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Data of Scholl's Normal Distribution

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Abstract

We have introduced two families of continuous distribution functions with not necessarily identical densities , which include the original distribution as a special case . Two sets of proposals are based on two parameters and are offered as an alternative to the skewed normal distribution and other assumptions in the statistical literature . The density functions for these new families are given by a closed expression that allows us to easily calculate probabilities, moments, and their associated quantities . The second family can show two sides and the fourth uniform center point (kurtosis) can be less than the normal distribution of skewness . Since the proposed second family could be biphyletic , we included two known datasets.

Keywords: logistic distribution , symmetric distribution , normal distribution , normal distribution.

البيانات الحقيقية للتوزيع الطبيعي لشول

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الخلاصة

لقد قمنا بتقديم عائلتين من دوال التوزيع المستمر مع عدم وجود كثافة متماثلة بالضرورة ، والتي تتضمن التوزيع الأصلي كحالة خاصة . تعتمد مجموعتان من المقترحات على معلمتين ويتم تقديمهما كبديل للتوزيع الطبيعي المنحرف والافتراضات الأخرى في الأدبيات الإحصائية . يتم تقديم دوال الكثافة لهذه العائلات الجديدة من خلال تعبير مغلق يسمح لنا بحساب الاحتمالات والعزوم والكميات المرتبطة بها بسهولة . يمكن للعائلة الثانية أن تظهر وجهين ويمكن أن تكون نقطة المركز الرابعة الموحدة (التفرطح) أقل من التوزيع الطبيعي للانحراف . وبما أن العائلة الثانية المقترحة يمكن أن تكون ثنائية النمط ، فقد قمنا بوضع مجموعتي بيانات معروفتين مع هذه الميزة كتطبيقات . نحن نركز على الحالة التي يكون فيها التوزيع الطبيعي هو التوزيع الأصلي ، ولكن نأخذ في الاعتبار التوزيعات الأصلية الأخرى ، مثل التوزيع اللوجستي .

الكلمات المفتاحية : التوزيع اللوجستي ، التوزيع المتماثل ، التوزيع الطبيعي ، التوزيع الطبيعي الشول

1. INTRODUCTION

Since the distant past, in statistical discussions of parametric households and their details, experts have discussed this topic, and in the past few years, this topic has been associated with great success . A large part of these studies is the family normal Scholl's distribution.

Scholl distributions are mostly used in medical and behavioral sciences where the true values of random variables have an asymmetric distribution . Due to the

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necessity of using Scholl's distributions in modeling and especially in analyzing medical data, they have been presented as one of the most useful distributions.
For the first time, we created a normal distribution by adding a skewness factor to the normal distribution . The t-chole distribution is a more general form of the normal chole distribution and allows the model to have tails in addition to skewness .

1. 1. The importance of research

Basic research forms the solid foundation for any development and progress, and provides the basis for the growth and excellence of societies . Imagine comprehensive development without this research; The idea is false and meaningless, and there is no doubt that it will lead nowhere, and sooner or later in the context of experimentation, especially long-term experiments, its invalidity will be revealed to everyone . Valuable research work is like fresh blood that flows through the veins of science and knowledge and gives it new life.
Scholl distributions are mostly used in medical and behavioral sciences where the true values of random variables have an asymmetric distribution . Due to the necessity of using Scholl's distributions in modeling and especially in analyzing medical data, they have been presented as one of the most useful distributions.
According to the cases mentioned in this research, we will express examples of real data of the normal function of the shul.

1. 2. The importance of research

In many statistical models, accepting the assumption of normality of data is a common assumption . This assumption may reduce the accuracy of the inference if the distribution of observations is asymmetric . Therefore in recent times, much research has been conducted to substitute a new class of distributions, including the distributions that have been presented and analyzed in this thesis, instead of the normal distribution . This replacement has achieved satisfactory results in many cases . This increased the degree of accuracy of statistical inferences and analyses . The applications mentioned in this section will prove this .

1. 3. research objectives

In the real world, every day with scientific advances, we deal with more diverse data in many application fields . In this regard, we need more and more flexible distributions for data modeling . In general, the new distributions are more flexible than the real data, having a higher degree of elongation and skewness . The family of distributions presented in this thesis was studied, which includes symmetric and asymmetric distributions with different degrees of skewness and kurtosis, which can be fit to different data by changing the parameters . These models show good fit with many data, especially economic and medical data
. is one of the few continuous distributions, which is very important in statistics Usually, . This distribution has characteristics that have increased its practical use in statistical analysis, the assumption of normality of the data is accepted and then

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In many cases in the practice of data distribution . statistical inferences are made the true values contain some skewness, and in this case, it is no longer possible In these cases, an . to use the normal distribution to analyze the data accurately attempt is often made to approximate the data distribution to a normal distribution I did the . using an appropriate transformation and then analyzing the data This work is not very convenient in practice because it reduces the . transfers accuracy of the analysis and on the other hand, finding such a transformation is sometimes a difficult task and in practice such a transformation may not be found Therefore, if an asymmetric distribution has the same properties as a normal . One such . distribution, it can play an essential role in analyzing asymmetric data recently introduced distribution that has attracted a lot of attention and has properties similar to the normal distribution is known as the Chole normal This asymmetric distribution has a parameter to adjust the skewness . distribution So if the value of this parameter is zero, the Chola normal distribution becomes . Therefore, to analyze data whose graph shows the amount . a normal distribution of skewness in the data a normal distribution can be more likely than a normal . distribution for this data

