

Analysis Frequency Modulation Mode-Locked Fiber Laser by Using ABCD Rule

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Abstract

In this paper the study of Frequency Modulation (FM) mode-locking fiber laser is presented. The time-domain ABCD law was employed to clarify the impacts of optical elements on the mode-lock pulses. ABCD matrices formalism in the time domain has been developed under a Gaussian paraxial approximation. Also, these matrices apply to the theory of the actively mode-locked fiber laser.

The numerical results are obtained by using MATLAB[®] software. The analysis was produced a shorter than 3.16ps duration pulses in laser mode locked with FM modulator driven at repetition frequency of 10GHz and cavity has anomalous dispersion of $-0.015\text{ps}^2/\text{m}$ and nonlinearity of $0.02\text{W}^{-1}\text{m}^{-1}$. The values of chirp are plotted versus average power with many values of dispersion. All relationship between the pulse laser parameters and other effective parameters like, second order dispersion, nonlinearity, average input power, repetition frequency and optical filter bandwidth are plotted and discussed.

Keyword: FM mode locking, Fiber laser, ABCD matrix rules.

تحليل التضمين الترددي لقفل النمط لليزر الليف البصري باستخدام قواعد ABCD

الخلاصة

في هذا البحث تم دراسة التضمين الترددي لقفل النمط لليزر الليف البصري باستخدام المجال الزمني لقانون ABCD لغرض توضيح تأثير العناصر البصرية على نبضة قفل النمط. تشكيل مصفوفات ABCD في المجال الزمني طورت باستخدام التقريب كاوسي. كذلك طبقت المصفوفات تطبيق في قفل النمط الفعال لليزر الليف البصري.

تم الحصول على النتائج العددية باستعمال برنامج MATLAB[®]. التحليلات أوصلتنا إلى فترة نبضة اقصر من 3.16ps في قفل النمط الترددي لليزر الليف البصري باستخدام التردد التكراري 10GHz و حجرة ذات تشتت شاذ مقداره $-0.015\text{ps}^2/\text{m}$ و لا خطية بمقدار $0.02\text{W}^{-1}\text{m}^{-1}$. القيم الزقزقة قد رُسمت إزاء معدل القدرة مع قيم مختلفة للتشتت. كل العلاقات بين متغيرات نبضة الليزر و المتغيرات الأخرى المثيرة مل التشتت من الدرجة الثانية و اللاخطية ومعدل القدرة و التردد التكراري و عرض حزمة المرشح البصري قد رُسمت ونوُقت.

1. Introduction

Mode locking is a phenomenon in which the cavity resonant frequencies of the laser are forced to have matched phase shifts. This effect causes the waves to constructively interfere to form

ultra-short pulses. The advance of ultrafast technique in the last decade has been able to develop lasers with a variety of features such as ultrashort pulse duration, high peak intensity and

high repetition rate. Ultrashort laser pulse with GHz repetition rate will be useful for many applications in areas such as optical communications [1].

The low-loss transmission window of optical fibers has a bandwidth comparable to that of a 100-fs pulse, and therefore ultrashort-pulse technology may play an important role in optical communications. Sub-picosecond pulses have already been used for laboratory experiments demonstrating fiber optic transmission of data at Tbit/s (10^{12} bit/s) rates. Ultrashort pulse sources with a high repetition rate are key components in optical time division multiplexing (OTDM) system [2]. Ultrashort pulses may also prove important in wavelength-division-multiplexing (WDM) systems in which the fiber bandwidth is carved up into different wavelength bands or channels [3]. Here ultrafast optics technology is important not only for pulse generation but also for signal processing, for data detection, and for the advanced metrology necessary for characterizing and optimizing ultrashort-pulse transmission [3-5].

Active mode-locking mechanisms are the result of a time varying optical or electronic exploitation of a material's response. FM modulation is discussed here as one types of active mode-locking techniques. However, actively mode-locked lasers need a radio-frequency (RF) signal generator and a phase/intensity modulator, which results in a complicated structure and higher cost. Passively mode-locked lasers based on Er :Yb : glass as a gain medium have high fundamental repetition rate. However, a high fundamental repetition rate fiber laser, in contrast to a glass- or crystal-based laser, would be advantageous due to its compact size, robustness, flexibility, low cost, and compatibility with other optical fiber systems [6].

To examine a complicated optical system with many components (such as an eye, a telescope), it can be cumbersome to calculate object and image distances for each component. When the paraxial regime and one axis are considered, the straightforward matrix method to simplify calculation is used [7].

The analyzing of a fiber laser mode locked simultaneously by FM modulator at higher harmonics of the cavity repetition rate is discussed. The set of ABCD matrices and q-parameters are programmed and analyzed to get predicted values of pulse width and chirp parameter versus the average optical output power. The performance analysis of FM mode locking fiber laser is done by using MATLAB[®] software.

This paper discusses FM mode locked fiber laser which is analyzed by using ABCD rules and q-parameters. Section 2 explains the modeling of mode-locked fiber lasers. Section 3 gives characterization of matrices of optical elements. In section 4, deals with simulation results of FM mode locking fiber laser. The summary of paper and present conclusion are discussed in section 5.

2. Modeling of FM Mode-locked Fiber Lasers

Mode locking occurs when phases of various longitudinal modes are synchronized such that the phase difference between any two neighboring modes is locked to a constant value ϕ such that $(\phi_n - \phi_{n+1} = \phi)$. Such a phase relationship implies that $(\phi_n = m\phi + \phi_0)$. The mode frequency ω_n can be written as $(\omega_n = \omega_0 + 2m\pi\Delta\nu)$. For simplicity, it can assume that all modes have the same amplitude E_0 , the sum can be performed analytically and the result is given by [8]:

$$|E(t)|^2 = \frac{\sin^2[(2M+1)pDnt + f/2]}{\sin^2(pDnt + f/2)} \quad (1)$$

Active mode locking requires modulation of either the amplitude or the phase of the intra-cavity optical field at a frequency fm equal to (or a multiple of) the mode spacing $\Delta\nu$. It is referred to as AM (amplitude modulation) or FM (frequency modulation) mode locking depending on whether amplitude or phase is modulated. One can understand the locking process as follows. Both the AM and FM techniques generate modulation sidebands, spaced apart by the modulation frequency fm . These sidebands overlap with the neighboring modes when $fm \approx \Delta\nu$. Such an overlap leads to phase synchronization.

