# A Review of Constructal Design for Different Shapes of Heat Exchangers

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## Abstract

The design of heat exchangers using constructal design principles plays a crucial role in enhancing thermal performance and optimizing heat transfer density, with significant implications for industrial applications such as power plants, chemical processes, and heating and cooling systems. Improving the efficiency of heat exchangers can result in substantial energy savings and lower operational costs. By incorporating complex shapes like finned tubes or corrugated plates, the surface area available for heat exchange is increased without significantly enlarging the exchanger's size, thus enhancing heat transfer density. Additionally, a balanced design ensures uniform heat distribution, reducing hot and cold spots and promoting more effective heat transfer. The flexibility in design allows for adaptation to various applications, whether for cooling or heating, making the exchanger more adaptable to practical and spatial requirements. Overall, employing different shapes in heat exchanger design significantly boosts their efficiency and heat capacity, leading to improved system performance, reduced operational costs, and lower energy consumption.

**Keywords:** Forward Kinematic, Jacobian, Nonlinear Controller, ode45, Solid Works.

## مراجعة التصميم الإنشائي لأنواع مختلفة من المبادلات الحرارية

سليمه راشد صالح ، أحمد وحيد مصطفى

**الخلاصة:**

تصميم المبادلات الحرارية باستخدام مبادئ التصميم البنائي يلعب دورًا حاسمًا في تعزيز الأداء الحراري وتحسين كثافة نقل الحرارة، مع آثار كبيرة للتطبيقات الصناعية مثل محطات الطاقة والعمليات الكيميائية وأنظمة التدفئة والتبريد. يمكن لتحسين كفاءة المبادلات الحرارية أن يؤدي إلى توفير عالي في الطاقة وتقليل تكاليف التشغيل. من خلال دمج الأشكال المعقدة مثل الأنابيب المزعنفة أو الصفائح المموجة، تزداد مساحة السطح المتاحة لتبادل الحرارة دون زيادة كبيرة في حجم المبادل، مما يعزز كثافة نقل الحرارة. بالإضافة إلى ذلك، يضمن التصميم المتوازن توزيعًا متساويًا للحرارة، مما يقلل من النقاط الساخنة والباردة ويعزز نقل الحرارة بشكل أكثر فعالية. تتيح المرونة في التصميم التكيف مع مختلف التطبيقات، سواء للتبريد أو التدفئة، مما يجعل المبادل أكثر تكيفًا مع المتطلبات العملية والمكانية. بشكل عام، يؤدي استخدام الأشكال المختلفة في تصميم المبادلات الحرارية إلى تحسين كفاءتها وقدرتها على نقل الحرارة، مما يؤدي إلى تحسين أداء النظام وتقليل تكاليف التشغيل واستهلاك الطاقة.

## 1. Introduction

Heat exchangers are a well-known technology used in the generation of energy, air cooling, and heat transfer systems. They also constitute one of the pillars of thermal science. Design with constructal theory (DCT) method, focuses primarily on geometric volume optimization, and it is used in many studies and articles on heat exchangers. The constructal design approach developed by Bejan[1], Bejan [2] and Kobayashi et. al. [3], is based on the principle of optimizing flow and transfer systems to improve performance and efficiency. On the other level non-constructal designs are frequently used in studies and articles. They addressed point-to-point and volume flow problems in other sciences besides engineering. The previous study on the constructal design of single and multi-scale tubes and fins is presented in this paper. Forced, mixed, and natural convection will be presented.

## 2. Constructal Designs of Single-Scale Tubes and Fins Cooled by Convection Heat Transfer.

Constructal design is a method for identifying the optimal configuration of flow systems. This approach is grounded in the principle that the evolution of a system's configuration should enhance the accessibility of the currents that flow through it. By systematically optimizing the architecture of flow paths, constructal design aims to improve the overall efficiency and performance of these systems. In this section, we will review the literature on the constructal design method for single scale systems, focusing on tubes and fins cooled by convection heat transfer. This review will cover configurations that utilize constructal theory, prioritizing optimized flow architecture over traditional flow nature considerations. We will also examine the endorsed types of convection used in these systems. The aim is to provide a comprehensive overview of the advancements and practical applications in this field.

