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Core Polarization Effects on the Inelastic Longitudinal C2 Form Factors of Open Sd-Shell Nuclei

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Abstract

Inelastic longitudinal $C2$ form factors for $0_1^+1 \rightarrow 2_1^+1$ and $0_1^+1 \rightarrow 2_2^+1$ transitions in open sd shell nuclei (^{22}Ne, ^{26}Mg and ^{30}Si) are discussed taking into accounot the effects of core polarization. These effects are calculated using the shape of Tassie model together with our derived form of the ground state two-body charge density distribution (2BCDD). Remarkable agreements are obtained between the calculated inelastic longitudinal $C2$ form factors and those of experimental data.

Keywords: Core polarization, Inelastic, Longitudinal and Charge

تأثيرات استقطاب القلب على عوامل التشكل للأستطارة الطولية غير المرنة C2 لنوى القشرة- sd

الخلاصة

تم حساب عوامل التشكل للاستطارة الطولية غير المرنة C2 للانتقالات $0_1^+1 \rightarrow 2_2^+1$ و $0_1^+1 \rightarrow 2_1^+1$ لنوى القشرة sd (^{22}Ne, ^{24}Mg و ^{30}Si) بالاعتماد على تاثيرات استقطاب القلب. التأثيرات تم حسابها بالاعتماد على Tassie Model ومن خلال اشتقاق صيغة لتوزيع كثافة الشحنة للحالة الارضية (2BCDD) . حصلنا على تطابق جيد بين النتائج العملية والنظرية لعوامل التشكل للاستطارة الطولية غير المرنة لهـه النوى .

1. Introduction

Charge density distributions, transition densities and form factors are considered as fundamental characteristics of the nucleus. These quantities are usually determined experimentally from the scattering of high energy electrons by the nucleus. The information extracted from such experiments is more accurate with higher momentum transfer to the nucleus. Various theoretical methods [1, 2, 3] are used for calculations of the charge density distributions, among them the Hartree-Fock method with the Skyrme effective interaction the theory of finite Fermi systems and the single particle potential method. Calculations of form factors [4] using the model space wave function alone is inadequate for reproducing the data of electron scattering. Therefore effects out of the model space, which is called core polarization effects, are necessary to be included in the calculations. These effects can be considered as a polarization of core protons by the valence protons and neutrons. Core polarization effects can be treated either by connecting the ground

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state to the J-multipole $n\hbar w$ giant resonances [4,5,6], where the shape of the transition densities for these excitations is given by Tassie model [7], or by using a microscopic theory [8,9] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Comparisons between theoretical and observed longitudinal electron scattering form factors have long Been used as stringent test of models of nuclear structure.

In this study, we have derived an expression for the ground state two - body charge density distributions (2BCDD) of light nuclei, based on the use of the two - body wave functions of the harmonic oscillator and the two–body correlation functions, which take account of the effect of the strong short range repulation and the strong tensor force in the nucleon-nucleon forces. Our aim is to investigate the inelastic longitudinal electron scattering form factors, where the deformation in nuclear collective modes (which represent the core polarization effects) is taken into consideration besides the shell model space transition density. Core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state 2BCDD. This study is devoted on $0_1^+1 \rightarrow 2_1^+1$ and $0_1^+1 \rightarrow 2_2^+1$ transitions in ^{22}Ne, ^{26}Mg and ^{30}Si nuclei.