In an effort to both understand and predict the behavior of fiber laser, it is felt that the insight obtained by developing a solid model for such laser system is invaluable. Modeling the FM mode locking fiber laser shown in Fig.(1) is suggested to understand mention behavior. For such a model to be physically sound it must include [4]:

i- The frequency modulator (FM) has a transfer function is:

$$\begin{aligned} \exp[jf(t)] &= \exp[jD_m \cos(w_M t)] \\ &\cong \exp[j(D_m - \frac{1}{2}D_m w_M^2 t^2)] \quad (2) \end{aligned}$$

The optical band pass filter, the transfer function is:

$$G(f) = \exp(-\frac{w^2}{2B_f^2}) \quad (3)$$

Where, B_f is the optical filter bandwidth.

ii- effect of dispersion on fiber is represent in frequency domain by:

$$\frac{\partial}{\partial z} u = \frac{i}{2} \beta_2 w^2 u \quad (4)$$

Where, β_2 is a second order dispersion and z is a transmission distance.

iii- fiber with nonlinearity which has transfer function as follow:

$$\frac{\partial}{\partial z} u = i g u_1^2 u \quad (5)$$

3. Characterization of Matrices of Optical Elements

The advantage of this matrix formalism is that any ray can be tracked during its propagation through the optical system by successive matrix multiplications. The transformation of the q-parameter of the Gaussian beam when it passes through the optical system is essential. This q-transformation is already given by applying Fourier optics but can also be derived by simple matrix multiplication [9, 10]

Compared with other complex numerical models, it is very convenient for easy derivation of analytical solution for understanding pulses characteristics. The pulse shape was assumed to be Gaussian which is defined by [2, 11]:

$$E(t) = \sqrt{P_o} \exp\left[-\frac{t^2}{2\tau^2}(1+iC)\right] \quad (6)$$

Where, P_o is the peak power, τ is the 1/e pulse width, C is the linear chirp parameter.

Therefore; the complex beam parameter is a complex number that specifies the properties of a Gaussian beam. It is usually denoted by q .

$$\frac{1}{q} = \frac{1+iC}{\tau^2} \quad (7)$$

If a Gaussian pulse passes through optical element, the q-parameter at the output (q_{out}) in term of input q parameter (q_{in}), will satisfy the following law [11]:

$$q_{out} = \frac{A \cdot q_{in} + B}{C \cdot q_{in} + D} \quad (8)$$

Now, each optical element which discussed in section 2 has respective ABCD matrix. These matrices can be written as follows:

i- The ABCD matrix for frequency modulator is [6]:

$$\begin{bmatrix} A_{FM} & B_{FM} \\ C_{FM} & D_{FM} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4iD_{FM}P^2 f^2 & 1 \end{bmatrix} \quad (9)$$

Where, Δ_{FM} is the phase modulation depth.

ii- The ABCD matrix for optical band pass filter is [2,11] :

$$\begin{bmatrix} A_{fl} & B_{fl} \\ C_{fl} & D_{fl} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{B_f^2} \\ 0 & 1 \end{bmatrix} \quad (10)$$

iii- The ABCD matrix for dispersion on fiber is:

$$\begin{bmatrix} A_{dis} & B_{dis} \\ C_{dis} & D_{dis} \end{bmatrix} = \begin{bmatrix} 1 & -ib_2 z \\ 0 & 1 \end{bmatrix} \quad (11)$$

iv- The ABCD matrix for fiber with nonlinearity is:

$$\begin{bmatrix} A_{nl} & B_{nl} \\ C_{nl} & D_{nl} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ i \frac{2gPz}{\sqrt{p}ft^3} & 1 \end{bmatrix} \quad (12)$$

For the Gaussian profiled pulses, the relation between the peak power and the average power is as followed:

$$P_o = \frac{\bar{P}}{\sqrt{p}ft} \quad (13)$$

Where, f is the repetition (modulation) frequency of the pulse train.

Similar to the idea of the split-step algorithm in numeric simulations [11], it can divide the cavity fiber into many segments and consider the dispersion and the nonlinearity separately in each segment. The length of one segment is L/n , for cavity length is L and number of segment is n .

The ABCD matrix represent whole elements of fiber laser are expressed in equation (14). It can predict many parameters of FM mode locking fiber laser by compute this ABCD matrix.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4i\Delta_{FM}P^2 f^2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{B_f^2} \\ 0 & 1 \end{bmatrix}$$

$$\prod_{k=1}^n \begin{bmatrix} 1 & 0 \\ i \frac{2gPL}{\sqrt{pn}ft(k)^3} & 1 \end{bmatrix} \begin{bmatrix} 1 & -ib_2 \frac{L}{n} \\ 0 & 1 \end{bmatrix} \quad (14)$$

From equation (7), we can write an expression for $q(k+1)$ as:

$$\frac{1}{t^2(k+1)} = REL \left[\frac{1}{q(k+1)} \right] \quad (15)$$

By using equation (8), we can express $q(k+1)$ as follows:

$$q(k+1) = \frac{A \cdot q(k) + B}{C \cdot q(k) + D} \quad (16)$$

4. Results and discussion

By solving the ABCD matrix and q -parameter of the FM mode-locking fiber laser in equations (14-16) numerically, the evolution of steady-state pulse parameters was predicted. This numerical analysis is done by using MATLAB[®] software. Fig.(2) illustrated the flow chart of proposed method to calculate pulse parameter. The true strength of ABCD rule lies in predicting the steady-state values for mode-locked pulses and chirp parameter. The results are presented in both anomalous ($D < 0$) and normal ($D > 0$) dispersion regimes. Fig.(3) demonstrates the strength of ABCD law by studying the effect of both anomalous and normal dispersions on pulse width. Data points marked with symbols were obtained using the q -parameter and ABCD rules for four cavity lengths. The pulse width is increasing slightly with increasing both the dispersion and cavity length in case of normal dispersion. While, the pulse width is decreasing both anomalous dispersion and cavity length are increasing.

