Analytical Solution for Anthropomorphic Limbs Model, (IK of Human Arm)

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Abstract

This paper considers a proposed algorithm for computation of the inverse kinematics(IK) model of the human arm. This algorithm introduces a new IK method suitable for reaching tasks performed by autonomous and interactive virtual humans. The basic problem is to pose the character in such a way that arm hand reaches the target (position and orientation) in space. The algorithm is composed of two phases. The first phase is the limitation of real task which concerning the human arm movement and the second phase presents the analytical solution for inverse kinematics problem (IKP) by trigonometric relations and algebraic solution according to limitation of joints. This algorithm is simulated by using MATLAB Ver. R2008a, and satisfied results are obtained, that explains the ability of the proposed algorithm to solve the inverse kinematics problem for real human arm.

الحل التحليلي لموديل الأطراف البشرية الافتراضية (الحركة العكسية للذراع البشري)

الخلاصة

يعرض البحث خوارزمية مقترحة لحساب الحركة العكسية لموديل الذراع البشري، تقدم الخوارزمية طريقة جديدة مناسبة للأداء وتمثيل حركات الذراع البشري الحقيقي.
المعضلة الأساسية في حساب الحركة للذراع البشري تتمثل في وصول اليد البشرية إلى موقع واتجاه نقطة الهدف. الخوارزمية المقترحة تتكون من طورين ، الأول يتلخص في معرفة المحددات الطبيعية لحركات الذراع البشري والطور الثاني يقدم الحل التحليلي لمعضلة الحركة العكسية باستخدام العلاقات المثلثية والمعادلات الجبرية لمفاصل الذراع البشري.
تم تمثيل هذه الخوارزمية باستخدام الماتلاب لعام 2009 وتم الحصول على نتائج مرضية توضح أمكانية حل مشكلة الحركة العكسية للذراع البشري باستخدام الحل التحليلي وفق المحددات الطبيعية للذراع البشري.

Introduction

The human arm mechanism consisting of the shoulder ball-and-socket joint with rotation axes for abduction-adduction, flexion-extension , and internal-external rotation of the upper arm, the elbow double-hinge joint with rotation axes for flexion-extension , and pronation-supination of the forearm and the wrist double-hinge joint with rotation axes for ulnar-radial deviation , and flexion-extension of the hand. For simplification will consider the elbow

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pronation-supination rotation as a wrist joint angle, since it only effects the hand orientation and not its position. The shoulder and elbow joints are connected through the upper arm segment with the length , and the elbow and the wrist joints through the forearm segment with the length[1]. The inverse kinematics problem (IKP) of the human arm can be stated as follows: given the position and the orientation of the hand, find the seven joint angles. Since the given position and the orientation of the hand specify six, rather than seven, independent quantities, the arm is a redundant system, and there are an infinite number of solutions for the joint angles. Therefore, the number of degrees of freedom (DOFs) of the upper extremity is such, that by using a rehabilitation robot attached to the hand, human arm joint angles are neither controllable nor observable.[2-3]. Several inverse kinematics algorithms have been proposed specifically for the human arm. The workspace of the mechanism was systematically analyzed by observed that the first two shoulder joints along with their joint limits restrict the tip of the elbow to lie on a spherical polygon. By intersecting the elbow swivel circle with the polygon it is possible to determine the legal elbow configurations as a function of the joint limits of the first two joints. Additionally, the twist induced by the third joint also restricts the elbow to lie on a circular arc. By taking the intersection of all sets of valid elbow arcs, and it derived the restrictions on the elbow position by the joint limits. The another solving by an analytic approach. The basic strategy of all solution in this approach is to reduce the degrees-of-freedom of the arm by one, where the idea of approach it is possible to obtain the closed-form equations that solve the inverse kinematics[4]. The latest approach dealing with human arm as anthropomorphic limb with all real movements path and explain an analytic solution for IKP of human by solving the angle of elbow joint by calculation the swivel angle and determine the elbow position, but this solution has high computation and complex and low accuracy[5] Kinematics is concerned with the motion of articulated structures The first element of such a chain is denoted as root, the last one is the end-effector. Each joint is capable of rotation around one or more axes. Each axis of rotation adds one degree of freedom, referred to as DOF. The links and joints of a kinematic chain may vary in size and possible angles of rotation, respectively[6]. In order to describe a kinematic chain, we are going to view the rotations performed by the joints of a chain separately from the setup of the chain, i.e. length of links, position and orientation of joints, etc. It is necessary to combine the kinematic chain with these additional rotations of the joints. This can be done, if all changes in orientation and position are put into the same format[7].In the space, translation and rotation can be expressed by homogeneous transformation matrices. In the 3D Euclidean space, these take the form of 4×4 matrices which can be

