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Approximate Solution For Multi Dimensional Delay Fredholm Integro Partial Differential Equation By Using B.Spline Polynomials

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Abstract

The main purpose of this work is to devote B.spline with two variables as basis functions to find approximate solution of Multi dimensional delay fredholm integro partial differential equations .Three numerical examples are given for determining the results of this method.

Keywords: Multi Dimensional Delay fredholm integro partial differential equation, B.spline**.**

الحلول التقريبية للمعادلات فريدهوم التكاملية التفاضلية التباطؤية الجزئية المتعددة الابعاد باستخدام متعددات الحدودالتوصلية

الخلاصة

الغرض من هذ البحث هو تكريس متعددات الحدودبرنشتاين كدوال اساسية لحساب الحل التقريبيي لهذة المعادلات واعطيت ثلاث أمثلة لتوضيح هذه الطريقة.

Introduction

Delay integro differential equation is an equation in which the unknown function appears under both the derivative and the integral sign,morever these equations are used as mathematical models of many problems such as physical problem and chemical analysis. The integro differential is said to be ordinary in case the ordinary derivatives occur in it. On the other hand ,if partial derivative are taken with respect to the unknown function then the integro differential equation is said to be partial .Also Multi dimensional delay integro partial differential equation can be classified into volterrra and fredholm of first and second kinds[1].

In this work can classify the Multi dimensional delay integro partial differential equations of first order as:

1. Multi dimensional delay integro partial differential equations in which the delay appears in unknown function f inside the integral signs .theses equations taken one the following forms

$$\frac{\partial f(x,y)}{\partial x \partial y} = g(x,y) + \int_c^d \int_a^b k(x,y,z,s,t) f(s-t_1, t-t_2)\, ds dt \quad \dots (1)$$

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2. Multi dimensional delay integro partial differential equations in which the delay appears in unknown function f inside the devrivative sign the following forms are such equations:

$$\frac{\partial f(x-t_1,y-t_2)}{\partial x\partial y}=g(x,y)+\int_c^d\int_a^b k(x,y,s,t)f(s,t)dsdt \quad ..(2)$$

3. Multi dimensional delay integro partial differential equations in which the delay appears in unknown function f inside both the devrivative and the integral sign. the following forms are such equations:

$$\frac{\partial f(x-t_1,y-t_2)}{\partial x\partial y}=g(x,y)+\int_c^d\int_a^b k(x,y,s,t)f(s-t_1,t-t_2)dsdt \quad \ldots (3)$$

Where k is known function of x and y called the kernel of the integral, g is known function of x and y, a, b, c, d are given constant, t_1, t_2 are known positive numbers and f is the unknown function that must be determined.

New formulation of Bernstein polynomial of two variable $B_{ij}^{n+m}(x,y)$ and the properties [2]

The new formula of Bernstein polynomials two variables of degree (n+m) can be established and given in the following formula

$$B_{ij}^{n+m}(x,y)=\binom{n}{i}\binom{m}{j}x^i y^j(1-x)^{n-i}(1-y)^{m-j}$$

Where

$$\binom{n}{i}\binom{m}{j}=\frac{n!}{i!(n-i)!}\frac{m!}{j!(m-j)!},$$ n,m are the degree of polynomials and i,j are the Index of polynomials and x,y are the variables.

Property (1):-

The Bernstein polynomial of degree n+m in terms of the power basis is given by the following formula:

$$B_{ij}^{n+m}(x,y)=\sum_{k=i}^{n}\sum_{l=j}^{m}(-1)^{k-i}(-1)^{l-j}\binom{n}{k}\binom{k}{i}\binom{m}{l}\binom{l}{j}x^k y^l$$

Property (2):-

The first derivative of $B_{ij}^{n+m}(x,y)$ polynomial with respect to x is a polynomial of degree (n+m-1) and is given by

$$\frac{\partial B_{ij}^{n+m}(x,y)}{\partial x}=n(B_{i-1}^{n-1}(x)-B_i^{n-1}(x))B_j^m(y)$$

Property (3):-

The first derivative of $B_{ij}^{n+m}(x,y)$ polynomial with respect to y is a polynomial of degree (n+m-1) and is given by

$$\frac{\partial B_{ij}^{n+m}(x,y)}{\partial y}=m(B_{j-1}^{m-1}(y)-B_j^{m-1}(y))B_i^n(x)$$

