2 Minimization via the Eigenproblem Algorithm [13]

Eq.(44) can be rewritten as:

$$(C(q^*) - \lambda^* I)q^* = 0 \quad ..(49)$$

Because the $q^* \neq 0$, therefore the homogeneous system above has nontrivial solution if and only if:

$$\det(C(q^*) - \lambda^* I) = 0 \quad ..(50)$$

where det(.) is the determinant of a matrix .

In fact , the solution of Eq.(50) , which is known as the characteristic equation of the matrix $C(q^*)$, gives four values of λ which are called eigenvalues, and the corresponding nontrivial solution vectors q's of the system Eq.(49) are called eigenvectors.

Recalling Eq.(48)

$$\min_q J(q)=\lambda_{min}\geq 0 \quad .... (51)$$

therefore, the smallest eigenvalue of C(q) will represent the minimum value of the cost function J(q) , and its corresponding eigenvector q will minimize J(q) .

From this idea, a solution that immediately comes to mind is the following. Guess an initial q and use it in Eq.(38) to compute **C(q)**. Then, find the eigenvalues and eigenvectors of **C(q)** and we select the eigenvector which is corresponding to smallest eigenvalue .Then , this eigenvector will be used as **q** for next iteration .

The eigenvalues associated with the equation $\mathbf{C(q)q = \lambda q}$ should be real and nonnegative since ¸ $\boldsymbol{\lambda = q^TC(q)q = J(q) \geq 0}$. Therefore, a necessary condition for the convergence of the above algorithm is that the matrix C(q) must be symmetric and nonnegative definite. Such a matrix has real nonnegative eigenvalues. The matrix C(q) is indeed symmetric because the matrices M^{ij} and qq^T that compose C(q) are symmetric. In addition the matrix C(q) is positive definite.

It was found that the convergence of the preceding algorithm is very slow and near the end it alternates between two values, none of which is the correct solution. It was observed, however, that the two values are almost symmetric about the correct solution. Therefore, the algorithm is modified in the following way. The solutions obtained from two successive iterations are averaged. The average solution is then fed into the iterative algorithm, which is run twice again. The results of these two iterations are averaged again and so on.

4- Integer Ambiguity Resolution

Since the receivers only measure the fractional part of the carrier phase, the range difference is ambiguous until the integer ambiguity resolution is solved. In this paper the attitude-independent algorithm refers to the method described by Crassidis [15] is presented. It is based on geometric inequality constraints to reduce the integer space search and on a cost-function to be minimized in order to determine the optimal solution. The algorithm can use three sightlines and then consider one baseline at a time (when three non-coplanar sightlines are available), or can use three baselines and then consider one sightline at a time (when three non-coplanar baselines are available). Since in this paper it is assumed that the baseline array is coplanar, due to the experiment configuration, a set of non-coplanar sightlines must be available.

4.1Geometric Constraint [15]

The first step involves reducing the integer search space by using a subset of only two sightlines simultaneously, instead of three. The reduced subset consists of the integers that pass the following geometric inequality for k^{th} baseline and differenced sightlines s^{12} and s^{13} :

$$[b_k \cdot (As^{12} \times As^{13})]^2 > 0 \quad \ldots (52)$$

This means that the $b_{k,}$ As^{12}, and As^{13} must not lie in the same plane. This constraint can be rewritten as attitude-independent as:

$$\|b_k\|^2 \|s^{12}\|^2 \|s^{13}\|^2 - \|b_k\|^2 (s^{12} \cdot s^{13})^2 - \|s^{12}\|^2 (\nabla\Delta\Phi_k^{13} - N_k^{13})^2 - \|s^{13}\|^2 (\nabla\Delta\Phi_k^{12} - N_k^{12}) + 2(\nabla\Delta\Phi_k^{12} - N_k^{12})(\nabla\Delta\Phi_k^{13} - N_k^{13})(s^{12} \cdot s^{13}) > 0 \quad ..(53)$$

Although double differences may increase the search space size twice as much as using single differences, a significant reduction of the search space size is achieved by using the constraint in Eq.(53). For example, with three sightlines (assuming that k denotes all possible integers associated with each baseline, where k is twice the number of the wavelength contained in the baseline) the search space required to determine the integers is on the order of $(2k)^3$; however, with the reduced subset using Eq.(53), the search space is now on the order of $3(2k)^2$. Furthermore, by taking the common integers corresponding to each baseline, found between any two sightline pairs, the search space becomes much smaller.

