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RESEARCH ARTICLE - MATHEMATICS

Inverse Chen Bayesian analysis for Parallel Redundant Stress –Strength Reliability

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Abstract

In this paper, a Bayesian analysis made to estimate the reliability of a parallel redundant system with independent stress and strength of Inverse Chen distribution(ICD). The analysis is focused on gamma-primed left-censored samples under four distinct loss functions. (squared error, Quadratic, weighted and Linear exponential loss function). A Monte Carlo simulation was conducted to compare the estimates using the mean square error to obtain the best estimate. It was found that the best estimators can be under squared error loss function.

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Keywords: inverse chen distribution ,Bayesian analysis ,left-censored , squared, Quadratic, weighted and Linear exponential loss function.

1. Introduction

Many researchers have studied deferent types of reliability and Bayes estimator of them , K. Kumar et.al.[1] were computed Bayesian and classical estimates and maximum likelihood estimators then corresponding asymptotic confidence intervals were developed for unknown parameters in the inverse Chen distribution under type II joint control.. Alwan and Karam [2] considered estimating the dependability of a parallel redundant system with separate Weibull-Ryleigh probability density functions for strength and stress, three approaches were used to estimate the dependability parameters: the maximum likelihood, moments, and percentiles methods in the end, the reliability estimate was determined, and the most accurate estimation technique for each scenario was provided, applying the mean squared error standards. The percentiles method was determined to be the most accurate estimating technique. Joshi and pandit [3] estimated the stress-strength of the (s-out –of –k) system for ICD. Karam and Jasem [4] with one known parameter and unknown parameter it is derived the reliability of n-cascade stress-strength system model based on the Gumbel Type-2 distribution.Mubarak and Ashraf [5] they were examined Bayesian estimates under the influence of the symmetric loss function, where comparisons between the different estimators were given considering that the problem of estimating parameters other than the inverse Chen distribution is obtained based on record values as upper record.In this article the reliability estimation of a parallel redundant stress-strength system by the unknown tow parameters from Inverse Chen distribution will be discussed, with cumulative distribution function (cdf) as follow [5], [6]

$$F(x) = e^{\alpha\left(1-e^{x^{-\beta}}\right)} , \; x > 0 \; ; \; \alpha, \beta \; > 0 \qquad (1)$$

And the probability density function (pdf)of Inverse Chen distribution (ICD) is

$$f(x) = \alpha\,\beta\,x^{-(1+\beta)}\,e^{x^{-\beta}}\,e^{\alpha\left(1-e^{x^{-\beta}}\right)} \quad , x, \alpha, \beta \; > 0 \qquad (2)$$

Where(α , μ) is the shape and β is the scale parameter.

2. The Parallel System Stress-strength Reliability

Let we define Y as the stress random variable such that $Y \sim ICD(\mu, \beta)$ and let X be define as strength random variable with $X \sim ICD(\alpha, \beta)$ then.

$g(y) = \mu\beta y^{-(1+\beta)} e^{y^{-\beta}} e^{\mu(1-e^{y^{-\beta}})}$ so that $F_x(y) = 1 - e^{\alpha(1-e^{y^{-\beta}})}$ for our real Y

$R(Y) = 1 - F_x(y) = e^{\alpha(1-e^{y^{-\beta}})}$ and

$$R_p(y) = 1 - [1 - R(y)]^k = 1 - \left[1 - e^{\alpha(1-e^{y^{-\beta}})}\right]^k$$

Using binomial expansion were,

$(x + y)^k = \sum_{i=0}^{k} C_i^k \; x^{k-i} \; y^i$ we can find

$$R_p(y) = \sum_{i=0}^{k} C_i^k \left(-e^{\alpha\left(1-e^{y^{-\beta}}\right)}\right)^i = 1 - \sum_{i=0}^{k} C_i^k \; (-1)^i e^{\alpha_i(1-e^{y^{-\beta}})}$$

