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Unscented Kalman Estimator for Estimating the State of Two-phase Permanent Magnet Synchronous Motor

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Abstract

This paper presents the unscented Kalman filters (UKF) for estimating the states (winding currents, rotor speed and rotor angular position) of two-phase Permanent Magnet Synchronous Motor (PMSM). The UKF is based on firstly specifying a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the Gaussian Random Variable (GRV), and when propagated through the true nonlinear system (motor model), capture the posterior mean and covariance accurately to the second order (Taylor series expansion). The results showed that the UK estimator could successively estimate the states of PMSM without need any Jacobian matrix.

Keywords: Two-Phase Permanent Magnet Synchronous Motor, unscented Kalman Filter, Modelling, Matlab.

مخمن كلمان المعدل لتحديد متغيرات محرك ذو طور ثنائي وذات تغذية دائمية

الخلاصة:

يتناول هذا البحث مرشح كلمان المعدل (UKF) لتحديد المتغيرات (تيارات الملف، الموقع الزاوي وسرعة الجزء الدوار) لمحرك ذو طور ثنائي وذات تغذية دائمية (PMSM). مرشح كلمان المعدل (UKF) يعتمد في البداية على تحديد اقل مجموعة من عينات النقاط المختارة بدقة. هذه العينات تمسك تماما بالمعدل الحقيقي والتباين لمتغير (Gaussian) العشوائي (GRV) وعندما تنتشر خلال النظام الغير خطي الحقيقي(نموذج محرك)، يمسك بالمعدل الجديد والتباين بدقة عالية لنظام ثنائي المرتبة (متسلسلة تايلر الموسعة). النائج تظهر بان مرشح كلمان المعدل (UKF) ينجح في تحديد المتغيرات لمحرك ذو طور ثنائي وذات تغذية دائمية بدون الحاجة إلى مصفوفة (Jacobian).

Introduction:

There are two reasons that one might want to know the states of a system:

- First, one might need to estimate states in order to control the system.
- Second, one might need to estimate system states because they are interesting in their own right. Some systems need to schedule their activities, to evaluate their performance or to predict failure probabilities.

The Kalman filter is a tool that can estimate the variables of a wide range of processes. The standard Kalman filter is an effective tool for estimation, but it is limited to linear systems. Most real-world systems are nonlinear, in

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which case standard Kalman filters do not directly apply [1,2].

The extended Kalman filter (EKF) is the most widely applied state estimation algorithm for nonlinear systems. However, the EKF can be difficult to tune and often gives unreliable estimations if the system nonlinearities are severe. This is because the EKF relies on linearization to propagate the mean and covariance of the state [3].

The unscented Kalman filter (UKF), proposed by Julier et al. [4], is an extension of the Kalman filter that reduces the linearization errors of the EKF. The use of the UKF can provide significant improvement over the EKF.

The basic difference between the EKF and UKF stems from the manner in which Gaussian random variables (GRV) are represented for propagating through system dynamics. In the EKF, the state distribution is approximated by a GRV, which is then propagated analytically through the first-order linearization of the nonlinear system. This can introduce large errors in the true posterior mean and covariance of the transformed GRV, which may lead to suboptimal performance and sometimes divergence of the filter. The UKF address this problem by using a deterministic sampling approach. The state distribution is again approximated by a GRV, but is now represented using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and, when propagated through the true nonlinear system, captures the posterior mean and covariance accurately to second order (Taylor series expansion) for any nonlinearity. The EKF, in contrast, only achieves first-order accuracy. No explicit Jacobian or Hessian calculations are necessary for the UKF [4,5].