2. Theory

The many particle reduced matrix elements of the longitudinal operator, consists of two parts; one is for the model space and the other is for core polarization matrix element[5,6]:

$$\left\langle f \left\| \hat{T}_J^L(t_Z,q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{T}_J^L}(t_Z,q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{T}_J^L}(t_Z,q) \right\| i \right\rangle \quad .....(1)$$

The model space matrix element has the form [10]:

$$\left\langle f \left\| \overset{ms}{\hat{T}_J^L}(t_Z,q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{ms}{\rho}_{J,t_Z}(i,f,r) \quad ....(2)$$

where $\overset{ms}{\rho}_{J,t_z}(i,f,r)$ is the transition charge density of model space given by [4]:

$$\overset{ms}{\rho}_{J,t_Z}(i,f,r) = \sum_{jj'} OBDM(i,f,J,j,j',t_z) \left\langle j \left\| Y_J \right\| j' \right\rangle R_{nl}(r)\, R_{n'l'}(r) \quad ...(3)$$

The core- polarization matrix element is given by[4]:

$$\left\langle f \left\| \overset{core}{\hat{T}_J^L}(t_Z,q) \right\| i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \overset{core}{\rho}_{J,t_Z}(i,f,r) \quad ....(4)$$

where $\overset{core}{\rho}_{J,t_Z}$ is the core-polarization transition density which depends on the model used for core polarization. To take the core-polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{Jt_Z}(i,f,r)=\rho^{ms}_{Jt_Z}(i,f,r)+\rho^{core}_{Jt_Z}(i,f,r) \quad .....(5)$$

where ρ^{core}_{J,t_Z} is assumed to have the form of Tassie shape and given by [7].

$$\rho^{core}_{Jt_z}(i,f,r)=N\frac{1}{2}(1+t_z)r^{J-1}\frac{d\rho(i,f,r)}{dr} \quad ...(6)$$

where N is a proportionality constant. It is determined by adjusting the reduced transition probability *B(CJ)* and can be given as:

$$N=\frac{\int_0^\infty dr r^{J+2}\rho^{ms}_{Jt_Z}(i,f,r)-\sqrt{(2J_i+1)B(CJ)}}{(2J+1)\int_0^\infty dr r^{2J}\rho(i,f,r)} \quad .. (7)$$

Here, $\rho(i,f,r)$ is the ground state two-body charge density distribution derived as follow; we have produced an effective two-body charge density operator by folding the two-body charge density operator with the two-body correlation functions $\tilde{f}_{ij}$ as[10]:

$$\hat{\rho}^{(2)}_{eff}(\vec{r})=\frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\tilde{f}_{ij}\left\{\delta\left[\sqrt{2}\vec{r}-\vec{R}_{ij}-\vec{r}_{ij}\right]+\delta\left[\sqrt{2}\vec{r}-\vec{R}_{ij}+\vec{r}_{ij}\right]\right\}\tilde{f}_{ij} \quad ....(8)$$

where $\vec{r}_{ij}$ and $\vec{R}_{ij}$ are relative and center of mass coordinates and the form of $\tilde{f}_{ij}$ is given by [11]:

$$\tilde{f}_{ij}=f(r_{ij})\Delta_1+f(r_{ij})\{1+\alpha(A)S_{ij}\}\Delta_2 \quad ....(9)$$

It is clear that eq. (9) contains two types of correlations:

1. The two body short range correlations (SRC) presented in the first term of eq. (9) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body functions with the exception of $^3S_1$ and $^1D_3$ states. It should be noted that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances,where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of $f(r_{ij})$ is given as [11]:

$$f(r_{ij})=\begin{cases}0 & for\ r_{ij}\le r_c\\ 1-\exp\{-\mu(r_{ij}-r_c)^2\} & for\ r_{ij}>r_c\end{cases} \quad ... (10)$$

where r_c (in *fm*) is the radius of a suitable hard core and $\mu = 25\ fm^{-2}$ [11] is a correlation parameter.

2. The two-body tensor correlations (TC) presented in the second term of eq.(9) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto $^1S_3$ and $^1D_3$ states only. S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{s}_i . \vec{r}_{ij})(\vec{s}_j . \vec{r}_{ij}) - \vec{s}_i . \vec{s}_j \quad \text{.... (11)}$$

The parameter $a(A)$ is the strength of tensor correlations and it is non zero only in the ${}^1S_3 - {}^1D_3$ channels.