4.2 Cost Function Minimization [15]

Next, the following negative-log-likelihood function is used :

$$J(n_k)=\frac{1}{2}\sum_{p=1}^{L}\left\{\sigma_k^{-2}(p)\left[\left\|S_k^{-1}(p)\Gamma_k(p)(\Phi_k(p)-n_k)\right\|^2 -\left\|b_k\right\|^2+\mathrm{trace}(S_k^{-1}(p))\right]^2+\log\sigma_k^2(p)\right\} \quad (54)$$

where

$$n_k=\left[N_k^{12}\quad N_k^{13}\quad N_k^{14}\right]^T \quad ..(55)$$

$$\Gamma_k=\left[w_k^{12}s^{12}\quad w_k^{13}s^{13}\quad w_k^{14}s^{14}\right] \quad ..(56)$$

$$\Phi_k=\left[\nabla\Delta\Phi_k^{12}\quad \nabla\Delta\Phi_k^{13}\quad \nabla\Delta\Phi_k^{14}\right]^T \quad ..(57)$$

$$S_k=w_k^{12}s^{12}(s^{12})^T+w_k^{13}s^{13}(s^{13})^T+w_k^{14}s^{14}(s^{14})^T \quad ...(58)$$

$$\sigma_k^2(p)=4(\Phi_k(p)-n_k)^T\Gamma_k^T(p)S_k^{-3}(p)\Gamma_k(p)(\Phi_k(p)-n_k)-\mathrm{trace}^2(S_k^{-1}(p)) \quad (59)$$

The symbol ***p*** denotes the variable at time t_p. Eq.(54) can be used to determine the double differenced integer ambiguities instantaneously (when L=1) or using a small amount of data (i.e. L > 1), by checking the remaining integers that pass the inequality condition in Eq.(53) .The integer set that minimizes Eq.(54) is chosen as the final solution.

5-Simulation Results

In this section, the results will be showed as obtained from applying the algorithm that shown in Figure(2). A real data recoded from an experiment of GPS attitude system which had been made by the **University of Calgary** on 29/06/2005, is used [16]. This data consists of five files; four for the data observations of the four GPS receivers (**Antenna1.obs**, **Antenna2.obs**, **Antenna3.obs** and **Antenna4.obs** files), and one for the ephemeris (**Antenna1.eph** file) that corresponds to master antenna (Antenna1).The configuration of the antenna array is shown in Figure(3). The recorded data is taken from a static case and the actual yaw, pitch and roll are (-113.9, -0.5 and -141) degree respectively. Some of the statistic analysis will be evaluated. Also the effects of the wrong integer ambiguities will be shown.

During the period of the data recorded (30 minutes), the satellites (3, 15, 18, 19, 21, and 22) are observed by the all receivers. Moreover, during this period (319099-320898 seconds in GPS time), (1800) epochs will be sampled. For each epoch, complete information for each satellite will be extracted (ephemeris data, GPS time, pseudorange, L1 carrier phase, and Doppler).

In this work, the baselines are coplanar therefore three sightlines non-coplanar case will be considered for the resolution integer ambiguity stage. Four of the observed satellites (15, 19, 21 and 22) will be used only in the calculation in order to perform the required three double differences equations to resolve the integer ambiguities (three integers for each baseline).

At first, we must resolve the integer ambiguities using the resolution integer ambiguity technique that has been described. Firstly, when the geometric constraint (Eq.(53)) is used, the search space for each baseline will be reduced as shown in Table (1). It is clear that a significant reduction in the integer sets is achieved by using this constraint.

After that, these integer sets that pass the geometric constraint will be as inputs to the cost function in Eq.(54).The integer sets corresponding to the minimum cost will be selected as the correct integers. The integer ambiguities are resolved for each baseline as shown in Table (2). These integers will be fixed during the period of simulation because there are no cycle slips.

Figure (4) and Table (3) show the comparison between the algorithms of attitude estimation (Single-point and Eigen-problem) using three baselines. In Table (3), the root mean square (RMS) is for the error between the estimated and the actual attitude parameters. Note that the Single-point algorithm converges exponentially to the correct solution and it is faster than others.

Obviously, the standard deviation (STD) and the accuracy of Single-point method for all attitude parameters (roll, pitch and yaw) are smaller than those of the other method. Actually, the main difference between the two algorithms is in the convergence to minimum point from any initial quaternion. Figure (5) shows the performance of the two methods. This example is taken from the data at epoch one when they are used to estimate the roll angle. In this test a large initial quaternion is chosen to demonstrate that there are cases where the Eigen problem algorithm converges to the wrong extreme point whereas the Single-point converges to the correct solution. Figure (6) shows another performance using initial condition which is different from that used in Figure (5). From this comparison, we note that the Single-point algorithm converges exponentially to the correct solution and it is faster than other.

In Figure (7), the residual error between the measured and estimated carrier phase double difference of Single-point test is shown. Since multipath errors still exit in the measurements, oscillations are shown in the residual errors. However, the residual errors are below 2-sigma (i.e. 2×0.0368). The assumed standard deviation for carrier phase double differenced is ($\sqrt{2}\times.026$).

To show the effect of the incorrect integer ambiguity on the attitude estimation, the integer of **SV15-SV19** corresponding to **baseline1** will be shifted by (+1) cycles. Figure (8) shows this effect on yaw, pitch and roll estimation. Obviously, there is a large error in roll angle, this is because the baseline1 represents roll axis.

6-Conclusions

Actually, the carrier phase measurement represents part of the range between the GPS receiver and viewed satellite in GPS signal wavelength because the integer ambiguity (the rest part of the range), which is unknown parameter, must be resolved firstly. Without resolving the integer ambiguity, the carrier phase measurements remain meaningless. Therefore, the integer ambiguity resolution is the key for the attitude estimation.

