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Effect of the Composite Material of the Car's Bumper on its Fundamental Natural Frequency and Response as A Result of Car Vibration

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Abstract

Effect of vibration on the car's bumper was studied in this paper. Composite materials are used in manufacturing the car's bumper. The methods that used in this investigation are Rayleigh's formula for lumped masses in addition to super position method and the mixture rule. By using these techniques it can be found the natural frequencies, mode shapes and deflection for the car's bumper. Different matrix material (resin), fibers and volume fraction are used in this investigation. MATLAB program are built in this study. The results of this program are compared with the results of the ANSYS 11 program. The comparisons show good agreement.

Keywords: car's bumper, vibration in composite materials, Rayleigh's formula for lumped masses, matrix resin with fibers, super position method.

تأثير المواد المتراكبة على التردد الطبيعي لمصدة السيارة واستجابتها كنتيجة لاهتزاز السيارة

الخلاصة

تم دراسة تأثير الاهتزاز على مصدة (دعامية) السيارة. يتم تصنيع مصدة (دعامية) السيارة من المواد المتراكبة. تم استخدام صيغة رايلي للكتل المتمركزة بالاضافة الى طريقة الموقع الممتاز super position وقانون الخلائط. بواسطة هذه التقنيات يمكن ايجاد التردد الطبيعي ونسق الاهتزاز والانحراف لمصدة (دعامية) السيارة. في هذا البحث تم استخدام انواع مختلفة من المواد الاساس (راتنج) والياف التقوية والكسور الحجمية كما تم بناء برنامج MATLAB لهذا الغرض. النتائج التي تم الحصول عليها من هذا البرنامج قورنت مع النتائج التي تم الحصول عليها من برنامج ANSYS 11 والمقارنة اظهرت تطابق جيد.

E	Young modulus of elasticity for composite material.	(MPa)
E_f	Fiber young modulus of elasticity.	(MPa)
E_m	Matrix young modulus of elasticity	(MPa)
g	Gravity.	(m/s^2)
m_i	Mass at each section.	(Kg)
P_i	Load at each section.	(N)
V_f	Fiber volume fraction.	----
V_m	matrix volume fraction.	----
y_i	Deflection at each section.	(m)
Greek Letters		

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ρ	Mass density for composite material.	(kg/m^3)
ρ_f	Fiber mass density.	(kg/m^3)
ρ_m	Matrix mass density.	(kg/m^3)
ω	Fundamental angular natural frequency	rad/sec

Introduction

The increasing application of the advanced composite materials in the design of principal structural components such as car's bumper makes the investigation of the dynamic characteristics (vibration) for these structures very important. The car's bumpers manufactured by using resin injection molding processes. With resin injection molding (see Figure (1)), the reinforcements (fibers) is put in place between the mold and counter mold then the resin is injected.

The studies related to dynamic behavior started since 1930. Recently, Akbarov et.al [1] studied the natural vibrations of composite materials having structures with small-scale curvatures. They used the continuum theory to develop an approach to the determination of the angular natural frequency and mode of composite structural elements with locally and regularly curved structures also they used the approach to examine the solution of a specific problem concerning natural vibration. They found that the fundamental frequency of the structural elements may be reduced considerably by the presence of curvature in the structure of the composite. Vibrations of the composite circular cylindrical shells Were investigated based on the first Love's approximation theory using the first-order shear deformation shell theory by Jafari et.al. [2]. They used the modal technique to develop the analytical solution of the composite cylindrical shell. The effect of some of the geometric parameters on the time response of the shells was studied. It was concluded that the dynamic responses were primarily governed by the natural period of the structure also the results showed that the natural frequencies inversely depend on the value of compressive load. Rangaswamy and Vijayarangan [3] made design optimization of composite drive shafts for power transmission applications. The one-piece composite drive shaft is designed to replace conventional steel drive shaft of cars using glass / epoxy and high modulus (HM) carbon/epoxy composites. A formulation and solution technique using genetic algorithms (GAs) for design optimization of composite drive shafts is presented. The purpose of using GA is to minimize the weight of shaft that is subjected to the constraints such as torque transmission, torsional buckling capacities and fundamental lateral natural frequency. The weight savings of the glass /epoxy and high modulus carbon/epoxy shaft were 48.36 % and 86.90 % of the steel shaft respectively. The effect of the boundary conditions on the dynamic stability of orthotropic composite material using a modified exact analysis was studied by Darvizeh et.al. [4]. It has been found that the frequency pattern was influenced by the change in the direction of orthotropic and the change in boundary conditions. Also, the influence of the boundary conditions was shown to increase the natural frequencies as travelling through the end conditions free–free and clamped–free, respectively. A numerical method based on the Rayleigh-Ritz method has been presented for the vibration of open cylindrical shells by Kandasamy and Singh [5]. The transient response of the shell is sought by transforming the equation of the motion to the state-space model, and then the state-space differential equations are solved using the Runge-Kutta algorithm. Trotsenko[6] considered a mechanical system consisting of a rigid body attached to one of the shell ends. A boundary eigen value problem describes the free vibrations of the ''body – shell'' system, and its approximate solution was determined. The exact solution of the above problem was constructed by replacing the shell with an equivalent Timoshenko beam. The effect of the rigid body on the system vibrations was estimated, and the accuracy of the beam approximation to the shell bending vibrations was studied. Analytical and experimental vibration analysis of glass fiber reinforced polymer (GFRP) composite beam studied by Adediran [7]. The composite beam assumed as prototype of bridge in which pedestrians impart to the model. For the dynamic test, hummer excitation is used to excite the beam at fixed locations. The model parameters are extracted from the time response using a time domain analysis, i.e. the stochastic identification technique. Euler-Bernoulli beam theory used in the investigation. Finite element models for different boundary conditions are constructed using commercial finite element software package ANSYS. The natural frequencies and the mode shapes for different boundary conditions are found. The results that obtained from analytical solution and dynamic tests (impact excitation) compared with the results obtained from the finite element method. A free vibration analysis of the composite shells with different boundary conditions was presented by Haftchenari et.al. [8], where they used the differential quadrature method (DQM). Equations of the motion are derived based on the first order shear deformation theory taking the effect of the shear deformation into account. By solving this algebraic system, the natural frequencies of the shells made of fibrous composite materials were evaluated. It has been found that DQM rapidly converges to accurate solutions. The DQM was found to be an excellent and sound technique in dealing with the stability problems of the composite shell structures. The feasibility of using the transfer matrix method to analyze a composite material was explored theoretically by Liang et.al. [9]. Governing equations of vibration for this system were expressed by the matrix differential equations, and the coefficient matrices and joining matrix were derived. After the relationship between the transfer matrix and the coefficient matrix was established, the fourth order Runge–Kutta method was numerically used to solve the matrix equation. The frequency equations and mode shapes were formulated in terms of the elements of the structural matrices. The three dimension finite element numerical simulation had validated the present formulas of the natural frequencies and mode shapes.

