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Effect of Differential Retardation Equations on Insect Life Cycle: Modeling and Analysis for Deeper Understanding

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Abstract

In this research, we explore the impact of using delay differential equations in analyzing and understanding actions within the framework of studying the life cycle of insects. It demonstrates how these equations can be used to predict insect population outbreaks and identify environmental conditions conducive to reproduction. In this paper, we rely on the growth function of the prey and predator equations to study the effect of delayed differential equations on the insect life cycle. The results and comparisons were obtained using MATLAB. Also, we give an applied example of the fruit fly and used numerical methods, namely Euler's method, to obtain accurate approximate values to study the phenomenon of the delay effect of the differential delay equations for the life cycle of the fruit fly. Finally, the new idea from this work is Include time delay effect where the Ordinary differential equations (ODEs) assume that changes occur instantaneously, which may not be accurate in many biological systems. While the delay differential equations (DDEs) allow the inclusion of time delays between different events, such as the time required for eggs to transform into larvae, larvae to pupae, and pupae to adults.

Keywords:
Delay differential equations; Modeling and Analysis; Insect Life Cycle.

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I. Introduction

This section is an application of delayed differential equations and their effects on the life cycle of insects. In this chapter, we showed the inverse relationship between lag equations and the growth and reproduction of insects starting from the egg stage [1,2].

Here are applied examples of delay differential equations in various fields:

Population Growth hysteresis differential equations can be used to model population growth, as the growth rate is affected by environmental factors and available resources. And environmental control, in the field of ecology, delayed differential equations can be used to model the effect of delays in an ecosystem's response to changes such as pollution or climate changes. And mathematical economics, in mathematical economics, slowing down differential equations can be used to model the effects of delays in economic responses to changes in markets and economic policies. And control systems, in control engineering lagged differential equations analysis is used to understand the stability and performance of dynamic systems with delay. And disease modeling, hysteresis differential equations can be used to model the spread of disease and the effect of delays in taking preventive measures or treatment. And communications Engineering, in the field of communications engineering delayed of differential equation can be used to analyze the performance of broadband communications systems and the effect of delay on data transmission. And modeling biology, these equations can be used to model the interactions of biological of biological systems such as the life cycle of organisms and the effect of delays in these processes. And control of robotic mechanisms, slowing down differential

equations can be used to model delays in the response of robots and improve their performance in controlling and interacting with the environment. Some insect species have a very long larval stage of many years and, by comparison, an extremely
short adult stage of just a few weeks or less [5].

Insect metamorphosis is the transformation of an immature larval individual into a reproducing adult of very different form, structure, and habit of life. It occurs to some extent in all but the most primitive insects and is under endocrine control. Insect metamorphosis differs from the ontogenetic processes of other animals only in degree: the larva and the imago are commonly far more different from one another than are the young and adults of most other organisms. The selective advantage of this "double life" is easy to perceive: the young insect can exploit one habitat, the mature insect another [8]. finally, insect develop during larval life either as imaginal discs or as small undifferentiated patches of larval epidermis [9].

Through the development of science, we were able to using delayed differential equations, we were able to improve the accuracy of insect life cycle modeling, understand the effect of time delays on population dynamics, and better analyze the stability and dynamic behavior of the system. These benefits could lead to practical applications in pest management and understanding environmental impacts on insect life cycles

2. principles and creating a mathematical model of the insect life cycle

The insect life cycle is a series of stages that an insect goes through from the beginning of its end. It usually consists of several stages, which may vary depending on the type of insect. Here we will give an overview of the insect life cycle and some examples of different types of insects and their life stages [3,4].

1. Eggs:
- At this stage, the mother insect lays eggs in a suitable environment.
- Example: Monarch butterfly eggs.

2. larva (larva):
- After a period of hatching, the larva emerges from the egg and begins to grow.
- Example: jaundice larva

Now, we will be creating a mathematical model of the insect life cycle

The life cycle of insects can be represented using a hysteresis differential equation. A common model used for this purpose is the Lotka-Volterra model.

Let us express the density of eggs by $E(t)$ and the density of large insects is by $A(t)$. We can use the following equations:

$$\frac{dE}{dt} = \beta A - \delta E$$
$$\frac{dA}{dt} = \alpha E - \gamma A \qquad (1)$$

where E is the number of eggs, β is the rate of egg reproduction, A is the number of large insects, δ is the death rate of eggs and α is the rate at which eggs transform into large insects and γ Death rate of large insects.

To illustrate how this model can be applied to the life cycle of an insect, let us assume that we have an insect that lays eggs and develops in to a large insect. We will use the Lotka-Volterra model containing a hysteresis term to illustrate the effect of hysteresis on the insect life cycle.