So, the overall real Y can be get as

$$R_0 = \int_0^{\infty} R_p(y) \; g(y) \; dy = \int_0^{\infty} (1 - \sum_{i=0}^{k} C_i^k \; (-1)^i e^{\alpha_i\left(1-e^{y^{-\beta}}\right)}) \, g(y) \, dy$$

$$= 1 - \sum_{i=0}^{k} C_i^k \; (-1)^i \int_0^{\infty} e^{\alpha_i\left(1-e^{y^{-\beta}}\right)} \; \mu\beta \; y^{-(1+\beta)} \, e^{y^{-\beta}} \, e^{\mu(1-e^{y^{-\beta}})} \, dy$$

$R_0 = 1 - \sum_{i=0}^{k} C_i^k \; (-1)^i \, \mu\beta \, \frac{1}{\beta(\alpha i+\mu)}$ Then

$$R_0 = 1 - \sum_{i=0}^{k} C_i^k \; (-1)^i \; \frac{\mu}{(\alpha i+\mu)}$$
(3)

3. Bayes analysis

The Bayes estimators of reliabilities R_0 are provided in this section based on a left censored sample and employ the (quadratic loss function, weighted loss function, linear exponential loss function and squared error loss functions) using gamma prior

3.1 The left censored sample.

From the invers chen distribution, a sample of size ‘’n’’ has been chosen, and least(minimal) ‘r ’ The suppressed observations, this implies that the final ($x_{r+1}, \dots, x_n$) Values that are arranged can only be seen. then the probability function for the left data observations $x_i, \dots x_n$, as used by Mitra and Kundu (2008) :[7]

$$L(\alpha|x) = f\left(X_{(r+1)}, \dots, X_{(n)}, \alpha, \lambda, \theta\right) = \frac{n!}{r!}\left(F\left(x_{(r+1)}\right)\right)^r f\left(x_{(r+1)}\right) \dots f\left(x_{(n)}\right)$$

$$= \frac{n!}{r!}\left[F_{\left(x_{(r+1)}\right)}\right]^r \prod_{i=r+1}^{n} f_{(x_i)} = \frac{n!}{r!}\left[e^{\alpha\left(1-e^{x^{-\beta}}\right)}\right]^r \prod_{i=r+1}^{n}\left[\alpha\beta\, x^{-(1+\beta)}\, e^{x^{-\beta}} e^{\alpha\left(1-e^{x^{-\beta}}\right)}\right]$$

$$= \frac{n!}{r!}\left[e^{\alpha r\left(1-e^{x_{(r+1)}^{-\beta}}\right)}\right] \alpha^{n-r}\beta^{n-r} \prod_{i=r+1}^{n} x_i^{-(1+\beta)} e^{\sum_{i=r+1}^{n} x_i^{-\beta}} \quad e^{\alpha\sum_{i=r+1}^{n}\left(1-e^{x_i^{-\beta}}\right)}$$

Let $D = \frac{n!}{r!}\, \beta^{n-r} \prod_{i=r+1}^{n} x_i^{-(1+\beta)} e^{\sum_{i=r+1}^{n} x_i^{-\beta}}$

$$L(\alpha|x) = D e^{\alpha r\left(1-e^{x_{(r+1)}^{-\beta}}\right)} \alpha^{n-r}\, e^{\alpha\sum_{i=r+1}^{n}\left(1+e^{x_i^{-\beta}}\right)}$$

$$= D \quad \alpha^{n-r} \quad e^{\alpha\left(r\left(1-e^{x_{(r+1)}^{-\beta}}\right)+\sum_{i=r+1}^{n}\left(1-e^{x_i^{-\beta}}\right)\right)}$$

And letting $\emptyset_x = r\left(1-e^{x_{(r+1)}^{-\beta}}\right) + \sum_{i=r+1}^{n}\left(1-e^{x_i^{-\beta}}\right)$ then we can written as:

$$L(\alpha|x) = D\, \alpha^{n-r}\, e^{\alpha\emptyset_x} \tag{4}$$

3.2 Bayes procedure

By determining the posterior function under the gamma prior function using the Bayes method:

$$\pi(\alpha) = \frac{b^{\alpha}}{\Gamma\alpha}\alpha^{a-1}\, e^{-\alpha b} \quad , \alpha > 0, a, b > 0 \tag{5}$$