Let's actually imagine a bell-shaped curve When we talk about normal distribution That there is symmetry around a number, which is called the mean of the Some of the distributions examined in this thesis have properties . distribution . similar to the normal distribution

Which are either skewed compared to the normal distribution or have wider tails, or have both characteristics together, that is, they are both skewed and have tails The important point about all distributions . wider than the normal distribution presented in this thesis is that the normal distribution is present as a major . component in all density functions of these distributions

This model of distributions In recent years recent studies have attracted much attention to the univariate Scholl standard normal distribution, which was first . was introduced (1931 . introduced by Azalini

Sports Hope 2-2

: The mathematical expectation of a random variable is defined as follows

$$E(X) = \int x f_x(x) dx$$

is the probability density function of the random variable X. For where $x f_x(x)$ *: discrete random variables, the above definition is rewritten as follows*

$$E(X) = \sum_{i=1}^{n} x_i P_X(x_i)$$

Variance is a number that shows how a series of data is spread around the mean It has a For Verda's definition, if we assume that the only variable . value It shows the average distribution of the population .distribution P(X) μ *and then : the average of this population is determined as follows*

$$Var(X) = \sigma^2 \equiv ((X - \mu)^2)$$

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In statistics and probability, the moment is a quantitative measure to describe the For example, the second moment represents . shape of a probability distribution the display of a number of samples in one dimension, and in higher dimensions Other moments . it represents the shape of an ellipse that estimates the data describe other aspects of the probability distribution, such as the deviation from . the mean
For higher order torques, the torque around the . The first moment is the average k -The central - . mean is usually calculated and is called the central torque moment of the random variable

$$\mu_k = E[(X - \mu)^k]$$

. E is mathematical hope is the average of the random variable and whereμ
In probability theory and statistics, a probability distribution function represents in the case of a discrete variable and with the) each value of a random variable probability of the variable's existence in a given period of time in the case of any The cumulative distribution of the probability of .(continuous random variable the presence of a random variable is a function of the range of that variable over so that events with a numerical value less than that are displayed ،[0,1] the interval : It is presented and detailed as follows .

$$F_X(x) = P_r[X \leq x]$$

identification will be
It is called discrete or Based on whether the variable is discrete or continuous . continuous
Normal distribution One of the most important Probability distributions Continuing on Probability theory is . The reason for this name, in addition to the importance of this distribution, is that the oscillation of many natural (physical) quantities around a fixed value follows this distribution . The reason is Limit theory Middle be.
based on The case end Central , total Independent random variables that each have a specific mean and variance. As the number of variables increases, their distribution is very close to the normal distribution . For example, although several factors affect the measurement error of a quantity (such as measuring device error, reading error, environmental conditions, etc.) , but mistake measurement in Measurements continuous, He has distribution natural will be Which around amount pinned, Which Same thing middle mistake Scatter was sampled last, to rise, weight or IQ is people.
Normal distribution, sometimes due to use Gauss and this in his work is called the Gaussian distribution, as this distribution is due to the form The probability density function is also known as the bell-shaped distribution.
The probability density function of this distribution has two parameters, one... average (μ) and else It is the standard deviation (σ) of the distribution . The probability density function curve for this distribution is symmetrical about its mean . when μ=0and σ=1This is the distribution Normal standard Named it will be

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With the placement $f = \emptyset$ *and* $G = \Phi$ *in* $w(x) = \lambda x$ *that (2 - 1) the relationship*
The density function $\emptyset$ *and* Φ *the standard normal distribution function,*
:We conclude the following definition respectively
$g(x) = 2f(x)G(w(x))$ *(1-2)*
The density function is symmetric about zero and And (kh) $G: R \longrightarrow [0,1]$ *the distribution function is a continuous random variable that is symmetric about zero and an* $w: R \longrightarrow R$. *odd function*
The random variable z has a normal distribution, if the shape of its : Definition density function is given by $\emptyset(z;\lambda)$*: We show that the matter is as follows*

$$\emptyset(z;\lambda) = 2\phi(z)\Phi(\lambda z) \qquad z,\lambda \in R$$

If the random variable z has a normal distribution with parameters λ *it is ، abbreviated by z* $Z \sim SN(\lambda)$ *This form of the density function was first introduced . . (1985) by Azzalini*
positive and) Histogram of normalized density of Chola for different values (negative λ *We have drawn it in the figure below*