The shortest pulses are obtained in the case of anomalous dispersion for small values of second order dispersion (β_2). For anomalous dispersion as shown in

Fig.(3-a), the pulse width becomes equal 1.76 ps for β_2 of $-0.025 \text{ ps}^2/\text{m}$ and cavity length of 2000m. But it can note the pulse width has larger value for smaller cavity length. Fig.(3-b) shows the pulse width is exceeds 3.16ps in the normal dispersion case. Also the pulse width becomes very high with larger cavity length and dispersion. It can note the pulse width approximately equal to 12ps at dispersion of $\beta_2 = 0.025 \text{ ps}^2/\text{m}$ and cavity length of 2000m. It can remember that the dispersion (D) is calculated by [12]:

$$D = -\frac{2pc}{l^2} b_2 \quad (17)$$

In This work, the effect of cavity length, second order dispersion, repetition frequency, input power and modulation depth on the output laser pulse are studied.

Fig.(4) demonstrates how the second order cavity dispersion (β_2) affect the chirp parameter (C) with various cavity length. The chirp parameter varies with β_2 in the anomalous and normal dispersion regimes. The chirp parameter (C) is positive with negative β_2 and vice versa. In case of large cavity length ($L=2000\text{m}$) and negative β_2 , the chirp parameter of the pulse becomes small (0.12).

The effect of nonlinear parameter γ on the pulse width and chirp parameter are shown in Fig (5) and Fig.(6) respectively. The pulse width (τ) is decrease with increase nonlinear parameter (γ) in both anomalous and normal dispersion regimes. Also the pulse chirp parameter is increase with increase nonlinear parameter (γ) in both anomalous and normal dispersion regimes

The relationships of the pulse width and the pulse chirp versus the average power (P) are shown in Fig.(7) and Fig.(8), respectively. From Fig.(7), It can find that the greater laser power is

favorable for obtaining shorter pulses in the anomalous dispersion region. In the normal dispersion region, the pulse width does not always decrease with the increasing of average power, but increases under a small average laser power. The relationship of the laser pulse chirp versus the average power (P) is shown in Fig.(8). It can note that, the pulse chirp parameter is becomes larger for average power more than 15mW in anomalous dispersion region. While, the pulse chirp parameter is always reduced when the average power (P) is increasing.

Fig.(9) shows the change of the pulse width and pulse chirp as a function of change the cavity length at various optical filter band width. It can see that the pulse width is decreasing with increase cavity length at normal dispersion of $\beta_2=0.015 \text{ ps}^2/\text{m}$ and nonlinearity $\gamma = 0.02 \text{ W}^{-1} \text{ m}^{-1}$. When the cavity length is increase, the chirp parameter began increase from certain value then it reach to a maximum value of 0.5 then return to decrease as cavity length increase.

Fig.(10) shows the effect of repetition (modulation) frequency on the pulse width of the stable output pulses. Larger modulation frequency leads to shorter pulse width. When, repetition frequency varies from 1GHz to 45GHz, the pulse width change from 3.16ps to 2.49ps at modulation depth of $\Delta_{\text{FM}} = 0.9$. The effect of bandwidth of optical filter is illustrated in Fig.(11). It seems that large bandwidth filter is preferable. When larger band width of filter is used, the output pulses become shorter. Therefore, it can be consider that the filter band width as main factor to influence on pulse width.

5. Conclusion

This paper discussed the predicted results of FM mode locking fiber laser by using ABCD matrix and q parameter. The shortest pulses are obtained in the

case of anomalous dispersion rather than for normal dispersion. Also, the pulse chirp always decreases while the average dispersion varies from the normal region to the anomalous region. In both dispersion regimes, the increase average power lead to output pulse with shorter width.

It is obvious that the repetition frequency affect strongly on pulse width where the pulse width is decrease with increase repetition frequency.

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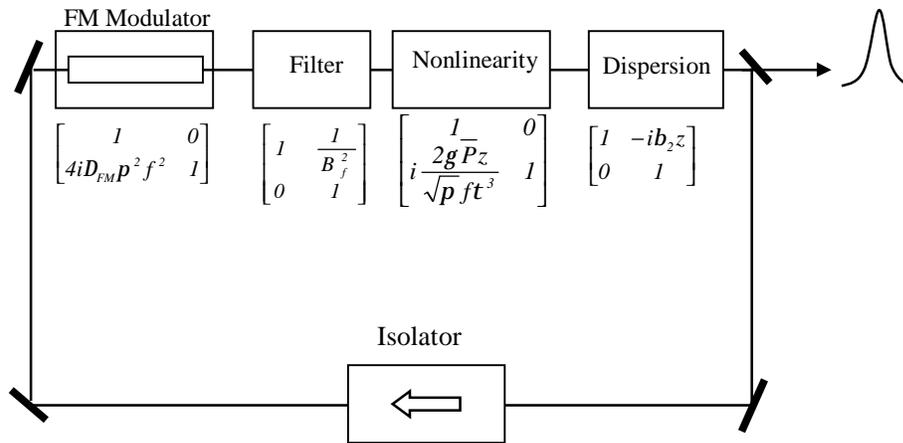


