

## Effect of the Number of Horizontal Construction Joints In Reinforced Concrete Beams

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### Abstract

In this paper some results from previous experimental test are adopted and analyzed using a nonlinear three-dimensional finite element ANSYS computer program (v.11) to investigate the effect of the presence of horizontal construction joints (H.C.J.) on the behavior of reinforced concrete (RC) beams.

Three beams having one, two and three (H.C.J.) that divide the beam into equal parts, as well as one reference beam without a joint were analyzed. The results obtained from the finite element analysis show very good agreement with the results obtained from the previous experimental test. The maximum differences in ultimate loads were about (8.2-10.4)% for all types of tested beams. The presence of one, two and three (H.C.J.) in RC beams under flexure gave a decrease in the value of the cracking load such that  $P_{cr}$  was (97%), (85%) and (80%) of (Bref). The respective ultimate load capacity  $P_u$  was (96%), (89%) and (84%) compared to (Bref).

**Keywords:** RC beams, horizontal construction joints, dowels, reinforcing steels, ANSYS, finite element method.

### تأثير عدد المفاصل الأفقية الإنشائية على البلاطات الخرسانية المسلحة

#### الخلاصة

في هذا البحث تم تبني وتحليل بعض النتائج العملية لفحوصات سابقة لتقصي تأثير وجود المفاصل الإنشائية الأفقية على سلوك العتبات الخرسانية المسلحة باستخدام طريقة العناصر المحددة ثلاثية الأبعاد لاخطية بالاستفادة من برنامج ANSYS (نسخة 11). تم دراسة تأثير عدد المفاصل الأفقية على الاحمال المسببة للشقوق والتحمل الأقصى للعتبات. تم تحليل ثلاثة من العتبات بوجود واحد، اثنين وثلاثة مفاصل إنشائية أفقية ضمن عمق العتبة بحيث تقسم العتبة الى اجزاء متساوية ، إضافة الى عتبة قياسية بدون مفصل. نتائج الدراسة أشارت الى تطابق النتائج النظرية مع العملية بالنسبة للحمل الأقصى بنسبة اختلاف تصل الى (8.2-10.4)% لكل انواع العتبات التي تم فحصها كما اوضحت النتائج الى ان وجود مفصل واحد ، مفصلين و ثلاثة مفاصل في العتبات الخرسانية المسلحة يؤدي الى تقليل في الاحمال المسببة للشقوق بنسبة (97%) ، (85%) و (80%) من العتبة القياسية بدون مفصل وتقليل في الاحمال القصوى بنسبة (96%) ، (89%) و (84%) من العتبة القياسية بدون مفصل.

### Introduction

Joints in concrete slabs and beams can be created by forming, tooling, sawing, and placement of joint formers, some forms of joints are: Contraction joints, Isolation or expansion joints and Construction joints.<sup>(1)</sup>

A construction joint is a plane surface between two sections of concrete; one placed against another that is already in place which has hardened to the extent that consolidation cannot be effected by vibration or re-vibration. Generally, construction joints can be classified according to the plane of joint construction into four groups; horizontal, vertical, longitudinal and transverse joints. In beams, there are horizontal and vertical joints, while columns and massive concrete structures have horizontal joints only. The terms "longitudinal" and "transverse" are most appropriate in each of slabs, bridge decks, pavements and tunnels. This type of joint is commonly termed as a "cold joint".<sup>(2)</sup>

Construction joints are nearly always the weakest points in a structure. Therefore, the formation of a good construction joint, having the capability of providing a well-bonded medium between the hardened and the fresh concrete, is the main problem remaining. Thus construction joints in

results, ultimate shear strength (Y) may be given by:

concrete structures should be placed where shear forces are expected to be low. Both the location and the size of joint should in general be chosen according to the type of structure to ensure good performance of the structure and to provide acceptable appearance.<sup>(3)</sup>

**Wuerpel C.**<sup>(4)</sup> carried out tests on 100 and 150 mm diameter concrete cores, containing horizontal construction joint planes, removed from hydraulic structures. The results indicate that: such joints are highly resistant to deterioration by frost action when carefully cleaned with air and water jets.

**Waters T.**<sup>(5)</sup> conducted a study on the tensile strength of concrete across construction joints. The results showed that the bond strength between the new and old concrete was increased when the old concrete had dried before the new concrete was placed.

**Saemann J. and Washa G.**<sup>(6)</sup> studied the strength of the joint between precast concrete beams and cast-in-place concrete slabs. The results showed that the ultimate shear strength of joints increased as the contact surface roughness increased from smooth to intermediate, as the amount of stirrup steel across the joint increased, and as the ratio of shear span to effective depth decreased, and on the basis of the test

$$Y = V_u = \frac{2700}{x + 6} + \frac{300P(33 - x)}{x^2 + 6x + 5} \dots \dots (1)$$

Where;  $P$ : represents the percent steel across the joint ( $\rho$ ) and  $x$ : represents the ratio of shear span to effective depth ( $a/d$ ).

**Paulay T. et.al** <sup>(7)</sup> carried out experimental tests that were conducted to examine performance of horizontal construction joints in cast-in-place reinforced concrete. It was found that adequately reinforced horizontal construction joints with a clean and rough surface, (to which the freshly placed concrete can bond), can develop interface shear strength equal to or larger than the diagonal tension capacity of the structure.

**ACI Committee 224 R** <sup>(1)</sup> report reviewed the state of the art on design, construction, and maintenance of joints in concrete structures subjected to a wide variety of use and environmental conditions. The option of eliminating joints was considered. Aspects of various joint sealant materials and jointing techniques were discussed in this report.

Some recommendations and suggestions for the use of construction joints were reported as follows;

1. The desirable location for joints placed perpendicular to the main reinforcement was at points of minimum shear or point of contraflexure.
2. Horizontal construction joints in beams and girders

are usually not recommended.