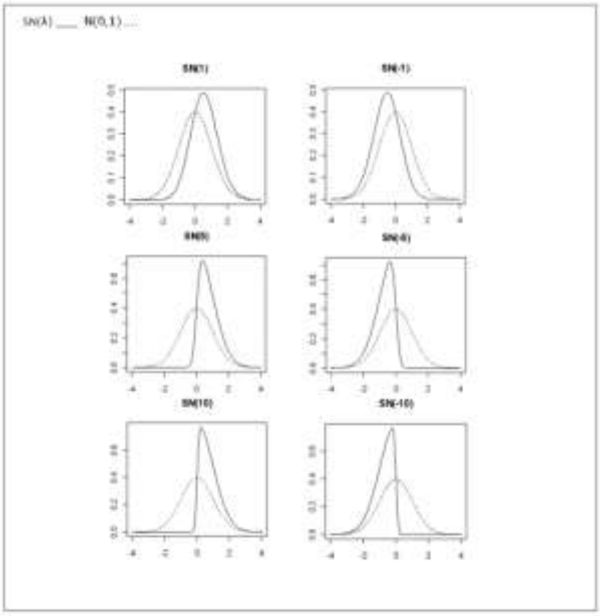

Figure 2-1. Graph of Chola normal density function for different values (positive and negative)λ

If **:1-2 Theorem** $Z \sim SN(\lambda)$ we have $X \sim N(0,1)$:

$$SN(0) = N(0,1)$$
$$-Z \stackrel{\text{def}}{=} SN(-\lambda)$$
$$|Z| \stackrel{\text{def}}{=} |X|$$

$$Z^2 \sim \chi_1^2$$

When then $Z \stackrel{\text{def}}{=} |X|$ that $\lambda \to +\infty$

When $\lambda \to -\infty$ then $Z \stackrel{\text{def}}{=} -|X|$

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. normal density function is strongly monocular Chole's

: guide

Using the definition of Scholl's normal density function, it can be$\Phi(0) = 1/2$. easily proven

Using the definition of Scholl's normal density function, it can be$\phi(-z) = \phi(z)$. easily proven

: According to the following theory

If$X_\pi \sim f_\pi$ and$X \sim f$ So it is an even function also $hh(X_\pi) \stackrel{\text{def}}{=} h(X)$.

. It's clear

According to the previous case, we have$Z^2 \stackrel{\text{def}}{=} X^2$ *because* $X^2 \sim \chi_1^2$ *So* $Z^2 \sim \chi_1^2$.

When$\lambda \to +\infty$: do we have

$$\phi(z;\lambda) = 2\phi(z)\Phi(\lambda z) = \begin{cases} 2\phi(z) & z > 0 \\ 0 & z < 0 \end{cases}$$

That the density function */X/* It is, as a result$Z \stackrel{\text{def}}{=} |X|$.

. It was proven as in Chapter Five

The density function *f* is strongly unimodal if and only if its logarithm is concave .

The following theorem states that a Scholl's normal random variable can be . written as a function of two independent normal random variables

The independent random variables are the normal *Fifth* and If *U* **:2-2 Theorem** standard and

$$Z = \frac{\lambda}{\sqrt{1+\lambda^2}}|U| + \frac{1}{\sqrt{1+\lambda^2}}V \qquad \lambda \in R$$

The random variable *Z* has a normal distribution with parametersλ.

: guide

We assume first$a = \frac{\lambda}{\sqrt{1+\lambda^2}}$ and$b = \frac{1}{\sqrt{1+\lambda^2}}$: as a result we have

$$P(Z_\lambda \le z) = E[P(Z_\lambda \le z)||U|]$$

$$= \int_0^{+\infty} P\left\{V \le \frac{z-au}{b}\right\} 2\emptyset(u)du = 2\int_0^{+\infty} \Phi\left\{\frac{z-au}{b}\right\}\phi(u)du$$

: We will get Now with the help of relationship$a^2 + b^2 = 1$

$$\frac{d}{dz}P(Z_\lambda \le z)$$

$$= 2\emptyset(z)\int_0^{+\infty} (2\pi b^2)^{-1/2} \exp\left(-\frac{(u-az)^2}{2b^2}\right)du$$

$$= 2\phi(z)\left\{1 - \Phi\left(-\frac{a}{b}z\right)\right\} = 2\phi(z)\Phi(\lambda z)$$

If *X* and *Y are* independent random variables of the standard **:2-3 Theorem** : normal distribution, then

$$X|(Y < \lambda X) \sim SN(\lambda)$$

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: guide

$$f_{X(Y<\lambda X)}(x) = \frac{f_x(x)P(Y<\lambda X|X=x)}{P(Y<\lambda X)} = 2\emptyset(x)\Phi(\lambda x)$$

: as a result of

$$X|(Y<\lambda X)\sim SN(\lambda)$$

Generalized normal distribution of chol 1-8-2

variable A random

$$f(x|\lambda_1,\lambda_2) = 2\phi(x)\Phi\left(\frac{\lambda_1 x}{\sqrt{1+\lambda_2 x^2}}\right) \quad x\in\mathbb{R}, \lambda_1\in\mathbb{R}, \lambda_2\geq 0$$

whereϕ and$\Phi$are the density function and distribution function for the standard . normal random variable, respectively

variable If the random

$$X\sim SGN(\lambda_1,\lambda_2)$$

It is one of [2]In the theory of statistics and probability, the normal distribution of Of course, this distribution is . the most important statistical distributions . or Gauss-Laplace distribution[3] sometimes called the Gaussian distribution [4]sometimes called a bell [5] Since this distribution has a bell-shaped curve, it is . curve