The results show that the accuracy of Single-point is better than other (i.e the RMS of yaw, pitch and roll respectively are 0.137, 0.079 and 0.197 degree for single-point and 0.146 , 0.117 and 0.247 degree for Eigenproblem) and is faster. It was shown that when the initial quaternion is far from the correct solution then Eigenproblem algorithm may converge to the wrong extreme point because the search direction may jump and miss the minimum.

The obtained standard deviation values in this paper are better than that obtained in Ref. [10] (in this paper the STD of yaw, pitch and roll are 0.029, 0.028 and 0.172 degree compared with 0.412, 0.394 and 0.378 degree in [10]).

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Table (1) Useful of the Geometric Constraint.

Baselines	Integer range (cycles)	Integer sets without constraint	Integer sets with constraint	Reduction %
1	-28:28	185193	23427	87.3
2	-102:102	8615125	1154150	86.6
3	-104:104	9129329	1433304	84.3

Table (2) The Resolved Integer Ambiguities for the Three Baselines.

Baselines	Integers		
	N^{15_19}	N^{15_21}	N^{15_22}
Baseline1	9	-1	3
Baseline2	4	1	6
Baseline3	6	-1	7

Table (3a) The Statistics of Yaw Estimation (degree).

Method	Min	Max	Mean	STD	RMS
Single-point	-113.8	-113.7	-113.8	0.029	0.137
Eigen problem	-113.9	-113.7	-113.8	0.036	0.146

Table (3b) The Statistics of Pitch Estimation (degree).

Method	Min	Max	Mean	STD	RMS
Single-point	-0.536	-0.330	-0.426	0.0281	0.079
Eigen problem	-0.557	-0.014	-0.403	0.061	0.117

Table (3c) The Statistics of Roll Estimation (degree).

Method	Min	Max	Mean	STD	RMS
Single-point	-141.6	-140.5	-141.1	0.172	0.197
Eigen problem	-141.4	-140.2	-140.9	0.225	0.247

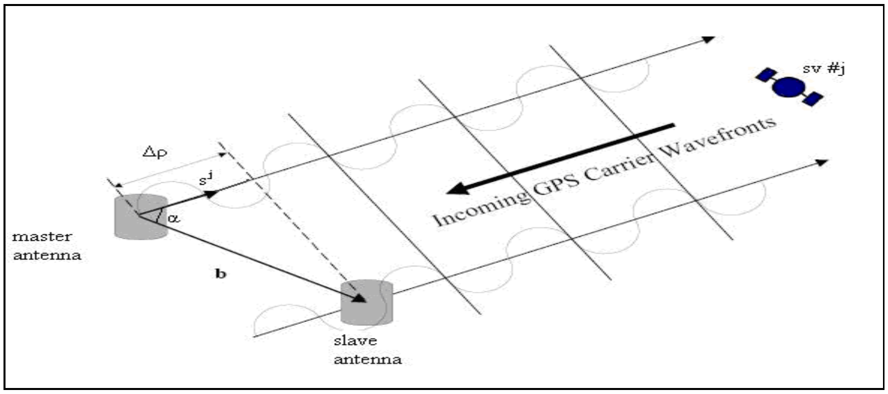

Figure (1) Interferometric Model.

