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On Fuzzy Ideal Homotopy Lifting Property

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ABSTRACT: In algebraic topology, F.I.H.L.P., or fuzzy ideal Homotopy Lifting Property is a key principle that offers a foundation for comprehending continuous mappings between topological spaces. A more detailed examination of maps that might not precisely retain topological structures is made possible by this characteristic, which expands the traditional idea of homotopy ilifting to a more adaptable andi versatile context. The abstract presented here investigates the fundamentals of the fuzzy ideal HomotopyiLiftingiProperty . along with itsitheoretical foundations, practical uses,i and relevance in modernimathematics. After providing a brief overview of FIHLP, we discuss its salient characteristics and make links to related ideas like topological invariance and homotopy theory. After that, we review significant findings and advancements in the discipline, emphasizing how FIHLPi interacts with diverse branches of mathematics such as algebraic topology, category theory, and differential geometry. Additionally, we go over the useful applications of FIHLPi in a variety of mathematical contexts, such as the analysis of topological data and the investigation of fiber bundles. We demonstrate the usefulness of FIHLPi in resolving theoretical issues and tackling practical difficulties by looking at specific situations and illuminating basic concepts. In addition, we examine unanswered concerns and directions for further study, speculating about possible expansions and improvements to FIHLP a theory to deepeni our comprehension of mappings and topological spaces.

Keywords: The following terms are used: fuzzy set, fuzzy covering space, fuzzyideal lifting function, fuzzy ideal fiber structure, fuzzy ideafibration, and fuzzy ideal homotopy covering property

1. INTRODUCTION AND OUR CONTRIBUTION

Fuzzy topology is a branch of topology and classical set theory that takes membership and uncertainty levels into account. Lotfi A. Zadeh first proposed fuzzy sets in 1965. They offer a mathematical foundation for dealing with ambiguity and imprecision in data representation. By allocating membership values between 0 and 1, fuzzy sets expand on classical sets by permitting elements to belong to a set to differing degrees [6]. Allen Hatcher introduced and expanded the theory of covering spaces. Elon Lages Lima investigated the idea of covering spaces and fundamental groups [7]. So, the idea of fuzzy covering spaces arises as a logical progression of traditional covering space theory, building upon the idea of fuzzy sets. Covering spaces are essential to comprehending the basic group and homotopy theory in conventional topology. This framework is expanded to include fuzzy sets via fuzzy covering spaces, which allows for a more adaptable handling of topological structures and mappings. [1][8][4],[5]. As a generalization of the classical lifting feature in covering space theory [2],[3], fuzzy ideal lifting is a key concept in the study of fuzzy ideal covering spaces. A fuzzy ideal lifting function captures the uncertainty present in the mapping between fuzzy ideal spaces by allocating fuzzy sets to points in the fuzzy ideal base space. This

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approach is fundamental to the definition of the fuzzy ideal Homotopy Lifting Property (FIHLP). The classical homotopy lifting property is extended to fuzzy settings by the fuzzy ideal homotopy lifting property, which pertains to a fuzzy ideal measure$(\mu_A, \mathbf{I})$. In summary, FIHLP preserves the fuzzy ideal structures created by the mapping by guaranteeing that homotopies between fuzzy ideal map lift to fuzzy ideal map in the covering space. This characteristic has important ramifications for comprehending the topology of fuzzy ideal spaces and is crucial for researching how fuzzy ideal mapping behave under continuous deformations. Fibrations are essential for capturing the local and global structure of spaces in fuzzy ideal topology. By extending the concept of a fibration to fuzzy ideal environments, a fuzzy ideal fibration offers a framework for examining how mappings between fuzzy ideal spaces behave. The characteristics of fuzzy ideal fibrations are described by a number of statements and theorems that clarify the relationship between fuzzy ideal lifting functions, fuzzy ideal maps, and fuzzy ideal covering spaces [2]. As a result, fuzzy ideal topology is based on the ideas of fuzzy ideal sets, fuzzy ideal covering spaces, fuzzy ideal lifting, FIHLP, and fuzzy ideal fibrations. It provides a flexible framework for researching topological spaces where uncertainty and imprecision are present. By bridging the gap between fuzzy ideal mathematics and classical topology, these ideas provide opportunities to investigate the complex interactions between structure and fuzziness in mathematical modeling and analysis.