The rules that govern most random phenomena in life follow the normal The [6]distribution, on the other hand according to the central limit theorem For . approximate distribution of other phenomena can also be considered normal this reason, the application of this distribution is wide in all fields from sociology . to medicine and engineering

The importance and application of the normal distribution due to the central limit This theorem states that for random variables that have finite . theorem is variance, the means of random samples of independent and identical (iid) random For this reason, the distribution of . variables tend to have a normal distribution) most physical quantities obtained as the sum of several independent processes .normal is assumed to be (for example, measurement error

Likewise, many other methods such as parameter fitting using least squares are These reasons determine the . used when the data distribution is normal .importance of normal distribution in data analysis

a German physicist, mathematician, and scientist, [7] ‹Karl Joves " ‹ 1809 In In his . investigated phenomena whose probability function was bell-shaped Theory of the Motion of Celestial Bodies in the Conic Sections " manuscript titled

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Normal distribution [2]
Graphical distribution [3]
Laplace Gauss [4]
Bell curve [5]
Central limit theorem [6]
Carl Gauss [7]

”.[9] he examined the error rate of the “least squares method ”[8] ،of the Sun .normal distribution" of payment " And the [10]Maximum health

They There are many cases in which experimental data are partially asymmetric appear specific and this is often the case, for example, with actuarial and financial data which, in addition to this feature, have heavy tails that reflect their se or) property means that the data cannot be adequately modeled by a Gaussian .(normal) distribution

For . In addition, binary distributions appear naturally in many different scenarios example, in specific disease patterns, and also in specific cancer incidence curves some cancer incidence curves, and (and also multilaterally) By bifurcating . studying them, doctors can improve their understanding of cancer, its development process, and its characteristics, and improve the possibility of identifying cancer and distinguishing a particular type of cancer from all other . types of cancer

Types of this occur, for example, in cases where there are two peaks of incidence . These cancers include Kaposi's sarcoma and Hodgkin's lymphoma . at each age in young people and : The second type of cancer has two peaks in incidence On the other hand, the normal distribution appears naturally . middle-aged adults in random frontier analysis, as it is assumed that the normal distribution It is the qualitative component and the normal distribution of . represents noise the mean to show the ineffective expression in the event that the researcher See all The sample of companies of interest . imposes ineffective behavior on him .[39] ،is, for example

presents a parametric model (using a finite mixture model) [40] ،More recently of zero incidences of inefficiency which can account for the presence of both efficient and inefficient firms in the sample with a scenario applied in two different ways so we try to access families of distributions that have a bias towards the distribution Natural, but at the same time in terms of the ability to adapt to the scenario and the two faces that appear in different situations that . acceptable seem more diverse, it seems

Although there are different summation sites for obtaining skewed distributions two well-known and generalizable) from non-skewed prime numbers procedures: The initial probability distribution provides symmetric or among ، [4 ، 3] asymmetric, there are cases that have been mentioned in works Here our attention turns to the toxic position for this purpose, which was .(others which is famous and discussed in many works on this topic . ، [5] introduced by and the cumulative (pdf) Let g and G be the probability density function As a . for the symmetric distribution. , respectively (cdf) distribution function : is said to have a skewed distribution if its pdf is given by random variable Z

$$f_z(z) = 2g(z)G(\lambda z), \qquad -\infty < z < \infty, \lambda \in \mathbb{R}.(2\text{-}1)$$

The theory of the movement of celestial bodies in conic sections surrounding the sun [8]
Least squares method [9]
Maximum probability [10]

This family of distributions has been extensively studied as an extension of the In . normal distribution using a shape parameter, α, that accounts for skewness are replaced by pdf and cdf of the standard this case, g and G in the relation)1-2(normal distribution and the resulting distribution is called the skewed normal the g to be It should be noted that it is not necessary for the function . distribution pdf is (1-2) given in Equation the pdf to ensure that G for cdf exact derivative of . . file, although it has not been studied in depth in the statistical literature we denote the family of ، [6] Following the notation described in reference pdf and cdf tert b where φ and Φ to (Φ(lectz g(z) = 2φ(z) distributions given by In addition, . are the standard normal distribution, with SN. = {SN(π) : α ∈ IR} when there is a random variable of normal distribution with location parameter .(σ ،SN(lect, μ Following σ>0, we will write gauge Factories and ∞ > μ > ∞−
In this thesis, a new generalization of the family of Chola distributions presented which includes the family of Chola Azalini ، is presented (1-2) in relation The method .(1-2) distributions as a special case of the meaning of the statement used is based on the interpretation of Azalini's proposal and the result presented Later, .(1-2) which led to the addition of a new parameter to the family ، [7] by from this new family, the second family, very similar to the first family, will be This new family of distributions can appear bimodal and the standard . introduced less than the skewness of the normal can be (kurtosis) fourth-center moment .(It can be positive or negative) . skewed distribution
In recent decades, starting with Azalini's proposal, several generalizations and for example,) extensions of the Cole's normal distribution have been presented The methods used . among others ، [9-11] For multivariate suffixes, see .([8] see in this paper can be considered an extension and replacement of the famous Chole see) whose properties have been considered ، ([5,12] see) normal distribution . It has been widely discussed .[14] and the corresponding estimation ، ([12,13] ، ERB distributions methods have been presented to obtain normal Other Green's the high density normal in ، [15] such as the method proposed by reference and the generalized normal [17] the reference proposal model ، [16] reference For a comprehensive . in the tides and ebbs ، [18-20] . distribution in references and comprehensive study of the normal COL distribution, see the latest reference .[21] book
a family of refers to In the case, the term Cole-normal In a simpler way continuous probability distributions on the real line with a density function of the form