The ground state two body charge density distribution $\rho_{ch}(r)$ is given by the expectation value of the effective two-body charge density operator of eq(8) and written as

$$\rho_{ch}(r) = \langle \psi | \hat{\rho}^{(2)}_{eff}(\vec{r}) | \psi \rangle = \sum_{i<j} \langle ij | \hat{\rho}^{(2)}_{eff}(\vec{r}) | [| ij \rangle - | ji \rangle] \quad \text{.... (12)}$$

where the two partical wave function is given by [12]

$$| ij \rangle = \sum_{JM_J} \sum_{TM_T} \langle j_i m_i \, j_j m_j | JM_J \rangle \times \langle t_i m_{t_i} \, t_j m_{t_j} | TM_T \rangle \times | (j_i j_j) JM_J \rangle | (t_i t_j) TM_T \rangle \quad \text{... (13)}$$

where J and M_J denote the total angular momentum and it's projection of a pair of particles formed by coupling j_i and j_j while T and M_T denote their total isospin and isospin projection formed by coupling t_i and t_j.

It is important to indicate that our effective two body charge density operator of eq(8) is constructed in terms of relative and centre of mass coordinates, therefore the space-spin part $| (j_i j_j) JM_J \rangle$ of the two partical wave function constructed in jj-coupling scheme must be transformed in terms of relative and centre of mass coordinates. This transformation can be achieved as follow:

1. Switching from jj to λS coupling schemes as [13]

$$| (j_i j_j) JM_J \rangle \equiv | (\lambda_i \tfrac{1}{2}) j_i , (\lambda_j \tfrac{1}{2}) j_j ; JM_J \rangle = \sum_{\lambda S} \hat{j}_i \hat{j}_j \hat{\lambda} \hat{S} \begin{Bmatrix} \lambda_i & \lambda_j & \lambda \\ \tfrac{1}{2} & \tfrac{1}{2} & S \\ j_i & j_j & J \end{Bmatrix} | (\lambda_i \lambda_j) \lambda (\tfrac{1}{2} \tfrac{1}{2}) S ; JM_J \rangle \quad \text{...(14)}$$

where the notation $\hat{A} = (2A+1)^{1/2}$ and the bracket $\begin{Bmatrix} \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \end{Bmatrix}$ is the 9-j symbol.

2. The Brody–Moshinsky transformation brackets [13] has been used to transform the spatial part of the two-body wave function $| (\lambda_i \lambda_j) \lambda \rangle$ in terms of relative and centre of mass coordinates.

$$| (\lambda_i \lambda_j) \lambda \rangle \equiv | n_i \lambda_i n_j \lambda_j ; \lambda \rangle = \sum_{n \lambda N L} \langle n \lambda , N L ; \lambda | n_i \lambda_i , n_j \lambda_j ; \lambda \rangle | n \lambda , N L ; \lambda \rangle \quad \text{....(15)}$$

Where the coefficient $\langle n \lambda , N L ; \lambda | n_i \lambda_i , n_j \lambda_j ; \lambda \rangle$ is an overlap integral and called a transformation bracket. For the purpose of extending the calculation to open shell nuclei we replaced the factors $\hat{j}_i$ and $\hat{j}_j$ in eq.(14) as

$$(2j_i+1)^{1/2} \Rightarrow \{\eta_{n_i l_i j_i}(2j_i+1)\}^{1/2}$$
$$(2j_j+1)^{1/2} \Rightarrow \{\eta_{n_j l_j j_j}(2j_j+1)\}^{1/2}$$

….(16)

where $\eta_{n_i l_i j_i}$ and $\eta_{n_j l_j j_j}$ are the occupation probabilities of the states $n_i l_i j_i$ and $n_j l_j j_j$, respectively. These parameters equal to (zero or 1) for closed shell nuclei while for open shell nuclei they are larger than zero or less than one (i.e. $0<\eta_{n_i l_i j_i}<1$) and ($0<\eta_{n_j l_j j_j}<1$).