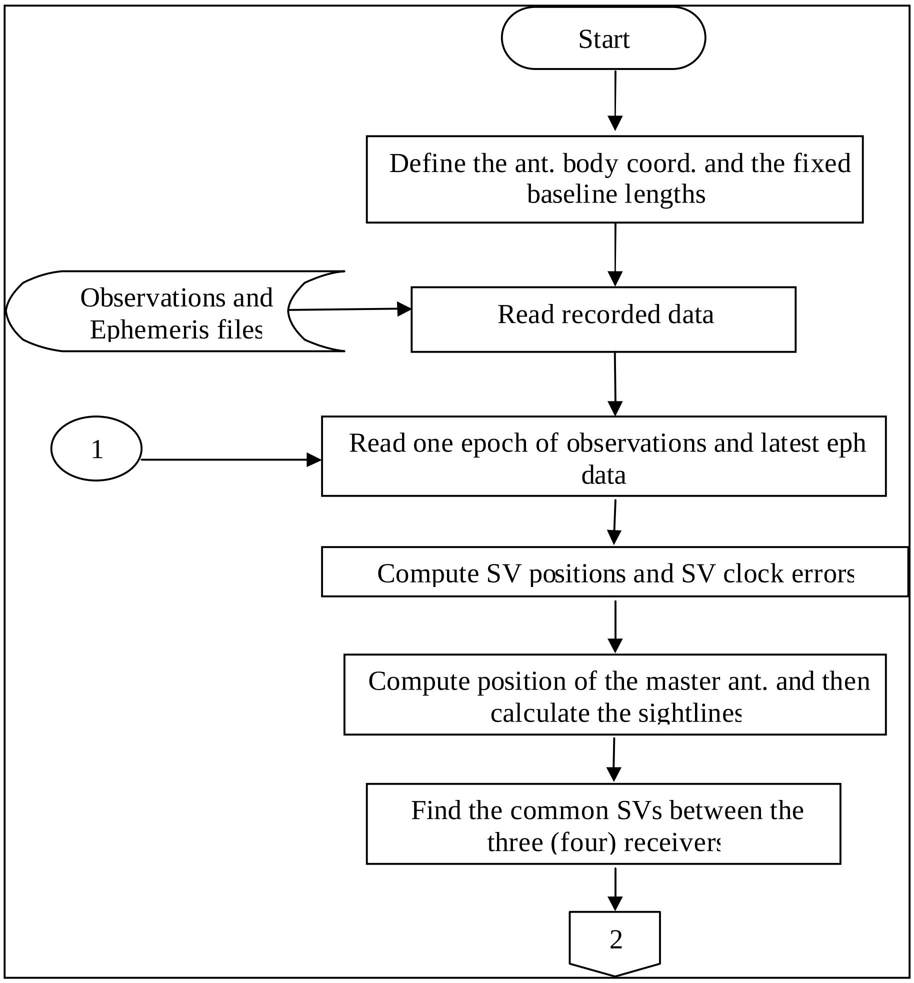

Figure (2) The Flowchart of the Attitude Determination system

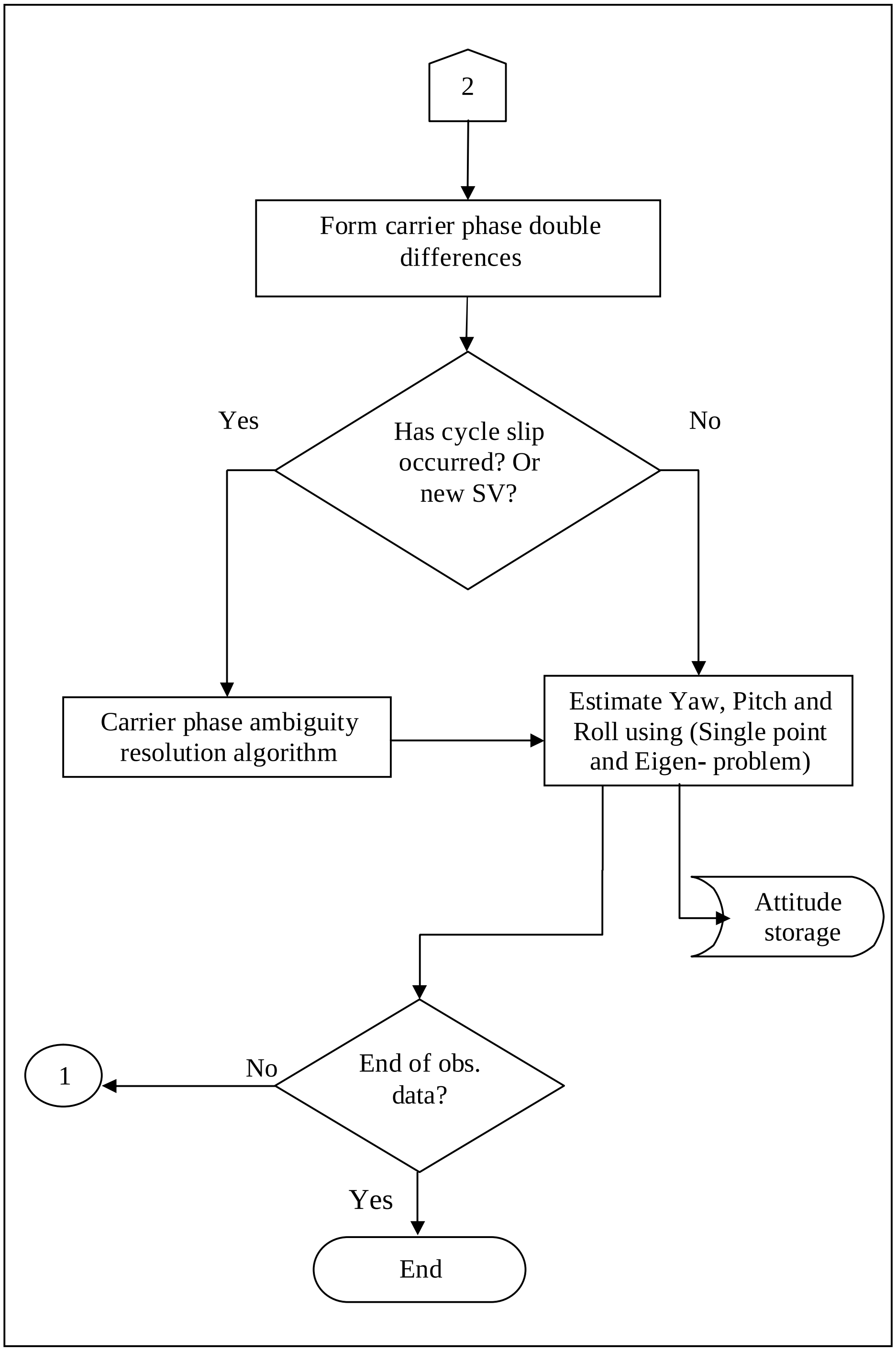

Figure (2) The Flowchart of the Attitude Determination system (continued).