Experimental study for the mechanical properties of epoxy nanocomposite made by Sarathi et.al. [10], where the fundamental characteristics of epoxy nanocomposites are analyzed through wide angle X-ray diffraction (WAXD) studies. They found the natural frequency for epoxy nanocomposite by using vibration tests, the vibration tests results indicate that natural frequency of vibration of the material increases by adding up to 5 wt% of nanoclay to base resin. In the present work, the dynamical behavior of the car's bumper structure is studied. Effects of many design parameters such as volume fraction, matrix and fiber material on the dynamic response of the car's bumper are investigated.

Modeling Of Car's Bumper

The bumper of the car which is treated in this investigation is illustrated in Fig. (1).

The matrix material that used in the composite material in this investigation are (phenolic or epoxy) while the fibers that used are (glass, copper, kevlar, boron). These composite materials will have new characteristics as high rupture strength, very good fatigue resistance and high elastic elongation.

For the purpose of the investigation the bumper can be reduced to supported beam with (U) cross-section area as illustrated in Fig. (2).

The car's bumper divided into (31) section as showing in Fig. (2). Where at each section there is a load result from the weight of this part, this load can be found as follows:

$$P_i = m_i * g \quad .....(1)$$

Mass Density And Young Modulus of Elasticity For The Composite Materials

The mass density and the young modulus of elasticity for the composite materials can be found by using the mixture rule. Where the mass density calculated by using the following:

$$\rho = \frac{\text{total mass}}{\text{total volume}} \quad (2)$$

The above equation can also be expanded as:

$$\rho = \frac{\text{mass of fiber}}{\text{total volume}} + \frac{\text{mass of matrix}}{\text{total volume}} \quad (3)$$

$$\rho = \frac{\text{volume of fiber}}{\text{total volume}} * \rho_f + \frac{\text{volume of matrix}}{\text{total volume}} * \rho_m \quad (4)$$

$$\rho = \rho_f * V_f + \rho_m * V_m \quad (5)$$

Similarly the equation of the young modulus of elasticity can be found, which gives:

$$E = E_f * V_f + E_m * V_m \quad (6)$$

Theoretical Analysis

The method that used in this investigation is Rayleigh's formula for lumped masses where in this method it can be found the fundamental natural frequency. Where Rayleigh suggested that the total energy within a vibrating system should remain constant and that energy interchange should be between the strain energy and kinetic energy. Hence the maximum strain energy will be equivalent to the maximum kinetic energy. Consider a uniform beam subjected to a vibratory force gives: [11]

$$\omega = \sqrt{\frac{g^* \sum_{i=1}^{n} m_i * y_i}{\sum_{i=1}^{n} m_i * y_i 2}} \qquad (7)$$

Deflection In Car's Bumper:

The deflection at each section of car's bumper comes from the compound effects between the load that applied at the same section and the effect of the other loads that applied on the other sections; therefore, the deflection at each section found in two steps:

1- Founding the deflection comes from the effect of the load that applied on the same section.

2- Founding the deflection comes from the effects of the loads that applied on the other sections.