$$\phi(z;\alpha) = 2\phi(z)\Phi(\alpha z), \qquad (-\infty < z < \infty),$$

represents the density and distribution (0,1) { N } ranking Φ(.) l and (.)where ϕ is a real parameter that adjusts the shape α K function of the accumulation, and is consistent with a more general 1 converges to (1) The fact that . of the density are identical functions for each Φ where φ and ،Azzalini)1985(result given by . that are not shifted 0 of the two identical distributions around

If for a He lends, and if Z {n})0,1(0 = α Immediately choose the distribution is shown - Z ~ S}N(-α(} {S}N(-α), then ~ Z ، (1) random variable with density Only positive values of this . Shown for several choices of α 1-2$\phi(z;\alpha)$ in Figure . parameter are taken into account because they are stated

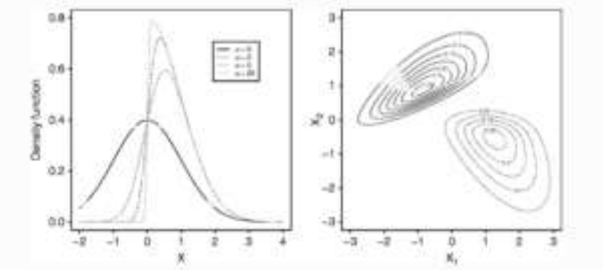

Figure 2-2 Some examples of the normalized density function for the numerical case (left side) and for the two variable cases in the form of contour plots (right).

As we see in the following result , multivariate extensions to univariate distributions are obtained in an easy way .

Case E 1-4 . Suppose let X and Y be random variables where X ~ N(0,1) and~N^((m)) (_0,Σ) _Ybe then,

$f(\underline{y} ؛\underline{\lambda},\alpha) = \frac{f\underline{Y}(\underline{y})}{\alpha}\int_{-\alpha}^{\alpha} F_X\left(z+\underline{\lambda}^T\underline{y}\right)dz$(4-1)

∋ α ∈ IR f- {0} Actual pdf 's Cheer k$\mathbb{R}^m$ $\underline{\lambda}$. He is

Without loss of generality , assume that X and Y are independent random **guide** symmetric fact and using the result given in this variables , taking into account .the theorem$X-\underline{\lambda}^T\underline{Y}$we get m 3-1

$$\alpha = \int_{-\alpha\alpha}^{\alpha} \Pr\left(X-\underline{\lambda}<z\right)dz$$

$$= \int_{R^m}[\int_{-\alpha}^{\alpha} \Pr\left(X<z+\underline{\lambda}^T\underline{y}\middle|\underline{Y}=\underline{y}\right)dz]f_{\underline{Y}}\left(\underline{y}\right)d\underline{y}$$

$$= \int_{R^m}[\int_{-\alpha}^{\alpha} \Pr\left(X<z+\underline{\lambda}^T\underline{y}\right)dz]f_{\underline{Y}}\left(\underline{y}\right)d\underline{y}$$

$= \int_{R^m} f\underline{Y}(\underline{y})(\int_{-\alpha}^{\alpha} F_X\left(z+\underline{\lambda}^T\underline{y}\right)dz)d\underline{y}$.

A only important feature of the distribution The .1 **Note**$\underline{Y}$That is, for every ،random variable$\underline{\lambda}\lambda^T\underline{y}$only thing needed to The . it is symmetric about zero This applies to the above case and the . X is symmetric about zero A distribute following case

Let X and Y be two **. 4-2 Case E**$X{\sim}N(0,1)$ random variables where and $\underline{Y}{\sim}N^{(m)}(\underline{0},\Sigma)$then

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$f(\underline{y} ؛\underline{\lambda}, \alpha) = f_{\underline{Y}}\left(\underline{y}\right)[F_X\left(\underline{\lambda}^T\underline{y} + \alpha\right) + F_X\left(\underline{\lambda}^T\underline{y} - \alpha\right)]$ (4-2)

∋ α ∈ IR f- {0} Actual pdf 's Cheer k$\mathbb{R}^m$ $\underline{\lambda}$. He is
as follows (4-1) The **evidence** comes from

$$\alpha = \int_{R^m} f_{\underline{Y}}\left(\underline{y}\right)(\int_{-\alpha}^{\alpha} F_X\left(z + \underline{\lambda}^T\underline{y}\right)dz)d\underline{y}$$

