Improving the Accuracy of Static Relative GPS Positioning using Genetic Algorithm

Dr. Farazdak Rafeek Yasien*

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Abstract

Over the years, the Global Positioning System (GPS) has evolved to become an important navigational and positional system and is widely used across the world. The system promises high accuracies if the navigational signals transmitted by the GPS satellites are observed accurately.

The modeling of a single point and relative point determination of user position includes pseudorange measurements. Taylor series is used to linearize the nonlinear model. Two methods are used to estimate the three dimensional user position: Recursive Least Square (RLS) method and continuous Genetic Algorithm (GA) method.

Real data is used and results show that the GA enhances the estimation of user position more than the RLS by high error minimization and the minimum number of available satellites needed. RLS with three satellites give an error that exceeds the allowable limits, while GA gives an acceptable error. Relative positioning method is more accurate than the point positioning method for both RLS and GA methods..

Keywords: Global Positioning System, Genetic Algorithm.

تحسين دقة موقع الإحداثيات الكروية النسبية باستخدام الخوارزمية الوراثية

الخلاصة

على مدى سنوات اصبحت منظومة الاحداثيات الكروية (GPS) المنظومة الملاحية لتحديد المكان الشائعة الاستخدام عبر العالم . تتصف المنظومة بدقة عالية فيما اذا كانت الاشارات الملاحية المرسلة من قبل الاقمار الصناعية تقاس بدقة.

اعتمدت نمذجة طريقة النقطة المنفردة والنقطة النسبية لايجاد موقع المستخدم على طريقة قياس المدى الكاذب. وقد استخدمت متوالية تايلر لتحويل النظام اللاخطي الى نظام خطي. وقد تم استخدام طريقتان لتحديد الاحداثيات الثلاثية لموقع المستخدم وهما طريقة المربعات الصغرى التكرارية RLS وطريقة الخوارزميات الوراثية GA.

تم استخدام بيانات حقيقية في المنظومة وقد اكدت النتائج انه يمكن تحسين المنظومة الملاحية باستخدام الخوارزميات الوراثية بصورة افضل من طريقة المربعات الصغرى التكرارية من جانبين: يكون تقليل الخطا عالي وعدد الاقمار الصناعية المطلوب تواجدها يكون اقل حيث ان طريقة RLS مع ثلاثة اقمار صناعية تعطي خطأ يتجاوز الحد المسموح بينما طريقة GA تعطي أخطاء مقبولة. طريقة النقطة النسبية اكثر دقة من طريقة النقطة المنفردة عند استخدام كلا الطريقتين RLS و GA.

1. Introduction:

The global positioning system (GPS) consists of a constellation of 24 operational satellites. These satellites are arranged so that four satellites are placed in each of the six orbital planes to ensure continuous worldwide converge. Positioning with GPS can be performed by either Single Point Positioning (SPP) method or Relative Point Positioning (RPP) method.

*Control and System Engineering Department, University of Technology/Baghdad

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

GPS single point positioning, also known as standalone or autonomous positioning, involves one GPS receiver; i.e., one GPS receiver simultaneously tracks four or more GPS satellites to determine its own coordinates with respect to the earth. To determine the receiver's point position at any time, the receiver gets the satellites coordinates through the navigation message, while the range is obtained from either the Coarse/Acquisition (C/A) code or the measured pseudorange. Both these methods are contaminated by the satellites and the receiver clock synchronization errors. Correcting the satellite clock errors is done by applying the satellite clock correction in the navigation message. The receiver clock error is an additional unknown parameter in the estimation process.

GPS relative positioning employs two GPS receivers simultaneously tracking the same satellites to determine their relative coordinates of the two receivers; one is selected as a reference, or base, which remains stationary at a site with precisely known coordinates, the other receiver is known as the rover or remote receiver. A minimum of four common satellites is required for RPP. However, tracking more than four common satellites simultaneously would improve the precision of the GPS position solution.