Bejan and Sciubba[4] Investigated the optimal spacing of maximum heat transfer from a package of parallel panels cooled by forced convection, where the study aimed to determine the optimal board-to-board spacing and associated features for packages cooled by forced convection. An order-of-magnitude analysis was conducted considering laminar flow and uniform board temperature to determine the optimal board-to-board spacing. The results showed that the optimal spacing was related to the plate length, property set, and pressure head across the stack. It is found that the maximum overall heat transfer rate is affected by the pressure head, the total flue thickness, and the maximum allowable temperature difference. In conclusion, choosing the optimal plate-to-plate spacing was critical to maximizing the overall heat transfer rate from the stack to the coolant stream.

Alebrahim and Bejan[5] used constructal method to study the three dimensions model of forced convection heat transfer to minimize the thermal resistance. Firstly, the conduction was presented in the heat mechanism. The low conductivity material (k0) and high conductivity inserts (kp) produced the heat generating volume. The disks were firstly mounted on a surface (stem) of kp material. Then, the interstitial spaces were examined and occupied by k0 material cooled by forced convection. The results included finding of the optimum of external shape, number of circular fins, and the ratio of the stem diameter to the fin thickness.

Almogbel and Bejan[6] used the constructal method to find the optimized assembly which arranged as trees with tapered pin fins of cylindrical assemblies. The optimization included maximizing the global conductance subject to fixed total volume and amount of fin material. The study presented responding the geometric optimum to variations in the residual variables of the design. These variables included the volume fraction of fin material, the free stream velocity, and the Prandtl number.

Bello and Bejan [7] Described numerically optimizing the geometry in rows of parallel heated plates with three modes of heat transfer: natural convection, forced convection, and mixed convection. The optimization included the distance between plates and the plate's number with fixing of volume. The results showed that limitations of natural and forced convection. Moreover, a correlation that describes the gap between the two limits were deduced.

Joucaviel et al. [8] The study investigated heat transfer density among revolving cylinders in a bundle cross-flow configuration, analyzing two rotational setups: counter-rotation and same-direction rotation. Using a numerical model, the author assumed constant, uniformly distributed wall temperatures higher than the inlet fluid temperature. The study optimized the non-dimensional distance between cylinders using three Bejan numbers (103, 104, and 105) to maximize heat density. The model was validated against non-rotating cylinder configurations. Results showed that counter-rotation was more effective than same-direction rotation, providing valuable insights into enhancing heat transfer density in rotating cylinder bundles in cross-flow setups.

Dos Santos et al. [9] studied numerically the characteristics of forced convection heat transfer in an assembly of rotated cylinders arranged in a cross flow. The aim of this study was maximizing the heat transfer density of this assembly. The study included two configurations: in the first configuration, the cylinders rotated in the same direction, while in the other, the cylinders rotated in opposite direction. The results showed that the second configuration produced higher efficiency than the first because of a smaller spacing for the flow.

The thermal behaviour of a sequentially arranged cylinder rotating in a counter mode configuration and undergoing natural convection cooling was systematically investigated by Page et al. [10] The primary objective was to optimize the rate of heat transfer density, seeking its maximum value. Numerical solutions for the flow and temperature fields were obtained by solving the governing equations using a simple algorithm and a finite volume code method. Special attention was dedicated to examining a range of Rayleigh numbers within the interval of 101 to 104. The study's conclusion revealed that an increase in the Rayleigh number corresponds to a decrease in the optimum spacing; ultimately leading to the attainment has the maximum possible heat transfer density.

In the study of Klein et al. [11] A dimensionless pressure gradient denoted as the Bejan number was introduced and specifically adapted for application when considering non-Newtonian fluids. Regarding non-Newtonian fluids, the Bejan number (Be) was defined by selecting a characteristic apparent viscosity within the composite Bingham number (Bi). The velocity scale in the Bejan number formulation was determined as the pressure drop raised to the power of

(1/2) and divided by the mass density, particularly in instances where the flow rate is unknown. Simulations were conducted to establish a suitable form for the Bejan number, serving as a viable alternative for experimental applications aimed at optimizing heat transfer in systems involving non-Newtonian fluids. This development contributes to the advancement of methodologies for characterizing and analysing heat transfer phenomena in non-Newtonian fluid systems.