The posterior function can be found form the relation:

$$p(\alpha|x) = \frac{L(\alpha|x)\ \pi(\alpha)}{\int_0^{\infty} L(\alpha|x)\ \pi(\alpha)\, d\alpha}$$

By using (4) and (5) we get :

$$p(\alpha|x) = \frac{D\ \alpha^{n-r}\ e^{\alpha\emptyset_x}\ \frac{b^{\alpha}}{\Gamma\alpha}\alpha^{a-1}\ e^{-\alpha b}}{\int_0^{\infty} D\ \alpha^{n-r}\ e^{\alpha\emptyset_x}\ \frac{b^{\alpha}}{\Gamma\alpha}\alpha^{a-1}\ e^{-\alpha b}\ d\alpha}$$

$$= \frac{D\ \frac{b^{\alpha}}{\Gamma\alpha}\ \alpha^{n-r+a-1}\ e^{-\alpha(b-\emptyset_x)}}{D\ \frac{b^{\alpha}}{\Gamma\alpha}\int_0^{\infty} \alpha^{n-r+a-1}\ e^{-\alpha(b-\emptyset_x)}\, d\alpha}$$

Using the following integration formula $\int_0^{\infty} y^{\alpha-1}\ \ e^{-y\beta}\, d\alpha \ = \frac{\Gamma\alpha}{\beta^{\alpha}}$ (6)

$$p(\alpha|x) = \frac{\alpha^{n-r+a-1}\, e^{-\alpha(b-\phi_x)}}{\Gamma n-r+a\ /(b-\phi_x)^{n-r+a}}$$

$$p(\alpha|x) = \frac{(b-\phi_x)^{n-r+a}}{\Gamma n-r+a}\ \alpha^{n-r+a-1}\ e^{-\alpha(b-\phi_x)} \quad (7)$$

3.2.1 Squared error loss function

The squared loss function is used by the Bayes estimator for α given as:[8]

$$\hat{\alpha}_s = E(\alpha|x) = \int_0^\infty \alpha\ p(\alpha|x)\ d\alpha \quad (8)$$

By comp.equ. (7) in (8) can we get:

$$\hat{\alpha}_s = \int_0^\infty \alpha \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a}\ \alpha^{n+a-r-1}\ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

$$\hat{\alpha}_s = \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a} \int_0^\infty \alpha^{n+a-r}\ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

Using equation (6) can we get:

$$\hat{\alpha}_s = \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a}\ .\frac{\Gamma n-r+a+1}{(b-\emptyset_x)^{n-r+a+1}}$$

Since $\Gamma\alpha = (\alpha-1)\,!$,we can get :

$$\hat{\alpha}_s = \frac{(n+a-r)}{(b-\emptyset_x)} \qquad ,\hat{\mu}_s = \frac{(m+a-r)}{\left(b-\emptyset_y\right)}$$

And the reliabilities estimation function in eq. (3) we get:

$$\hat{R}_0 = 1 - \sum_{i=0}^{k} c_i^k\ (-1)^i\ \frac{\hat{\mu}_s}{\hat{\alpha}_{is}+\hat{\mu}_s}$$

3.2.2 Quadratic loss function

The Quadratic loss function is used by the Bayes estimator for α. which given as:[9]

$$\hat{\alpha}_Q = \frac{E(\alpha^{-1}|x)}{E(\alpha^{-2}|x)} \quad (9)$$

$$E(\alpha^{-1}|x) = \int_0^\infty \alpha^{-1}\ p(\alpha|x)\ d\alpha$$
(10)

By comp. equ. (7) in (10) we get

$$E(\alpha^{-1}|x) = \int_0^\infty \alpha^{-1} \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a}\ \alpha^{n+a-r-1}\ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

$$= \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a} \int_0^\infty \alpha^{n+a-r-2}\ \ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

$$= \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a} \cdot \frac{\Gamma n-r+a-1}{(b-\emptyset_x)^{n-r+a-1}}$$

$$E(\alpha^{-1}|x) = \frac{(b-\emptyset_x)}{(n-r+a-1)} \tag{11}$$

$$E(\alpha^{-2}|x) = \int_0^\infty \alpha^{-2}\ p(\alpha|x)\ d\alpha \tag{12}$$

We put equation (7) in (12) we can get

$$E(\alpha^{-2}|x) = \int_0^\infty \alpha^{-2} \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a}\ \alpha^{n+a-r-1}\ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