Then we can use equations to determine how the two numbers evolve over time. We will use Eller's method integration with a time step $\Delta(t)$.

Now, we will repeat the following steps to determine the evaluation of the system over period of time.

a. Start from time $t = 0$

b. uses the above equations to find the rate of change in E and A

c. use Euler's method to update $E(t + \Delta t) = E(t) + \frac{dE}{dt} \times \Delta t$, and the same for $A(t + \Delta t)$

d. repeats this process until we reach the specified final time

now, let us give simplified steps to calculate egg and insect values over several time periods.

suppose we have a type of insect that is affected by a slowdown in the processes of reproduction and transformation due to negative environmental effects over time,

We will use the following mathematical model to determine the evolution of insects over time using Euler's method to calculate the following values.

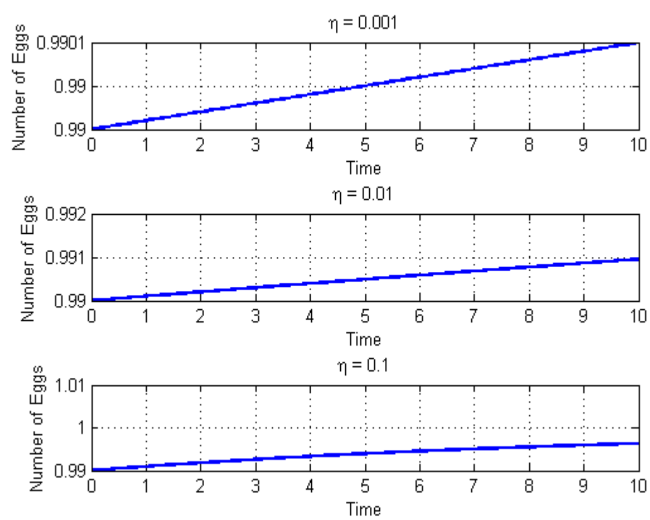

Fig 1. calculate the hysteresis coefficient when taken $\eta = (0.001, 0.01, 0.1)$

Here we noticed that when different values of the hysteresis coefficient are taken $\eta = (0.001, 0.01, 0.1), \beta = 0.02, \delta = 0.01, A = 50$

t It changes from 0 to 10, The relationship will be inverse with the number of eggs over time.

Now, the results of the two different models with different treatments and parameters will be compared and we will see the difference in the number of eggs over time where, $(\beta_1, \delta_1, \eta_1) = (0.02, 0.01, 0.001)$ for the first model $\frac{dE}{dt} = \beta A - \delta E \times e^{-\eta_1 t}$ and $(\beta_2, \delta_2, \eta_2) = (0.03, 0.015, 0.01)$ for second model $\frac{dE}{dt} = \beta A - \delta E \times e^{-}\eta_2 t$

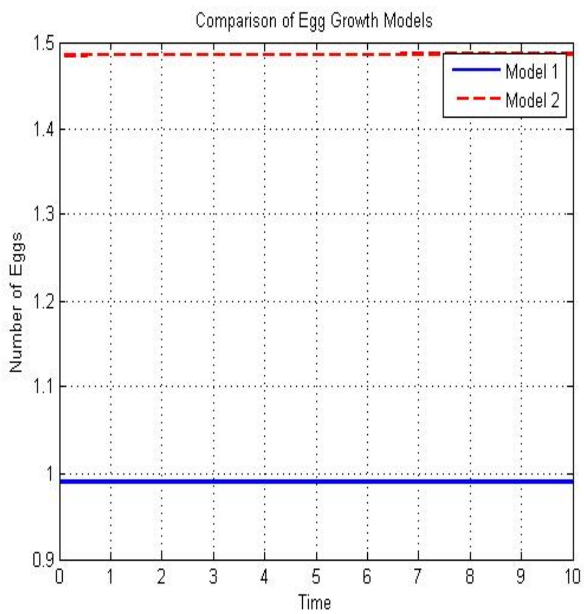

Fig 2. comparison of growth models where, $(\beta_1, \delta_1, \eta_1) = (0.02, 0.01, 0.001)$ for the first model and $(\beta_2, \delta_2, \eta_2) = (0.03, 0.015, 0.01)$ for second model.