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concatenated simply by matrix multiplication. Since it is essential to combine the frame transitions with other translations and rotations, foremost the rotations performed by the joints, the rotation matrices and translation vectors have to be transformed to homogeneous transformation matrices. To transform a 3 × 3 rotation matrix or a 3 × 1 translation vector into homogeneous transformation matrices, they are positioned in a 4 × 4 matrix where the remaining entries are filled with the identity matrix. A rotation matrix R replaces the upper left part of the 4 × 4 matrix, a translation vector T replaces the three upper entries of the last column[8]:

$$\left.\begin{aligned} rot_{hom} &= \begin{bmatrix} & R_{3\times3} & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ tran_{hom} &= \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & t_{3\times1} \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ rot_{hom}.tran_{hom} &= \begin{bmatrix} R_{3\times3} & & & t_{3\times1} \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}\right\} \quad \ldots.(1)$$

demonstrated, asingle homogeneous transformation matrix can be equipped with both a translational and a rotational part. This enables a composition of the rotations performed by the joints and the frame transitions caused by the nature of the kinematics chain.

$$\left.\begin{aligned} A_i &= Rot_{x,a_i} Trans_{x,a_i} Rot_{z,q_i} Trans_{z,d_i} \\ &= \begin{bmatrix} \cos q_i & -\sin q_i & 0 & b_i \\ \cos a_i \sin q_i & \cos a_i \cos q_i & -\sin a_i & -d_i \sin a_i \\ \sin a_i \sin q_i & \sin a_i \cos q_i & \cos a_i & -d_i \cos a_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}\right\} \ldots \quad (2)$$

The kinematics analysis of an n-link manipulator can be extremely complex and the conventions introduced below simplify the analysis considerably. Moreover, they give rise to a universal language with which robot engineers can communicate. A commonly used convention for selecting frames of reference in robotic applications is the Denavit-Hartenberg, (DH) convention. In this convention, each homogeneous transformation Ai is represented as a product of four basic transformations[9]

where the four quantities θ_i α_i, a_i, d_i, are parameters associated with link (*i*) and joint (*i*). The four parameters a_i, α_i, d_i, and θ_i in equation(2) are generally given the names link length, link twist, link offset, and joint angle, respectively. These names are derive from specific aspects of the geometric relationship between two coordinate frames, as will become apparent below. Since the matrix A_i is a function of a single variable, it turns out that three of the above four quantities are constant for a given link, while the fourth parameter, θ_i for a revolute joint and d_i for a prismatic joint, is the joint variable.

In order to obtain a systematic representation of the workspace produced by the motion of a point of interest (typically called a point on the end-effector), we will use the (D-H) method adopted from the field of robotics Consider Fig.1. where three consecutive links are shown[10].

Let Z_{i-1} and Z_i represent fixed axes at either end of link *i*-1, about which or along which links *i*-1 and *i*

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move, respectively. Let axes X_{i-1} be defined from Z_{i-1} to Z_i and perpendicular to both. Let Y_{i-1} represent the unique axis that together with X_{i-1} and Z_{i-1} completes a right-hand Cartesian coordinate system. Let Z_i represent a vector from O_{i-1} parallel to Z_i . Let X_{i-1} a vector from O_i parallel to X_{i-1} as illustrated in Fig.2.