$= \int_{R^m} f_{\underline{Y}}\left(\underline{y}\right)[\int_{-\alpha}^{0} F_X\left(z + \underline{\lambda}^T\underline{y}\right)dz + \int_{0}^{\alpha} F_X\left(z + \underline{\lambda}^T\underline{y}\right)dz]d\underline{y}$(4-3)

and (4-3) on both sides of relation α If we now take the derivative with respect to . d apply the Fundamental Theorem of Calculus , we obtain

$$\int_{R^m} f_{\underline{Y}}\left(\underline{y}\right)\left[F_X\left(\underline{\lambda}^T\underline{y} + \alpha\right) + F_X\left(\underline{\lambda}^T\underline{y} - \alpha\right)\right]dy = 1.$$

Real pdf . (4-2) Therefore
we get (17) in 0 = α Note that if we set

$$f\left(\underline{y};\underline{\lambda}\right) = 2f_{\underline{Y}}\left(\underline{y}\right)F_X(\underline{\lambda}^T\underline{y}),$$

Chole relationship to - It was one of the first multivariate models of the normal . [11 ، 10] For example , see . appear in the literature

The density of the bivariate model of the generalized normal . 1-4 Figure for some parameters. Inspection of Figure (BGSN) relationship shows a decrease model shows a more interesting set of BGSN that the density of the confirms 1-4 It is expected to be elastic . The . possible shapes than many of its competitors . model is useful in fitting the model to different data sets

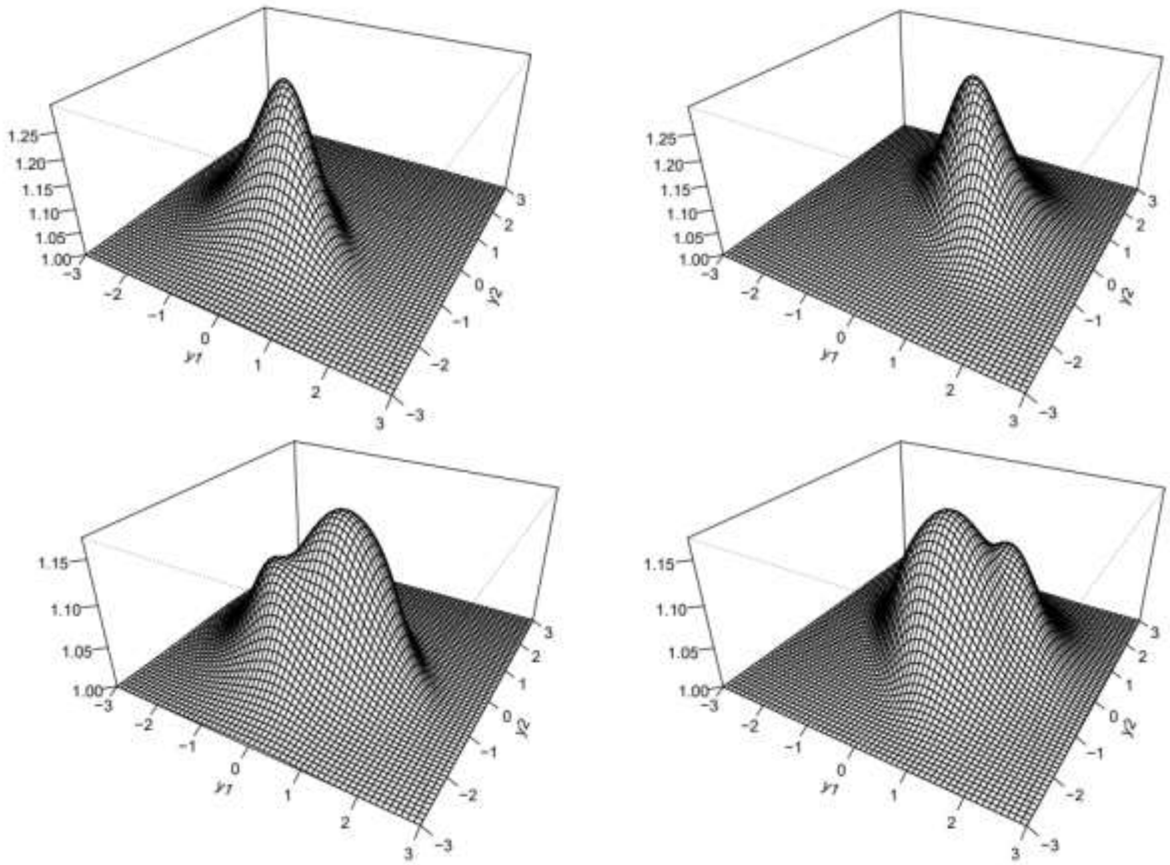

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Figure 1-4 . density l BGSN for : α1 = α2 = -3 and α = 1 (top left panel). α1 = lect2 = 3 and α = 1 (top right panel). α1 = lect2 = -3 and α = 5 (lower left panel). α1 = lect2 = 3 and α = 5 (lower right panel).