Definition 1.1[9] Suppose X is a non-empty set which is an object collection. The fuzzy set A in X is of order pairs $A = \{(x, \mu_A(x)) \mid x \epsilon X, 0 \leq \mu_A(x) \leq 1\}$, where $\mu_A : X \to [0,1]$ is called the "membership function". and each $x \epsilon X$ the value of $\mu_A(x)$ is called the grade of membership of $x \; in \; A$.

Definition 1.2[10]**:** A fuzzy point p in X is a particular fuzzy set with a membership function defined by: $p(x) = \begin{cases} \mu & if\ x = y \\ 0 & if\ \ x \neq y \end{cases}$

where $0 < \mu \leq 1$. P is considered to possess support m, value μ and it is denoted by p_m^μ or p(m,μ).

Definition 1.3[11]**:** Assume that all fuzzy subsets on the discourse universes X and Y belong to the classes F(X), F(Y). A fuzzy function is a mapping f from F(X) into F(Y) if, for any fuzzy subsets A; A′ ∈F(X) and B;B′ ∈F(Y) that are f-related with A;A′, respectively, A =A′ ⇒B=B′. holds true.

Definition 1.4[12]**:** A map $\mathbb{I}: I^x \to I$ is referred to as topological space X a fuzzy ideal if it fulfills the following conditions:

- $(O_1)\mathbb{I}(\underline{0}) = 1$,
- (0_2) $\mu_A \leq \mu_B \to \mathbb{I}(\mu_A) \geq \mathbb{I}\ (\mu_A)$ *for all* $\mu_A, \mu_B \in I^x$,
- (O_3) $\mathbb{I}\ (\mu_A \vee \mu_B) \geq \min(\mathbb{I}\ (\mu_A), \mathbb{I}\ (\mu_B))$ for all $\mu_A, \mu_B \in I^x$.

Such that $(\mu_X, \tau^*, \mathbb{I})$ is FuITs.

Denoted by iff $I_1\ (\mu_A) \leq I_2\ (\mu_A)\ \forall \lambda \in I^x$, we obtain I_1 is finer than I_2 (I_2 is coarser than I_1) if I_1 and I_2 are fuzzy ideals on X. Fuzzy ideal topological space is the name given to the triple (X, τ, I). Articulate the fuzzy ideal I° by:

$$\mathbb{I}^\circ(\mu_A) = \begin{cases} 1 & at\ \mu_A = \bar{0} \\ 0 & otherwise \end{cases}$$

Definition 1.5: Let $p: (\mu_x, \mathbf{I}) \to (\mu_y, \mathbf{I})$ be a fuzzy ideal continuous and surjective. Let $(\mu_u, \mathbf{I}) \subset (\mu_y, \mathbf{I})$ be a fuzzy open set in $(\mu_y, \mathbf{I})$. We say $(\mu_u, \mathbf{I}) \subset (\mu_y, \mathbf{I})$ is evenly fuzzy ideal cover by p if $p^{-1}\ (\mu_{u,}\ \mathbf{I})$ is a union of an arbitrary number of disjoint fuzzy open sets $\mu_{v\alpha}$ such that for all α, the function $p|\mu_{u\alpha}: \mu_{u\alpha} \to \mu_u$ is a fuzzy ideal homeomorphism.

Definition 1.6: A fuzzy ideal fibration structure $(\mu_E, \mathrm{p}, \mathrm{I}, \mu_B)$ consists of two fuzzy ideal space μ_E, μ_B and a fuzzy ideal continuous surjective p: $(\mu_E, \mathbf{I}) \to (\mu_B, \boldsymbol{I})$. The fuzzy ideal space $(\mu_E, \mathbf{I})$ is called a fuzzy ideal total (or fuzzy ideal fibered) space; P is termed the projection, and $(\mu_B, \boldsymbol{I})$ is the fuzzy ideal base space for each $u_0 \in (\mu_y, \mathbf{I})$. Let $u_0 \in (\mu_B, \boldsymbol{I})$ then $F: p^{-1}\ (u_0)$ and F is called fuzzy ideal fiber on u_0. We refer to $(\mu_E, \mathrm{p}, \mathrm{I}, \mu_B)$ as a fuzzy ideal f iber structure over P.