**ACI 318M-08 Building Code** <sup>(8)</sup> suggested the following formula for estimating the ultimate shear force across an interface;

$$V_n = \mu A_{vf} \times f_y \quad \dots\dots (2)$$

Where:  $V_n$  = nominal shear strength,

$\mu$  = coefficient of friction along the interface and

$A_{vf}$  = area of shear-friction reinforcement.

**Djazmati Basel and Pincheira Jose A.** <sup>(9)</sup> presented the test results of unreinforced concrete construction joints subjected to in-plane shear forces. The main purpose of the study was to determine whether concrete foundations cast in multiple pours with horizontal construction joints could offer the same initial (uncracked) stiffness of those cast monolithically. It is concluded that members with a properly prepared and moist-cured joint offer the same initial stiffness as that of a member cast monolithically.

**Ismail A.Kh.** <sup>(10)</sup> carried out an experimental work of ten simply supported RC beams to study the behavior of RC beams having (H.C.J.). The results of this series of tests have indicated that the presence of (H.C.J.) in a RC beams leads to a decrease in its cracking and ultimate loads and increase in its ultimate deflection while no appreciable change in the value of the beam deflection at first crack can be expected.

**Research Significance**

The aim of this research is to compare the test results of three existing simply supported RC beams with horizontal construction joint and one reference beam without any joint, adopted from Ref. (10), with the theoretical results of the same beams analyzed using the finite element software ANSYS.

All the tested beams had been designed to fail in flexure and had the same amount and type of longitudinal and transverse reinforcement as well as similar concrete properties, and the same rectangular cross-section of dimensions (150mm) width, (250mm) depth and an overall length of (2m). The geometry and loading conditions for this studied specimens are detailed in (Fig. - 1).

The beams differs in the number of (H.C.J.) used, one of the beams was without any joint and it was used as a reference beam (Bref), while (B1) has one horizontal joint placed at the mid depth of the beam, (B4) has two horizontal joints placed at 1/3 and 2/3 of the depth of the beam and the last beam (B5) has three horizontal joints placed at 1/4, 1/2, and 3/4 of the depth of the beam, the types of the studied beams are shown in (Table - 1).

The concrete properties for the tested beams were established by casting three (150mmx150mmx150mm) cubes and three (300mmx150mm) cylinders as well as

(100mmx100mmx400mm) prisms, all of which were tested at the age of 28 days to determine their compressive strength ( $f_{cu}$ ), ( $f_c'$ ) and the modulus of rupture ( $f_r$ ); the average mean values of the beams were ( $f_{cu}=32\text{MPa}$ ), ( $f_c'=26\text{MPa}$ ) and ( $f_r=3.9\text{MPa}$ ). the mechanical properties of the reinforcing steel bars used are shown in (Table - 2).

**Finite Element Analysis Approach**

The finite element method (FE) is a numerical analysis technique that can be applied to obtain solutions to a variety of engineering problems, it can be used for linear and nonlinear analysis of RC structures.

ANSYS (ANalysis SYStem) computer program (v.11) is used for the model analysis. The status transition of concrete from an uncracked to cracked state and the nonlinear material properties of concrete in compression and steel as it yields cause the nonlinear behavior of the structures under loading. Newton-Raphson equilibrium iteration is used to solve nonlinear problem in ANSYS. In a linear analysis the size of the load increment does not affect the results at all.

However, for a nonlinear analysis, in which FE structures start cracking and behave nonlinearly under a sufficiently large load, the load applied to the structures must be increased gradually to avoid non-convergence. Tolerances in both force and displacement criteria may have to be gradually increased during

the loading history to attain convergence<sup>(12)</sup>.

### Material Modeling:

1. **Modeling of Concrete:** Three dimensional brick element (SOLID65) was used to model the concrete with reinforcing bars (rebars). The element is defined by eight nodes having three degrees of freedom at each node: translations in x, y, and z-directions. The most important aspect of this element is the treatment of nonlinear material properties. The concrete is capable of cracking (in three orthogonal directions), crushing, plastic deformation, and creep. The rebars are capable of tension and compression, but not shear. They are also capable of plastic deformation and creep. The element is defined by eight nodes and by the isotropic material properties. The geometry, node locations, and the coordinate system for this element are shown in (Fig. - 2). The element, using a  $2 \times 2 \times 2$  set of Gaussian integration points, is defined by eight nodes having three translational degrees of freedom at each node as detailed in (Fig. - 3).

A typical uniaxial compressive stress - strain curve is shown in (Fig.- 4a). It can be noted that the concrete behaves as a linear elastic material when the stress level is less than about 30 percent of the uniaxial compressive strength ( $f'_c$ ), then

the curve shows a gradual increase in curvature up to about  $(0.75 f'_c - 0.9 f'_c)$ , thereafter the stress-strain curve descends (after reaching  $f'_c$ ) until failure occurs due to the crushing of concrete at the ultimate strain ( $\epsilon_u$ ). (Fig.- 4b) shows the post cracking model for the

concrete, where  $S_n$  and  $e_n$  are the stress and the strain normal to the cracked plane,  $e_{cr}$  is the cracking strain associated with cracking stress  $S_{cr}$  equal to  $f_t$  and  $a_1$  and  $a_2$  are the tension stiffening parameters.  $a_1$ , represents the rate of stress release as the crack widens, while  $a_2$  represent the sudden loss of stress at instant of cracking<sup>(12)</sup>.

2. **Modeling of Steel:** Modeling of steel in connecting with the finite element analysis of RC members is much simpler than modeling of concrete. The uniaxial stress-strain relation for steel was idealized as a bilinear curve, representing elastic-plastic behavior with strain hardening, this relation is assumed to be identical in tension and in compression as shown in (Fig. - 5).