2. number of photos

(3-13) presented in Equation GSN model In this section, three examples of the were made and the results were compared with the flexible epsilon model of Chole In the first example one of them . [33] presented in reference Normal (FESN) - MN) in the second) was presented , with a combination of two normal models in the third [34] presented by reference FSN model example and the flexible Three density to be the third b by it . example

$$1 \quad f(y;\mu,\sigma,\lambda,\varepsilon) = \begin{cases} \frac{1}{2\sigma c_\lambda}\phi\left(\frac{y-\mu}{\sigma(1+\varepsilon)} - \lambda\right) & if \;\; y < \mu \\ \frac{1}{2\sigma c_\lambda}\phi\left(\frac{y-\mu}{\sigma(1+\varepsilon)} + \lambda\right) & if \;\; y \geq \mu \end{cases}$$

$$2 \quad f(y;\mu,\mu_1,\sigma,\sigma_1,p) = \frac{p}{\sigma}\phi\left(\frac{y-\mu}{\sigma}\right) + \frac{1-p}{\sigma_1}\phi\left(\frac{y-\mu}{\sigma_1}\right)$$

$$3 \quad f(y;\mu,\sigma,\lambda,\alpha) = \frac{2}{\sigma}\phi\left(\frac{y-\mu}{\sigma}\right)\Phi(\lambda\left(\frac{y-\mu}{\sigma}\right) + \alpha\left(\frac{y-\mu}{\sigma}\right)^3)$$

where φ(.) and Φ(.) represent the density and distribution functions of the standard ‹ (o)normal distribution , co = 1 - Φ
. ε < 1 and 0 ≤ p ≤ 1 > 1- ‹ σ, σ1 + μ μ1, α, ∈ R α ∈ t

We use these three models because they have been used in the applied statistics A . literature to explain bivariate experimental data . We chose the MN model class model was used to model two- dimensional data sets , and we chose the - Epsilon the It is one of the first two - dimensional expansions of . FESN model . chose the FSN model we metric distribution family , and - Kevil

1 Example

patients and is available 315 The data for this example is a set of fiber levels for and contains 14 http://Lib.stat.cmu.edu/datasets/Plasma_Retinol online at) analysis we will only use A variable called fiber For . for each patient variables of this variable may be associated levels Low . (grams of fiber consumed daily Descriptive statistics for the . with an increased risk of certain types of cancer Tables B1 and B2 , the deflection and In . 1-4 data set are presented in Table that the data shows a high level of Note. elongation of the sample are shown . flexibility

Descriptive statistics for the fiscal year : Fiscal year . 1-4 Table

n	$\bar{y}$	s^2	b_1	b_2
315	12.789	28.411	1.147	5.425

estimated values of the parameters of the two models are shown in Table 4-2 with standard errors (SE) in parentheses . The table also includes the maximum log likelihood function (l_max), the Akaike information criterion (AIC) and the consistent Akaike information criterion (CAIC), which were respectively proposed in references [35,36]. Model with AIC or Lower CAIC , higher value model is preferred .

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Table 2-4 . Parameter estimation (SE) of the models FESN and GSN

Parameter	FESN	GSN
μ	7.176 (0.405)	6.714 (0.329)
σ	4.396 (0.446)	8.076 (0.406)
λ	−0.288 (0.235)	9.692 (2.510)
n	-	2.486 (0.764)
ε	−0.695 (0.048)	-
ℓ_{max}	−949.438	−945.575
AIC	1906.916	1899.150
CAIC	1925.926	1918.160

As is . 2-4 The histograms of the fitted data and densities are shown in Figure GSN is considered better than these two models due clear , the distribution of A All calculations here were . to the reflection of the nature of the experimental data v.11.0 and WinRATS v.7.0. These codes are Mathematica performed using . available upon request

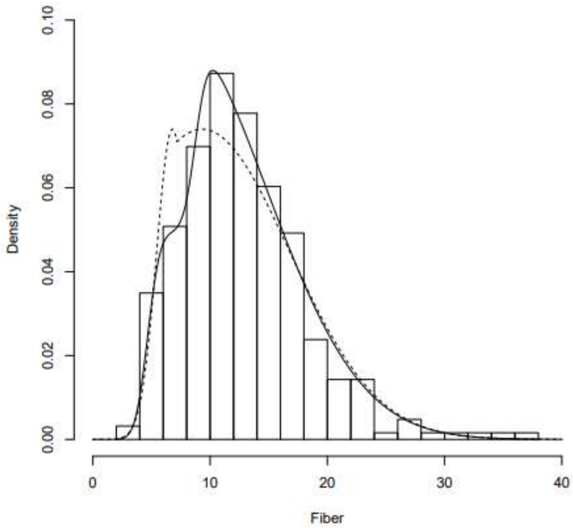

Figure 2-4 . Touz A FESN (China Line) and Tozyi A GSN (solid line) for fiber data .

2 Example

listed in the Cream Cheese Database which can be found M - Sweet is We are free . at http://www.models.kvl.dk/research/data/Cream/ Find Index.asp

. M- Sweet offers summary statistics for the data set provides 4-4 Table

Table 3-4 . M- Al-Helou : Descriptive statistics .

n	$\bar{y}$	s^2	b_1	b_2
240	3.276	1.964	0.882	5.049

Table 4-4 displays the parameter estimates (SE) for both the MN and GSN models . It can be seen that the probability of entering the system is higher for the GSN model compared to the MN model . Benchmark: AIC and CAIC are used again to compare the three estimated models , and it can be seen that the GSN model provides the best fit (the smaller AIC and CAIC).