Figure (1): Modeling the FM Mode Locking Fiber Laser

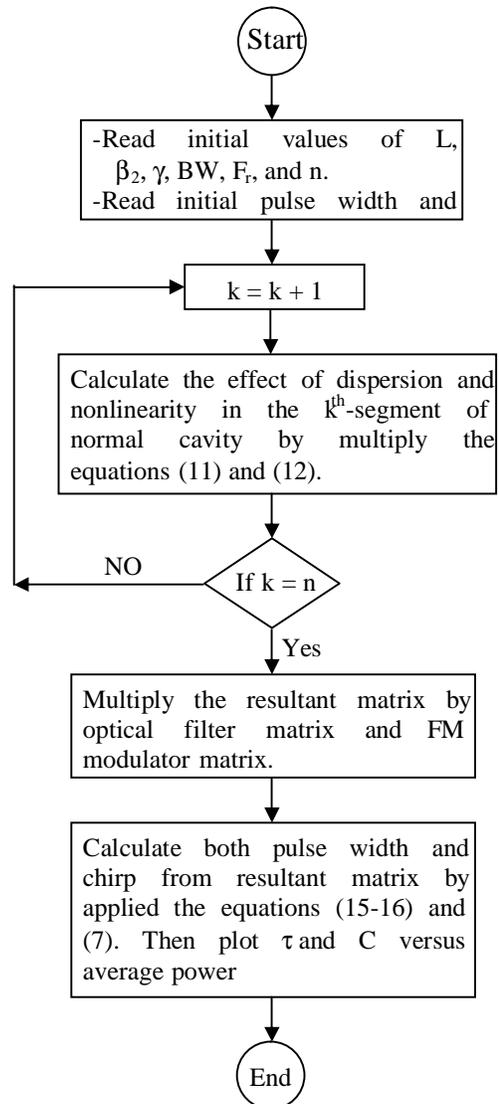
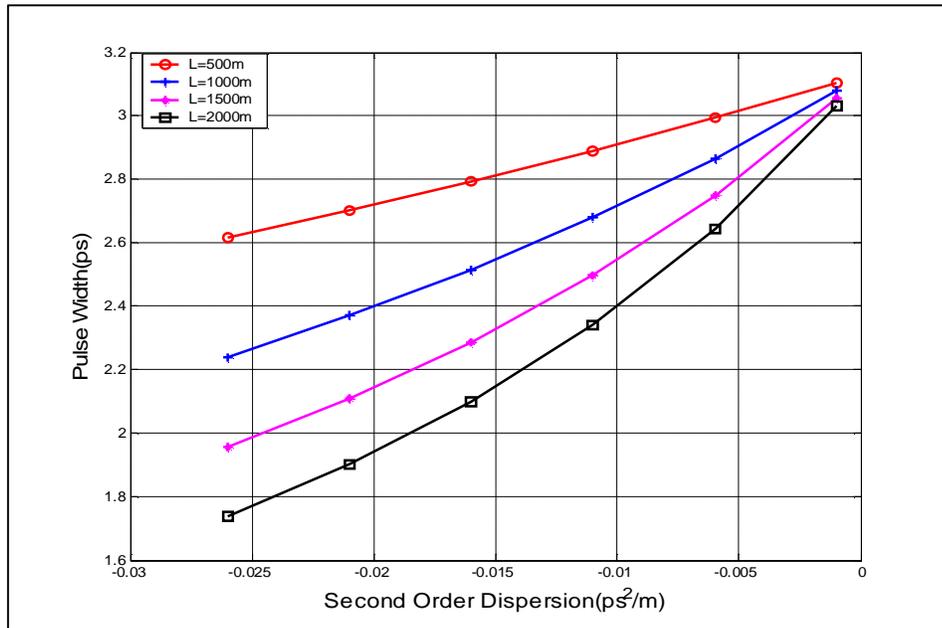
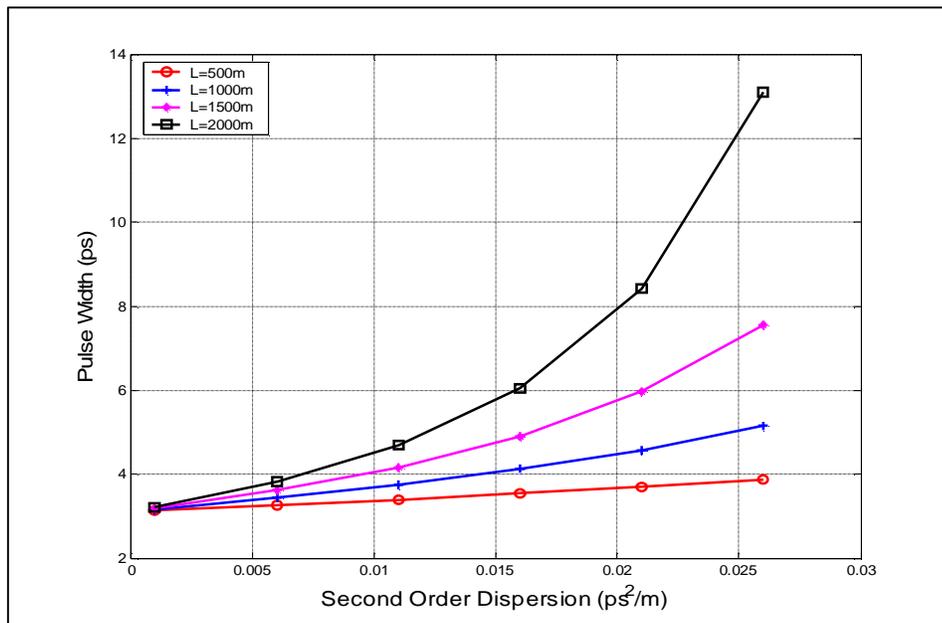


Figure (2): Flow chart of proposed program to plot the pulse width and chirp versus average power



(a)



(b)

Figure (3): Influence of second order dispersion on pulse width with various cavity length.