In the present research, the reinforcement is represented by

using “link elements” (Discrete Representation). The reinforcement in the discrete model uses one dimensional bar or beam elements that are connected to concrete mesh nodes as shown in (Fig. - 6). Therefore, the concrete and the reinforcement mesh share the same nodes and the same occupied regions. Full displacement compatibility between the reinforcement and concrete is a significant advantage of the discrete representation. Their disadvantages are the restriction of the mesh and the increase in the total number of elements. The LINK8, 3-D spar element, is used to represent the reinforcing steel bar.

### 3. Interface Finite Element Idealization:

Under static loading, a construction joint is modeled by a medium of negligible thickness called an “Interface”, which represents two surfaces that are in a state of physical contact but may slide relative to each other<sup>(13)</sup>. When structural members deform under external vertical loads, large horizontal (shears) forces are developed that act on the planes of contacts. Shear force may be transferred by means of friction and by the dowel action of crossing bars. Two combined interface models are used in this study. The first interface is capable of supporting

only compressive forces in the direction normal to the interface surface and Coulomb shear friction in the tangential direction. While, the second uses the normal and tangential (or dowel) stiffness of the transversely crossing bars.

- a. **Shear – Friction Modeling:** The behavior at interface between structural materials involves relative translational motions under static loading<sup>(11)</sup>. In the finite element method, an interface or joint elements are used in order to account for the relative motions and associated deformation modes.

A three-dimensional point-to-point contact element<sup>(12)</sup> is used to model the nonlinear behavior of the surface between two concretes cast at different times. This model also includes the definition of the stress transfer. The element joins two surfaces that may maintain or break physical contact and may slide relative to each other. Also, the element is capable of supporting only compression in the direction normal to the interface between the two surfaces and Coulomb shear-friction in the tangential direction.

The 3-D point-to-point contact element has three degrees of freedom at each node (u, v and w) in the element coordinate system. The orientation of the interface is defined by the node locations. The interface is assumed

to be perpendicular to I–J line, as shown in (Fig. - 7).

In the basic Coulomb friction model, the two contacting surfaces can carry shearing stress up to a certain magnitude across their interface before they start sliding relative to each other. This state is known as sticking. Once the shearing stress is exceeded, the two surfaces will slide relative to each other. This state is known as sliding. The sticking–sliding calculations determine when a point is transferred from sticking to sliding or vice versa<sup>(12)</sup>. The force–deflection relationships for the interface element can be separated into normal and tangential (sliding) directions as shown in (Fig. - 8).

#### b. Dowel Action Modeling

It is necessary to include the shear transfer mechanism of the dowel bars that are crossing the joint. The dowel action (shearing and flexure of the bars) will contribute to the overall shear stiffness at the joint interface. To include this mechanism, the nonlinear spring element is used<sup>(12)</sup>. This element is a unidirectional element with nonlinear generalized force-deflection capability. The element has longitudinal capability with up to three degrees of freedom at each node (translations in the nodal x, y and z directions). The element is defined by two nodes. The geometry, node locations, and the nonlinear force-deflection for this element are shown in (Fig. - 9).

#### 4. Construction Joint Modeling: .

Construction joints in beams,

columns, and walls can present a potential weakness if large shear forces need to be transmitted across them<sup>(13)</sup>. The shear capacity of the interface could be influenced by the type of surface penetration used for the joint. Shear may be transferred by means of friction when there is normal compressive stress acting on the interface. This normal stress may be due either to an externally imposed load or to reinforcing bars crossing the interface<sup>(14)</sup>. The interaction, or stress transfer between two concretes cast at different time obviously occurs via the following components<sup>(15)</sup>: chemical adhesion, friction caused by direct bearing of small asperities projecting from the faces of the joint and dowel action of the reinforcement crossing the joint.

#### Finite Element Modeling

The parameters used in this research to represent the finite element models which includes the material properties and nonlinear solution parameters are summarized in (Table-3) and (Table-4)<sup>(16)</sup>.

The finite element mesh for all of the tested beams are shown in (Fig. - 10).

#### Comparison of Experimental and Theoretical Results

At the nonlinear stage of loading the FE approach was still reasonably close to the experimental results, which indicates that the model used was efficient. Figs. (11) to (14) show Load-Deflection curves for all

analyzed beams. The experimental and theoretical cracking and ultimate loads are summarized in (Table - 5). The table indicates that the difference was between (8.2-10.4)%, showing an acceptable percentage difference.

#### Discussion of Theoretical Results

Theoretical load-deflection curves for all types of tested beams (Fig. – 15) show a significant drop in post cracking loads, especially in beams with more than one joint. This drop is due to horizontal cracks occurring parallel to the lower horizontal joint and running along the span of the beam in its middle third at a load level of 50 kN. This is shown clearly in beams (B<sub>4</sub>) and (B<sub>5</sub>).

(Fig. – 16) shows the influence of the number of joints on the cracking load compared to (B<sub>ref</sub>), where the drop in the load was about (3%), (15%) and (20%) for (B<sub>1</sub>), (B<sub>4</sub>) and (B<sub>5</sub>) respectively. This difference in the cracking loads is due to the horizontal cracks that occurred below the lower joint in (B<sub>4</sub>) and (B<sub>5</sub>).

The same result was obvious in the ultimate loads as shown in (Fig.– 17) in comparison with (B<sub>ref</sub>) by (4%), (11%) and (16%) for (B<sub>1</sub>), (B<sub>4</sub>) and (B<sub>5</sub>) respectively.