Here in this figure we notice the inverse relationship between the hysteresis coefficient, Egg reproduction rate and Egg death rate for two different values in different models

Now, Under the parameters $(\beta, \delta, \alpha, \gamma, \eta) = (0.02, 0{,}01, 0.001, 0.03, 0.001)$ and initial condition at the beginning $(E(0), A(0)) = (1000, 50)$ and $t_0 = 0, t = 1$, We find,

$$\frac{dE}{dt} = 0.02 \times 50 - 0.01 \times 1000 \times e^{-0.001\times 1} \cong 1 \qquad (3)$$

$$\frac{dA}{dt} = 0.01 \times 1000 - 0.03 \times 50 \times e^{-0.001\times 1} \cong -1 \qquad (4)$$

$$E = 1000 + 1 \times 1 = 1001 \qquad (5)$$

$$A = 50 - 1 \times 1 = 49 \qquad (6)$$

Value can be changed η to see the effect of hysteresis integration on the dynamics. We can see the difference between the following two images,

And under the parameters $(\beta, \delta, \alpha, \gamma, \eta) = (0.02, 0{,}01, 0.001, 0.03, 0.001)$, $(\beta, \delta, \alpha, \gamma, \eta) = (0.02, 0{,}01, 0.001, 0.03, 0.1)$ and initial condition at the beginning $(E(0), A(0)) = (1000, 40)$ and $t_0 = 0, t = 1$,

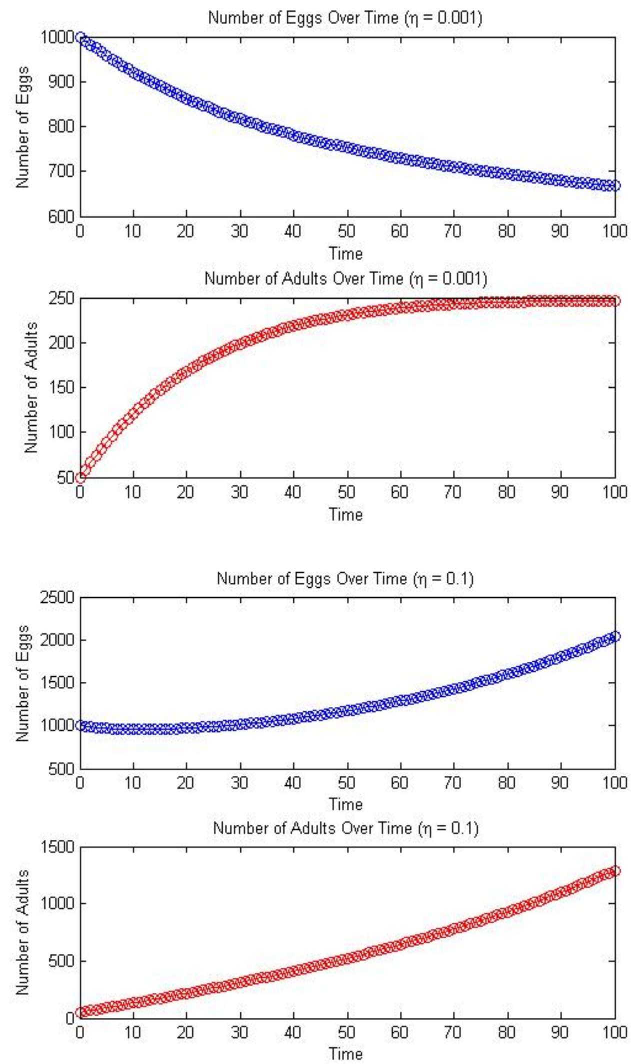

Fig 3. effect of difference values of hysteresis integration on numbers of eggs and adults under the parameters $(\beta, \delta, \alpha, \gamma, \eta)(0.02, 0{,}01, 0.001, 0.03, 0.001)$, $(\beta, \delta, \alpha, \gamma, \eta) = (0.02, 0{,}01, 0.001, 0.03, 0.1)$.

Hence, we note if we reduce the deceleration factor, we will have a greater acceleration effect. This means that insects will reproduce and mutate at a faster rate over time. This could be a representation of favorable environmental conditions that help accelerate reproductive processes.

On the other hand, if the value of the hysteresis coefficient increases

We will have a greater deceleration effect. This means that reproduction and metamorphosis will occur more slowly, which reflects unfavorable environmental conditions that cause insects to reproduce and metamorphose at a slower rate.