2. Calculation of User Position:

The unknown user position in three dimensions (x,y,z) and the known jth satellite positions (x_j, y_j, z_j) within the Earth Centered Earth Fixed (ECEF) Cartesian coordinate system are shown in Fig. (1). The vector r represents the positions of the satellite relative to the coordinate origin, R is the vector that represents the distance between the user and the center of the earth, and **P** represents the vector offset from the user to the satellite.

The satellite to user receiver vector is given by [1]:

$$P = r - R \qquad (1)$$

The measurement of the propagation time Δt and multiplying it by the speed of light (c) give the distance P. The timing relationships are shown in Fig.(2) where T_s is the system time at which the signal left the satellite, T_a is the system time at which the signal reached the user receiver, δ_t is the offset of the satellite clock from system time, t is the offset of the receiver clock from system time.

Thus the difference between the satellite and the receiver clock offset denoted by the pseudorange (r), then Eqn.(1) can be written as [1]:

$$r - c\,t = | r - R | \qquad (2)$$

In three dimensional coordinates

system the pseudo range is given by [2]

$$r_j = \sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2} + c\,t \qquad (3)$$

Where j represent the satellite used. The approximate pseudorange is estimated by

$$\hat{r}_j = \sqrt{(x_j - \hat{x})^2 + (y_j - \hat{y})^2 + (z_j - \hat{z})^2} + c\,\hat{t} \qquad (4)$$

This equation is difficult to solve and a linearized version can be used to solve the user position through iteration by using Taylor series with first order approximation. Hence the pseudorange error is given by

$$\hat{r}_j - r_j = \frac{x_j - \hat{x}}{\hat{r}_j}\Delta x + \frac{y_j - \hat{y}}{\hat{r}_j}\Delta y + \frac{z_j - \hat{z}}{\hat{r}_j}\Delta z - c\Delta t \quad (5)$$
$$= A_{xj}\Delta x + A_{yj}\Delta y + A_{zj}\Delta z + c\,\Delta t$$

Where

$$\hat{r}_j = \sqrt{(x_j - \hat{x})^2 + (y_j - \hat{y})^2 + (z_j - \hat{z})^2}$$

$$A_{xj} = \frac{x_j - \hat{x}}{\hat{r}_j}$$

$$A_{yj} = \frac{y_j - \hat{y}}{\hat{r}_j}$$

$$A_{zj} = \frac{z_j - \hat{z}}{\hat{r}_j}$$

The A_{xj}, A_{yj}, and A_{zj}, terms in Eqn. (5) denote the direction cosines of the unit vector pointing from the approximate user position to the jth satellite. Four unknowns Δx, Δy, Δz and Δt, can be solved by making ranging measurements to four satellites. The unknown quantities can be determined by solving the set of linear equations below:

$$\begin{bmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \\ \Delta r_4 \end{bmatrix} = \begin{bmatrix} A_{X1} & A_{y1} & A_{z1} & 1 \\ A_{x2} & A_{y2} & A_{z2} & 1 \\ A_{x3} & A_{y3} & A_{z3} & 1 \\ A_{x4} & A_{y4} & A_{z4} & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ -c\Delta t \end{bmatrix} \quad (6)$$

or

$$\Delta r = A\,\Delta X$$

Where

$$\Delta r = \begin{bmatrix} \Delta r_1 \\ \Delta r_2 \\ \Delta r_3 \\ \Delta r_4 \end{bmatrix}, \quad \Delta X = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ -c\Delta t \end{bmatrix}$$

$$A = \begin{bmatrix} A_{X1} & A_{y1} & A_{z1} & 1 \\ A_{x2} & A_{y2} & A_{z2} & 1 \\ A_{x3} & A_{y3} & A_{z3} & 1 \\ A_{x4} & A_{y4} & A_{z4} & 1 \end{bmatrix}$$

The pseudo inverse of A matrix can be used to obtain the solution

$$\Delta X = A^{-1}\Delta r \quad (7)$$

This linearization scheme will work well as long as the displacement (Δx, Δy, Δz) is within close proximity of the linearization point.