#### Conclusions

Results obtained from the FE model regarding cracking and ultimate loads, compare well with the results of the experimental data for the chosen beams, verifying the accuracy and the validity of the adopted model. Therefore, the nonlinear FE method of analysis may be considered as a

powerful and relatively economic tool for analyzing RC beams with (H.C.J.). The use of interface elements in connecting the concrete brick elements at the location of the construction joint is necessary to simulate the weakness of the joint and to assess the way the stresses will transfer through that joint.

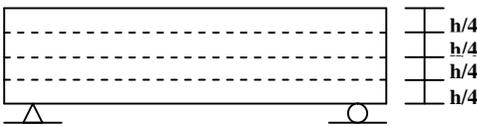
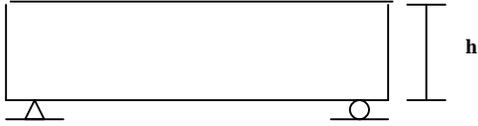
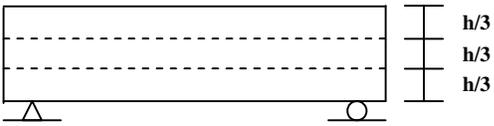
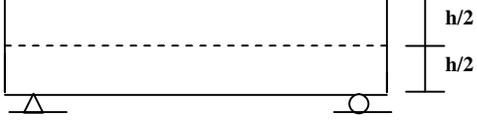
The presence of one, two and three (H.C.J.) in RC beams under flexure gave a decrease in the value of the cracking load such that  $P_{cr}$  was (97%), (85%) and (80%) of (B<sub>ref</sub>). The respective ultimate load capacity  $P_u$  was (96%), (89%) and (84%) compared to (B<sub>ref</sub>).

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**Table ( 1) Summary of the tested beams**

| Beam Mark   | No. of joints | location of joints*      | Beam shape   |
|---|---------------|--------------------------|--|
| Bref  | –             | –                        |    |
| B1  | 1             | $h/2$                    |   |
| B4  | 2             | $h/3$<br>$2/3h$          |  |
| B5  | 3             | $h/4$<br>$h/2$<br>$3/4h$ |  |
| * The locations of joints are measured from bottom face of beam |               |                          |  |

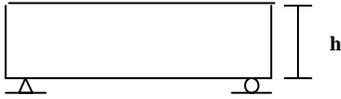
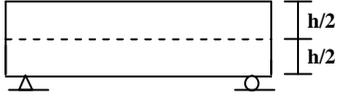
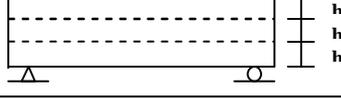
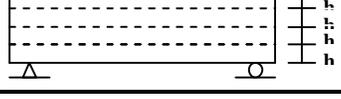
| Table ( 2) Properties of Steel Reinforcement |               |                            |                            |                        |
|--|---------------|----------------------------|----------------------------|------------------------|
| Type of Reinforcement Bars                   | Diameter (mm) | $f_y$ (N/mm <sup>2</sup> ) | $f_u$ (N/mm <sup>2</sup> ) | E (N/mm <sup>2</sup> ) |
| Shear Reinforcement                          | 5             | 380                        | 425                        | 200000                 |
| Longitudinal Bars (in tension)               | 12            | 425                        | 530                        | 200000                 |
| Longitudinal Bars (in compression)           | 8             | 420                        | 510                        | 200000                 |

| Table ( 3) Material property parameters |       |                         |                    |
|---|-------|-------------------------|--------------------|
|   | Name  | Definition              | Value              |
| Concrete                                | $E_c$ | Young's modulus (MPa)   | $4700 \sqrt{f'_c}$ |
|   | $f_t$ | Tensile strength (MPa)  | $0.33 \sqrt{f'_c}$ |
|   | $\nu$ | Poisson's ratio         | 0.2*               |
| Interface                               | $\mu$ | Coefficient of friction | 1*                 |
| Steel                                   | $E_s$ | Young's modulus (MPa)   | 200000*            |
|   | $\nu$ | Poisson's ratio         | 0.3*               |

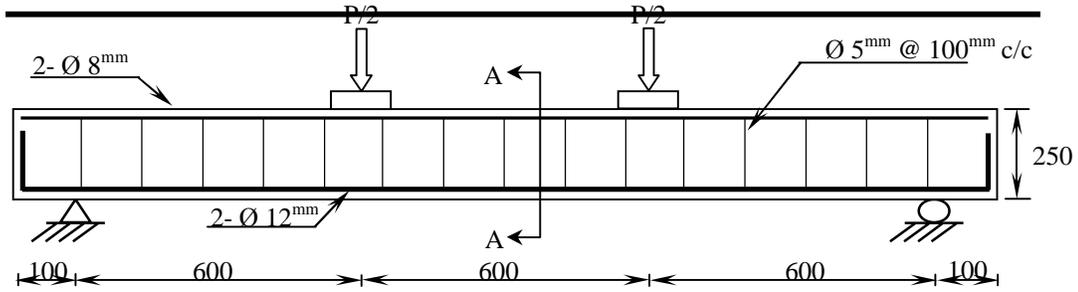
\* Assumed values

| Table (4) Nonlinear solution parameters |  |              |
|---|--|--------------|
| Name                                    | Definition   | Value        |
| $f_{cb}$                                | Ultimate biaxial compressive strength  | $1.200 f'_c$ |
| $\sigma_h^a$                            | Hydrostatic stress   | $1.157 f'_c$ |
| $f_1$                                   | Ultimate compressive strength for a state of biaxial compression superimposed on ( $\sigma_h^a$ )  | $1.450 f'_c$ |
| $f_2$                                   | Ultimate compressive strength for a state of uniaxial compression superimposed on ( $\sigma_h^a$ ) | $1.725 f'_c$ |
| $\alpha_1$                              | Tension stiffening parameters  | $60^*$       |
| $\alpha_2$                              |  | $0.6^*$      |
| $\beta_o$                               | Shear transfer parameters  | $0.0 - 1.0$  |
| $\beta_c$                               |  | $0.0 - 1.0$  |
| $E_w$                                   | Steel hardening parameter  | $0.02 E_s^*$ |
| *Assumed values                         |  |              |