$$= \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a} \int_0^\infty \alpha^{n+a-r-3}\ e^{-\alpha(b-\emptyset_x)}\ d\alpha$$

$$= \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma n-r+a} \cdot \frac{\Gamma n-r+a-2}{(b-\emptyset_x)^{n-r+a-2}}$$

$$E(\alpha^{-2}|x) = \frac{(b-\emptyset_x)^2}{(n-r+a-1)(n-r+a-2)} \tag{13}$$

We put equation (11) and (13) in (9) we can get:

$$\hat{\alpha}_Q = \frac{(b-\emptyset_x)}{(n-r+a-1)} \Big/ \frac{(b-\emptyset_x)^2}{(n-r+a-1)(n-r+a-2)}$$

$$\hat{\alpha}_Q = \frac{(n-r+a-2)}{(b-\emptyset_x)} \quad , \hat{\mu}_y = \frac{(m-r+a-2)}{\left(b-\emptyset_y\right)}$$

And the reliabilities estimation function in eq. (3) we get:

$$\hat{R}_{0Q} = 1 - \sum_{i=0}^{k} c_i^k\ (-1)^i\ \frac{\hat{\mu}_Q}{\hat{\alpha}_{iQ} + \hat{\mu}_Q}$$

3.2.3 The Weighted loss function

The weighted loss function is used by the Bayes estimator for α. Which given as[8]:

$$\hat{\alpha}_w = \left(E(\alpha^{-1}|x)\right)^{-1} \tag{14}$$

By making up for it (11) in (14) we get

$$= \frac{1}{(b-\emptyset_x)/(n-r+a-1)}$$

$$\hat{\alpha}_w = \frac{(n-r+a-1)}{(b-\emptyset_x)} \quad , \hat{\mu}_w = \frac{(m-r+a-1)}{(b-\emptyset_y)}$$

And the reliabilities estimation function in eq. (3) we get:

$$\hat{R}_{0w} = 1 - \sum_{i=0}^{k} c_i^k \ (-1)^i \ \frac{\hat{\mu}_w}{\hat{\alpha}_{iw} + \hat{\mu}_w}$$

3.2.4. Linear exponential loss function

The Bayes estimator for α using Linear exponential as loss function given as:[10]

$$\hat{\alpha}_L = \frac{1}{c} \ln E(e^{-c\alpha} \,|x) \tag{15}$$

$$E(e^{-c\alpha}|x) = \int_0^{\infty} e^{-c\alpha} \ p(\alpha|x) d\alpha$$

$$= \int_0^{\infty} e^{-c\alpha} \ \frac{(b-\phi_x)^{n-r+a}}{\Gamma n - r + a} \ \alpha^{n-r+a-1} \ e^{-\alpha(b-\phi_x)} \ d\alpha$$

$$= \frac{(b-\phi_x)^{n-r+a}}{\Gamma n - r + a} \int_0^{\infty} \alpha^{n-r+a-1} \ e^{-\alpha(b-\emptyset_x+c)} \ d\alpha$$

$$= \frac{(b-\emptyset_x)^{n-r+a}}{\Gamma \mathrm{n} - \mathrm{r} + \mathrm{a}} \ . \frac{\Gamma n + a - r}{(b-\phi_x)^{n+a-r}}$$

$$E(e^{-c\alpha}|x) = \frac{(b-\phi_x)^{n-r+a}}{(b-\phi_x+c)^{n-r+a}} \tag{16}$$

By compensating (16) in (15) we get:

$\hat{\alpha}_L = \frac{-1}{c} \ ln\left(\frac{(b-\phi_x)^{n-r+a}}{(b-\phi_x+c)^{n-r+a}}\right)$ Then the estimates will be as

$$\hat{\alpha}_L = \frac{-1}{c} \ ln\left(\frac{b-\phi_x}{b-\phi_x+c}\right)^{n+a-r}, \hat{\mu}_L = \frac{-1}{c} \ ln\left(\frac{b-\phi_y}{b-\phi_y+c}\right)^{m+a-r}$$