A double difference can be expressed as a difference between two single differences which is the difference in observation measured for two satellites by two receivers. The double difference pseudorange measurements for two receivers (a and b) is given by [3]:

$$\Delta r_d = \Delta r_j^{\,a} - \Delta r_j^{\,b} \quad (8)$$

Where Δr_d is the double change in the range measurements. The difference of the direction matrix is denoted by:

$$A_d = A_a - A_b \quad (9)$$

Where A_d is the difference between the parameters of the two direction cosine matrices. These equations can be put in matrix form by making the definitions:

$$\Delta r_d = A_d \, \Delta X_d \qquad (10)$$

Where

$$\Delta r_d = \begin{bmatrix} \Delta r_{d1} \\ \Delta r_{d2} \\ \Delta r_{d3} \\ \Delta r_{d4} \end{bmatrix}, \quad \Delta X_d = [\Delta X_a - \Delta X_b]$$

This equation has the solution

$$\Delta X_d = A_d^{-1} \, \Delta r_d \qquad (11)$$

3. GPS Errors and Biases:

The GPS pseudorange is affected by several types of random errors and biases (systematic errors). These errors may be classified as [4]; errors at the satellites (orbital errors, satellite clock errors and effect of selective availability), errors at the receivers (receiver clock errors, multipath error and receiver noise), and errors due to signal propagation (the delays of the GPS signal as it passes through the ionospheric and tropospheric layers of the atmosphere). Some of these errors and biases can be eliminated or reduced through appropriate combinations of the GPS observables.

4. Recursive Least Square (RLS) Algorithm:

It is an iterative method that provides the coordinate of the user position and clock bias from the given information. This iterative method extracts the GPS user position when at least four satellites and the corresponding pseudo ranges are known. Refer to Eqn. (7) the solution may be written as

$$\Delta X = [A^T A]^{-1} A^T \Delta r \qquad (12)$$

In RLS method, the estimated parameters are improved with each new data. The new estimate of the parameter is equal to the old estimate plus correction terms. The correction term depends linearly on the error between the estimated and its prediction parameter.

5. Continuous GAs:

The GA is a global search method that operates on a population that is created from the approximate coordinates of the GPS receiver, denoted by potential solution applying the principle of survival of the fittest which uses minimum error distance and maximum fitness to produce better and better approximations to a solution.

GAs can be characterized in terms of eight basic attributes: (1) the genetic representation of candidate solutions, (2) the population size, (3) the evaluation function, (4) the genetic operators, (5) the selection algorithm, (6) the generation gap, (7) the amount of elitism used, and (8) the number of duplicates allowed [5].

For most GAs, candidate solutions are represented either by binary or real coded chromosomes. Real coded representations typically allow for more accurate solutions, because the chromosomes do not have to be decoded prior to the evaluation of the cost function [6].

Starting from an initial population of strings, the GA uses these operators, under specified selection rules to a state that maximizes the "fitness", i.e., minimizes the cost function, to calculate successive generations. The population size is the number of individuals that are allowed in the population maintained by a GA. Since our aim is to determine the user position in three coordinate systems then the number of parameters in a population are three. Fitness is normally defined as a function that takes as its single parameter the individual and returns a real number representing the fitness value of that individual. The evaluation function of a GA is used to determine the fitness of an individual.

GA researchers have developed a variety of selection algorithms that provide the type of harmony between the selective pressure and diversity needed to enable GAs to search efficiently and robustly. There are basically three types of selection algorithms: proportionate selection, linear rank selection, and tournament selection. In this work a proportionate selection will be used. In proportionate selection, individuals are selected based on their fitness relative to all other individuals in the population. This process of selecting parents is similar to spinning a roulette wheel to determine which individual is chosen to be a parent. The better is an individual's fitness the bigger is the piece of the roulette wheel that is taken up by the individual and the greater is the probability that it will be selected as a parent. One advantage of using proportionate selection is that its selective pressure varies with the distribution of fitness within a population.