Note that the four parameters , d , a , , completely define the relation between any two consecutive frames. These values are entered in a table, which is typically known as the DH Table. The overall Denavit-Hartenberg coordinate transformation matrix from frame *i* coordinate system relative to the frame *i-1* coordinate system is then given by matrix A_i same as equation(2).

Similarly, for an *n*-DOF model of a limb, the global joint and end-effector frames using equation(3) are restated using *n*-homogeneous transformation matrices.

$A_1, A_1, A_1, A_1, \mathbf{L}, A_n$ The transformation matrix from the end-effector frame to global frame is then obtained by pre-multiplying each matrix in series as[11-12]:

$$A_0^n(q) = A_1(q_1)A_2(q_2)A_3(q_3)\,\mathbf{L}\,\mathbf{L}, A_n(q_n) \quad (3)$$

Mathematical Model of Human Arm

The development of a high-DOF, kinematics is discusses human model that can be used to predict realistic human arm postures. For this purpose, at last the model is built upon the six-DOF spine and five-DOF shoulder . This model falls short of realistic arm postures, however, as a result of spine rigidity and unrealistic skin deformations with shoulder movements. The goal of this work, therefore, is to develop an improve model that leads to more natural movement and allows for more realistic skin deformations in the shoulder. From this description one may deal with anthropomorphic arm by 7-DOF and assume the origin at shoulder joint[13-14].

The first joint is the shoulder joint s with 3 DOFs. The elbow joint e has only one DOF. The wrist joint w is of the same type as the shoulder joint s and also has 3 DOFs. Note that the arrow at the end of the chain indicates the end-effectors orientation and is not another link.

It can be focused on a kinematic chain that is formed after a human arm. This means the kinematic chain has 3 joints with spherical joints as shoulder and wrist joint and a hinge joint as the elbow joint. The spherical joints have 3 DOFS while the hinge joint has only one DOF, giving a total of 7 DOFs for this kinematic chain, see Fig.3.

Forward Kinematic Matrices can be setup of the kinematic chain of our interest has seven DOFs and three joints, i.e. shoulder, elbow and wrist. The shoulder joint is positioned at the origin of the world frame. The following equations can all be modified to match the case of an arbitrarily positioned shoulder. The specifications of the arm can be assume to know form of the translation and rotation from one joint frame to another. The entire arm is therefore expressed as two frame transition in

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between the three rotations of the respective joints. As mentioned before, may be used the 4×4 homogeneous matrices to express the joint rotations. The homogeneous transformation matrices for the frame transitions are set up with (D-H) parameters. The rotation and the translation of the frames are described by writing the D-H parameters for all joints [15-16].

Fig.4. shows the complete kinematics chain, including the frame transitions. The transition from the world frame to the frame of the end-effector is achieved by concatenating all seven transformation matrices. The sequence shows an instance of the kinematics chain where one after another the joints are being altered[19].

Every joint has transformation matrix same as matrix in equation(2).Given that the first joint of our kinematics chain is located at the origin of the world (the shoulder of arm) frame, the position and orientation of the end-effector is given by the concatenation of all the homogeneous transformation matrices. The forward kinematics is achieved by the whole multiplication of matrices ($A_1, \ldots, A_n$) i.e.,

$$G = A_1 A_2 A_3 A_4 A_5 A_6 A_7 \qquad \ldots\ldots(4)$$

G matrix represents the orientation and position of the end-effector in the world frame(the wrist of arm). With the rotational part R_g and the translational part t_g, the matrix G takes the form[17-18].

$$G = \begin{bmatrix} & R_g & & t_g \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots..(5)$$

Proposed Algorithm of Analytical Solution

The IK will be considered now. The explanation of solution for the seven joint angles of the kinematic chain introduced in the previous section analytically is present. The proposed algorithm can be explain in the following steps and the Fig.6.