Motor state estimation

The main objective of the work is to use the UKF for estimating the states of a two-phase permanent magnet synchronous motor. The state estimation may be necessary to regulate them with a control algorithm, or to know the position or velocity of the motor for some other reason. Let's suppose that it is possible to measure the motor winding currents, and we want to use the UKF to estimate the rotor position and velocity. The system equations are [4,6]

$$\dot{i}_a = \frac{-R}{L} i_a + \frac{w\, l}{L} \sin q + \frac{u_a + \Delta u_a}{L}$$
$$\dot{i}_b = \frac{-R}{L} i_b + \frac{w\, l}{L} \cos q + \frac{u_b + \Delta u_b}{L}$$
$$\dot{w} = \frac{-l}{2J} i_a \sin q + \frac{3l}{2J} i_b \cos q - \frac{Fw}{J} + \Delta a$$

$$\dot{q} = w$$
$$y_1 = i_a + v_a$$
$$y_2 = i_b + v_b$$

Where i_a and i_b are the currents in the two motor windings, q and w are the angular position and velocity of the rotor, R and L are the motor winding's resistance and inductance, l is the flux constant of the motor, F is the coefficient of viscous friction that acts on the motor shaft and its load, J is the moment of inertia of the motor shaft and its load. u_a and u_b are the voltages that are applied across the two motor windings, Δu_a and Δu_b are noise terms due to errors in u_a and u_b. Δa is a noise term due to uncertainty in the load torque, y_1 and y_2 *are* the measurements.

It is assumed that the measurements of the two winding currents may be performed by sense resistors. The measurements are distorted by measurement noises v_a and v_b, which are due to things like sense resistance uncertainty, electrical noise or quantization errors.

The unscented transformation

The unscented transformation (UT) can be summarized as follows [4,5,6].

1. Beginning with an *n*-element vector x, with known mean $\bar{x}$ and covariance **P**, and with given a known nonlinear transformation $y = f(x)$, one can estimate the mean and covariance of y, denoted as $\bar{y}_u$ and P_u.

2. Forming $2n$ sigma point vectors $x^{(i)}$ as follows:

$$x^{(i)} = \bar{x} + \tilde{x}^{(i)} \qquad i = 1,\mathbf{K}, 2n$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP}\right)_i^T \qquad i = 1,\mathbf{K}, n$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP}\right)_i^T \qquad i = 1,\mathbf{K}, n$$

where $\left(\sqrt{nP}\right)$ is the matrix square root of nP such that $\left(\sqrt{nP}\right)^T\left(\sqrt{nP}\right) = nP$, and $\left(\sqrt{nP}\right)_i$ *is* the ith row of $\left(\sqrt{nP}\right)$.

3. Transforming the sigma points as follows:

$$y^{(i)} = f(x^{(i)}) \qquad i = 1,\mathbf{K}, 2n$$

4. Approximate the mean and covariance of y as follows:

$$\bar{y}_u = \frac{1}{2n}\sum_{i=1}^{2n} y^{(i)}$$

$$P_u = \frac{1}{2n}\sum_{i=1}^{2n}\left(y^{(i)} - y_u\right)\left(y^{(i)} - y_u\right)^T$$

The block diagram illustrating the steps in performing the UT is shown in Figure (1).

To apply the UKF to the motor, one need to define the states of the system. The state vector x can be defined as

$$x = [\, i_a \quad i_b \quad w \quad q \,]^T$$

The unscented transformation is the milestone of the UKF algorithm, which is abbreviated as follows [7, 8, 9].

1. The system equations are given as

$$x_{k+1} = f(x_k, u_k, t_k) + \omega_k$$

$$y_k = h(x_k, t_k) + \upsilon_k$$

$$\omega_k \approx (0, Q_k)$$

$$\upsilon_k \approx (0, R_k)$$

Where Q, R are the covariance of the process noise w_k and the measurement noise v_k, respectively.

2. The UKF is initialized as follows.

$$\hat{x}_0^+ = E(x_0)$$

$$P_0^+ = E\left[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T\right]$$

3. The following time update equations are used to propagate the state estimate and covariance from one measurement time to the next.