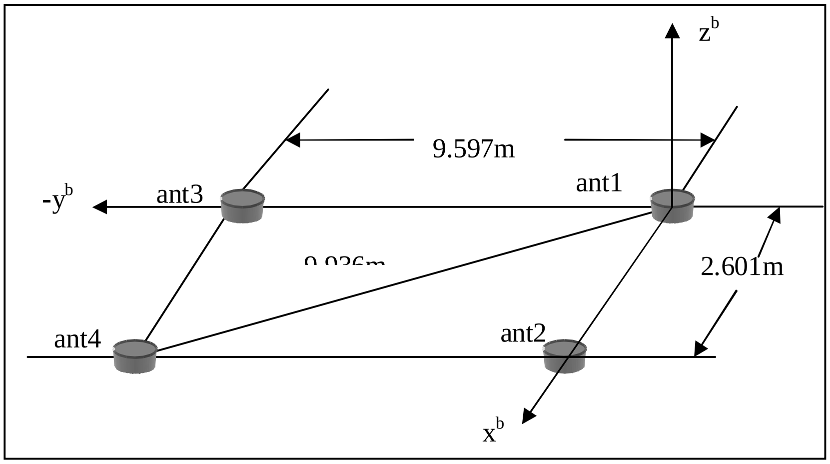

Figure (3) Antenna Body Coordinates

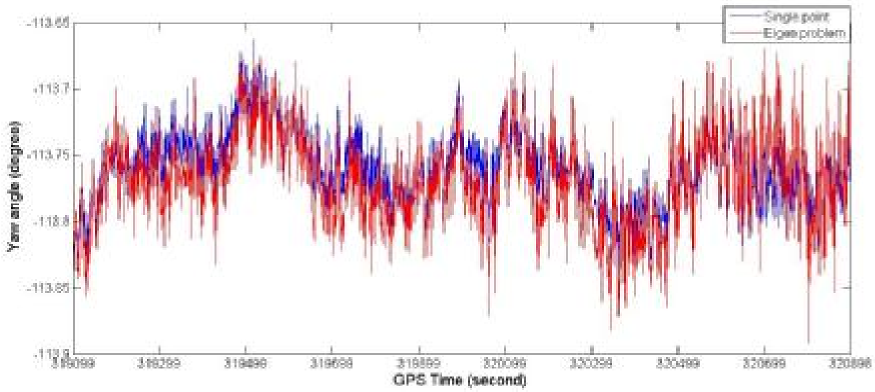

Figure (4a) Yaw Estimation

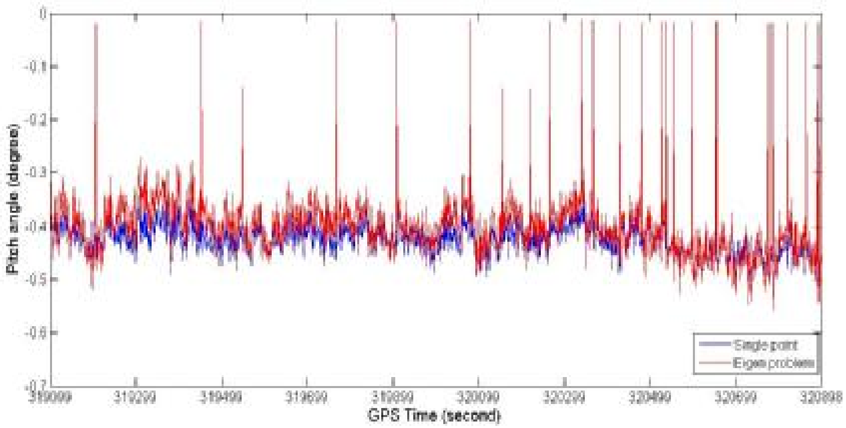

Figure (4b) Pitch Estimation

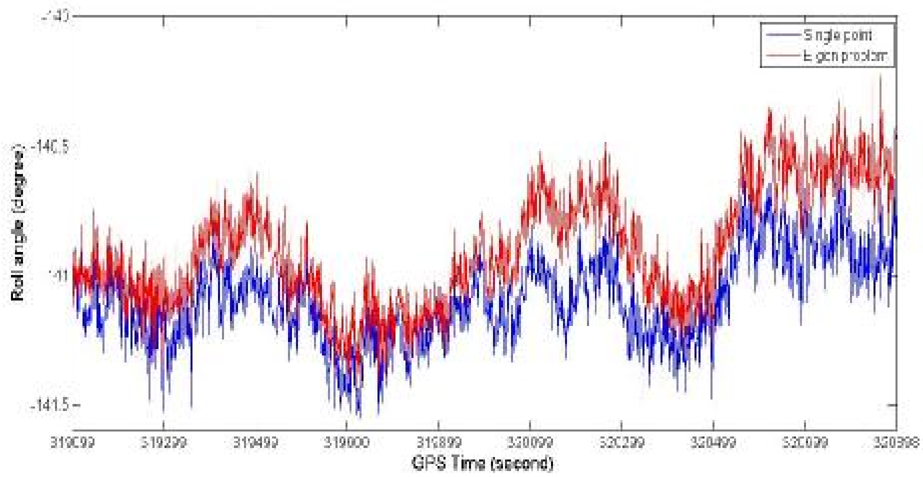

Figure (4c) Roll Estimation

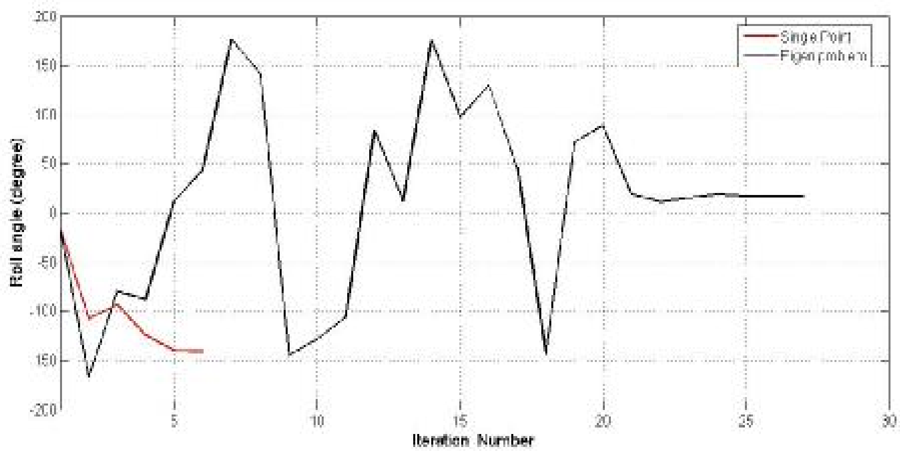

Figure (5) Performance Comparison to Estimate the Roll Angle.