Razera et al. [12] Investigated the Constructal Design principles were employed to analyze the mixed convection from triangular fin inside a square cavity. The flow was initiated by moving the upper wall or lid. Understanding how the dimensionless convective heat transfer coefficient (Nusslet Number) was influenced by fin geometry and area ratio($\phi$) was the primary goal. Further, while keeping a constant Prandtl number (Pr = 0.71 in all cases), the impact of Rayleigh (RaH) and Reynolds (ReH) numbers on thermal performance and ideal geometries was examined. Using a code based on the Finite Volume Method (FVM), the conservation equations governing mass, momentum, and energy were numerically solved. Research results show that when Reynolds and Rayleigh numbers rise and fin area ratio ($\phi$) falls, thermal performance improves as well. When comparing the most effective rectangular and a triangle fins at RaH = 105, it can be shown that the rectangular fin performs better at higher ReH values than the triangular fin, with the former showing an improvement of up to 8% for low Reynolds numbers (ReH < 200).

Feijó et al. [13] Studied numerically a two-dimensional channel with two triangular fins exposed to forced convection heat transfer in laminar flow. The geometry of the first fin was evaluated using the Constructal Design approach. With different first channel fin dimensions, the primary goals were to reduce the pressure differential between the channel's intake and outlet flows and increase the rate of heat transfer. Three restrictions pertaining to channel area, fin area, and the maximum occupancy area of each fin were taken into consideration during the analysis, which was conducted under constant Reynolds (ReH = 100) and Prandtl (Pr = 0.71) numbers. The system displays three degrees of freedom, and the channel's height-to-length ratio (H/L = 0.0625) was fixed. The other two degrees of freedom correspond to the height-to-width ratios of the downstream fin (H4/L4), which was fixed at H4/L4 = 1.11, and the upstream fin base (H3/L3). The study investigated the fraction area of the upstream fin at three different values ($\phi1$ = 0.1, 0.2, and 0.3). Using numerical simulations with the finite volume technique (FVM), the findings revealed that the suggested multi-objective was best satisfied when ($\phi1$ = 0.2) which represents the most advantageous trade-off between pressure difference and heat transfer. Significantly, with a performance ratio of 25.2 times between the best and performance for fluid dynamics case, the value of ($\phi1$ = 0.1) indicated exceptional fluid dynamics performance. On the other hand, the optimal thermal performance was attained when ($\phi1$ = 0.3) with the best case performing 65.75% better than the worst.

Hermany et al. [14] Investigated the viscoplastic fluid flow characteristics within elliptic tubes under forced convection were examined using the constructal design methodology. Determining an optimal solution was the primary objective of this analysis ellipse aspect ratios that enhance heat transfer rates while minimizing frictional losses, under conditions of constant ellipse area, Prandtl number (Pr=1) and Reynolds number (Re=1). The range of the power-law index (n = 0.4 to 1) was utilized to characterize fluid shear thinning behaviour. The results showed that there was a direct relationship between Nusselt Numbers along with the aspect ratio, showing an increase in the heat transfer effectiveness with increasing aspect ratio approaching unity. Additionally, the use of elliptic tubes with mild viscoplastic fluids was identified as a viable approach to reduce pressure drop, thereby lowering pumping power requirements.

Mustafa. [15] The study investigated heat transfer density in forced convection-cooled rhombic tubes using a constructal design methodology. Parallel rhombic tubes were arranged within a set volume, aligned horizontally, with vertical positioning and spacing adjusted for optimal heat transfer to the coolant. The tubes were kept at a uniform temperature, and a constant pressure drop induced steady, two-dimensional, incompressible, and laminar forced convection. The method of finite volume was utilized to solve the governing equations, and the SIMPLE algorithm. The Bejan number (Be) ranged from 103 to 105, with the vertical axis (B) between 0.2 and 2, using air as the operating fluid (Prandtl number =0.71). Results indicated that optimal spacing decreased with increasing Bejan numbers, while maximum heat transfer density increased. The study highlighted the significant impact of the Bejan number and tube bluntness on flow structure, including separation and vortex formation around the tubes at optimal spacing.