And the reliabilities estimation function in eq.(3) we get:

$$\hat{R}_{0L} = 1 - \sum_{i=0}^{k} c_i^k \ (-1)^i \ \frac{\hat{\mu}_L}{\hat{\alpha}_{iL} + \hat{\mu}_L}$$

4. Simulation study

A simulation study of size (1000) was used to estimate the shape parameters (α) using four loss function that were derived in the section 3 to obtain the best estimate using the mean square error and Monte Carlo simulation using the MATLAB (2013) program. four sample sizes were used: small, a little more, medium large (15,30,50,100), value(c=1) and the prior distribution (a, b). From equation (1), we let $U = F(x)$ where uniformly distributed over (0,1). And the random sample is generated by:

$$U = e^{\alpha\left(1-e^{x^{-\beta}}\right)} \Rightarrow lnU = \ lne^{\alpha\left(1-e^{x^{-\beta}}\right)} \Rightarrow lnU = \alpha\left(1-e^{x^{-\beta}}\right)$$

$$1 - e^{x^{-\beta}} = \frac{lnU}{\alpha} \Longrightarrow 1 - \frac{lnU}{\alpha} = e^{x^{-\beta}} \qquad \rightarrow ln\left(1 - \frac{lnU}{\alpha}\right) = lne^{x^{-\beta}}$$

$x^{-\beta} = ln\left(1 - \frac{lnU}{\alpha}\right)$ therefore, we get : $X = \left(ln\left(1 - \frac{lnU}{\alpha}\right)\right)^{\frac{-1}{\beta}}$

Table 1. The best estimate when(α,β,μ)=(2,1,2.9)and (a,b,k,c,r)=(3,2.5,4,1,2) R_0= 0.8829

(n ,m)	BS	BQ	BL	BW	best
(15,30)	0.0026	0.0023	0.7796	0.0025	BQ
(30,15)	0.0059	0.0080	0.0137	0.0068	BS
(50,15)	0.0064	0.0099	0.0137	0.0079	BS
(50,30)	0.0027	0.0031	0.0137	0.0029	BS
(15,50)	0.0069	0.0116	0.0137	0.0089	BS
(30,50)	0.0016	0.0015	0.7796	0.0015	BQ,BW
(100,15)	0.0089	0.0137	0.0116	0.0069	BW
(15,100)	0.0018	0.0025	0.7796	0.0021	BS
(100,30)	0.0025	0.0033	0.0137	0.0028	BS
(30,100)	0.0012	0.0013	0.7796	0.0012	BS,BW
(100,50)	0.0013	0.0015	0.0137	0.0014	BS
(50,100)	0.0011	0.0010	0.7796	0.0011	BQ

Table 2. The best estimate when(α,β,μ)=(1.5,0.6,0.1)and (a,b,k,c,r)=(1.5,3,4,1,2) R0= 0.1270

(n ,m)	BS	BQ	BL	BW	best
(15,30)	0.0033	0.0045	0.0161	0.0038	BS
(30,15)	0.0027	0.0021	0.7614	0.0024	BQ
(50,15)	0.0018	0.0014	0.7621	0.0016	BQ
(50,30)	0.0011	0.0009	0.6669	0.0010	BQ
(15,50)	0.0033	0.0052	0.0161	0.0041	BS
(30,50)	0.0013	0.0015	0.0161	0.0014	BS
(100,15)	0.0019	0.0015	0.7621	0.0016	BQ
(15,100)	0.0029	0.0051	0.0161	0.0038	BS
(100,30)	0.0008	0.0006	0.7621	0.0007	BQ
(30,100)	0.0011	0.0015	0.0161	0.0013	BS
(100,50)	0.0005	0.0004	0.7621	0.0005	BQ
(50,100)	0.0006	0.0007	0.0161	0.0007	BS

Table 3. The best estimate when(α,β,μ)=(2,1.1,0.3)and (a,b,k,c,r)=(1.5,3,4,1,2) R_0= 0.2575