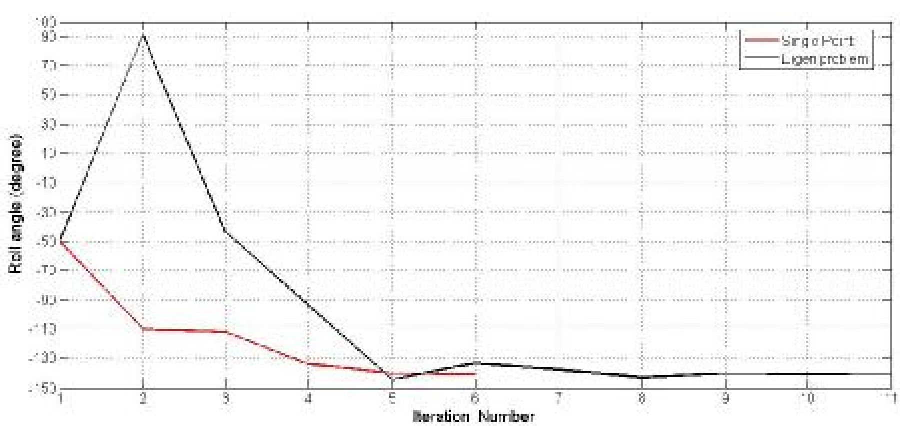

Figure (6) The Performance Using Different Initial Quaternion

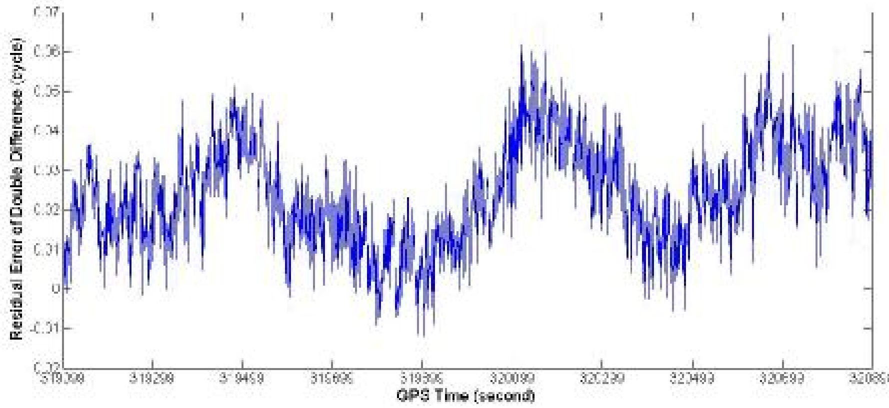

Figure (7a) The Residual Error of $\Delta\nabla\tilde{\Phi}_1^{15-19}$

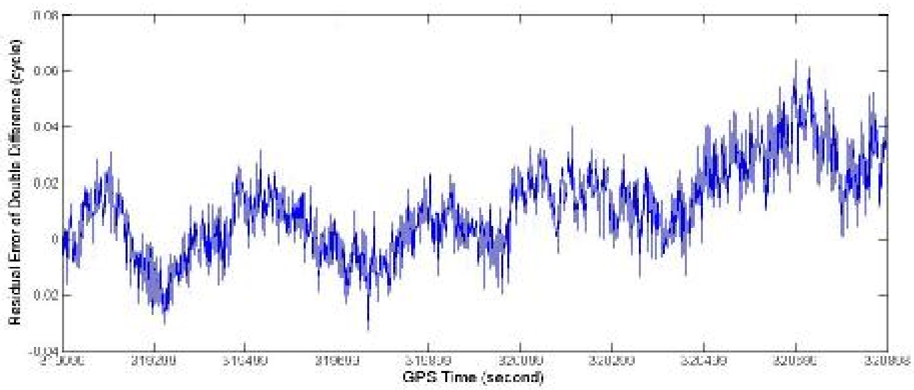

Figure (7b) The Residual Error of $\Delta\nabla\tilde{\Phi}_1^{15-21}$

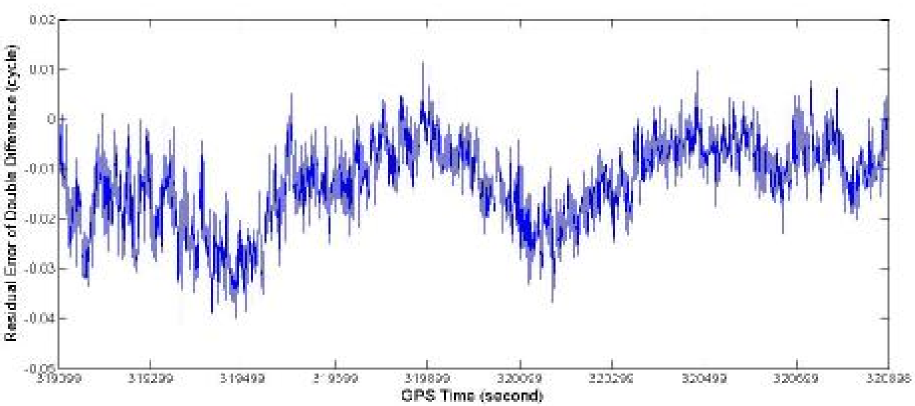