Mustafa et al. [16] Investigated the optimal distance within a constrained volume using constructal theory for a diamond-shaped tube array. A numerical analysis was conducted on free convection cooling with isothermal tubes arranged in a row shape, employing a SIMPLE algorithm using a finite volume method to solve the governing equations and collocated mesh grid. The investigation spanned a Rayleigh number range (103 ≤ Ra ≤ 105) and a diamond axis ratio range of (0 ≤ e ≤ 0.5), with air as the fluid having a Prandtl number of (0.71). Grid independence tests revealed that the heat density converged to constant values using a (50×50) control volume, with a percentage difference of (1.5%). Upstream and downstream extension distances were set to (0.5) and (2), respectively. Validation of the numerical results against previous literature showed acceptable agreement with a percentage difference of (0.77%). The study concluded that with an increasing Rayleigh number, the optimal distance decreased accordingly. Additionally, it was found that the maximum heat density occurred at an axis ratio of zero, behaving like a flat fin, while the minimum value was observed at an axis ratio of (0.5), and exhibiting behaviour similar to a rhombic tube.

A single row of heat exchangers heated elliptical tubes operating was created in free convection by Mustafa and Zahi [17] The optimal spacing in between elliptic tubes in free convection cooling were investigated numerically, following Bejan's constructal theory. Elliptic tubes that are isothermal were arranged within a particular volume, with spacing among chosen to maximize heat transfer density. The finite volume approach was utilized to solve the governing equation, utilizing that SIMPLE algorithm with a collocated grid for velocity-pressure coupling. Considering a Rayleigh number range (103 ≤ Ra ≤ 105) and axis ratios (0 ≤ ε ≤ 0.5) for air as (Pr=0.71) the working fluid, it was found was the optimal spacing decreased with increasing Rayleigh numbers across all axis ratios. Furthermore, the maximum heat transfer density increased with higher Rayleigh numbers for all axis ratios, peaking at ε = 0 (flat plate) and reaching its lowest value (circular tube) at ε = 0.5. Importantly, that optimal spacing remained constant at a given Rayleigh number regardless of the axis ratio.

Built a single row of flat tubes in a heat exchanger for three constant Bejan values (Be=103, 104, and 105) by Mustafa and Ghani [18] investigated the density of forced convection heat transfer rate from isothermal flat tubes using contractual theory. Placed in a cross flow in a vertical direction. The volume of the domain where the tubes were positioned was used as the study's constraints. This volume had two degrees of freedom: the distance between the tubes and the flatness of the tubes. Prandtl number was (Pr = 0.71), and Bejan number was modified from 103 to 105. The range of tube flatness was 0 ≤ F ≤ 0.8. It was demonstrated that, for a constant Bejan number, a reduction in tube flatness caused a decrease in the maximum heat transfer density.

Mustafa et al. [19] used the constructal law to develop a collection of vertical flat tubes that were cooled by natural convection and arranged in a finite size space. The size of the area where the tubes are located served as a design limitation. The space's independence was defined by the separation of the tubes. The ideal spacing between the tubes was found by using the constructal law. The Rayleigh numbers (Ra=103, 104, and 105) were determined. The circular tube's dimensionless tube diameter (diameter/height) was modified from (D*=0.2) to (D*=1). The wall temperature of each tube was the same after heating. The Prandtl number = 0.72 of the air used to cool the tubes was known. The finite volume method was utilized to solve the mass, momentum, and energy conservation equations for a steady, two-dimensional, and incompressible flow. The outcome shown that for all tube diameters, the ideal or best distance at a given Rayleigh number stays constant. The outcome also demonstrated that, in order to facilitate heat transfer from the tubes to the coolant, small- diameter tubes have to be more common than large- diameter tubes for a given Rayleigh number and space size.