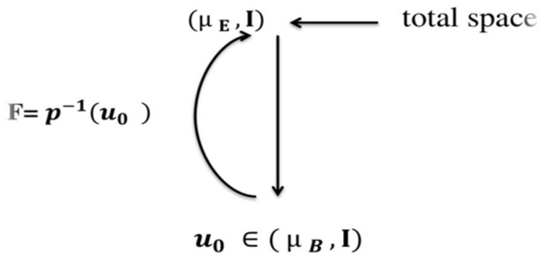

FIGURE 1: F$UZZY\ \ ideal\,fiber\ \ structure$

Definition 1.7: consider P: $(\mu_E, \boldsymbol{I}) \to (\mu_B, \boldsymbol{I})$ act as a map, also p has $the\ fuzzy$ ideal $homotopy$ lifting $(\mathrm{F.I.H.L.P})$ w.r.t $(\mu_x, \mathbf{I})$ if given a fuzzy ideal map $v: (\mu_x, \boldsymbol{I}) \to (\mu_E, \boldsymbol{I})$ and a fuzzy ideal homotopy $h_t: (\mu_x, \mathbf{I}) \to (\mu_B, \boldsymbol{I})$ such that $P \circ v = h_0$. so, there $is\ a\ fuzzy$ ideal homotopy $h_t^*: (\mu_x, \mathbf{I}) \to (\mu_B, \boldsymbol{I})$ such that: "1.$h_0^* = v(x)$" "2. $p \circ h_t^*(x) = h_t$ for all $x \in (\mu_x, \mathbf{I})$ and $t \in I$."

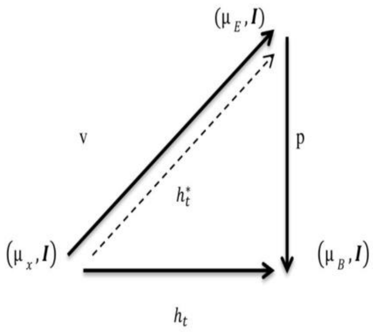

FIGURE 2 .F.I.H.L.P

Definition 1.8: Let $(\mu_B, \boldsymbol{I})$ $(\mu_E, \boldsymbol{I})$ two fuzzy ideal topological spaces. The fuzzy ideal fiber structure $(\mathrm{F.I.H.L.P})$ $is\ called$ a $fuzzy\ ideal\ \ fiber\ space$ or a fuzzy $ideal\ fibration$ $(F.I.F.S)$ for the class $\mathcal{R}$ of the fuzzy idea spaces if p has a fuzzy ideal $homotopy\ lifting$ (covering) $property$ $(F.\mathrm{I.H.}L.\mathrm{P})$ for all $(\mu_{\mathrm{x}}, \mathbf{I}) \in \boldsymbol{\mathcal{R}}$.

Example 1.9: Consider $\mathrm{P} : (\mu_E, \boldsymbol{I}) \times \mathrm{F} \to (\mu_B, \boldsymbol{I})$ $be\ a\ fuzzy$ ideal projection. Also, p is a fuzzy ideal fibration. Additionally, $b \in (\mu_B, \boldsymbol{I})$, the fuzzy ideal fiber over $(\mu_B, \boldsymbol{I})$ $is\ a\ fuzzy$ ideal $homeomorphic\ to\ F$. A fuzzy ideal $fibration\ can\ be$ applied to fuzzy ideal $lift\ a\ fuzzy$ ideal path in $(\mu_B, \boldsymbol{I})$ to a fuzzy ideal path into $(\mu_Z, \boldsymbol{I})$ as demonstrated by the next theorem.

Theorem 1.10: Consider $(\mu_E, \boldsymbol{I})$, $(\mu_B, \boldsymbol{I})$ two fuzzy ideal topological spaces. If $\mathrm{P} : (\mu_E, \boldsymbol{I}) \to (\mu_B, \boldsymbol{I})$ is a $(F.I.F.S)$, so, any fuzzy ideal path v within μ_B togetherwith $v(0) \in p(\mu_Z, \boldsymbol{I})$can be considered a fuzzy ideal lifted to a fuzzy path in $(\mu_Z, \boldsymbol{I})$.

proof: Let One fuzzy ideal point space is denoted by p. Suppose v is a fuzzy ideal where a fuzzy ideal point $e \in (\mu_E, \boldsymbol{I})$ such that $p(e) = v(0)$ In line with a fuzzy ideal map: $p \to (\mu_E, \boldsymbol{I})$ like that $p(v(0) = v(\alpha, 0))$, where $\alpha \in p$. There $is\ a\ fuzzy\ ideal\ path$ u in$(\mu_E, \boldsymbol{I})$ such that $u(0) = e$ alongwith pu = v, indicating that u is a $fuzzy\ ideal$ lifting of v. This is $because\ p$ is a fuzzy ideal fibration, which $has\ the\ fuzzy$ ideal homotopy lifting property.