Property (4):-

The second derivative of $B_{ij}^{n+m}(x,y)$ polynomial of to variable (x,y) is a polynomial of degree(n+m-2).

$$\frac{\partial^2 B_{ij}^{n+m}(x,y)}{\partial x\partial y}=nm(B_{i-1}^{n-1}(x)-B_i^{n-1}(x))(B_{j-1}^{m-1}(y)-B_j^{m-1}(y))$$

Expansion Methods:

Expansion methods are important methods used to approximate the solutions of the Multi dimensional delay integral equations [3, 4, 5].

Consider the Multi dimensional linear fredholm delay integro partial differential equations

$$\frac{\partial f(x,y)}{\partial x \partial y} = g(x,y) + \int_c^d \int_a^b k(x,y,z,s,t) f(s-t_1, t-t_2)\, ds\, dt \quad \ldots (4)$$

The expansion methods are based approximating the solution f of eq.(4) as:-

$$f(x,y) = \sum_{K=1}^{N} c_K B_K(x,y) \quad \ldots(5)$$

By substituting eq (5) into eq (4) one can obtain

$$\sum_{K=1}^{N} c_K f_k(x,y) = g(x) + R(x,y,c_1,c_2,\ldots,c_K) \quad \ldots(6)$$

Whe $R(x,y,c_1,c_2,\ldots,c_K)$ is the error function and

$$f_K(x,y) = \frac{\partial B_K(x,y)}{\partial x \partial y} - g(x,y) - \int_c^d \int_a^b k(x,y,s,t) B_K(s-t_1, t-t_2)\, ds\, dt \quad \ldots(7)$$

The main point here is how we can find N-condition to get N-equations reqaired for determining the N-coefficients of the approximated solution.

In this work, we will present areview of some of these methods that will be used to solve the Multi dimensional delay integro partial differential equations namely (collection and Galerkin's) methods.

The collocations Method:-

The collocation method is one of the most common methods used to approximate the solutions of one and two dimensional integral and intgro differential equations with or without delay. The collocation method requires that $R(x_i, y_j)=0$

Where

$(x_i, y_j) \in D, D = \{(x,y) | a \le x \le b, c \le y \le d\}$

for all i=1, 2… n , j =1, 2...m

Therefore

$$\sum_{K=0}^{N} c_K f_K(x_i, y_j) = g(x_i, y_j),\ i = 1,\ldots,n,\ j = 1,\ldots,m$$

We can get system of NLinear equations with N unknown $C_1, C_2, \ldots C_N$ which can be solved by using Matlab language to find valued of C_k.

Galerkin'n Method:-

Galerkin'n method is also one of the most important methods that can be used to approximate the solutions of one and two dimensional integral delay differential equations [4].the Galerkin'n method requires that the error function $R(x,y,c_1,c_2,\ldots,c_N)$ to be orthogonal to N linearly independent functions φ_j, $j=1,\ldots,n$. In other words, we set:

$$\int_c^d \int_a^b R(x,c_1,c_2,\ldots,c_N)\varphi_j(x)\,dx = 0,\quad j=1,2\ldots,N.$$

Therefore

$$\sum_{k=1}^{N} c_K \int_c^d \int_a^b f_K(x,y)\varphi_j(x,y)\,dx\,dy = \int_c^d \int_a^b g(x,y)\varphi_j(x,y)\,dx\,dy,\ j = 1,\ldots,$$

By evaluating the above equation for each j=1,…,N one can get a linear system of N equations with N unknowns $c_1,\ldots,c_N$ which can be solved by using Matlab language.

Numerical Examples:

Example 1: Consider the multi dimensional fredholm delay integro partial differential equation of first order:

$$\frac{\partial f(x-1/2,y-1/2)}{\partial x\partial y}=\frac{7}{4}x+\frac{7}{4}y-2+\int_0^1\int_0^1(sx+ty)f(s-1,t-1)dsdt$$

And the exact solution is taken to be

$f(x,y)=x^2+y^2$.