Table (5) Theoretical Results for Tested Beams

| Beam Mark | Beam shape  | P <sub>cr</sub> (kN) |       | Difference Percent % | P <sub>u</sub> (kN) |       | Difference Percent % |
|-----------|---|----------------------|-------|----------------------|---------------------|-------|----------------------|
|           |   | Exp.                 | Theor |                      | Exp.                | Theor |                      |
| Bref      |    | 28                   | 25.3  | 9.6                  | 81                  | 72.8  | 10.1                 |
| B1        |    | 26.7                 | 24.5  | 8.2                  | 78                  | 69.9  | 10.4                 |
| B4        |    | 24                   | 21.5  | 10.4                 | 71                  | 64.8  | 8.7                  |
| B5        |  | 22.5                 | 20.2  | 10.2                 | 67                  | 61.5  | 8.2                  |

Effect of the Number of Horizontal Construction Joints in Reinforced Concrete Beams



Note: All dimensions are in millimeters

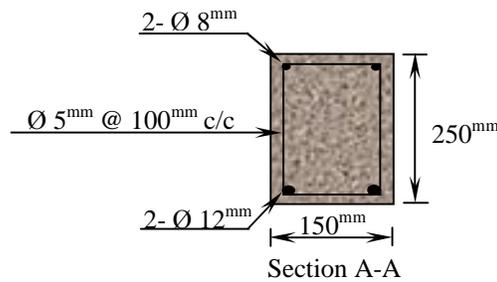


Figure (1) Details & Description of Tested Beams

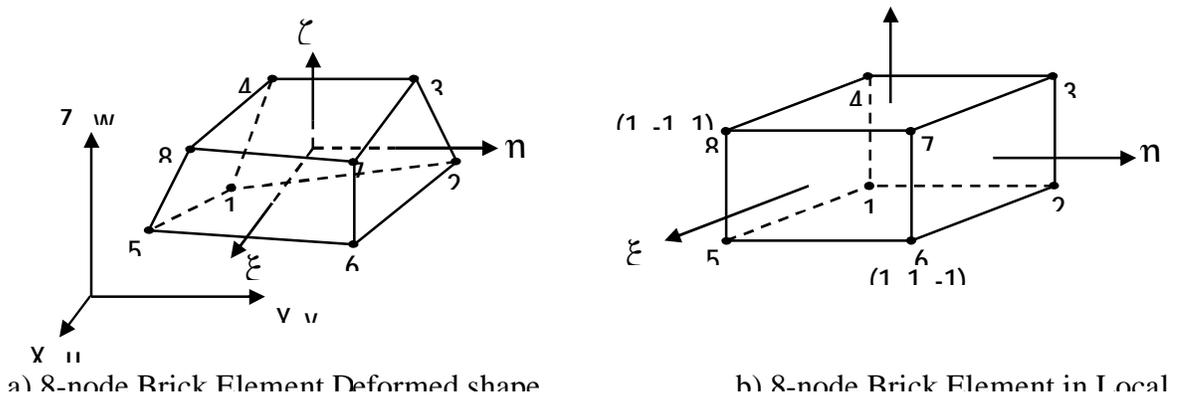


Figure (2) Three-Dimensional 8-Node Brick Element (Solid 65)<sup>(12)</sup>

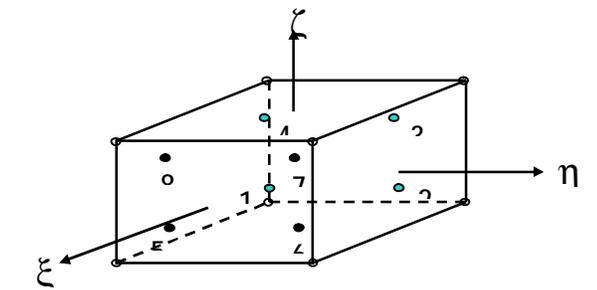


Figure (3) Distribution of Integration Points <sup>(12)</sup>

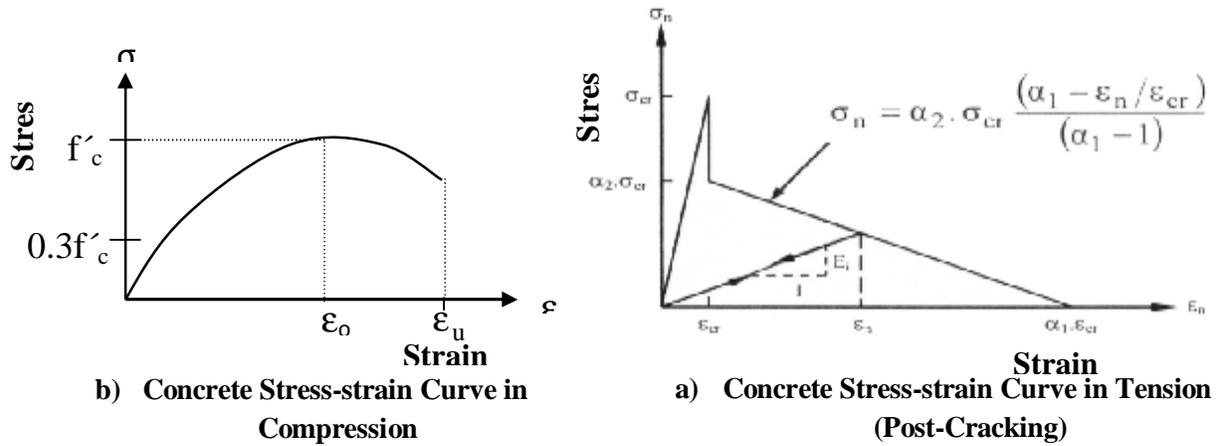


Figure (4) Concrete Stress-strain Curve <sup>(12)</sup>

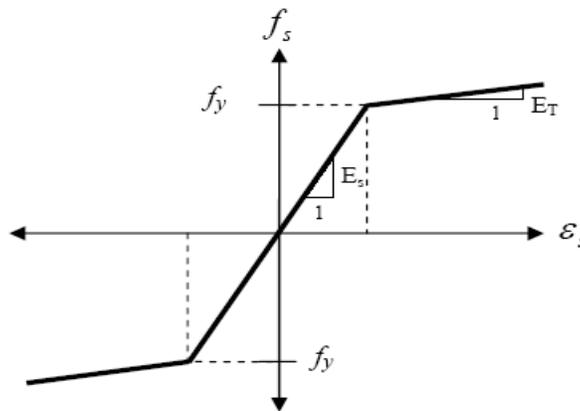


Figure (5) Steel Stress-Strain Curve with Strain Hardening <sup>(12)</sup>

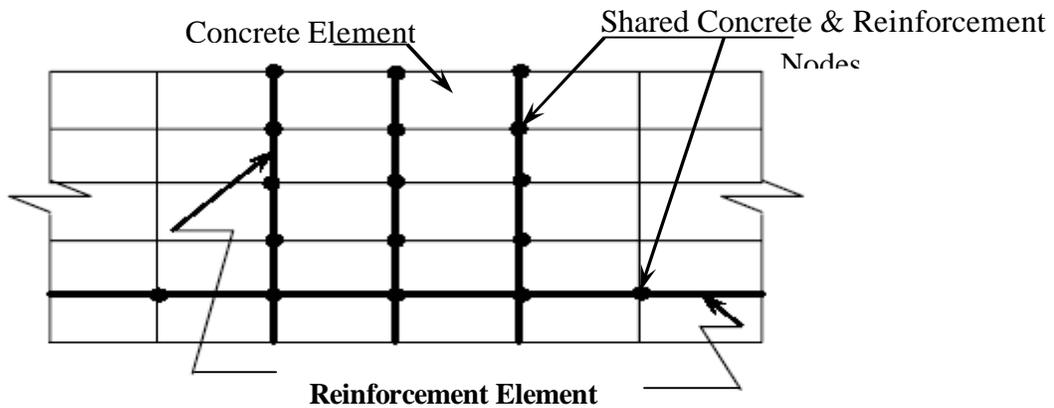
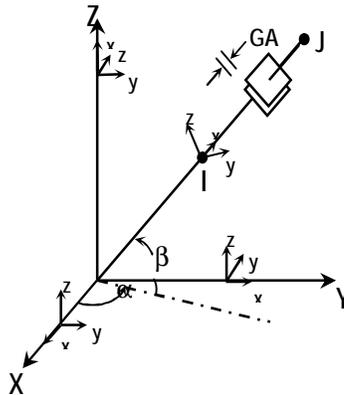