2. UNIQUE FUZZY IDEAL PATH LIFTING

Definition 2.1: A fuzzy ideal map P : $(\mu_E, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I})$ It is alleged to have a unique fuzzy ideal path lifting $(U.F.I.P.L)$ if for a fuzzy ideal paths u and v in $(\mu_E, \boldsymbol{I})$ such that $pu = pv$ and $u(0) = v(0)$, We have $u = v$.

Lemma 2.2: The fuzzy ideal path lifting of a fuzzy ideal covering map is unique.

Lemma 2.3: Consider P : $(\mu_E, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I})$has unique fuzzy ideal path lifting. So,p has The unique fuzzy ideal lifting $property\ for\ fuzzy$ ideal path connected spaces.

proof: Let μ_k be a fuzzy ideal path connected. Let $u, v :\ (\mu_K, \boldsymbol{I}) \rightarrow (\mu_E, \boldsymbol{I})$ be fuzzy ideal maps such that $pu = pv$. Let $k_0 \in (\mu_K, \boldsymbol{I})$ such that $u(k_0) = v(k_0)$. We must demonstrate that $u = v$. Let k be an arbitrary fuzzy ideal element of $(\mu_K, \boldsymbol{I})$and let h be a fuzzy ideal path in $(\mu_K, \boldsymbol{I})$ starting at

$u = v$. The $following\ theorem$ demonstrates the particular $connection\ between$ a lifting fuzzy ideal path and a fuzzy ideal fibration.

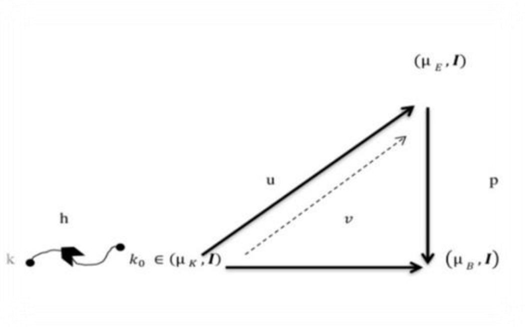

FIGURE 3: U. F. I. p. L

Theorem 2.4: If and only if each fuzzy ideal fiber has no non − null fuzzy ideal path, then a fuzzy ideal fibration has a unique fuzzy ideal path lifting.

proof: Consider $(\mu_E, \boldsymbol{I})\ alongwith\ (\mu_B, \boldsymbol{I})$ constitute any fuzzy ideal topological spaces.
Let P : $(\mu_E, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I})\ be\ a\ fuzzy\ ideal\ fibration$ with unique fuzzy ideal path lifting. Let $r \in (\mu_B, \boldsymbol{I})$ and suppose that u be a fuzzy ideal path in the fuzzy ideal fiber $p^{-1}(r)$. Let v be a null fuzzy ideal path in $p^{-1}(r)$ such that $u(0) = v(0)$. So, $pu = pv$ this implies to $u = v. Then\ u$ is a null fuzzy ideal path. On the contrary, suppose that P : $(\mu_E, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I})$ is such a fuzzy ideal fibration that each fuzzy ideal fiber has no fuzzy ideal path that is not null.
Consider $u\ alongwith\ v$ be fuzzy ideal paths within $(\mu_E, \boldsymbol{I})$ such that $pu = pv$ and $u(0) = v(0)$. For $t \in C$, let h_t be the fuzzy ideal bath in $(\mu_E, \boldsymbol{I})\ as$ defined by:

$$h_t(s) = \begin{cases} u\big((1-2s)t\big) & , \quad 0 \leq s \leq \frac{1}{2} \\ v(2s-1)t) & , \quad \frac{1}{2} \leq s \leq 1 \end{cases}$$