The longitudinal form factor is related to the charge density distribution through the matrix elements of multipole operators $\hat{T}_J^L(q)$[4].

$$\left|F_J^L(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \times \left|\left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle\right|^2 \left|F_{cm}(q)\right|^2 \left|F_{fs}(q)\right|^2$$

….(17)

where Z is the proton number in the nucleus and $F_{cm}(q)$ is the centre of mass correction, which removes the spurious state arising from the motion of the center of mass when shell model wave function is used, and given by [11]:

$$F_{cm}(q) = e^{q^2b^2/4A} \quad …..(18)$$

where A is the nuclear mass number and b is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the finite size correction, considered as a free nucleon form factor and assumed to be the same for protons and neutrons, and it takes the form[11]:

$$F_{fs}(q) = e^{-0.43q^2/4}$$

…(19)

3. Results, Discussion and Conclusion:

We first discuss the effects of two-body SRC and TC on the ground state 2BCDD in the open sd-shell nuclei ^{22}Ne, ^{26}Mg and ^{30}Si. The parameters required in the calculations of 2BCDD's such as the occupation probabilities η's of the states, the values of $\alpha\ (A)$ and the values of $\hbar\omega$ which are chosen in such away that to reproduce the root mean square radii of considered nuclei are presented in table 1. The occupation probabilities are determined from the comparison between the calculated and experimental charge densities at $r=0$ (i.e. $\rho_{\exp}(r=0)$). The dependence of $\rho_{ch}(r)$ (in fm^{-3}) on r (in fm) for ^{22}Ne, ^{26}Mg and ^{30}Si nuclei are displayed in Fig. 1. The dashed and solid distributions are the calculated $\rho_{ch}(r)$ without including the effects SRC and TC (r_c =0 and $\alpha\ (A)$=0) and with including the effects of SRC and TC (r_c =0.5 fm and $\alpha\ (A) \neq 0$), respectively. These distributions are compared with those of experimental data [14], denoted by dotted symbols. It is clear that the dashed distributions deviate from the experimental data especially at small r. Introducing the effects of SRC and TC tends to remove these deviations from the region of small r as seen in the solid duistributions. It is evdient from these figures that the calculated 2BCDD's represented by the solid curves are in excellent agreement with those of experimental data hence they coincide with each other throughout the whole range of r(fm). Considering the effect of higher state occupation probabilities and the effects of SRC and TC are generally, essential in getting good agreement between the calculated result and experimental data.

Core polarization effects on the inelastic longitudinal C2 form factors for $0_1^+1 \rightarrow 2_1^+1$ and $0_1^+1 \rightarrow 2_2^+1$ transitions in some open sd shell nuclei, are discussed. The core polarization effects on the form factors are based on the Tassie model [7] together with the calculated 2BCDD. We adopt the universal sd *(USD)* interaction of Wildenthal [4] to generate the sd model space matrix elements, using the shell model code OXBASH [15]. The inelastic longitudinal C2 form factors for the transitions to the 2_1^+1 and 2_2^+2 states are displayed in Figs. 2 to 4. The dash-dotted curves represent the contribution of the model space where the configuration mixing is taken into account, the dashed curves represent the core polarization contribution where the collective modes are considered and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects. The experimental data are represented by solid circles.

The C2 form factor of the lowest 2^+1 states (1.275 MeV and 4.457 MeV) in ^{22}Ne nucleus are shown in Fig.2 and compared with the experimental data of Ref.[16]. The sd-shell model space calculation fails to describe the data in q-dependent form factor. The core-polarization effects give a strong modification to the form factors, where the core polarization effects enhance the form factors at the first maximum and bring the calculated values very close to the experimental data. The experimental data for the second 2^+1 state is well described by the *sd*-shell model, and the core polarization effect enhances the form factor.

The C2 form factors of the lowest 2^+1 states (1.809 MeV and 2.938 MeV) in ^{26}Mg nucleus are shown in Fig. 3. The model space calculations underestimate the data at all region of q. The core-polarization effects give a strong modification to the form factors, where the core polarization effects enhance the form factors and bring the calculated values very close to the experimental data. The experimental data for the second 2^+1 state is mostly determined by the core polarization form factor, where the *sd*-shell contribution is small. The data are slightly over estimated when core polarization effect is included. The core polarization effects play the major role for this transition.