Figure (6) Discrete Representation of Reinforcement <sup>(12)</sup>



Figurer (7) – 3-D Point-to-Point Contact Element <sup>(12)</sup>

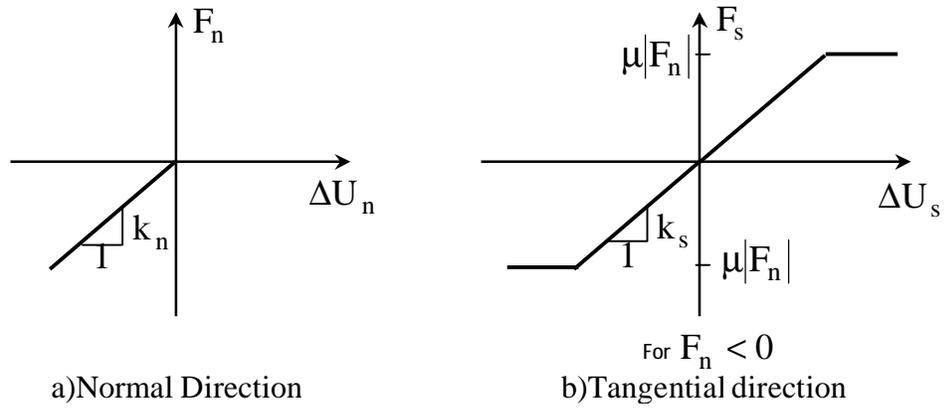


Figure (8) Interface Force – Deflection Relationship <sup>(12)</sup>

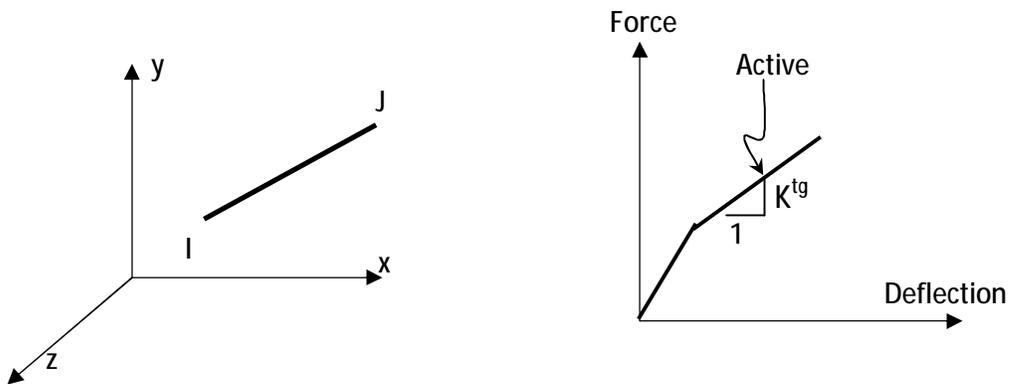


Figure (9) Nonlinear Spring Element <sup>(12)</sup>

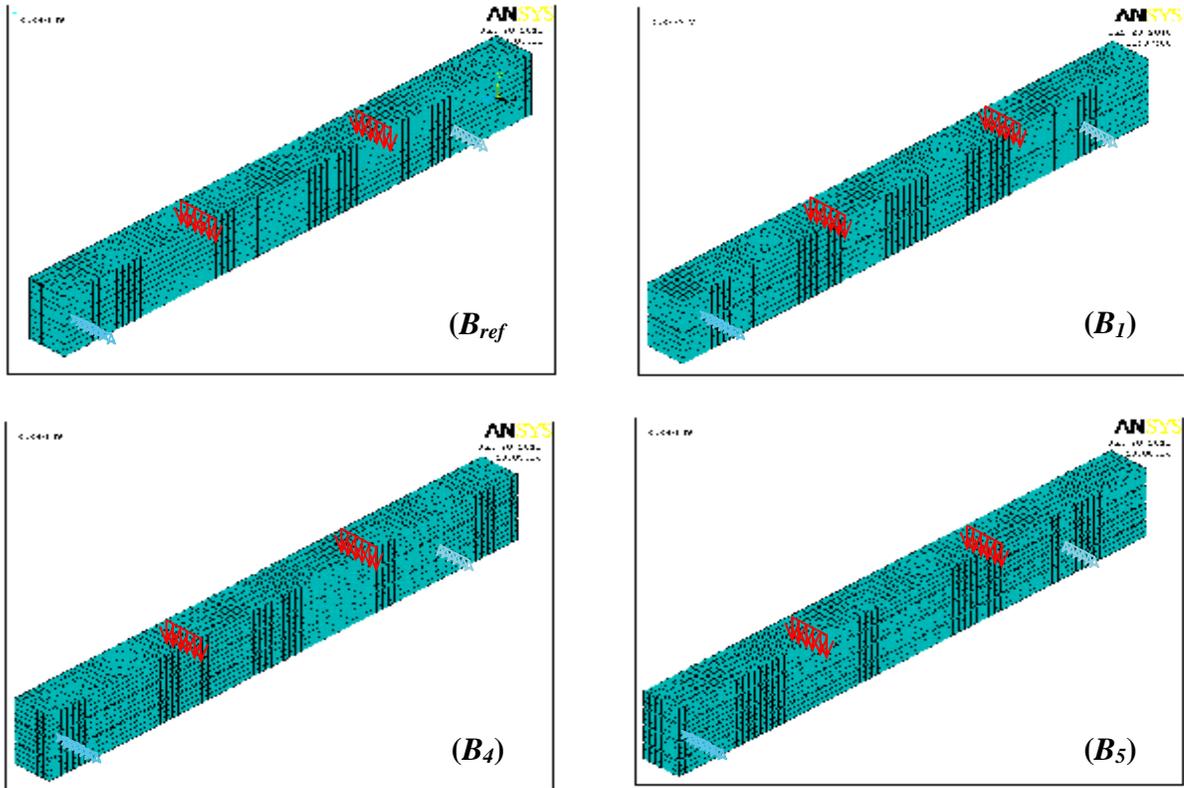


Figure (10) Finite Element Mesh for the Specimens

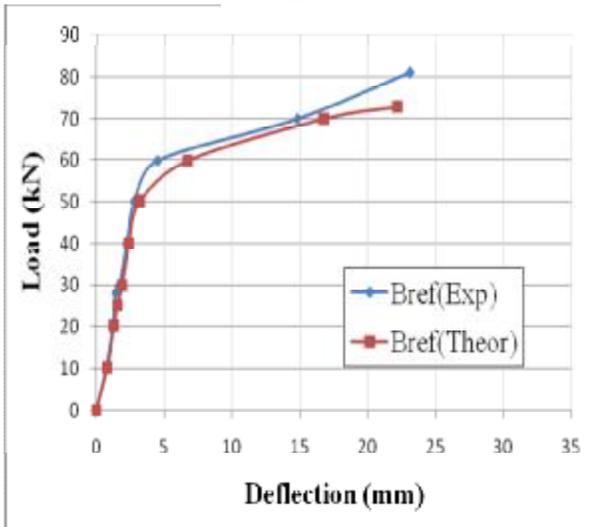


Figure (11) Load-Deflection Curves for beam ( $B_{ref}$ )

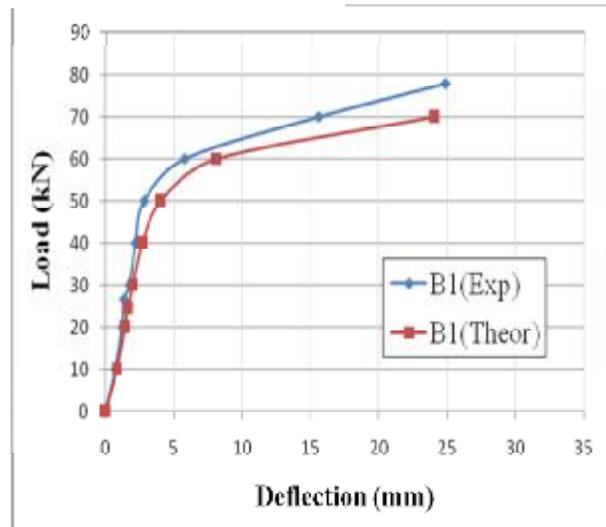


Figure (12) Load-Deflection Curves for beam ( $B_1$ )