Offspring are created as a result of applying genetic operators to individuals that are selected to be parents. There are basically two types of operators used in genetic algorithms: crossover and mutation [5]. Crossover operators create offspring by recombining the chromosomes of the selected parents with a probability proportional to their fitness. The crossover operator randomly exchanges substrings between two parent chromosomes to create two offspring The most common type of crossover operator is called single point crossover [5]. This operator takes two parents and randomly selects a single point between two genes to cut both chromosomes into two parts. The crossover operator then takes the first part of the first parent and combines it with the second part of the second parent to create the first child. Single point crossover can be applied to real coded representations.

Mutation is used to make small random changes to a chromosome in an effort to add diversity to the population. The mutation operator enhances the ability of the GA to find a near optimal solution to a GPS problem by maintaining a sufficient level of genetic variety in the search for the best solution.

In this work, the parameters for the GA are set as: population size equal to 1000, mutation probability equal to 0.05, crossover probability equal to 0.7, and the maximum generation equal to 5.

6. Conversion from ECEF Coordinates into East, North, Up (ENU) Coordinates:

The relation between ECEF coordinates and the ENU coordinates may be determined by using appropriate direction cosine matrix which is a function of the azimuth angle λ and pitch angle φ as shown in Fig.(3). The baseline of satellite with length P and azimuth angle Ψ measured from the north in clockwise direction in horizontal plane Θ is the inclination angle of baseline in the horizontal plane. The direction cosine matrix is giving by [7,8]

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECEF} \quad (13)$$

7. Accuracy Measurements:

The dilution of the precision term is used to measure the accuracy of user position. The position dilution of precision PDOP is defined as [1,8]

$$PDOP = \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (14)$$

or

$$PDOP = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2} \quad (15)$$

where σ is the measured rms error of the pseudorange, which has a zero mean, while σ_x, σ_y, and σ_z are the measured rms errors of the user position in the xyz directions. σ_E, σ_N, and σ_U are measured rms errors of the user position in the ENU directions. The smallest PDOP value means the best satellite geometry for calculating user position. As a general rule, PDOP values larger than 5 are considered poor [8].

8. Results and Conclusions:

The objective of the present work is to build a navigation algorithm to initiate the construction of the GPS receiver with 4 and 5 satellites.

Fig.(4) and Fig.(5) show the SPP error using both RLS and GA methods. One can conclude from these figures that increasing the number of satellites (4 to 5) for each strategy would decrease the positioning error to a large extent. Furthermore, it is clear from these figures that the GA method would enhance the positioning accuracy more than that produced from its counterpart RLS method.

In Fig.(6) and Fig.(7) , the SPP method has been replaced by an RPP one. Again, increasing the number of satellites (4 to 5) would improve the accuracy of the GPS system for both RLS method and GA method. It is clear from Fig.(4) to Fig.(7) that the RPP method gives better results (less PDOP) than that with SPP method. The special case of 3 satellites has been simulated using GA with SPP and RPP methods. As shown from Fig.(8), the position error has been much decreased with RPP than with SPP method. The error response of GA method with SPP and RPP are shown in Fig.(9). Error minimization occurs at second generations.

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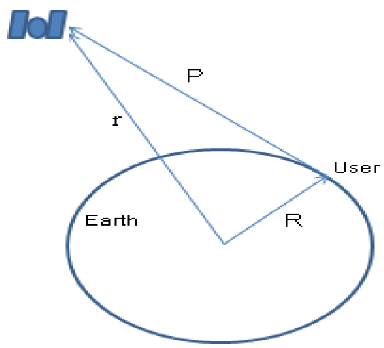

Figure (1) The GPS Vector Position.