The C2 form factors of ^{30}Si nucleus are shown in Fig.4 for the lowest $2^{+}1$ states (1.809 MeV and 2.938 MeV). The model space calculation fails to describe the data through out all range of q values. The core polarization effects enhance the form factors and lead to give a satisfactroy description with the data.

It is concluded that the core polarization effects, which represent the collective modes, are essential in obtaining a remarkable agreement between the calculated and experimental $C2$ form factors of the open sd -shell nuclei.

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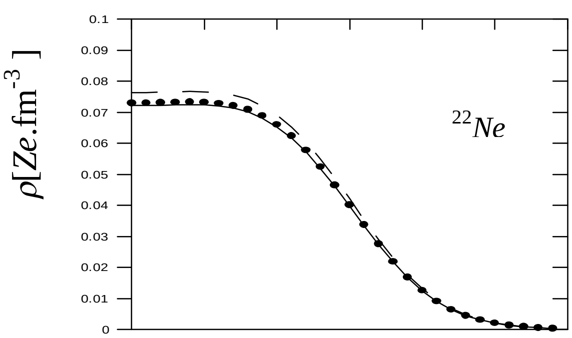
Table (1) Parameters to the ground state 2BCDD's for some open shell nuclei.

Nucleus	**h**w (MeV)	$h_{1S_{1/2}}$	$h_{1P_{3/2}}$	$h_{1P_{1/2}}$	$h_{1d_{5/2}}$	$h_{2s_{1/2}}$	$r_{\exp}(0)$ (fm^{-3}) **[14]**	$a(A)$
^{22}Ne	**12.75**	**1**	**1**	**1**	**0.275**	**0.175**	**0.073**	**0.083**
^{26}Mg	**12.70**	**1**	**1**	**1**	**0.611**	**0.165**	**0.075**	**0.080**
^{30}Si	**12.5**	**1**	**1**	**1**	**0.866**	**0.40**	**0.076**	**0.077**