These deflections can be found by using the formulas for the deflection of beams [12] which is given bellow:

$$y_{kx} = \frac{P_j l}{E}\left[\frac{Ld}{3I} + \frac{l(3d-l)}{6I}\right] \qquad (8)$$

$$y_{mx} = \frac{P_j c}{E}\left[\frac{Ll}{3I} + \frac{c(3l\ c)}{6I}\right] \qquad (9)$$

$$y_{rx} = \frac{P_j L l c}{6EI} \qquad (10)$$

$$y_{qx} = \frac{P_j lab(L+b)}{6EIL} \qquad (11)$$

$$y_{kj} = \frac{P_j ghd(L+h)}{6EIL} \qquad (12)$$

$$y_{nj} = \frac{P_j hf(L^2-h^2-f^2)}{6EIL} \qquad (13)$$

$$y_{jj} = \frac{P_j g^2 h^2}{3EIL} \qquad (14)$$

$$y_{qj} = \frac{P_j go(L^2-g^2-o^2)}{6EIL} \qquad (15)$$

$$y_{rj} = \frac{P_j ghe(L+g)}{6EIL} \qquad (16)$$

Then it must be using the super position method to found the total deflection on each section. Where in this method the deflection at any section (i) found by the formula:

deflection at any section (y) = deflection at the same section with respect to the first load + deflection at the same section with respect to the second load + deflection at the same section with respect to the third load +……………………………….

Results And Discussion

1- The effect of the composite material on the deflection

Figures (4 to 7) represent the deflection for different kinds of the composite material. For all figures it can be notice that the maximum deflection lies in the mid length of the car's bumper that's will make the car's bumper may be brake in this point, while there are two points have zero deflection that because there are two brackets in these points supports the car's bumper and prevent it from moving in these points. Also from these figures it can be notice that the effect of the composite material on the deflection dependent on many variables as follows:

a) Effect of fiber's material on the deflection:

Figure (4) represent the effect of changing fiber's material on the deflection, where fig.(4a) shows the deflection for epoxy matrix while fig.(4b) shows the deflection for phenolic matrix, different fiber's material are used (glass, copper, Kevlar, boron). In these figures it can be notice that when using the matrix without fibers (alone epoxy or phenolic) the deflection will be very high comparing with the other cases that used the fibers, that's because the fibers will increase material rigidity hence decrease its deflection. Also it can be notice that using the copper as fiber gives the composite material deflection more than the other fiber's material, while using the boron as fiber gives the composite material deflection less than the other fiber's material, that's because the modulus of elasticity for copper is less than the modulus of elasticity for boron which make the composite material for (epoxy-copper or phenolic-copper) has modulus of elasticity less than the composite material for (epoxy-boron or phenolic-boron) and since increasing the modulus of elasticity leads to increasing the stiffness material hence decrease the deflection; therefore, its prefer using the boron as a fiber's material to give small deflection through the vibration comparing with the other fiber's material (glass, copper, Kevlar).

b) Effect of volume fraction on the deflection:

Figures (5 & 6) represent the effect of changing volume fraction on the deflection, where fig.(5) shows the deflection for epoxy matrix with different volume fraction and fiber's material while fig.(6) shows the deflection for phenolic matrix with different volume fraction and fiber's material. For all these figures it can be notice that when increasing the volume fraction of the fibers for the same composite material the deflection will decrease, that's because increasing the volume fraction for fibers leads to increasing the percentage of fibers in the composite materials in turn increases the modulus of elasticity for composite material which is found by using equation (6) and shown in tables (I & II) hence decreasing the deflection.

c) Effect of matrix material on the deflection:

Figure (7) represent the effect of changing matrix material on the deflection, this figure shows that for all fiber's material when using phenolic as a matrix the deflection will be higher than using epoxy as a matrix, that's because phenolic has modulus of elasticity (E) less than epoxy that's leads to give it flexibility more than epoxy; therefore, its prefer using the epoxy as a matrix material to give small deflection through the vibration of car's bumper.

2- The effects of the composite material on the natural frequency:

Figures (8 to 13) and tables (I & II) represent the natural frequency for different kinds of composite materials. In these figures it can be notice that all graphs start from the same point that's because the first point represent the matrix without fibers (alone epoxy or phenolic), the natural frequency in this point is the least natural frequency comparing with the other cases that used the fibers, that's because the fibers will increase its stiffness and since the stiffness has direct proportion with the natural frequencies $\left(w_n = \sqrt{\text{stiffness/mass}}\right)$; therefore, the fibers will increase the natural frequency. In addition to before it can be notice from these figures and tables that the maximum natural frequency happened when using the boron as a fiber's material because the boron has the maximum modulus of elasticity comparing with the other fibers used which leads to make the composite material has the maximum modulus of elasticity and that's leads to increase its stiffness hence increase the natural frequencies (as explained before). While using the copper as a fiber's material gives least natural frequency comparing with the other fiber's materials, that's because the copper has minimum modulus of elasticity comparing with the other fibers used which leads to make the composite material has the minimum modulus of elasticity and in turn decreases the natural frequencies. Also from these figures and tables it can be notice that the effect of the composite material on the natural frequency dependent on many variables as follows:

a) Effect of changing the modulus of elasticity on the natural frequency:

Figures (8 & 9) represent the effect of changing the modulus of elasticity on the natural frequency, where fig.(8) shows the relation for epoxy matrix (by using MATLAB and ANSYS program) while fig.(9) shows the relation for phenolic matrix (by using MATLAB and ANSYS program), different fiber's material are used (glass, copper, Kevlar, boron). From these figures it can be notice that increasing the modulus of elasticity for composite material will increase the natural frequency. Also these figures show that it can be give the maximum modulus of elasticity for composite material by using boron as a fiber's material while it can be give the minimum modulus of elasticity for composite material by using glass as a fiber's material. From these charts it can be found the natural frequency for the composite material at any modulus of elasticity.

b) Effect of changing volume fraction on the natural frequency:

Figures (10 & 11) represent the effect of changing the volume fraction on the natural frequency, where fig.(10) shows the relation for epoxy matrix (by using MATLAB and ANSYS program) while fig.(11) shows the relation for phenolic matrix (by using MATLAB and ANSYS program), different fiber's material are used (glass, copper, Kevlar, boron). From these figures it can be notice that increasing the volume fraction for fibers will increase the natural frequency that's because increasing volume fraction for fibers will increase the stiffness of composite material and in turn increases the natural frequency (as explained before). From these charts it can be found the natural frequency for the composite material at volume fraction range from (0%) to (50%).

c) Effect of changing matrix material on the natural frequency:

Figures (12 & 13) represent the effect of changing the matrix material on the natural frequency, where fig.(12) shows the relation between the natural frequency and the modulus of elasticity while fig.(13) shows the relation between the natural frequency and the volume fraction. From these figures it can be notice that using epoxy as a matrix gives natural frequency higher than using phenolic as a matrix. Also these figures show that it can be give the maximum modulus of elasticity and maximum natural frequency for composite material by using epoxy as a matrix material with boron as a fiber's material (epoxy-boron) while it can be give the minimum modulus of elasticity for composite material by using phenolic as a matrix material with glass as a fiber's material (phenolic-glass) and minimum natural frequency by using phenolic as a matrix material with copper as a fiber's material (phenolic-copper).

3- Comparison between MATLAB program and ANSYS 11 program

Figures (14 & 15) and tables (I & II) represent the comparison between MATLAB program by using Rayleigh's formula for lumped masses and super position method and ANSYS program by using finite element method, where fig.(14) shows the relation between the natural frequency and the modulus of elasticity while fig.(15) shows the relation between the natural frequency and the volume fraction for epoxy and phenolic matrix with different fiber's material (glass, copper, Kevlar, boron). From these figures and tables it can be notice that there is good agreement between two programs.

Conclusions

1- The minimum natural frequency happened when using the matrix material without fibers (alone epoxy or phenolic) but in the same time it gives the maximum deflection comparing with the other cases that used the fibers.

2- Using the boron as a fiber's material give small deflection through the vibration comparing with the other fiber's material (glass, copper, Kevlar).

3- The deflection will decrease when increasing the volume fraction of fibers for the same composite material.

4- Using the epoxy as a matrix material give small deflection through the vibration comparing with the phenolic.

5- Using the boron as a fiber's material give the maximum natural frequency for the composite material while using the copper as a fiber's material give the minimum natural frequency for the composite material.

6- Increasing the modulus of elasticity or increasing the volume fraction of fibers will increase the natural frequency for composite material.

7- It can be give the maximum modulus of elasticity for composite material by using boron as a fiber's material while it can be give the minimum modulus of elasticity for composite material by using glass as a fiber's material.

8- Using epoxy as a matrix gives natural frequency higher than using phenolic as a matrix in composite material.

9- Its prefer using epoxy as a matrix with boron as a fiber's material (epoxy-boron) to give the maximum modulus of elasticity and maximum natural frequency for composite material while using phenolic as a matrix with glass as a fiber's material (phenolic-glass) to give the minimum modulus of elasticity for composite material and using phenolic as a matrix with copper as a fiber's material (phenolic-copper) to give the minimum natural frequency.

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Table (I)

Composite material	Volume fraction (%)	Modulus of elasticity (Mpa)	Density (Kg/m^3)	Rayleigh's method (by using Matlab) Fundamental natural frequency (Hz)	Finite element method (by using Ansys 11) Fundamental natural frequency (Hz)
Epoxy	0.0	4500	1200	321.278	320.37
Epoxy-Glass	10	12650	1330	511.664	496.69
	20	20800	1460	626.211	615.28
	30	28950	1590	707.932	697.26
	40	37100	1720	770.528	766.72
	50	45250	1850	820.520	821.75
Epoxy-Copper	10	16550	1960	482.099	475.2
	20	28600	2720	537.978	530.26
	30	40650	3480	567.030	560.79
	40	52700	4240	584.909	579.53
	50	64750	5000	597.036	596.17
Epoxy-Kevlar	10	17050	1225	618.956	614.01
	20	29600	1250	807.340	805.84
	30	42150	1275	953.915	950.98
	40	54700	1300	1076.187	1072.2
	50	67250	1325	1181.964	1177.1
Epoxy-boron	10	44050	1340	951.232	945.38
	20	83600	1480	1246.919	1243.8
	30	123150	1620	1446.524	1444.8
	40	162700	1760	1595.157	1594.3
	50	202250	1900	1711.723	1711.5

Table (II)

Composite material	Volume fraction (%)	Modulus of elasticity (Mpa)	Density (Kg/m^3)	Rayleigh's method (by using Matlab) Fundamental natural frequency (Hz)	Finite element method (by using Ansys 11) Fundamental natural frequency (Hz)
Phenolic	0.0	3000	1300	252.031	251.21
Phenolic -Glass	10	11300	1420	468.016	461.43
	20	19600	1540	591.880	585.79
	30	27900	1660	680.164	672.37
	40	36200	1780	748.187	743.54
	50	44500	1900	802.914	798.49
Phenolic -Copper	10	15200	2050	451.763	447.61
	20	27400	2800	518.994	515.35
	30	39600	3550	554.114	550.78
	40	51800	4300	575.833	571.36
	50	64000	5050	590.623	589.42
Phenolic -Kevlar	10	15700	1315	573.261	570.67
	20	28400	1330	766.654	759.83
	30	41100	1345	917.119	912.35
	40	53800	1360	1043.488	1040.59
	50	66500	1375	1153.786	1149.86
Phenolic -boron	10	42700	1430	906.592	903.92
	20	82400	1560	1205.778	1201.38
	30	122100	1690	1410.199	1406.55
	40	161800	1820	1564.298	1562.87
	50	201500	1950	1686.499	1686.14