Abbas et al. [20] applied constructal design to optimize forced convection cooling for longitudinally finned tubes. In a two-dimensional domain with parallel finned tubes, two degrees of freedom were considered: the length of the longitudinal fin and the tube-to-tube spacing. The setup included three fin positions: front, back, and both front and back of the tube. The goal was to optimize the heat transfer density from the tubes to the cold cross flow, driven by a constant pressure difference. The study solved dimensionless continuity, momentum, and energy equations using the finite volume method, assuming steady, incompressible flow and constant surface temperature. The Bejan number (103 to 105) and fin lengths (Lf = 0, 0.2, 0.4) were varied, with air as the coolant (Prandtl number 0.71). Results indicated that optimal fin positioning and Bejan numbers could maximize heat transfer by optimizing the spacing between finned and unfinned tubes.

Mustafa and Ismael [21] Investigated the optimum distance between finned tubes for free convection cooling within a defined volume, there was a row of finned tubes that were isothermal, and the spacing between them was determined in accordance with Bejan's constructal theory. This theory dictates that tube spacing should be selected to maximize heat transfer density. The numerical simulations utilize a collocated grid for the coupling between pressure and motion in the SIMPLE algorithm, which uses the solution to the governing equations using a finite volume approach. With air serving as the working fluid (Pr = 0.71), the study taken into account several Rayleigh numbers in range (103 ≤ Ra ≤ 105) and tube locations (0.25 ≤ δ ≤ 0.75). The findings indicated that the optimal spacing diminishes while the Rayleigh number increased for each tube positions. Furthermore, for every tube position, the maximum HTD (Heat Transfer Density) increased in the Rayleigh Number increased. In particular, tube position (δ = 0.75) exhibited the highest heat transfer density at Ra=105, whereas tube position (δ = 0.25) exhibited the lowest. Surprisingly, even when tube location was varied at a fixed Rayleigh number, the ideal spacing was stayed constant.

Anas and Mussa [22] applied constructal design theory to design a single-row cross-flow heat exchanger with wing-shaped tubes oriented in two directions relative to the free-stream. The wings were isothermally heated. The design aimed to optimize the wing-to-wing spacing to maximize heat transfer density from the wings. Two ratios of wing thickness to chord length, 0.2 and 1, were investigated, along with three Bejan numbers (Be = 103, 104, and 105) for each thickness ratio. The results indicated that for all wing thicknesses and Bejan numbers, the maximum heat transfer density was higher in the left flow direction than in the right flow direction.

Mustafa et al. [23] used constructal design method to investigate that maximizing that of volumetric heat transfer density in cross-flow from radially finned tubes. Cross-air flow was installed into a row of radially finned tubes. The air cross-flow cooled the tubes and radial fins while heating them to the same temperature. A finite pressure different generated the cross-air flow. Be = 103 and Be = 105 were the two dimensionless pressure differences (Bejan number) that were taken into account. It is essential to identify the constructal design method's objective function, degrees of freedom, and constraints. Maximizing the heat transfer density derived from the finned tubes was the aim of the work. The fin tip-to-fin tip spacing, number of fins, tube diameter, fin thickness, and angle between the fins were the degrees of freedom. The space that the finned tubes occupied was limited in both height and length. The method of finite volume was applied to resolve the steady, 2-D (Two Dimensional), and incompressible energy and flow equations driven by pressure. For (Be = 103 and 105), respectively, the dimensionless fin tip-to-fin tip spacing ranges were ($0.2 \leq S \leq 1$) and ($0.05 \leq S \leq 0.3$). It changed to have (N=2, 4, 6, 8, 10, and 12) fins. The diameter of the dimension-less tube was modified to (D=0.25, 0.5, and 0.75). Changes were made to the dimensionless fin thickness as (T = 0.001, 0.01, and 0.05). The outcomes indicated considering that the maximum volumetric heat transfer density (HTD) for both (Be=103) and (Be=105) was highest for (N=2) and decreased with an increase in fin number. In addition, the vertical fins at (N = 4, 8, and 12) were when the minimum values of the maximum volumetric heat transfer density occur.

Razera et. al [24] studied the effect of the gap between the two parts of elliptic cross-sectional shape on the process of forced convection heat transfer. Additionally, the study investigated the effect of horizontal-to elliptical cross-section's vertical axis ratio on the density of heat transfer maximization. For Pr=0.72, the Bejan number was represented as the pressure differential throughout the flow. In comparison to the lower level examples examined, it was shown that the ideal arrangements increased the heat transmission density by 50% to 97%.