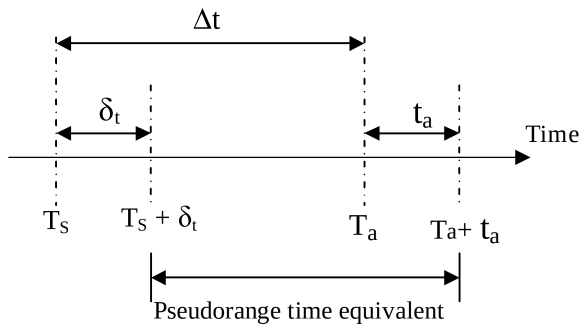

Figure (2) Range Measurement Timing Relationship.

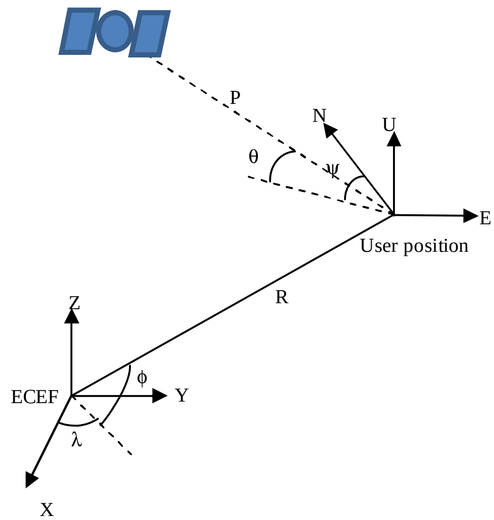

Figure (3) ECEF and ENU Coordinates.

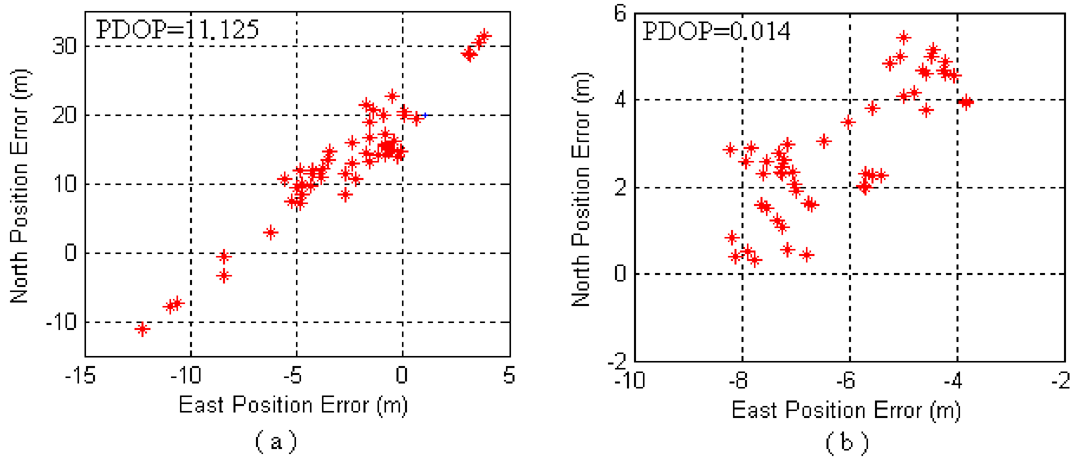

Figure (4) SPP with 1 Reciver and 4 satellites (a) RLS (b) GA.