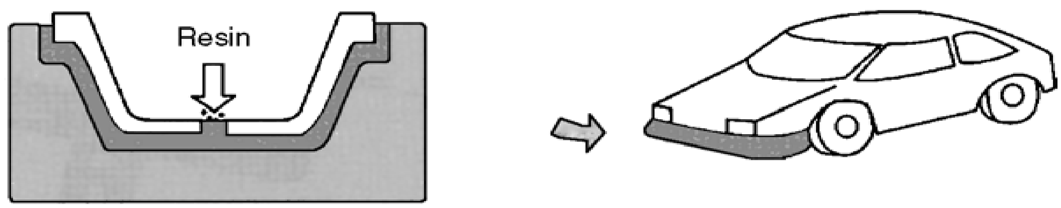


Figure (1) manufacturing the bumper of the car

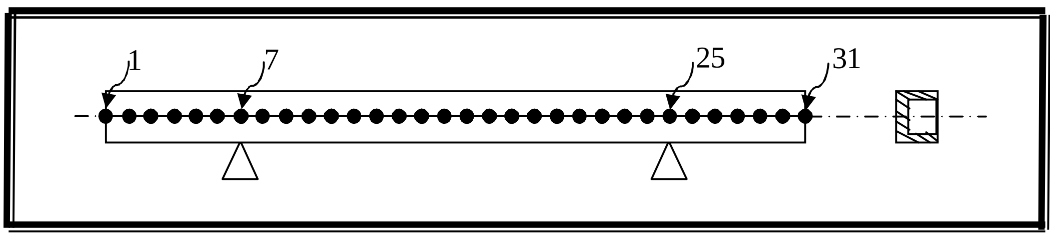


Figure (2) modeling the bumper of the car

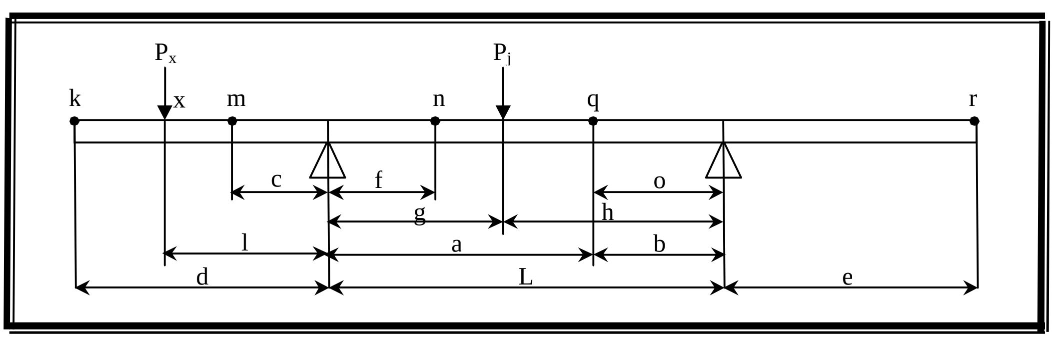
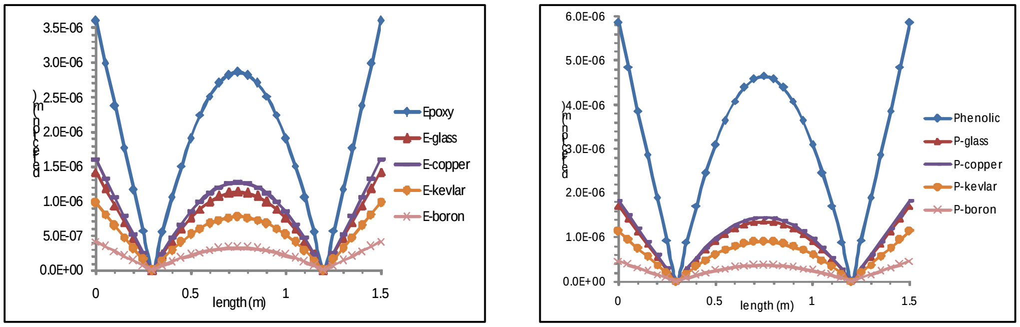
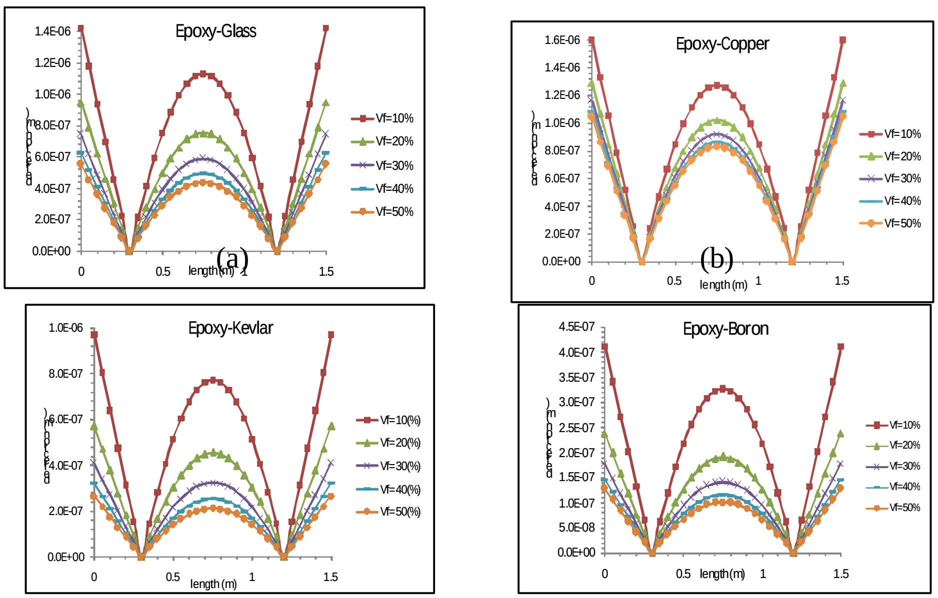