- a) Anomalous dispersion
- b) Normal dispersion

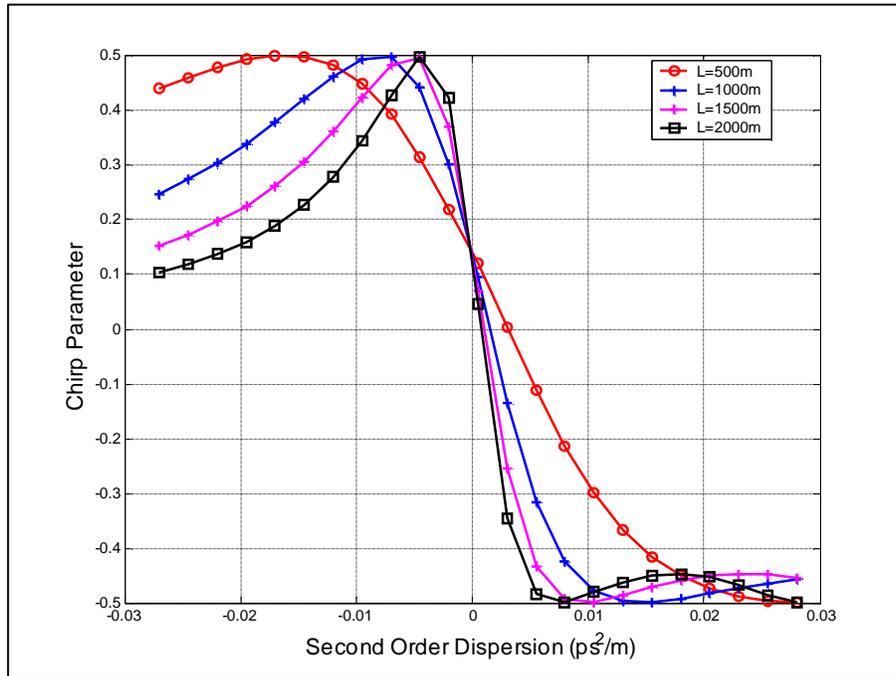


Figure (4): Illustrated effect of second order dispersion on chirp parameter.

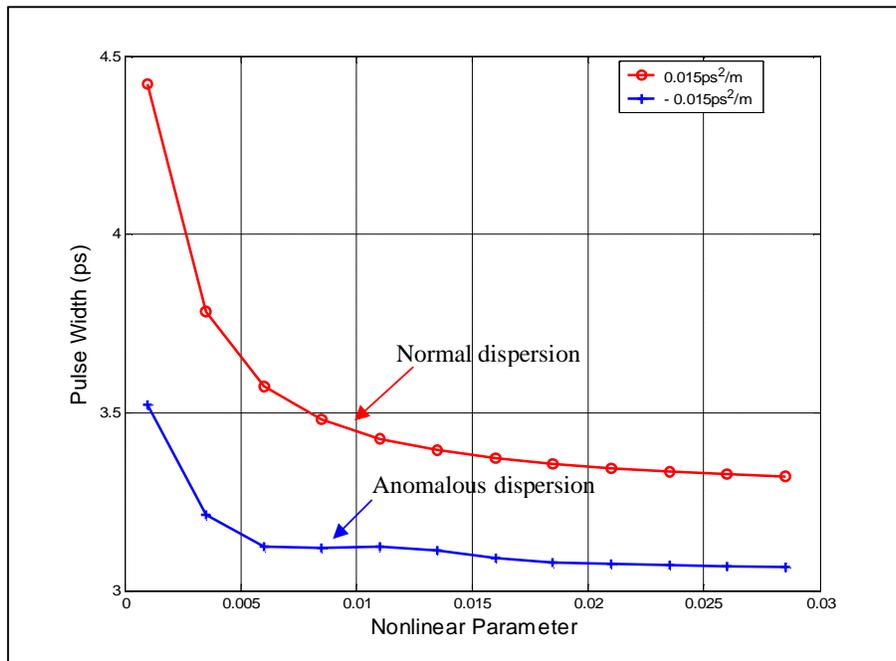


Figure (5): Affect of nonlinear parameter on the pulse width at anomalous and normal dispersion regimes.