Step one: the starting point is setting the shoulder joints as origin point and then define the D-H parameters for all seven joints and we will get seven transformation matrix has the form same as matrix A_i in equation(2),where (i=1,….,7).And then define the position and orientation of end-effector to get the G matrix which define in equation(5) and then calculate the three lengths: the first Lt_g which represents the distance between end effector and shoulder joint:

$$Lt_g = norm(\overrightarrow{t_g}) \qquad \ldots\ldots\ldots\ldots\ldots(6)$$

The second length (L_a) represent the length of upper arm and it equal to a_3 in D-H parameter. The third length (L_b) represent the length of lower arm and it equal to a_4 in the D-H parameters.

Step two: As (Lt_g ,L_a ,L_b) where compute from step one the expected form of human arm may be considered as in Fig.5.

The angle of elbow joint (θ_4) can be calculate by:

$$q_4 = p \mp \cos^{-1}\left(\frac{L_a^2 + L_b^2 - Lt_g^2}{2L_a L_b}\right) \ldots(7)$$

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From experience concept the reality constrain on the value of elbow angle for human arm ($15^{\circ} < \theta_4 < 160^{\circ}$), will be taken the first value of θ_4 (i.e. θ_4=pi-cos…).

Step three: It is well known the position of wrist (which represent the position of end effector) depends only upon the first four joints and this fact can be written in the equation :

$$A_1 A_2 A_3 A_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = t_g \quad \ldots\ldots(8)$$

We know the first three matrices (A1,..A_3) represents the transformation of shoulder to elbow while A_4 represent the transformation from elbow to wrist.

Step four: In this step the reality of human arm for a given D-H parameters is tested, which describe shape with 7-DOF in space. It may describe human arm form in space, that can do some jobs with acceptance error or it can represent the manipulator with 7-DOF. This checking is achieved by the solution of equation(8), if it has a unique solution by comparison of two sides of the equation. This case is for human arm and has the angle of shoulder represent by relation between the position of end-effectors in (x,y,z) axes. If equation(8) has no solution, that means it represents a manipulator with 7-DOF as human arm, and all these cases can be solved by using any other method for IK solution, which explained in[4,7,9]. Usually numerical methods are used if and only if all transformation matrices has no singularity.

Step five: Now we can find the rotation matrix of wrist joint by finding the rotation matrix of shoulder joint using the angles (θ_1 , θ_2 , θ_3) and the rotation matrix of elbow joint by using θ_4 ,then used the following form:

$$R_g = R_1 R_2 R_3 R_4 R_5 R_6 R_7 \ldots\ldots(9)$$

Considering that: $R_w = R_5 R_6 R_7$ ....(10)

Then, $R_w = R_4' R_3' R_2' R_1' R_g$ ....(11)

By using the comparison between two sides we can get the formula for (θ_5 , θ_6 , θ_7) Finally, the computed values of joint angles (θ_1,…., θ_7) can check the accuracy of the solution by applying these values in forward kinematics formula which described in equation(4) and checking the error in the position of end-effector.

Case Study

The proposed algorithm is simulated using MATLAB Ver.R2008a with many cases and get satisfied result for solution of IKP. Now the following case will be considered, where the human arm has the following D-H parameters that described in table 2.