Now, here we will take different values of parameters for model 1 (β_1= 0.02; δ_1= 0.01; A_1= 80 ; η_1 = 0.001) and Parameters for model 2 (β_2= 0.025; δ_2= 0.015; A_2 = 40; η_2= 0.0005) and compare them with the number of eggs

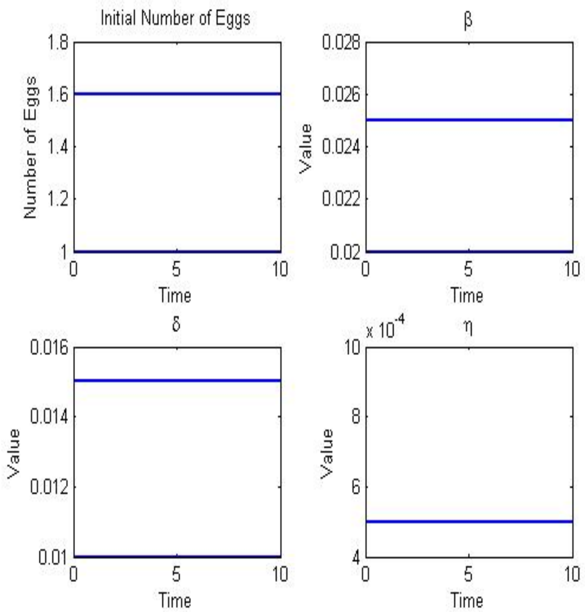

Fig 4. Compare parameters for model 1 (β_1= 0.02; δ_1= 0.01; A_1= 80 ; η_1 = 0.001) and Parameters for model 2 (β_2= 0.025; δ_2= 0.015; A_2 = 40; η_2= 0.0005) with the number of eggs

From the results displayed in the graph, we can see the differences in egg growth between the two different models. Here are some points to note:

- **Effect of growth and hysteresis parameters:** Different values of growth and hysteresis parameters led to changes in growth speed and expected egg quantity over time. It can be seen how increasing values of growth coefficients β and lower values of hysteresis coefficients δ may accelerate egg growth, while different values may reduce growth.

- **Effect of initial values:** The expected quantity of eggs was affected by the initial values of large insects A.Increase value A The second model resulted in a smaller amount of eggs compared to the first model.

- **Differences in the effect of the hysteresis coefficient:** We also noticed that changing the value of the hysteresis coefficient η has a noticeable effect on egg growth. Smaller value for ($\eta = 0.0005$ in the second model) led to an increase in the amount of eggs compared to a larger value of (η =0.001 in the first model).

We conclude from that:

The deceleration factor in the model affects the insect life cycle adversely.

Hysteresis factor η affects the processes of reproduction and model transformation. The higher the value η, the hysteresis effect increased.

The effect of the hysteresis factor is inverse on reproduction and transformation rates meaning that as time (t) increases, the hysteresis effect increases, which reduces the rates of reproduction and transformation. This reflects the natural idea that the hysteresis effect increases over time.

Therefore, if it is a value η high, increasing the deceleration effect will result in a greater slowdown in the reproductive and metamorphosis processes, so we will have less effect in increasing the number of eggs and large insects over time.

In another place,
We will illustrate the effect of the hysteresis coefficient through some cases:

1. The η Small (accelerative effect): when it is the η Small, the hysteresis effect is weak. This means that reproduction and growth processes occur more quickly, and the hysteresis effect is less. This can be rapid growth in the right conditions

2. The η Large (hysteresis effect): when it is the η Larger, the hysteresis effect increases. This means that the processes of reproduction and growth accelerate slowly, and the deceleration effect is greater. This can be slower growth in less favorable conditions. the notion of model organism has been crucial in the development of biology and mathematics as a science in the 20th century. A model organism is one which allows us to analyze a particular problem in the hope that the answer it gives us will be general and perhaps universal. Thus, peas and plants in general, were essential to develop the key notions of heredity, finches for evolution, and bacteria for unravelling the molecular nature of the gene, the genetic code, and the fabric of metabolism. With few exceptions model organisms come and go and are limited to specific fields and moments but one has had a constant presence in the 20th century and has made significant contributions to multiple areas of biology and mathematics: the fruit fly Drosophila melanogaster.

Drosophila was introduced as an experimental animal at the beginning of the 20th century, probably around 1901 in the context of evolutionary [10].

Now, we will give an applied example of the effect of delay in the differential delay equations for the life cycle of the Drosophila melanogaster and another model that contains a specific delay. And we will assume that the fruit fly goes through three main stages: the egg stage, the larval stage, and the adult stage. We will also assume that there is a small time delay between each stage. For ease of calculation, we will assume that the fruit fly undergoes optimal growth and that the growth ratio between the three stages is constant.