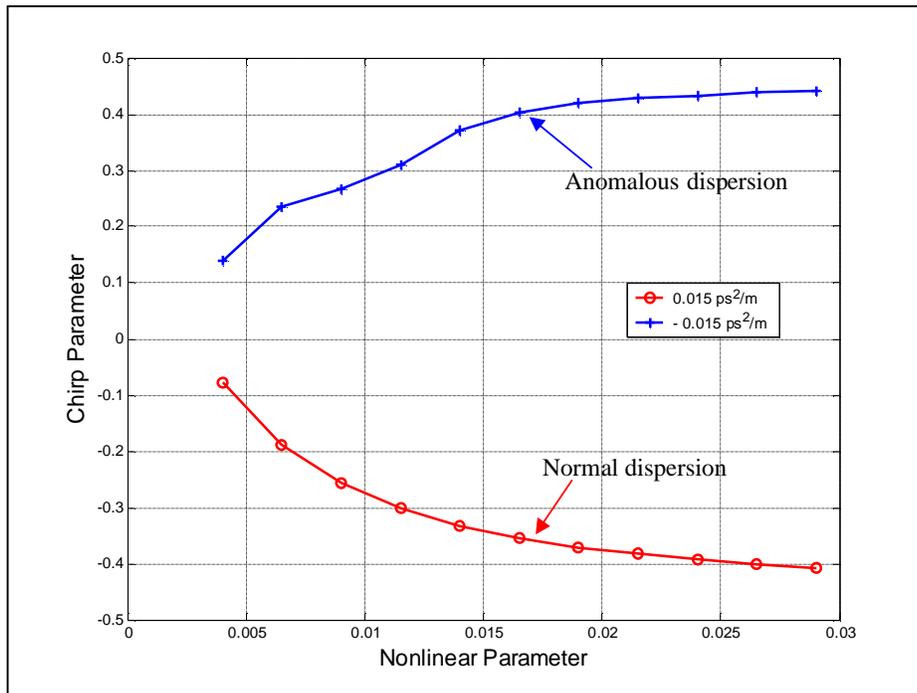
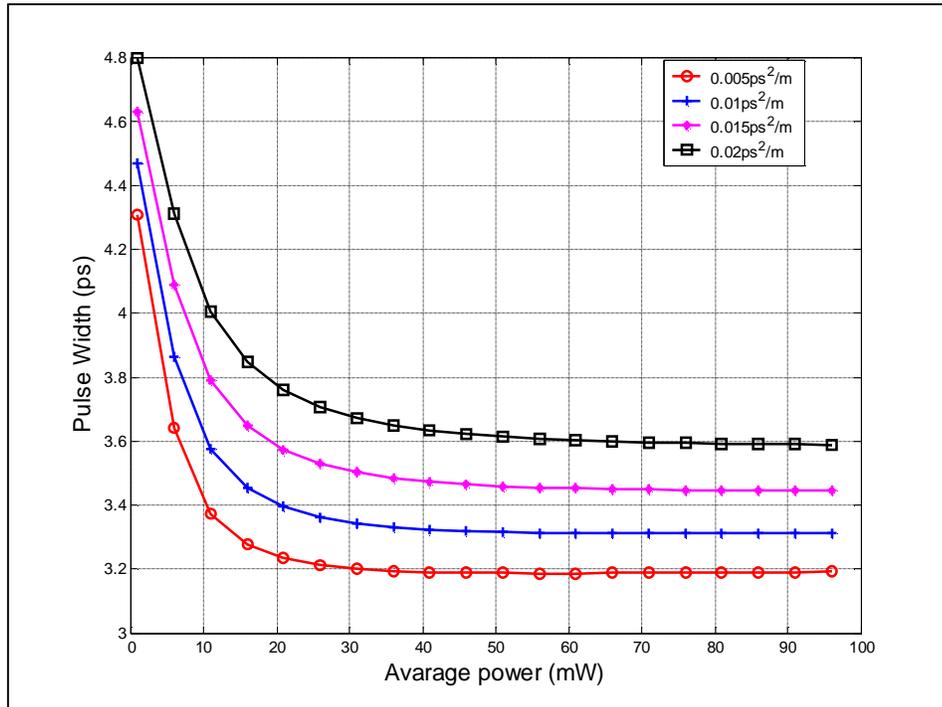
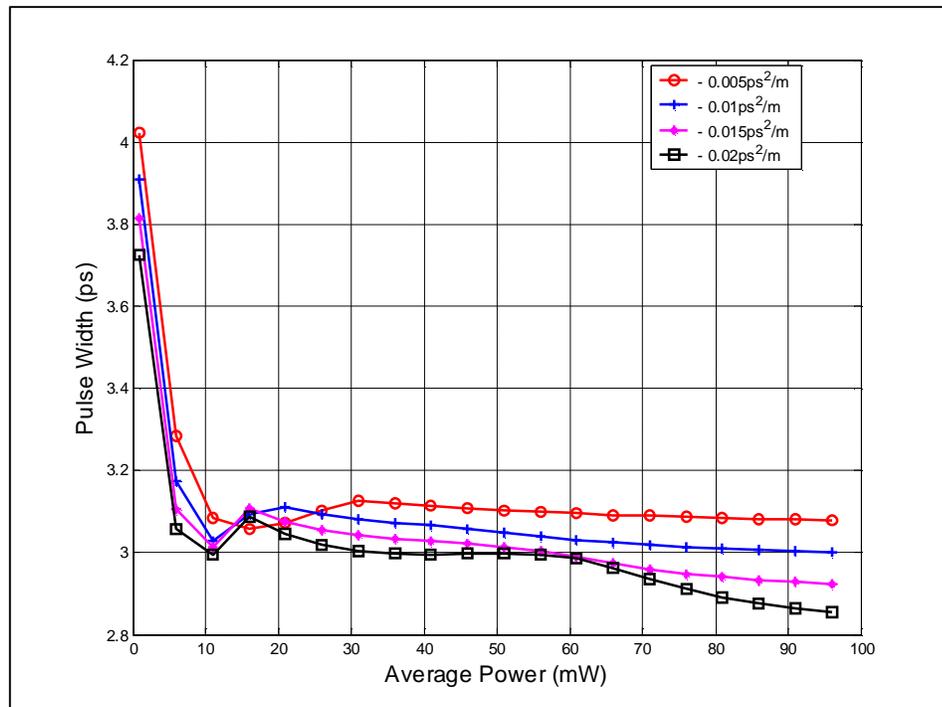


Figure (6): Affect of nonlinear parameter on the pulse chirp parameter at anomalous and normal dispersion regimes.