Figure (3) loads and dimensions of the car's bumper



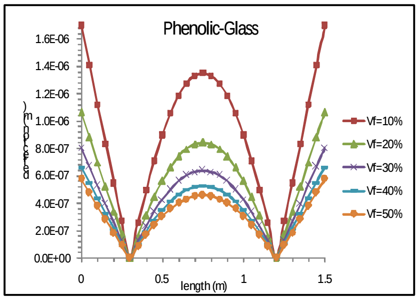
(a) (b)

Figure (4) the deflection for epoxy and phenolic with different fiber's materials

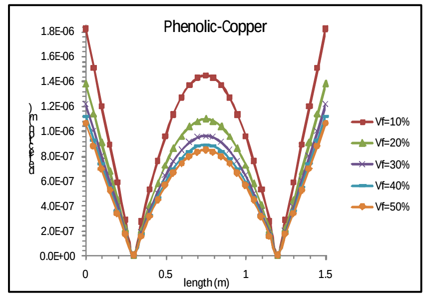


(c) (d)

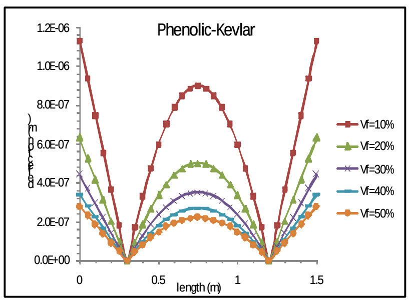
Figure (5) the deflection for epoxy with different volume fraction



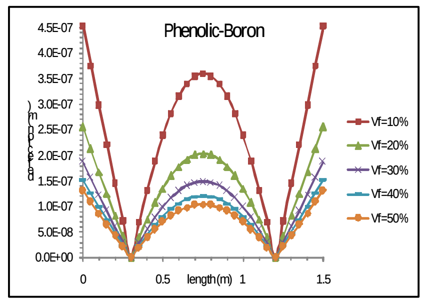
(a)



(b)



(c)



(d)

Figure (6) the deflection for phenolic with different volume fraction

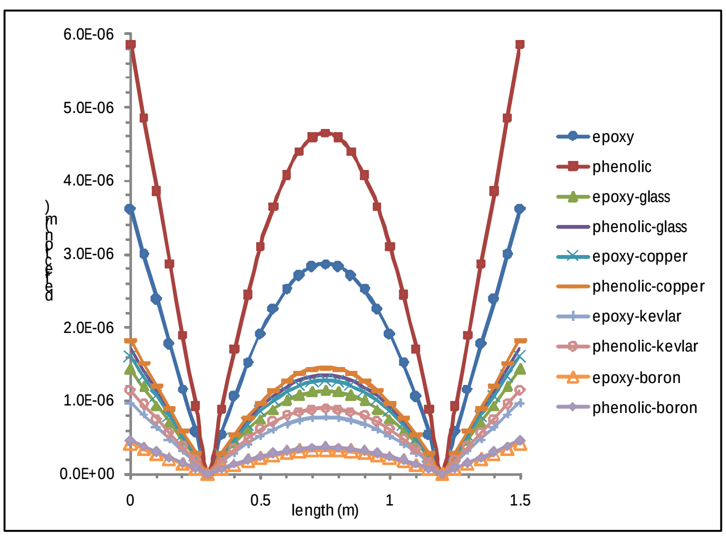
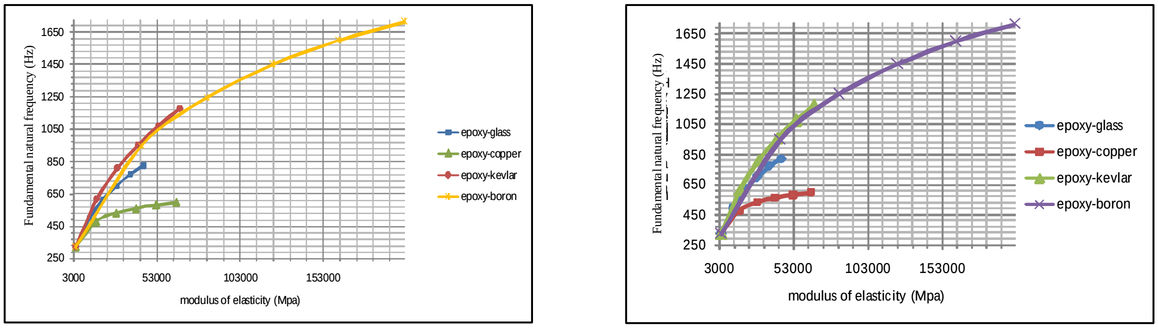
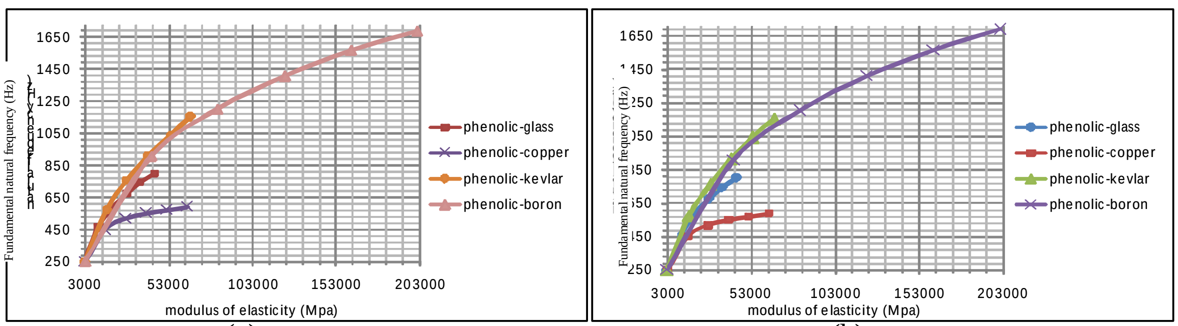