The following position of end-effector is considered [29.71 , 13.45 , 23.09] with the orientation at(π , π /2 , π /4). All initial value of angles joint are zero. When the proposed algorithm steps are applied the following are obtained:

L_a=20 cm ; L_b=25 cm; while Lt_g=39.95 cm;

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The rotation matrix and position of end-effector can be represent by the matrix:

$$G_{end-effector} = \begin{bmatrix} 0 & 0.7071 & -0.7071 & 29.71 \\ 0 & -0.7071 & -0.7071 & 13.45 \\ -1 & 0 & 0 & 23.09 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q_2 = \tan^{-1}\left(\frac{-a_4 \cos(q_3)}{a_3 t_{gy}}\right)$$

Apply equation(7) (θ_4) is calculated with the value of =(1.5728 rad). And applying the equation(8) the following equations are obtained:

$$(-c_1 s_2 c_3 - c_1 s_3) a_4 - c_1 s_2 a_3 = t_{gx}$$
$$-c_2 c_3 a_4 - c_2 a_3 = t_{gy}$$
$$(-s_1 s_2 c_3 + s_1 s_3) a_4 - s_1 s_2 a_3 = t_{gx}$$
$$where: \quad s_i = \sin(q_i), \quad c_i = \cos(q_i)$$

It is quit clear that the above equations have unique analytical solution where:

$$q_1 = \tan^{-1}\left(\frac{t_{gz}}{t_{gx}}\right)$$

$$q_3 = \cos^{-1}\left(\tan(q_1) \cdot \frac{a_3 t_{gz} - a_3 t_{gx}}{a_4 t_{gx} - a_4 t_{gz}}\right)$$

When applying the considered D-H parameters in these equations the following values for shoulder joint are obtained:

θ_1=0.7854 rad , θ_2=-0.6155 rad , θ_3=0.4243 rad

To find the value of angles of wrist joint , with the considering of R_w as in equation(10) the following is obtained:

$$R_w = \begin{bmatrix} c_5 s_6 c_7 - s_5 s_7 & -c_5 s_6 s_7 - s_5 c_7 & c_5 c_6 \\ -c_6 c_7 & c_6 s_7 & s_6 \\ -s_5 s_6 c_7 - c_5 s_7 & s_5 s_6 s_7 - c_5 c_7 & -s_5 c_6 \end{bmatrix}$$

Computing the matrix R_w using equation(11) the following is obtained:

$$R_w = \begin{bmatrix} 0.5785 & 0.0015 & 0.8157 \\ 0.5661 & 0.7192 & -0.4028 \\ -0.5873 & 0.6947 & -0.4152 \end{bmatrix}$$

By comparing the internal values and relations for the matrix R_w the wrist joint angles may be calculated as follows:

$$q_5 = \tan^{-1}\left(\frac{-R_w(3,3)}{R_w(1,3)}\right); q_5 = 0.4708\, rad.$$

$$q_7 = \tan^{-1}\left(\frac{R_w(2,2)}{-R_w(2,1)}\right), q_7 = 0.4146\, rad.$$

$$q_6 = \tan^{-1}\left(\frac{R_w(2,3)}{-R_w(3,3)/\sin(q_5)}\right), q_6 = 0.904\, rad.$$

Now, applying the computed values for angle joints(θ_1--- θ_7) the forward kinematic may be computed using equation(4);

$$G = \begin{bmatrix} 0 & 0.7071 & -0.7071 & 29.712 \\ 0 & -0.7071 & -0.7071 & 13.452 \\ -1 & 0 & 0 & 23.094 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

When comparison between the end-effector matrix and the forward matrix by using the solving values of joints we can calculate the error and

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can checking the accuracy of algorithm:

$$\left[e_{t_{gx}}, e_{t_{gy}}, e_{t_{gz}}\right] = \left[2.4\times10^{-3}, 2\times10^{-4}, 4\times10^{-4}\right]$$

Conclusions

This paper introduces a proposed algorithm to solve the IKP for Anthropomorphic Limbs by analytical solution with high reality, while the latest analytical solution introduced in 2008[5] has high and complex computation with low accuracy, that can not be implemented practically.

From the obtained results, it can be seen, that it can be implemented practically, because of using very simple equations for the solution of IKP.

By another setting of D-H parameters, the Human Arm may able to do another movement. This fact is very important by decreasing the dependency of changing of D-H parameters on the ability of Human Arm to do any real movements.