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Table 4-4 . Parameter estimation (SE) of the models MN and GSN

Parameter	MN	GSN
μ	2.115 (0.115)	2.577 (0.126)
σ	0.498 (0.104)	1.564 (0.091)
λ	-	3.780 (0.852)
α	-	3.685 (0.944)
μ_1	3.727 (0.176)	-
σ_1	1.376 (0.084)	-
p	0.280 (0.077)	-
AIC	828.214	826.585
CAIC	850.617	844.507

. 3-4 Finally, the histogram of the processed data and density is shown in Figure

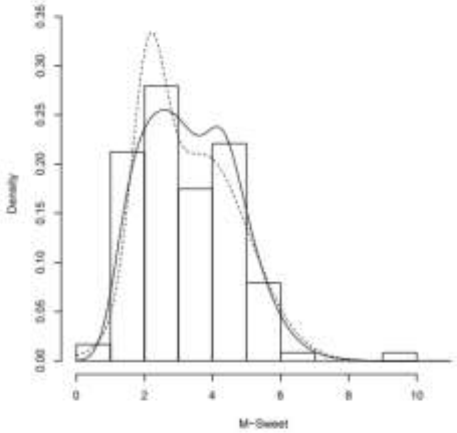

Figure 3-4 . Touz A MN (Chinese calligraphy) and Tozyi A GSN (solid line) for data M- Sweet .

3 Example

Finally , data on age and recurrence of cancer classified as Kaposi's sarcoma were This is the type of cancer that can cause lumps in the skin and lymph . collected organs can cause undiagnosed fissures . Data were collected from Other . nodes Department of Health Statistics ، ONS) the Office for National Statistics website It can be seen that the . A visible . in the Appendix A1 and appear in Table (years and also for people 25 . infection rate is high in people aged approximately -1995 records were taken during the years years . These aged approximately 60 summary shows 5-4 Table . and relate to different regions of Britain 2016 . statistics for the Kaposi's sarcoma dataset

descriptive statistics : Kaposi's sarcoma . 5-4 Table

n	$\bar{y}$	s^2	b_1	b_2
29131	45.396	416.487	0.313	1.936

The two fitted models are shown in Figure 4-4 and the corresponding estimated values can be seen in Table 4-6. The GSN model provides a better fit to the data, because... . AIC and CAIC are smaller

Table 6-4 . Parameter estimation (SE) of the models FSN and GSN .

Parameter	FSN	GSN
μ	17.896 (0.119)	37.039 (0.139)
σ	34.245 (0.171)	22.052 (0.105)
λ	6.016 (0.118)	4.898 (0.118)
α	−1.007 (0.090)	5.525 (0.138)
AIC	255,356.2	253,832.6
CAIC	255,393.3	253,869.7

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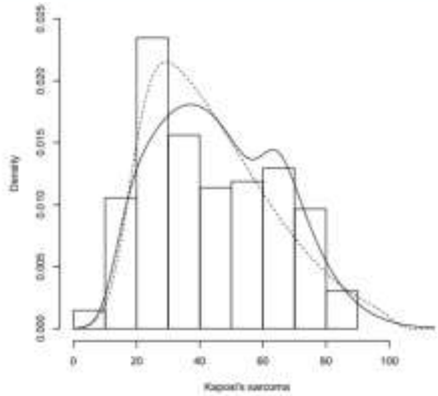

Figure 4-4 . Touz A FSN (China Line) and Tozyi A GSN (solid line) for Kaposi's sarcoma data .

2. Results

distributions that can be considered Chule have proposed two families of skewed Chule distribution for matching alternatives to the well-known normal . data

We can ask whether it is . Future research can address the following topics normal distribution - possible to generalize the proposed method for the elastic With . to obtain what should be applied more flexibly [9] described in reference regard to the standard components , the following model can be considered , . which is the average of two densities . Arnold and Beaver

$$f_X(x) = \frac{1}{2}\phi(x)\left[\frac{\Phi(\lambda x + \alpha_1)}{\Phi\left(\frac{\alpha_1}{\sqrt{1+\lambda^2}}\right)} + \frac{\Phi(\lambda x - \alpha_2)}{\Phi\left(\frac{-\alpha_2}{\sqrt{1+\lambda^2}}\right)}\right],$$

where $\lambda, \alpha_1, \alpha_2 \in \mathbb{R}$.

Note that this model cannot be obtained by methods similar to those used in this is a simpler and more flexible ، However . (9-3) developing the model But when we define it as a mixture . development than the Arnold Beaver model with equal weights , it makes sense to add more flexibility by taking into account . . unequal weights

$$f_X(x) = \phi(x)\left[\gamma\frac{\Phi(\lambda x + \alpha_1)}{\Phi\left(\frac{\alpha_1}{\sqrt{1+\lambda^2}}\right)} + (1-\gamma)\frac{\Phi(\lambda x - \alpha_2)}{\Phi\left(\frac{-\alpha_2}{\sqrt{1+\lambda^2}}\right)}\right],$$

. ∋ γ [1 ، 0] Where

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