Figure (7) effect of changing matrix material on the deflection for different fiber's materials



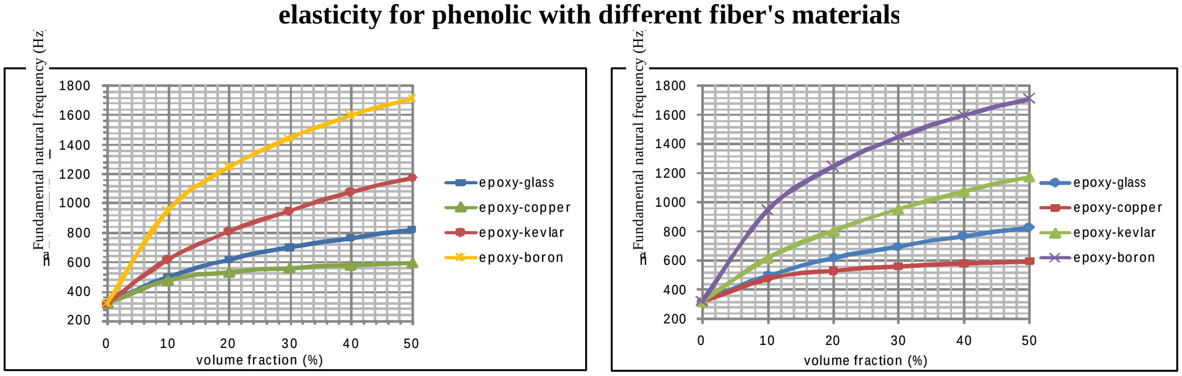
(a) (b)

Figure (8) relation between the fundamental natural frequency and the modulus of elasticity for epoxy with different fiber's materials



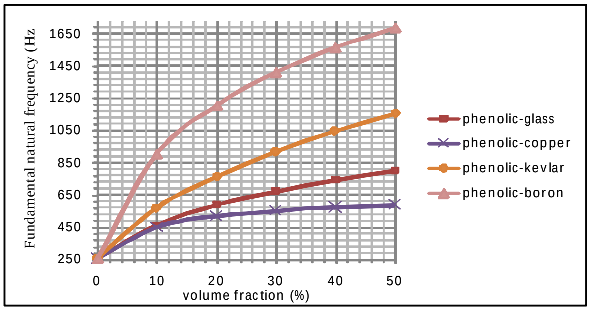
(a) (b)

Figure (9) relation between the fundamental natural frequency and the modulus of elasticity for phenolic with different fiber's materials

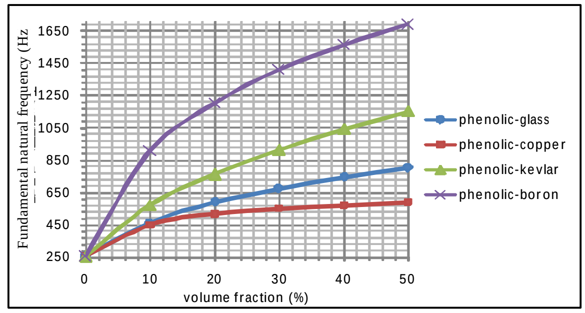


(a) (b)

Figure (10) relation between the fundamental natural frequency and the volume fraction for epoxy with different fiber's materials



(a)



(b)

Figure (11) relation between the fundamental natural frequency and the volume fraction for phenolic with different fiber's materials