## 3. Constructal Designs of Multi-Scale Tubes and Fins Cooled by Convection Heat Transfer.

In this section, we will review the literature on the constructal design method for multi-scale systems, specifically focusing on tubes and fins cooled by convection heat transfer. This review will delve into configurations that utilize constructal theory, prioritizing optimized flow architecture over traditional flow nature considerations. Additionally, we will examine the endorsed types of convection used in these systems. Our objective is to provide a comprehensive overview of the advancements and practical applications in this field, highlighting the innovative approaches and benefits of using constructal theory in multi-scale systems.

Bejan and Fautrelle [25] Applied the constructal design method for optimizing a multi-scale blades in cross flow. The system entails a situation of steady utilizing parallel isothermal blades for forced convection occupying the designated volume. Constructal theory was harnessed to optimize the spacings between neighbouring blades of progressively diminishing scales, with the central aim of achieving peak heat transfer density. The layout incorporates smaller blades strategically positioned within regions of fresh fluid, flanking the extremities of the longer blades, boundary layer. The total pressure differential was constant across the system. As the number of scales increases, the flow velocity decreases, culminating in an elevation of the volume-averaged heat transfer density. Notably, there existed a minimal (threshold) scale length, beneath which heat transfer surfaces no longer manifest as distinct (slender) boundary layers. The proposed constructal multi-scale algorithms were derived from fundamental principles, setting them apart from methodologies in fractal geometry where algorithms were typically assumed rather than derived. Furthermore, the concept was expansible to natural convection, facilitating the development of multi-scale flow structures optimized for maximal heat flux density in a manner analogous to forced convection scenarios.

Bello and Bejan [26] Studied the levels of parallel plates with ideal spacing in a domain populated by heat-producing parallel plates under laminar forced convection. The approach entails the introduction of successive generations of smaller plates into each inlet region, as these smaller plates possess slender boundary layers that can conform to the unused (isothermal) inlet flow. The study focuses on increasing heat transfer density through the insertion of multiple length scales of plates in the entrance region. Numerical simulations were conducted to optimize cooling electronics package and heat exchanger flow structures and in the range of Begin number were m = 0.78 (log10 Be − 3) was use to approximate (104 < Be < 1011 ).

Results showed progressively higher heat transfer densities with diminishing returns as there are more length scales available. The study demonstrated the potential for developing new internal flow structures using multi-scale non uniform flow structures.

Bello-Ochende and Bejan [27] Applied the Constructal Theory was to a series of circular cylinders arranged in a multi-scale configuration exposed to cross-flow with up to four degrees of freedom. The flow pattern was characterized by laminar forced convection at a specified Bejan number. The arrangement consisted of cylinders of various sizes distributed non-uniformly within a constrained volume. Smaller cylinders were placed at the inlet of the vertical row of larger cylinders, utilizing the unused flow area. This configuration included three sizes of small cylinders and up to four degrees of freedom. A numerical solution employing the finite element method was employed for the study. Initially, the study focused on a large scale (single scale) with a single diameter size, where the constraints were defined by the total bundle height, diameter, and cylinder width under a given pressure difference. The subsequent optimization phase involved a more intricate multi-scale configuration. Smaller diameter cylinders were introduced at intermediate distances between the large-scale cylinders, acting as constraints for the problem. Two various degrees of freedom were identified, shown by the dimensionless diameter of the small cylinder and the dimensionless spacing between the large-scale cylinders. A third round of optimization added smaller diameter cylinders, further increasing the degrees of freedom to four. All four optimization rounds were conducted over a range of Bejan numbers

(103 ≤ Be ≤ 106) and a Prandtl number of 0.72, with the objective of maximizing heat density within the constrained volume. It was concluded that the flow structure became less permeable with an increase in the number of fins, resulting in a decrease in flow rate. Simultaneously, the overall the solid structure's heat transfer rate density increased. Additionally, the increase in heat transfer density was found to correlate with the non-uniform the length scale distribution throughout the space that is available.