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Table (1) Numeric value for D-H parameter

Frame (joint)	θ_i *(rad)*	d_i *(cm)*	a_j *(cm)*	α_j *(rad)*
1	θ_1	0	0	Pi/2
2	θ_2	0	0	pi/2
3	θ_3	0	20	-pi/2
4	θ_4	0	25	Pi/2
5	θ_5	0	0	-pi/2
6	θ_6	0	0	pi/2
7	θ_7	0	0	-Pi/2

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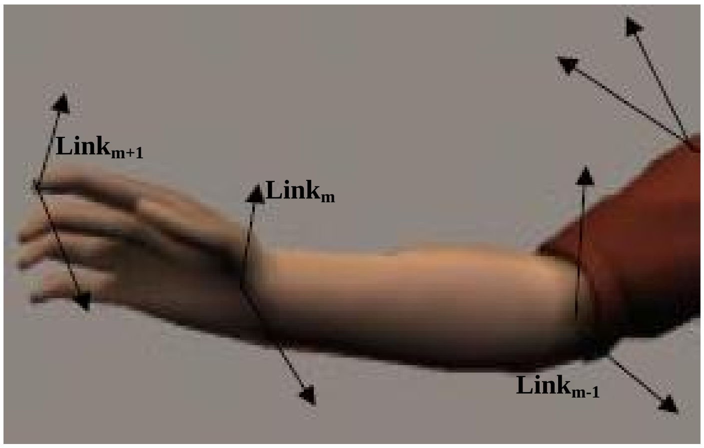

Figure (1) define the joint reference frames for the D-H representation.

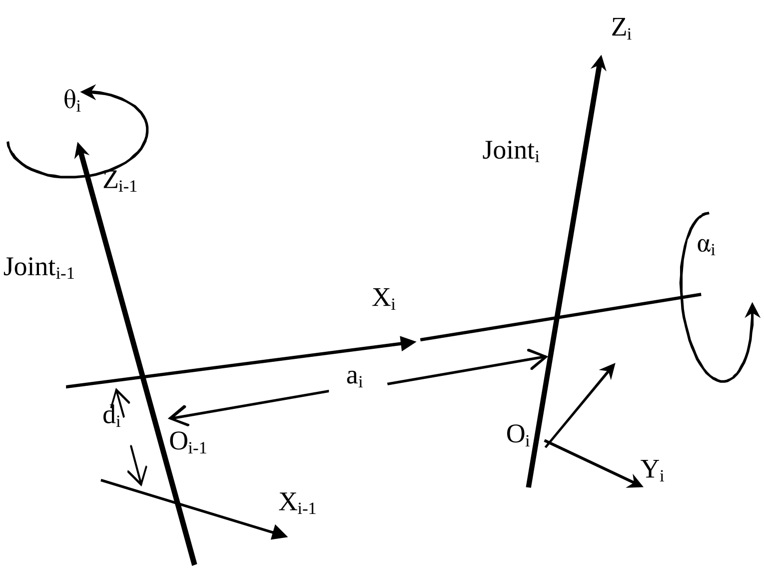

Figure (2) the relation between two consecutive coordinates

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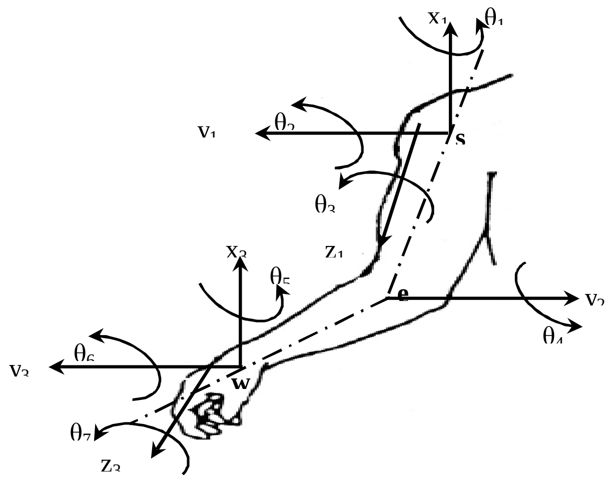