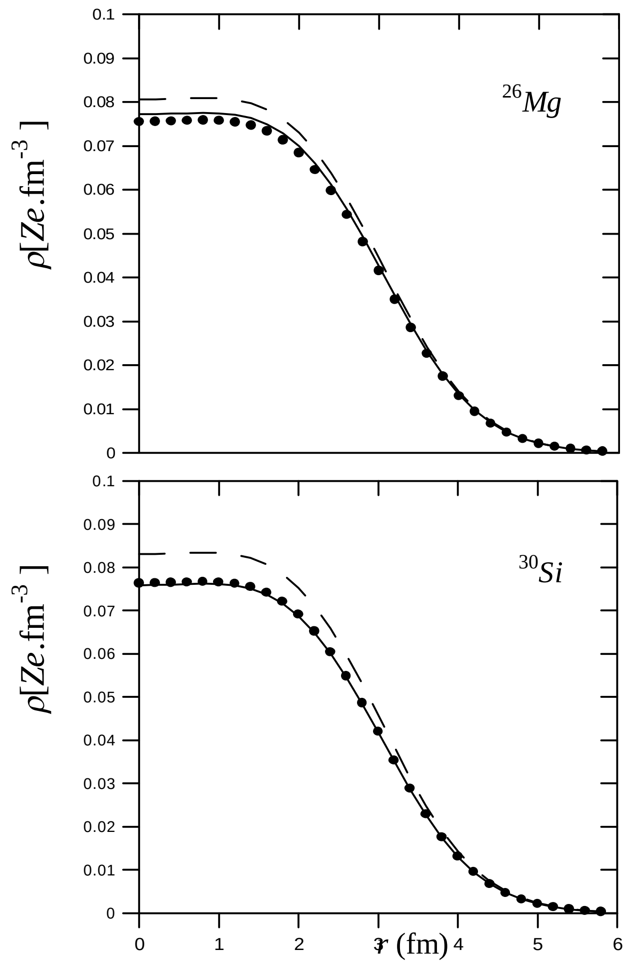
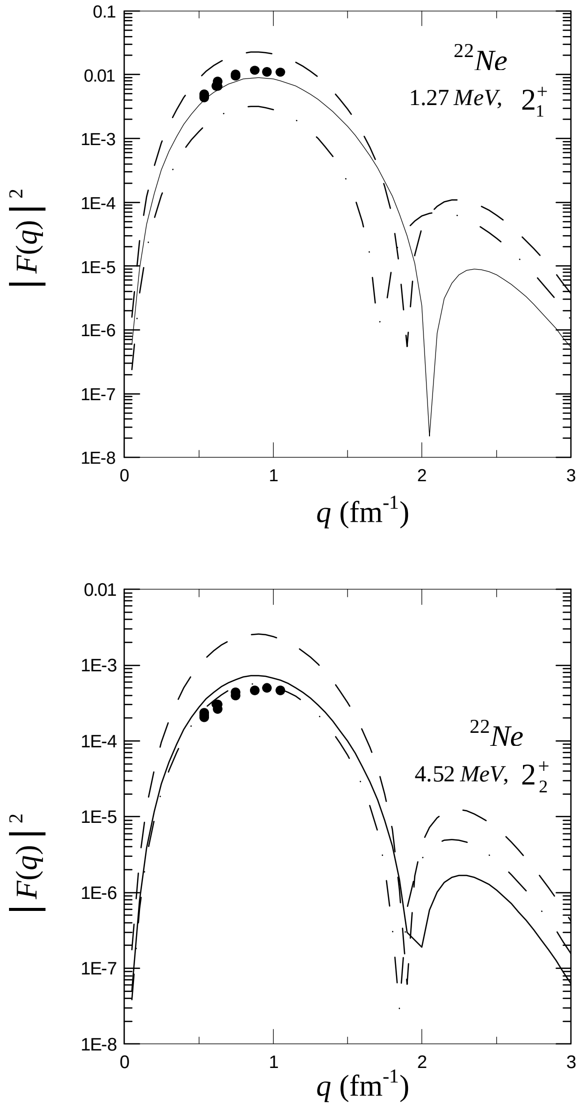


Figure (1) Dependence of the 2BCDD on r(fm) for ^{22}Ne, ^{26}Mg and ^{30}Si Nuclei respectively. The dotted symboles are the experimental data of Ref [14].



q (fm^{-1})

Figure (2) The Coulomb $C2$ form factors for the transitions to the 2^+1 (1.275 MeV and 4.457 MeV) states in ^{22}Ne. The experimental data are taken from Ref. [16].