Figure (7c) The Residual Error of $\Delta\nabla\tilde{\Phi}_1^{15-22}$

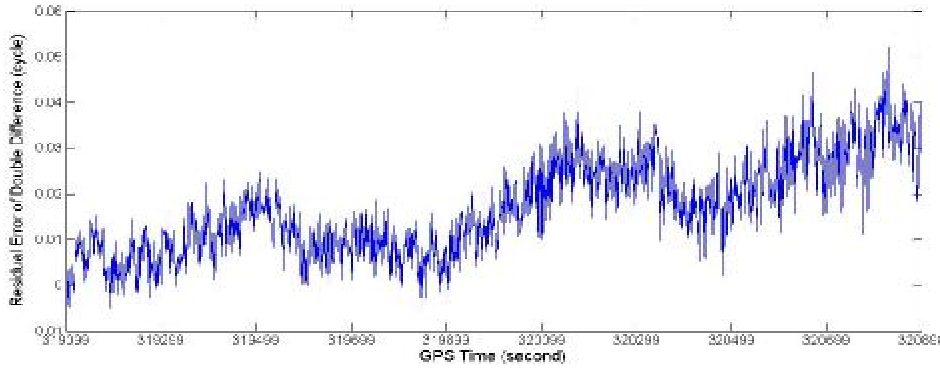

Figure (7d) The Residual Error of $\Delta\nabla\tilde{\Phi}_2^{15-19}$

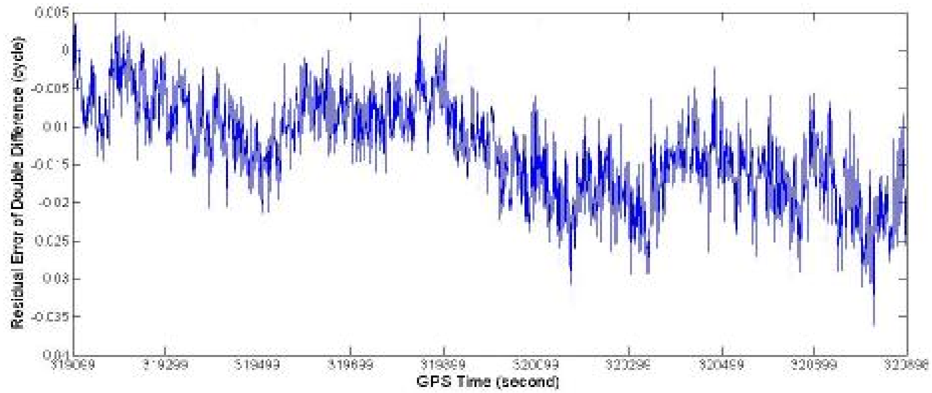

Figure (7e): The Residual Error of $\Delta\nabla\tilde{\Phi}_2^{15-21}$

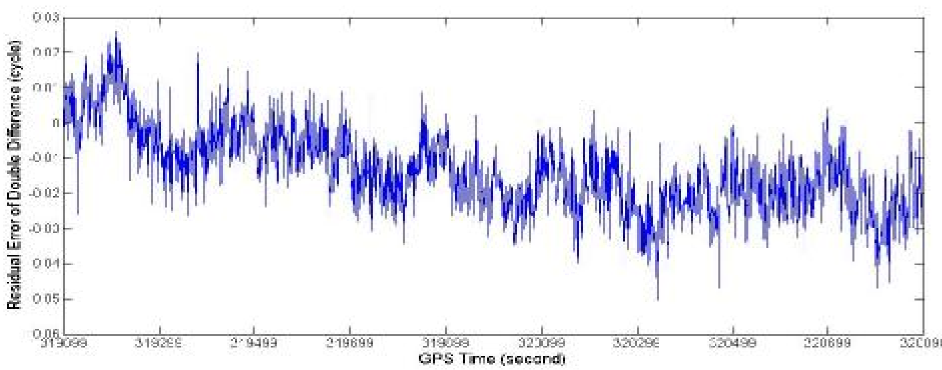