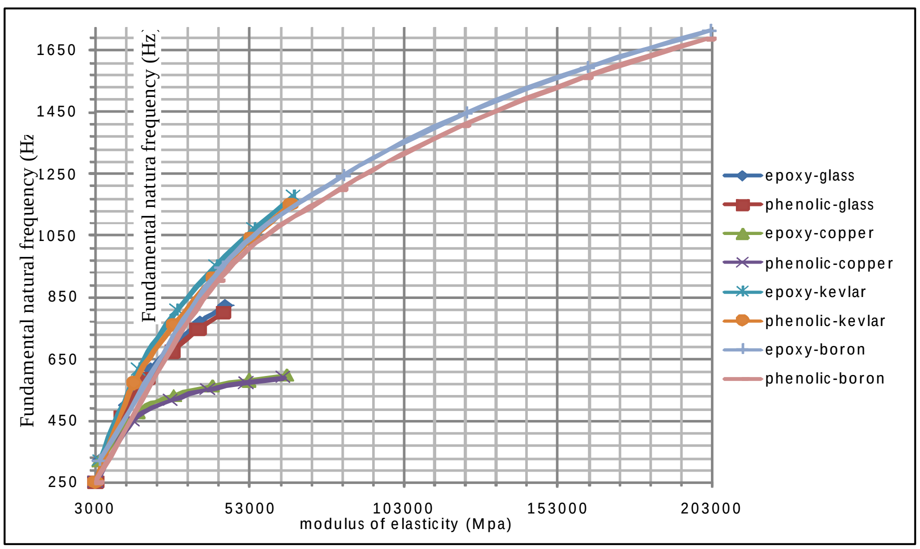


Figure (12) effect of changing matrix material on the fundamental natural frequency and the modulus of elasticity with different fiber's materials

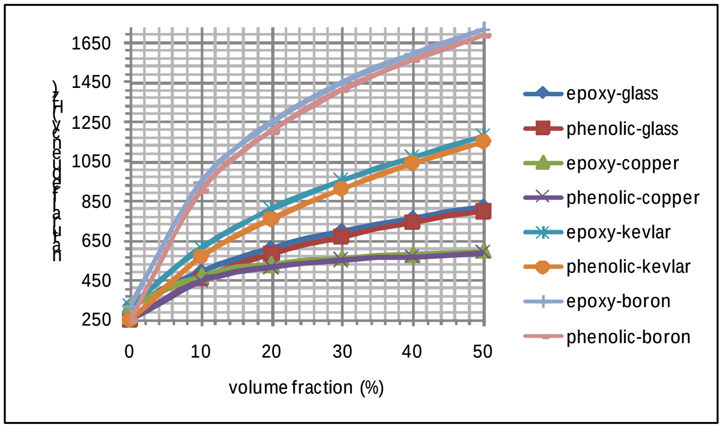
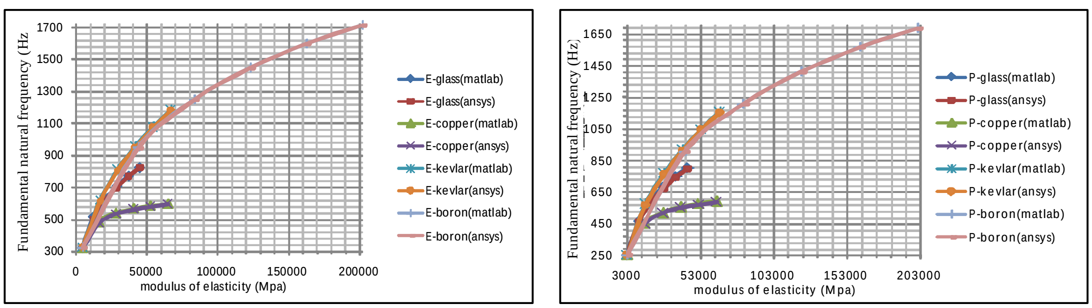
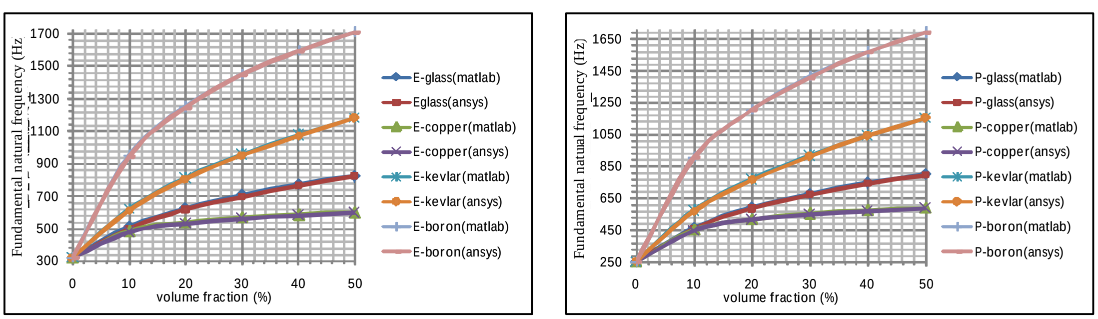


Figure (13) effect of changing matrix material on the fundamental natural frequency and the volume fraction with different fiber's materials



(a) (b)

Figure (14) comparison between MATLAB and ANSYS for the fundamental natural frequency and the modulus of elasticity for epoxy and phenolic with different fiber's materials



(a) (b)

Figure (15) comparison between MATLAB and ANSYS for the fundamental natural frequency and the volume fraction for epoxy and phenolic with different fiber's materials