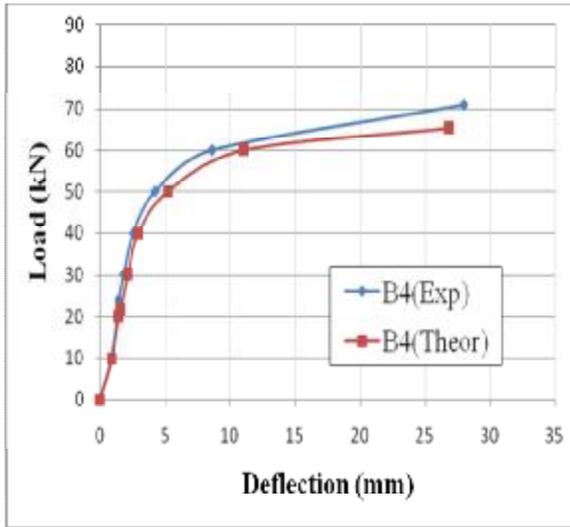


Figure (13) Load-deflection curves for beam ( $B_4$ )

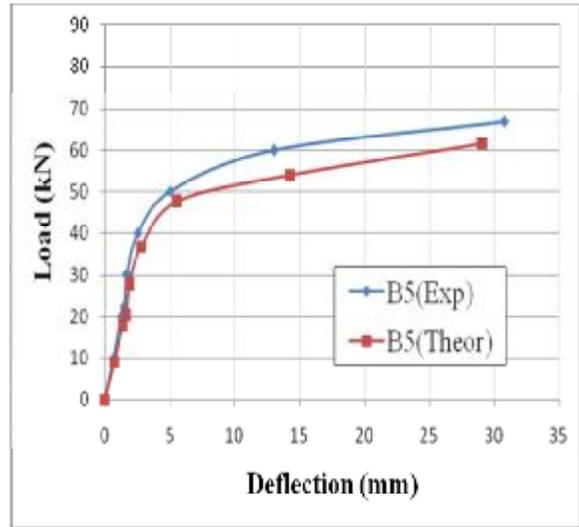


Figure (14) Load-deflection curves for beam ( $B_5$ )

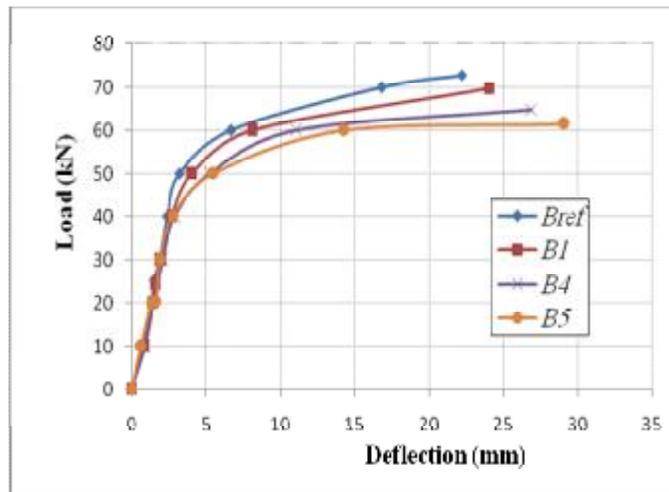


Figure (15) Theoretical Load-Deflection Curves for all Beams

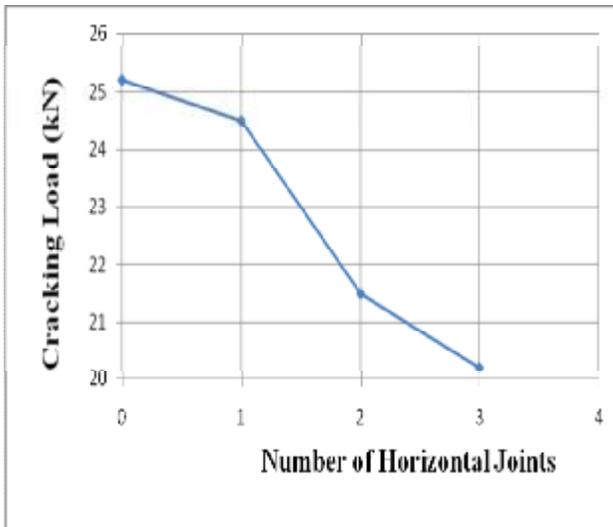


Figure (16) Effect of No. of Joints  
on Cracking Load

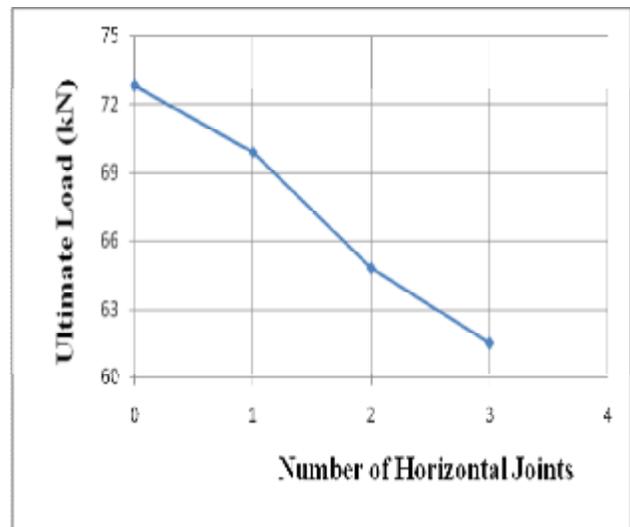


Figure (17) Effect of No. of Joints  
on Ultimate Load