Let's define the variables:

$\big(N_1(t), N_2(t), N_3(t)\big) =$ (number of eggs per time t, number of larvae per time t , number of adult flies per time t)

$(r_1, r_2, \tau_1, \tau_2)$ = (the growth rate from the egg stage to the larval stage, the growth rate from the larval stage to the adult stage, the delay between the egg stage and the larval stage, the delay between the larval stage and the adult stage)
Now, we will create a model of delayed ordinary differential equations related to the life cycle of the fruit fly.

$$\frac{dN_1}{dt} = -r_1 N_1(t) \qquad (7)$$

$$\frac{dN_2}{dt} = r_1 N_1(t-\tau_1) - r_2 N_2(t) \qquad (8)$$

$$\frac{dN_3}{dt} = r_2 N_1(t-\tau_2) \qquad (9)$$

To solve this model, we can use analytical or computational solution methods, such as Euler's method or others, to obtain the evolution of the insect population over time.
We will solve this model using Euler's method for illustration purposes, where we approximate the transition between time steps by a small step.
Let's start applying the steps:
Choose values for rates and delays, and determine the time step Δt, then we choose initial conditions, finally we will apply a certain algorithm to calculate $N_1(t), N_2(t), N_3(t)$ for each time value.
Let,

$$(r_1, r_2, \tau_1, \tau_2, \Delta t,) = (0.1, 0.2, 1, 2, 1)$$

with initial conditions,

$$\big(N_1(0), N_2(0), N_3(0)\big) = (1000, 0, 0)$$

Suppose we want the insect to evolve over a certain period of time($t = (0{:}10)$)
Now, we will impose the following algorithm for each specific time value under the initial conditions given previously:

$$N_1(0.1) = N_1(0) - 0.1 * r_1 * N_1(0) \qquad (10)$$
$$= 1000 - 0.1 * 0.1 * 1000$$
$$= 900$$

$$N_2(0.1) = N_2(0) + 0.1 * r_1 * N_1(0-\tau_1) - 0.1 * r_2 * N_2(0) \qquad (3.11)$$
$$= 0 + 0.1 * 0.1 * 1000 - 0.1 * 0.2 * 0$$
$$= 100$$

$$N_3(0.1) = N_3(0) + 0.1 * r_2 * N_2(0-\tau_2) \qquad (11)$$
$$= 0 + 0.1 * 0.2 * 0$$
$$= 0$$

We repeat the steps for each time step until we reach the required time($t = 10$)
After repeating the steps for each time step, we obtained the following results:

Table 1. Calculate $N_1(t), N_2(t), N_3(t)$ for each time value)$t = (0{:}10)$).

t	$N_1(t)$	$N_2(t)$	$N_3(t)$
0.1	900	100	0
1.0	348.6785	229.6816	16.3861
2.0	121.5766	322.8849	37.618
3.0	42.3908	361.1791	89.3136
4.0	14.7506	377.8084	189.6318
5.0	5.1423	382.3222	338.8941
6.0	1.7927	378.8712	490.6279
7.0	0.6247	371.2503	637.1554
8.0	0.2183	360.8207	774.8617
9.0	0.0760	350.6312	902.1559
10.0	0.0266	340.4722	1028.0097

Hence, the final result of the insect population at the end of the time period($t = (0{:}10)$)$(N_1(10.0), N_2(10.0), N_3(10.0)) = (0.0266, 340.4722, 1028.0097)$

These are the approximate numbers of insects of the species N_1, N_2 and N_3 at the time $t = (0{:}10)$ according to the delay differential equation model we used
From the solution we conducted using a delay differential equation model to study the insect life cycle, we can conclude several things:

- **The effect of delay between life stages** : The model explained how the delay between life stages has a significant impact on the insect's life cycle. Delaying the start of the second and third stages has been shown to lead to changes in insect numbers in each stage.
- **Growth of the insect population in stages** : Thanks to the model, we can understand how the number of insects in each stage develops over time. In this case, the number of insects increased in the last stage while it decreased in the first and second stages.
- **The importance of understanding the life cycle for effective control** : A good understanding of the insect life cycle helps in developing effective strategies to combat it. This understanding may help determine appropriate time periods for insect control interventions or the use of biological methods to reduce insect populations.
- **Future projections** : The model can be used to predict future insect populations based on different environmental conditions and possible interventions. This can help in developing sustainable strategies to control insect populations.

Conclusions

The incorporation of delayed differential equations into insect life cycle modeling represents an important development in mathematical modeling of biological systems. This approach allows for a more accurate representation of complex biological processes and enhances our understanding of insect population dynamics and the impact of temporal factors on them. Through these models, we can improve pest management and develop effective strategies to control insect populations, enhancing general understanding of ecosystems and the interaction between organisms and their environments.

Future prospects:
This research can be expanded to include more complex models that take into account additional factors such as multiple environmental interactions, climate changes, and human interventions. This research could have a significant impact on environmental science, agriculture, and natural resource management.

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