This allows us to derive a fuzzy ideal pathh_t in $(\mu_E, \boldsymbol{I})$ between u(t) and v(t) Ph_t becomes a fuzzy ideal closed path in $(\mu_B, \boldsymbol{I})$ which is a fuzzy ideal homotopic relative to C to the null fuzzy ideal path at $p(u(t))$. From the fuzzy ideal homotopy lifting property of p, We observe the existence of a fuzzy ideal map $\acute{F} :\ C \times C \rightarrow (\mu_E, \boldsymbol{I})$ such that
$\acute{F}(\acute{t}, 0) = h_t(\acute{t})$ and $\acute{F}$ fuzzy ideal maps $(0 \times C) \cup (C \times 1) \cup (1 \times C)$ to the fuzzy ideal fiber $p^{-1}(p(u(t)))$. Using a hypothesis, so $p^{-1}(p(u(t)))$ has no non-null fuzzy ideal paths. So, $\acute{F}$ fuzzy ideal maps $0 \times C, C \times 1$ alongwith $1 \times C$ to a single fuzzy ideal point, which suggests $F'(0,0) = F'(1,0)$. Thus: $h_t(0) = h_t(1)$ and $u(t) = v(t)$.

Theorem 2.5: Consider $(\mu_E, p, I, \mu_B)$and (μ_K, q, I, μ_C) possess a unique fuzzy ideal path lifting of a fuzzy ideal fibration.
then $((\mu_E, \boldsymbol{I}) \times (\mu_K, \boldsymbol{I}), p \times q, I, (\mu_B, \boldsymbol{I}) \times (\mu_C, \boldsymbol{I}))$ is also fuzzy ideal fibration with a unique fuzzy ideal path lifting.

proof: $(\mu_E, \boldsymbol{I})$, $(\mu_B, \boldsymbol{I})$, $(\mu_K, \boldsymbol{I})$ and $(\mu_C, \boldsymbol{I})$ are fuzzy ideal topological spaces. Let $u :\ (\mu_Z, \boldsymbol{I}) \rightarrow (\mu_E, \boldsymbol{I})$ and

$\acute{u}$: $(\mu_Z, \boldsymbol{I}) \longrightarrow (\mu_K, \boldsymbol{I})$ be any fuzzy ideal maps. Define $u^* : (\mu_Z, \boldsymbol{I}) \rightarrow (\mu_E, \boldsymbol{I}) \times (\mu_K, \boldsymbol{I})$ be a fuzzy ideal map by $u^* \ (z) = (u(z), \ \acute{u}(z))$ and h_t: $(\mu_Z, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I})$ and $\acute{h}_t : (\mu_Z, \boldsymbol{I}) \rightarrow (\mu_C, \boldsymbol{I})$ be any fuzzy ideal maps. Define $h_t^* : (\mu_Z, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I}) \times (\mu_C, \boldsymbol{I})$ by $h_t^*(z) = \big(h(z), \acute{h}(z)\big)$ $longwith$ $(p \times q) \circ u^* = \acute{h}_0$. Because p, q are fuzzy ideal fibrations, Afterward, there is u_t: $\mu_Z \rightarrow \mu_E$ such that

$p \circ u_t = h_t$, $u_0 = u$ and $\acute{u}_t : \mu_Z \rightarrow \mu_K$ such that $q \circ \acute{u}_t = h_t$, $\acute{u}_0 = \acute{u}$. Right now, for h_t^*There is $u_t^* : \mu_Z \rightarrow \mu_E \times \mu_K$ define as $u_t^*(z) = (u_t(z), \ \acute{u}_t(z))$ such that**:**

"1. $(p \times q) \circ u_t^* = h_t^*$ "

"2. $u_0^* = u$"* . After that, $p \times q : (\mu_E, \boldsymbol{I}) \times (\mu_K, \boldsymbol{I}) \rightarrow (\mu_B, \boldsymbol{I}) \times (\mu_C, \boldsymbol{I})$ has fuzzy ideal homotopy lifting property with respect to: $(\mu_Z, \boldsymbol{I})$ So, it is a fuzzy ideal fibration with a $unique$ fuzzy $ideal\ path\ lifting$.