(a) To propagate from time step *(k - 1)* to *k*, first choose sigma points $x_{k-1}^{(i)}$, with appropriate changes since the current best guess for the mean and covariance of x_k are $\hat{x}_{k-1}^+$ and P_{k-1}^+:

$$\hat{x}_{k-1}^{(i)} = \hat{x}_{k-1}^+ + \tilde{x}^{(i)} \qquad i = 1,\mathbf{K}, 2n$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_{k-1}^+}\right)_i^T \qquad i = 1,\mathbf{K}, n$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_{k-1}^{+}}\right)_i^T \qquad i = 1,\mathbf{K},n$$

(b) The known nonlinear system equation $f(x_k, u_k, t_k)$ is used to transform the sigma points into $\hat{x}_k^{(i)}$ vectors with appropriate changes since our nonlinear transformation is $f(x_k, u_k, t_k)$ rather than $h(x_k, t_k)$:

$$\hat{x}_k^{(i)} = f(\hat{x}_{k-1}^{(i)}, u_k, t_k)$$

(c) Combine the $\hat{x}_k^{(i)}$ vectors to obtain the *priori* state estimate at time *k*.

$$\hat{x}_k^{-} = \frac{1}{2n}\sum_{i=1}^{2n}\hat{x}_k^{(i)}$$

(d) The a *priori* error covariance is estimated. However, one should add Q_{k-1} to the end of the equation to take the process noise into account:

$$P_k^{-} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^{(i)} - \hat{x}_k^{-}\right)\left(\hat{x}_k^{(i)} - \hat{x}_k^{-}\right)^T + Q_{k-1}$$

4. Now that the time update equations are done to implement the measurement update equations:

(a) Choose sigma points $x_k^{(i)}$ with the current best guess for the mean and covariance of x_k are $\hat{x}_k^{-}$ and P_k^{-}:

$$\hat{x}_k^{(i)} = \hat{x}_k^{-} + \tilde{x}^{(i)} \qquad i = 1,\mathbf{K},2n$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_k^{-}}\right)_i^T \qquad i = 1,\mathbf{K},\ n$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_k^{-}}\right)_i^T \qquad i = 1,\mathbf{K},n$$

This step can be omitted if desired. That is, instead of generating new sigma points one can reuse the sigma points that were obtained from the time update. This will save computational effort, but would sacrifice performance [10, 11].

(b) The known nonlinear measurement equation $h(x_k, t_k)$ is used to transform the sigma points in $\hat{y}_k^{(i)}$ vectors (predicted measurements):

$$\hat{y}_k^{(i)} = h\left(\hat{x}_k^{(i)}, t_k\right)$$

(c) The $\hat{y}_k^{(i)}$ vectors are combined to obtain the predicted measurement at time *k*.

$$\hat{y}_k = \frac{1}{2n}\sum_{i=1}^{2n}\hat{y}_k^{(i)}$$

(d) The covariance of the predicted measurement is estimated. However, one should add R_k to the end of the equation to take the measurement noise into account:

$$P_y = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{y}_k^{(i)} - \hat{y}_k\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T + R_k$$

(e) Estimation of the cross covariance between $\hat{x}_k^{-}$ and $\hat{y}_k$.

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^{(i)} - \hat{x}_k^{-}\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T$$

(f) The measurement update of the state estimate can be performed using the normal Kalman filter equations [12]:

$$K_k = P_{xy}P_y^{-1}$$

$$\hat{x}_k^{+} = \hat{x}_k^{-} + K_k(y_k - \hat{y}_k)$$

$$P_k^{+} = P_k^{-} - K_k P_y K_k^T$$

Results

It will be assumed that the measurement noise terms, v_{ak} and v_{bk}, are zero-mean random variables with standard deviations equal to 0.1 amps. The control inputs (winding voltages) are equal to

$$u_{ak} = sin(2p\ kT)$$
$$u_{bk} = cos(2p\ kT)$$

The voltages that are applied to the winding currents are equal to these values plus Δu_{ak} and Δu_{bk}, which are zero-mean random variables with standard deviations equal to 0.001 amps. The noise due to load torque disturbances Δa has a standard deviation of 0.05 rad/sec^2.

Even though the measurements consist only of the winding currents, one can use UKF to estimate the rotor position and velocity.

The initial conditions of the system and the estimator are given as

$$x_0 = [0 \quad 0 \quad 0 \quad 0]^T$$
$$\hat{x}_0^+ = x_0$$
$$P_0^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A rectangular integration with a step size of T=1 msec is used to simulate the system (the extended Kalman filter and the unscented Kalman filter) for 2 seconds.