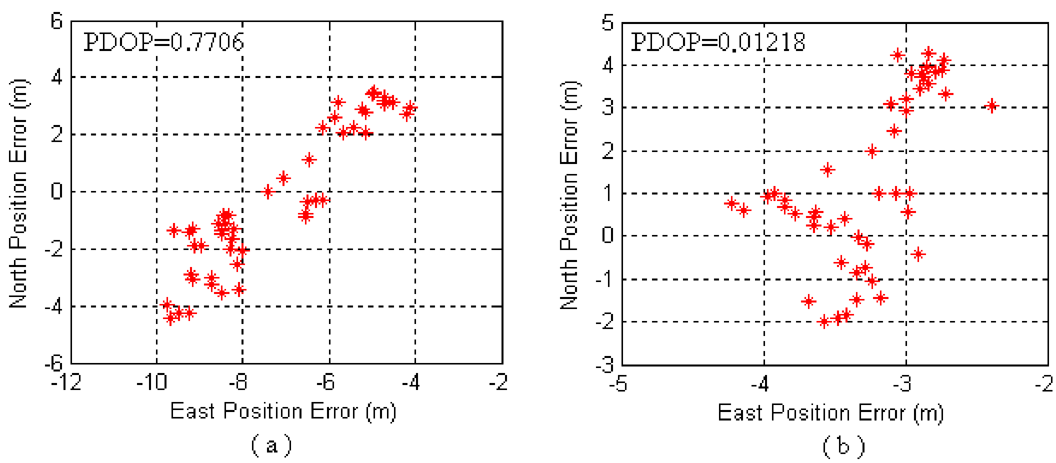

Figure (5) SPP with 1 Reciver and 5 satellites (a) RLS (b) GA.

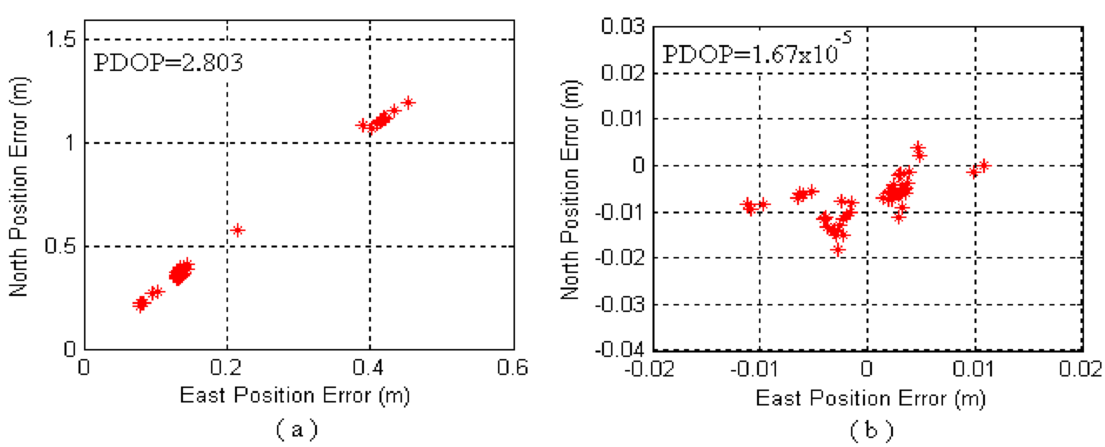

Figure (6) RPP with 2 Recivers and 4 satellites (a) RLS (b) GA.

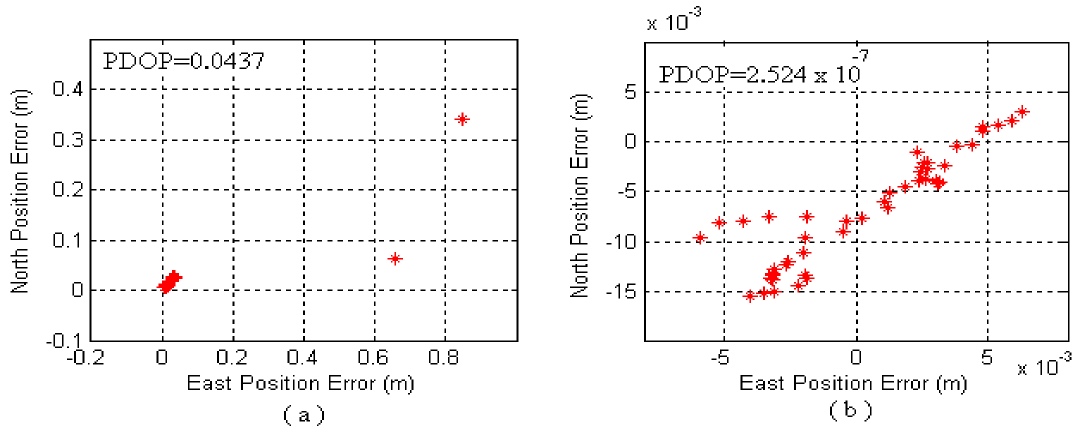

Figure (7) RPP with 2 Recivers and 5 satellites (a) RLS (b) GA.