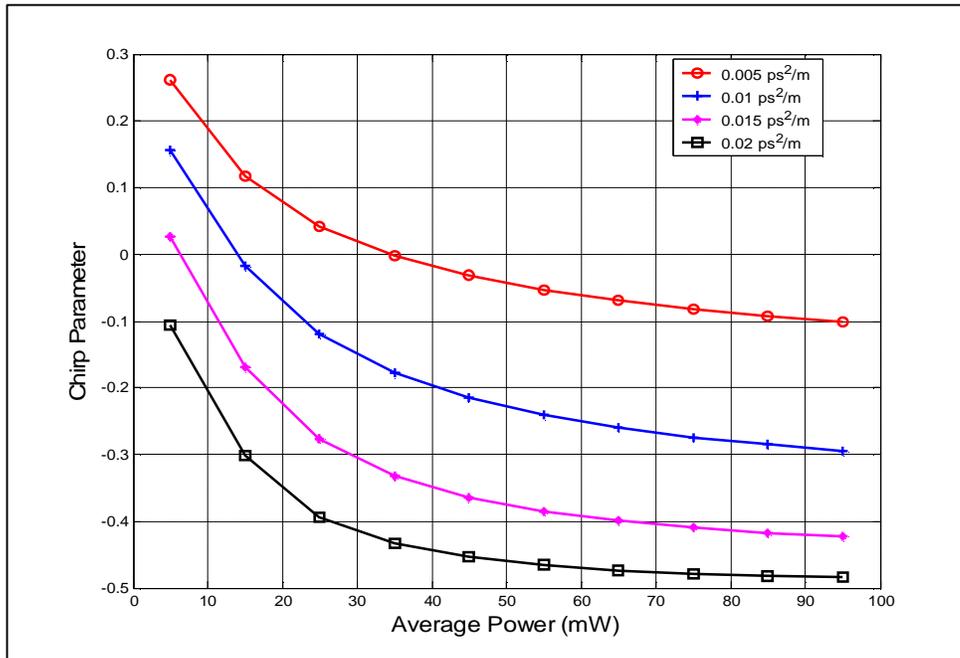


(a)

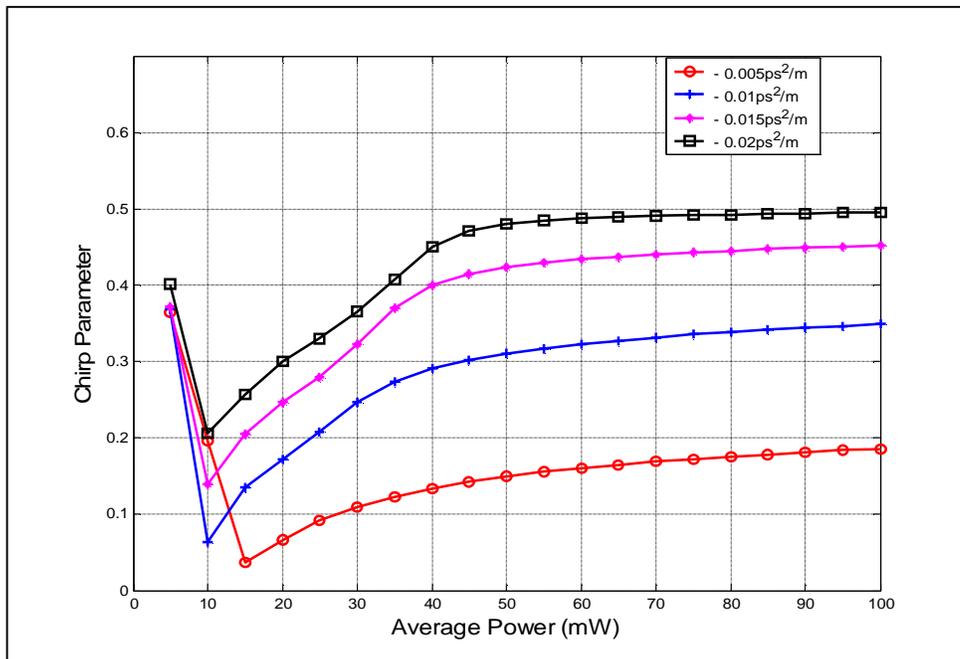


(b)

Figure (7): Pulse width versus the average power
a) Normal dispersion,
b) Anomalous dispersion.

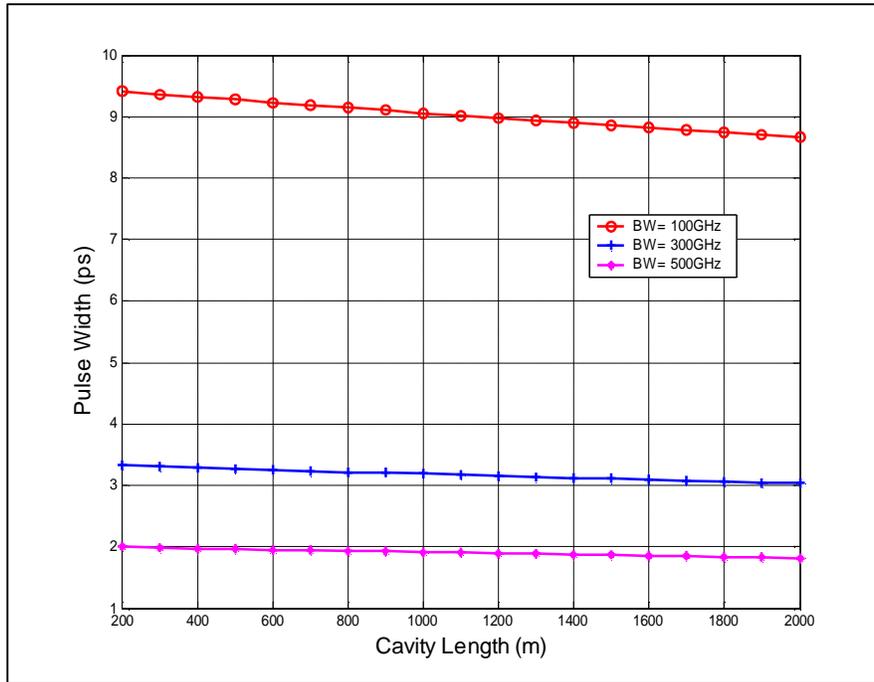


(a)