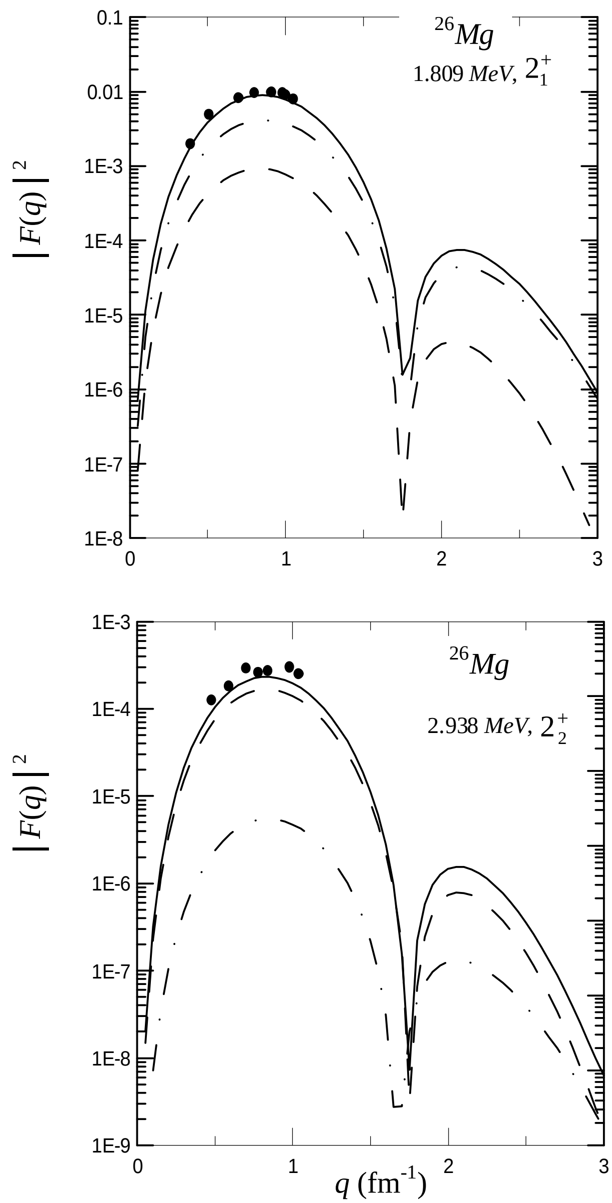


Figure (3) The Coulomb $C2$ form factors for the transitions to the 2^+1 (1.809 MeV and 2.938 MeV) states in ^{26}Mg. The experimental data are taken from Ref. [17].

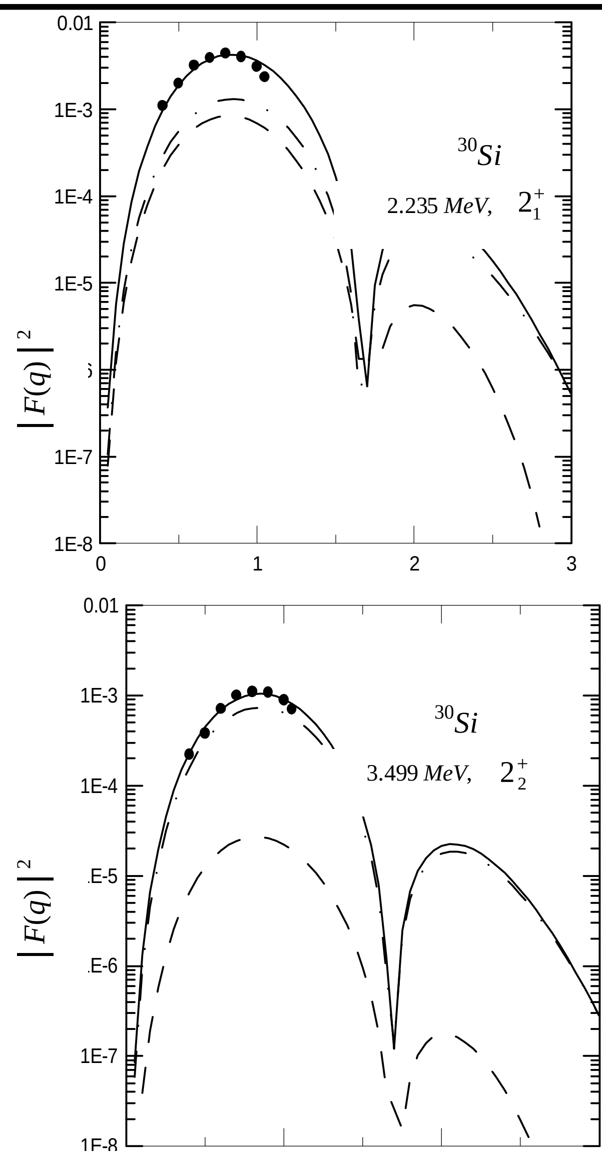


Figure (4) The Coulomb $C2$ form factors for the transitions to the 2^+1 (2.235 MeV and 3.499 MeV) states in ^{30}Si. The experimental data are taken from Ref. [18].