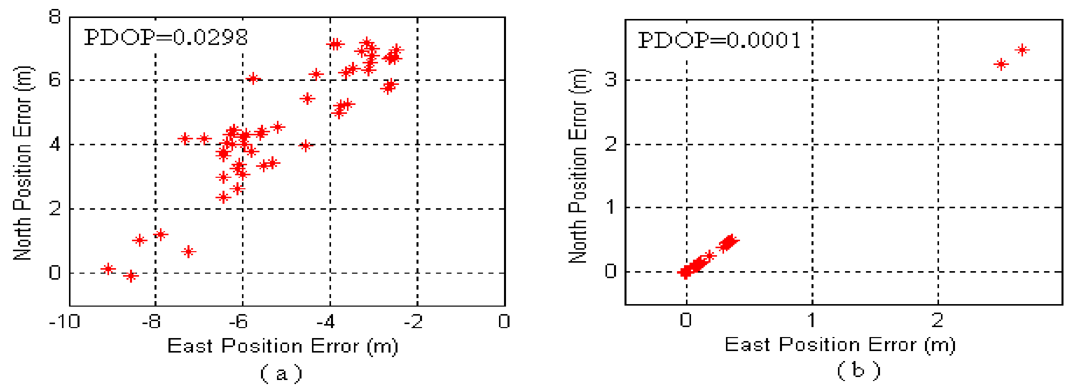

Figure (8) GA with 3 satellites (a) SPP (b) RPP.

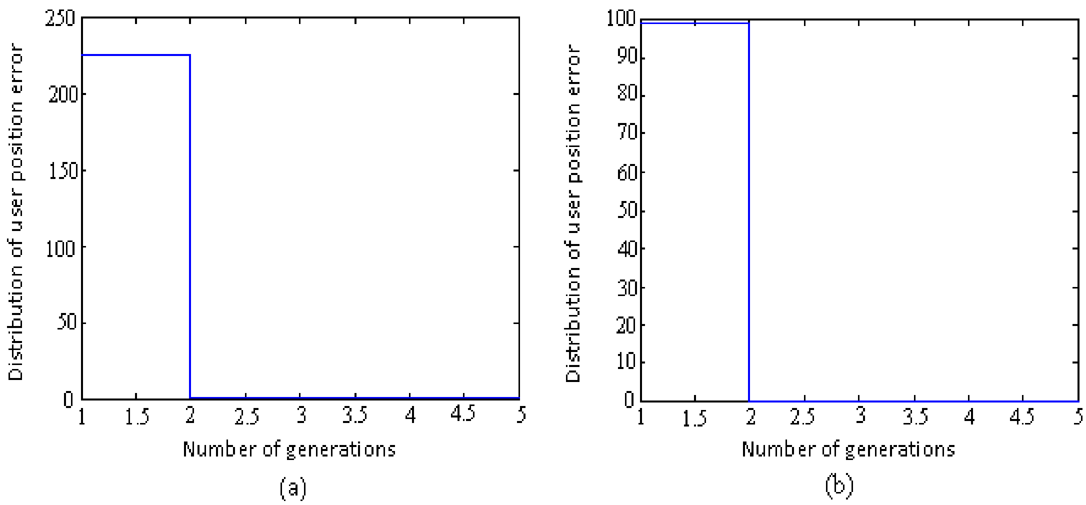

Figure (9) Error Distribution with GA (a) SPP (b) RPP.