This problem is solved using collection method and Galerkin's method with the aid of B-spline functions as basis functions, the solution is obtained as shown in table (1),with N= 10, h=0.1 and x_i=ih&yi=ih i=0,1,..,N

Example 2: Consider the multi dimensional fredholm delay integro partial differential equation of first order:

$$\frac{\partial f(x,y)}{\partial x\partial y}=\frac{23}{12}x+\frac{23}{12}y+\int_0^1\int_0^1(sx+ty)f(s-1/2,t-1/2)dsdt$$

And the exact solution is taken to be

$f(x,y)=x^2+y^2$.

This problem is solved using collection method and Galerkin's method with the aid of B-spline functions as basis functions, the solution is obtained as shown in table(2),with N= 10, h=0.1 and x_i=ih&yi=ih i=0,1,..,N

Example 3: Consider the multi dimensional fredholm delay integro partial differential equation of first order:

$$\frac{\partial f(x,y-1/2)}{\partial x\partial y}=\frac{3}{2}y-\frac{1}{2}x-\frac{3}{2}+\int_0^1\int_0^1(sx+ty)f(s,t)dsdt$$

And the exact solution is taken to be

$f(x,y)=x+y^2$.

This problem is solved using collection method and Galerkin's method with the aid of B-spline functions as basis functions, the solution is obtained as shown in table (3), with N= 10, h=0.1 and x_i=ih&yi=ih i=0,1,..,N

Conclusions:

1- The expansion method including (collocation and Galerkin's) methods with the aid of B-spline polynomials as a basis function which are used in this paper have proved their effectiveness in solving(Multi dimensional fredholm delay integro partial differentialequations) numerically and finding accurate results.

2- B-spline function depends on N as N increased, the error term is decreased.

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Table (1)

Presents acomparison the exact solution and approximate solution by using collection and Galerkin's methods with aid B-spline functions.

x	*y*	Exact Solution	Collection with(B.spline)	Galerkin's with(B.spline)
0.0	***0.0***	**0**	**0**	**0**
0.1	***0.1***	**0.02**	**0.02**	**0.02**
0,2	***0,2***	**0.08**	**0.08**	**0.08**
0.3	***0.3***	**0.18**	**0.18**	**0.18**
0.4	***0.4***	**0.32**	**0.32**	**0.32**
0.5	***0.5***	**0.50**	**0.50**	**0.50**
0.6	***0.6***	**0.72**	**0.72**	**0.72**
0.7	***0.7***	**0.98**	**0.98**	**0.98**
0.8	***0.8***	**1.28**	**1.28**	**1.28**
0.9	***0.9***	**1.62**	**1.62**	**1.62**
1	***1***	**2**	**2**	**2**

Table (2)
Presents acomparison the exact solution and approximate solution by using collection and Galerkin's methods with aid B-spline functions

x	*y*	Exact Solution	Collection with(B.spline)	Galerkin's with(B.spline)
0.0	*0.0*	0	0	0
0.1	*0.1*	0.02	0.02	0.02
0,2	*0,2*	0.08	0.08	0.08
0.3	*0.3*	0.18	0.18	0.18
0.4	*0.4*	0.32	0.32	0.32
0.5	*0.5*	0.50	0.50	0.50
0.6	*0.6*	0.72	0.72	0.72
0.7	*0.7*	0.98	0.98	0.98
0.8	*0.8*	1.28	1.28	1.28
0.9	*0.9*	1.62	1.62	1.62
1	*1*	2	2	2

Table (3)
Presents acomparison the exact solution and approximate solution by using collection and Galerkin's methods with aid B-spline functions.

x	*y*	Exact Solution	Collection with(B.spline)	Galerkin's with(B.spline)
0.0	*0.0*	0	0	0
0.1	*0.1*	0.110	0.110	0.110
0,2	*0,2*	0.240	0.240	0.240
0.3	*0.3*	0.390	0.390	0.390
0.4	*0.4*	0.560	0.560	0.560
0.5	*0.5*	0.750	0.750	0.750
0.6	*0.6*	0.960	0.960	0.960
0.7	*0.7*	1.190	1.190	1.190
0.8	*0.8*	1.440	1.440	1.440
0.9	*0.9*	1.710	1.710	1.710
1	*1*	2.000	2.000	2.000