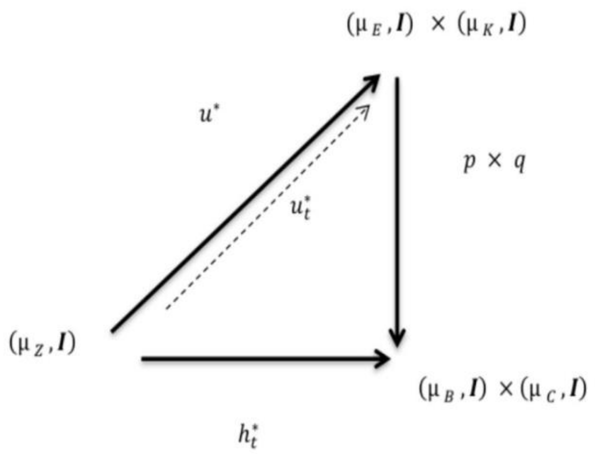

FIGURE 4: The product Fuzzy Ideal Fibretion

2.THE FUZZY IDEAL LIFTING FUNCTION

Definition 3.1: Let (μ_E, p, I, μ_B) is a fuzzy ideal fiber structure and let $(\mu_A, \boldsymbol{I}) : \big(\alpha : I \rightarrow (\mu_B, \boldsymbol{I})\big)$, $\Omega_p \subset (\mu_E, \boldsymbol{I}) \times (\mu_A, \boldsymbol{I})$ be a fuzzy ideal subspace $\Omega_p = (u, \alpha) \in (\mu_E, \mathbf{I}) \times (\mu_A, \boldsymbol{I}) | p(u) = \alpha(0)$ $regarding\ the\ Cartesian\ product$. A fuzzy $ideal\ lifting\ function$ for (μ_E, p, I, μ_B) is a continuous fuzzy ideal map $\lambda : \Omega_p \rightarrow (\mu_L, \boldsymbol{I})$ such that:
$\lambda(u, \alpha)(0) = u$ and $p \circ \lambda(u, \alpha)(t) = \alpha(t)$ for each $(u, \alpha) \in \Omega_p$ and $t \in I$, We state that λ is a regular fuzzy ideal if $\lambda(u, \alpha)$ is a $constant\ fuzzy$ ideal path.

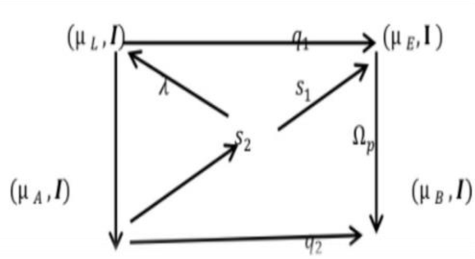

FIGURE 5: F.I.L.F

Thus, a fuzzy ideal lifting function associated with each $u \in (\mu_E, \mathbf{I})$ and a fuzzy ideal path $\alpha\ in\ (\mu_B, \mathbf{I})$ starting at (u), a fuzzy ideal path $\lambda(u, \alpha) in\ \mu_E$ starting at u, that is lift of α. Since the c-fuzzy ideal topology is used in $(\mu_L, \boldsymbol{I})$

, the fuzzy ideal continuity of λ is a fuzzy ideal equivalence to that of the fuzzy ideal associated $\lambda : \Omega_p \times I \rightarrow$ $(\mu_E, \mathbf{I})$, by sampling: $\lambda(u, \alpha) \in (\mu_L, \boldsymbol{I})$ and $\lambda(u, \alpha): I \rightarrow$ $(\mu_E, \mathbf{I}), \lambda(u, \alpha)(0) = u,\ p \circ \lambda(u, \alpha)(t) = \alpha(t).\ p^*(\alpha): I \rightarrow (\mu_B, \mathrm{I}), p^*(\alpha)(t) = p(\alpha(t)),\ \lambda : \Omega_p \times I \rightarrow$ (μ_E, I) such that $\lambda(u, \alpha)(t) = (\lambda(u, \alpha))(t)$.