(n ,m)	BS	BQ	BL	BW	best
(15,30)	0.0119	0.0159	0.0663	0.0137	BS
(30,15)	0.0068	0.0054	0.5514	0.0060	BW
(50,15)	0.0059	0.0048	0.5514	0.0053	BQ
(50,30)	0.0035	0.0031	0.5514	0.0033	BQ
(15,50)	0.0103	0.0158	0.0663	0.0127	BS
(30,50)	0.0044	0.0051	0.0663	0.0047	BS
(100,15)	0.0046	0.0041	0.5514	0.0042	BQ
(15,100)	0.0104	0.0174	0.0663	0.0134	BS
(100,30)	0.0026	0.0023	0.5514	0.0024	BQ
(30,100)	0.0038	0.0051	0.0663	0.0044	BS
(100,50)	0.0016	0.0014	0.5514	0.0015	BQ
(50,100)	0.0021	0.0024	0.0663	0.0022	BS

Table 4. The best estimate when(α,β,μ)=(1.9,0.5,2.3)and (a,b,k,c,r)=(1,3,2,1,3) R_0= 0.7182

(n ,m)	BS	BQ	BL	BW	best
(15,30)	0.0062	0.0077	0.5158	0.0068	BS

(30,15)	0.0127	0.0176	0.0794	0.0148	BS
(50,15)	0.0159	0.0244	0.0794	0.0196	BS
(50,30)	0.0051	0.0059	0.0794	0.0055	BS
(15,50)	0.0077	0.0112	0.5158	0.0092	BS
(30,50)	0.0036	0.0038	0.5158	0.0037	BS
(100,15)	0.0199	0.0318	0.0794	0.0251	BS
(15,100)	0.0093	0.0145	0.5158	0.0117	BS
(100,30)	0.0059	0.0077	0.0794	0.0067	BS
(30,100)	0.0036	0.0045	0.5158	0.0040	BS
(100,50)	0.0027	0.0030	0.0794	0.0028	BS
(50,100)	0.0020	0.0022	0.5158	0.0021	BS

Table5. The best estimate when(α,β,μ)=(2.5,1.3,1)and (a,b,k,c,r)=(1,3,2,1,3) R_0= 0.4048

(n ,m)	BS	BQ	BL	BW	best
(15,30)	0.0039	0.1638	0.0042	0.0037	BW
(30,15)	0.0074	0.0075	0.3543	0.0073	BW
(50,15)	0.0066	0.0086	0.3543	0.0073	BS
(50,30)	0.0045	0.0043	0.3543	0.0044	BQ
(15,50)	0.0221	0.0324	0.1638	0.0267	BS
(30,50)	0.0076	0.0088	0.1638	0.0082	BS
(100,15)	0.0079	0.0121	0.3543	0.0096	BS
(15,100)	0.0238	0.0369	0.1638	0.0296	BS
(100,30)	0.0033	0.0039	0.3543	0.0036	BS
(30,100)	0.0075	0.0097	0.1638	0.0085	BS
(100,50)	0.0022	0.0023	0.3543	0.0023	BS
(50,100)	0.0033	0.0038	0.1638	0.0035	BS

5. Discuss the results

In tables (1) and (2) when different sample sizes were taken as shown above values were taken (α,β,μ)=(2,1,2.9)=(1.5,0.6,0.1) respectively and the parameters was also reduced(a) and raised (b) the best estimate was obtained (BS)as for the second table (BQ,BS), because it achieved the lowest (MSE).

In the table (3) it we took (α,β,μ)=(2,1.1,0.3) the changes (a=1.5,k=4, r=2) that occurred while holding the other values constant. we obtained the best estimate(BS) ,because it achieved the lowest (MSE) .

In the table (4) the values of (α,β,μ)=(1.9, 0.5, 2.3) were taken ,and the values of (a=1, k=4,r=2) were changed while holding the other values constant .the best estimate for all sample sizes was obtained (BS) because achieved it the lowest (MSE).

In table (5) we took (α,β,μ)=(2.5,1.3,1)and the values (a=1,k=2,r=3)we obtained the best estimate (BS) too because it achieved the lowest (MSE).

6. Conclusion

By observing the results in the tables above and using the mean square error, the best estimate was obtained, which is the least mean square error for left data and Bayes estimators (BS, BQ, BW) as arranged above under Bayes estimators.

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