Figure (2) shows the true and estimated winding currents, rotor velocity and rotor position of synchronous machine. One can easily see that the UKF could estimate all the states of the motor.

Figure (3) shows the standard deviation of state estimation errors obtained from the filter with six sigma points (since we chose $W(0) = 0$).

The **P** matrix quantifies the uncertainty in the state estimates. In other words, the **P** matrix should give us an idea of how accurate our estimates are. Figure (4) gives the behavior of the sum of diagonal elements (trace) of matrix **P** for the UK filter. The figure shows that the confidence of the filter with its estimates is low at start of filtering, and then the filter will become more certain with its estimate as time pass beyond 0.2 seconds.

Conclusions:

- The simulated results shows that the unscented filter could successively estimate all motor states.
- The UKF does not use Jacobians. For systems with analytic process and measurement equations, it is easy to compute Jacobians. But some systems are not given in analytical form and it is numerically difficult to compute Jacobians.
- The confidence of the filter with its estimates has been measured by the trace of the covariance matrix **P**. The transient behavior of this matrix shows that the filter is not certain of its estimates, while the steady state of matrix **P** will settles to a low value (0.5). This indicates that the filter will have more accurate estimations than its transient phase.
- One can figure out the rotor position and velocity without using an encoder. Instead of an encoder to get rotor position, one just needs a couple of sense resistors to measure the winding currents, and a Kalman filter algorithm.

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Appendix

Table (1) The Machine parameters

Parameter	***value***
Winding resistance	*1.9 ohm*
Winding inductance	*0.003 H*
Motor constant	*0.1 sec*
Moment of inertia	*0.00018*
Coefficient of viscous friction	*0.001*
Input frequency	*1 Hz*