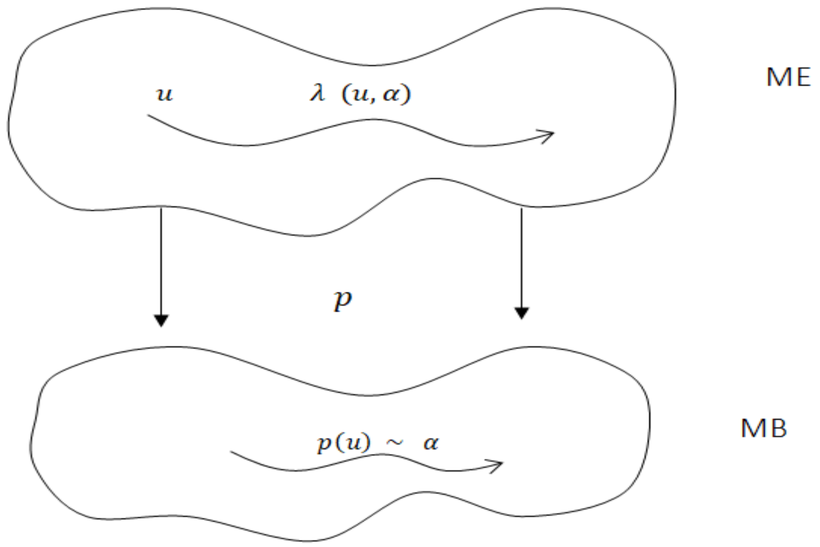

FIGURE 6: Fuzzy Ideal Path

Remark 3.2: A fuzzy ideal map $p(\mu_E, \mathrm{I}) \rightarrow (\mu_B, \mathbf{I})$ is a fuzzy ideal fibration if *and only* if there exists a fuzzy ideal lifting function for p.

Theorem 3.2: The fuzzy ideal *fiber structure* (μ_E, p, I, μ_B) is a fuzzy ideal f ibration if *and only* if a fuzzy *ideal lifting function exists*

proof Suppose that the fuzzy ideal fibration p is fuzzy. Let $(\mu_A, \mathbf{I}) = \Omega_p$ and $v : \Omega_p \rightarrow (\mu_E, \mathrm{I})$ and $h_t : \Omega_p \rightarrow (\mu_B, \mathbf{I})$ defined by $v(u, \alpha) = u$ and $h_t(u, \alpha) = \alpha(t)$. Then$h_0(u, \alpha) = \alpha(0) = p(u) = p \circ v(u, \alpha)$

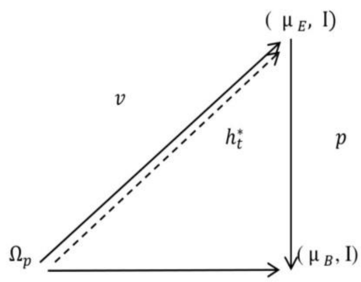

FIGURE 7: fuzzy ideal fibration

There exists a fuzzy ideal map $h_t^*: \Omega_p \rightarrow (\mu_E, \mathrm{I})$ be a fuzzy ideal homotopy lifting such that $h_0^*(u, \alpha) = v(u, \alpha) = u$ and $p \circ h_t^* = h_t$

h_t^* defines a fuzzy ideal *lifting function* λ for p by $\lambda(u, \alpha)(t) = h_t^*(u, \alpha)$ λ is a fuzzy ideal lifting function which is whenever h_t^* is stationary with h_t. Conversely, if p has a fuzzy ideal lifting function Let $v : (\mu_A, \mathbf{I}) \rightarrow (\mu_E, \mathrm{I})$ be given and $h_t : (\mu_A, \mathbf{I}) \rightarrow (\mu_B, \mathrm{I})$ is *a fuzzy* ideal homotopy so, $p \circ v = h_0$, for all $x \in (\mu_A, \mathbf{I})$ let $\alpha_x: I \rightarrow (\mu_B, \mathrm{I})$ be identified by $\alpha_x(t) = h_t(x)$ *Defined a* fuzzy ideal map $h_t^*: (\mu_A, \mathbf{I}) \rightarrow (\mu_E, \mathbf{I})$ as follows: $h_t^*(x) = \lambda(v(x), \alpha_x)(t)$ then

h_0^* (x) = $v(x)$

$p \circ h_t^* = h_t$

Therefore p has a fuzzy ideal fibration.

4.CONCLUSION

The following results are from this paper: We provided examples to demonstrate fuzzy covering space and fuzzy sets. according to homotopy theory. Fuzzy ideal homotopy lifting property andifuzzy idealifibration were also defined in a new way. As shown, a fuzzy ideal fibration also possesses. A uniqueifuzzy ideal path lifting. Our notion of the fuzzy ideal lifting function has been expanded, and we demonstrate that the fuzzy ideal fiberistructure is a fuzzy idealifibration.

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