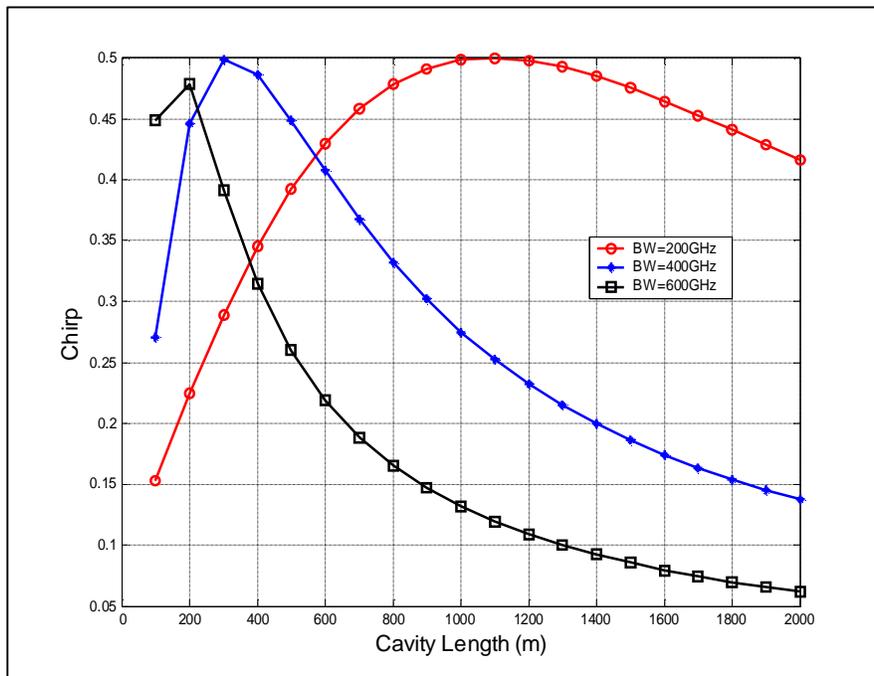


(b)

Figure (8): Chirp parameter versus the average power
a) Normal dispersion,
b) Anomalous dispersion.



(a)



(b)

Figure (9): Laser characteristics versus the cavity length in the normal dispersion region. (a) Pulse width. (b) Pulse chirp.

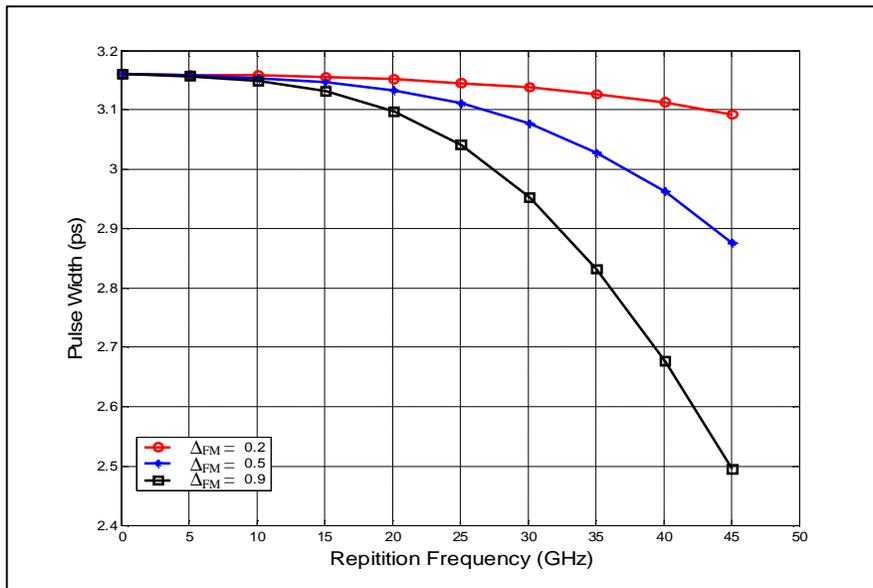


Figure (10): Pulse width versus the repetition (modulation) frequency with various frequency modulation depth.

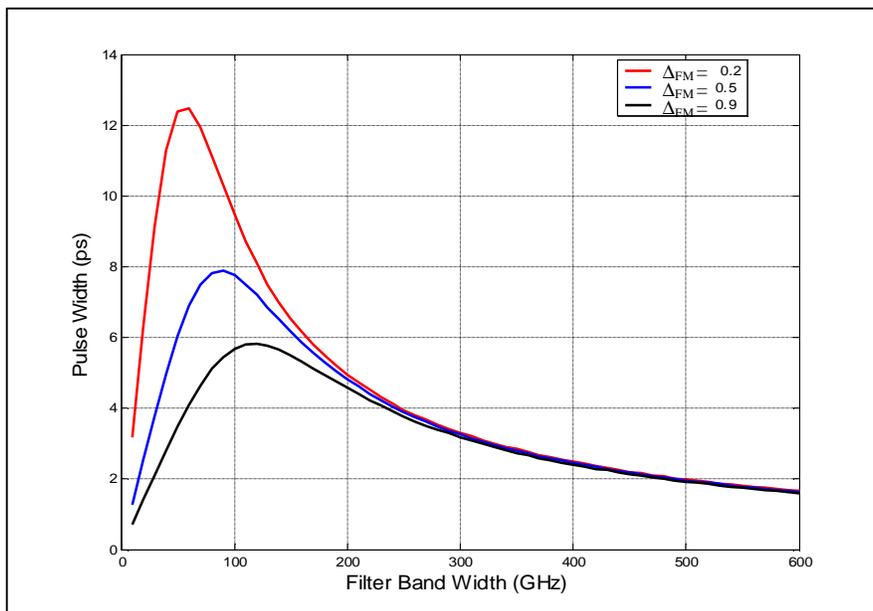


Figure (11): Pulse width versus the optical filter band width with various frequency modulation depth.