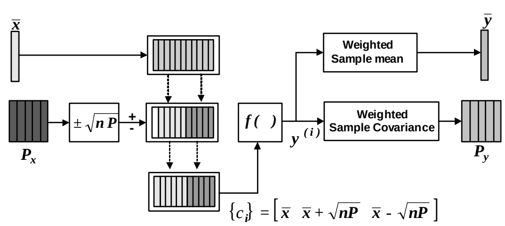

Figure (1) Steps of performing UT

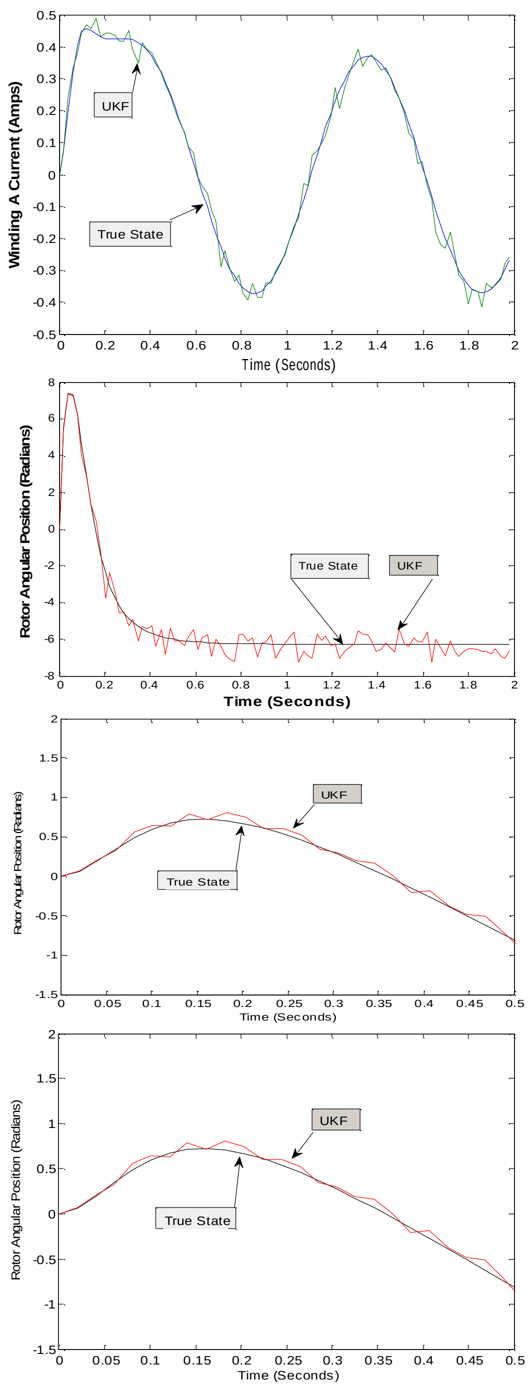

Figure (3) Currents, velocity and position estimation error magnitudes resulting from filter

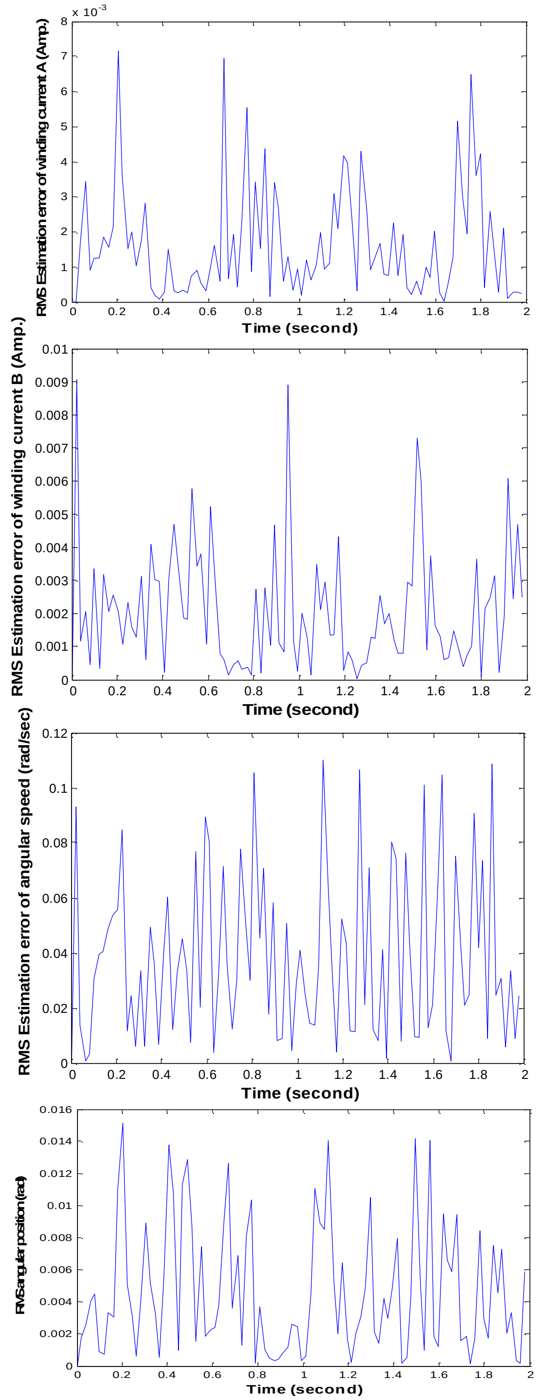

Figure (2) True and estimated variables of two-phase PM synchronous motor with UK filter

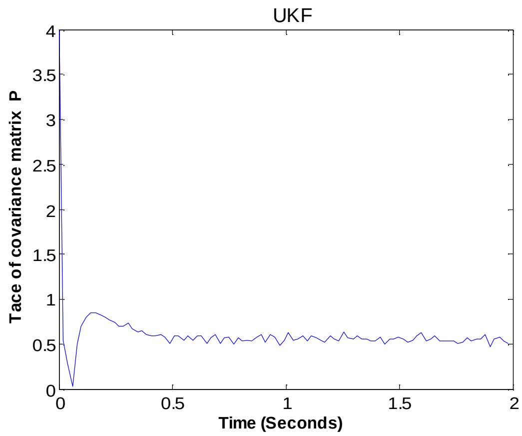

Figure (4) Behavior of covariance matrices P of UKF.