Figure (3) kinematics chain with 3 joints

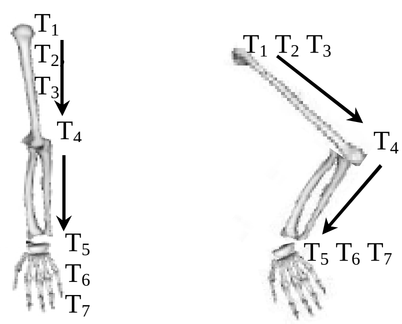

Figure (4) complete kinematics chain with 7 DOFs

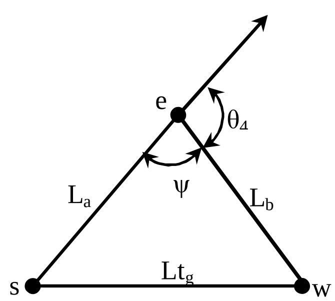

Figure (5) The trigonometric structure of joints human arm

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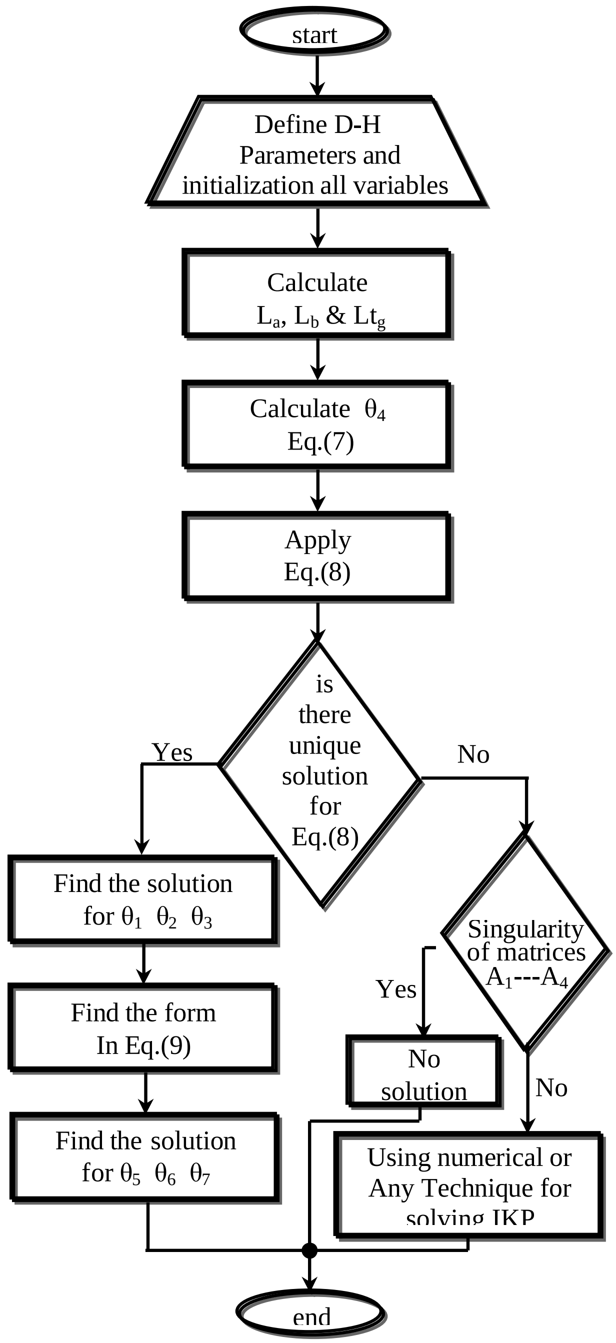

Figure (6) The proposed algorithm for IKP of human arm

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