Figure (7f) The Residual Error of $\Delta\nabla\tilde{\Phi}_2^{15-22}$

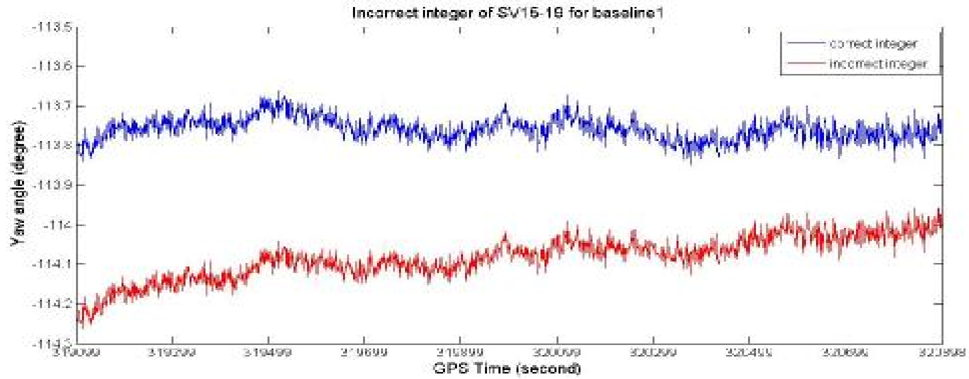

Figure (8a) Yaw Estimation Using Incorrect Integer.

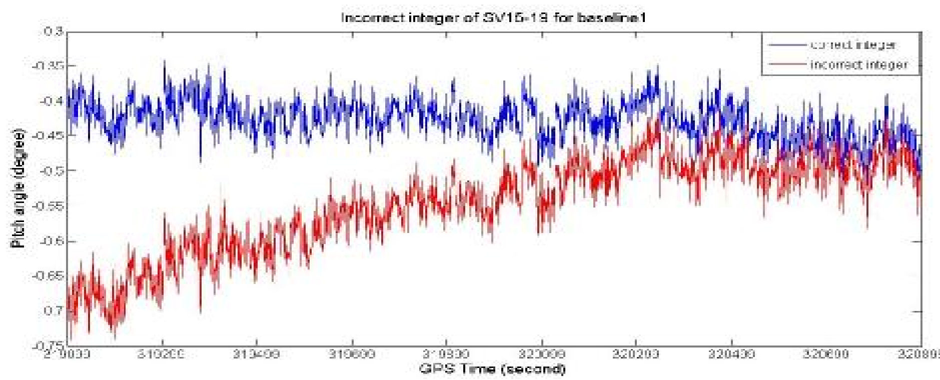

Figure (8b) Pitch Estimation Using Incorrect Integer.

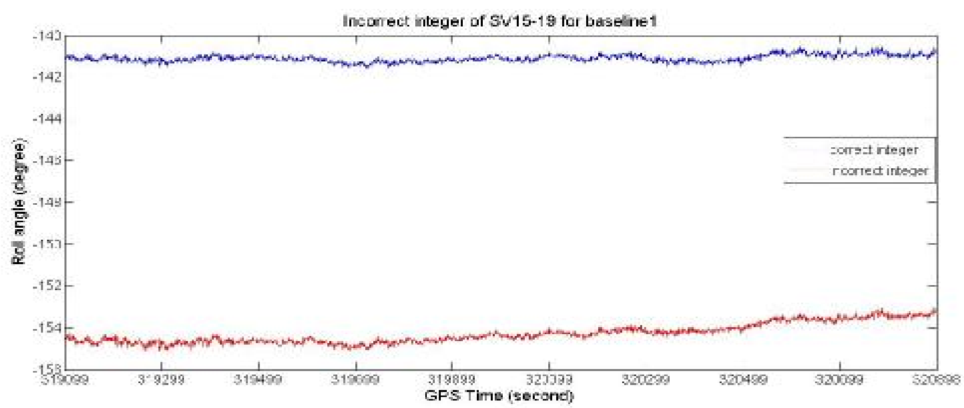

Figure (8c) Roll Estimation Using Incorrect Integer.