Bello and Bejan [28] The study investigated methods to incrementally raise and maximize convective heat transfer density inside the presence of natural convection within cylindrical assemblies. Utilizing a constructal design approach, small cylinders of varying diameters were strategically inserted at the entrance plane of constraints, particularly within the region of unheated fluid (i.e., unused flow). The optimization procedure involved one and two degrees of freedom by employing one and two cylinder sizes for multi-scale optimization criteria. Numerical solutions were obtained using finite element code. The numerical study explored Rayleigh numbers within the range of (103 ≤ Ra ≤ 105). The initial optimization stage focused on adjusting the spacing between large-scale cylinders with one degree of freedom to identify the optimal spacing for maximizing heat transfer density (HTD). In the subsequent stage, the complexity of the configuration was increased by introducing a small cylinder at the entrance of the constraint volume and between the large-scale cylinders to utilize the unused coolant fluid efficiently. The analysis revealed that as the complexity increased with the placement of small cylinders within the unheated area, the spacing between large-scale cylinders also increased. However, as the flow rate intensified, the spacing decreased, while the diameters of the cylinders exhibited minimal changes throughout the optimization process.

Silva and Bejan [29] Provided a novel design approach to develop a multi-scale configuration under natural convection conditions, aiming to maximize heat transfer density within a specified constraint. This configuration featured equidistantly spaced, vertically arranged heated blades. The spacing between these heated blades was systematically adjusted to optimize the removal of heat density. The author suggested implementing constructal theory to enhance heat transfer density by introducing additional fins with shorter lengths into the regions of unheated fluid, thereby maximizing thermal conductance within the constraint volume. The numerical investigation utilized Rayleigh numbers ranging from 107 to 108. It was concluded that upon optimizing the complicated nature of the flow structure, the rate of heat transfer density reached its maximum.

Bello-Ochende et. al [30] Constructal design method utilized to determine the optimal arrangement of two rows of pin-fins for the maximization of total thermal conductance rate. Conductance the fins occurs via laminar forced convection, exposed to a uniform and isothermal free-stream. The optimization was carried out under the constraint of a fixed total volume of fin materials. The dimensions of the optimized arrangement were determined by balancing the conduction along the fins with the transversal convection. The resulting flow structure displays multiple scales distributed non-homogeneously throughout the flow domain. Numerical analyses were presented, investigating the influence of the optimal configuration's Reynolds number and the thermal conductivity ratio. The numerical outcomes were in good agreement with predictions derived from scale analysis. Moreover, the findings suggest that the structure of flow achieves optimal performance when the diameters and heights of the fins were non-uniform.

Bello-Ochende et. al [31] Three dimensional study was conducted during a parallel fin system designed for heat generation. These fins were arranged in a stacked configuration, forming a fluid channel driven by the differential pressure across the fins. The numerical simulation involved multiple trials with varying fin spacings to determine the optimal spacing that would yield maximal heat transfer density, marking the first stage of the optimization process. In the second optimization stage, shorter fins were introduced between the original large-scale fins to occupy the regions of unheated fluid and enhance the overall thermal conductance. The third phase of optimization aimed to optimize the heat transfer density rate within a specified constraint volume, utilizing the outcomes from the previous rounds. The governing equations were solved for a three-dimensional, steady-state, incompressible, laminar flow with constant Newtonian fluid properties. The fin width was standardized to unity, while the thickness varied within the range of 0.01 to 0.05. The driving pressure difference was characterized by the Bejan number, ranging from 105 to 107. The author concluded that the obtained results were in good agreement with the expected analytical predictions.

Bello-Ochende et. al [32] A numerical analysis was conducted with the intention of maximizing the rate of heat transfer density from a heat exchanger with a crossflow ubjected to a fixed pressure drop within a constrained volume. This study involved the use of rotating multi-scale tubes under laminar forced convection conditions of steady-state nature. Two configurations were considered: the first configuration included two large-scale cylinders along with two small-scale cylinders inserted at the midpoint between the large ones, all aligned along the same centerline. The second configuration retained the same large and small-scale cylinders with the same spacing criteria, but they were not aligned on the same centerline; instead, the leading edges of the four cylinders were aligned in the same direction. The author investigated counter-spinning and co-spinning configurations for both stationary and rotational cases, aiming to optimize the heat transfer density rate through the optimization of cylinder diameters and spacing between them. Furthermore, the study explored the influence of the location of rotational centers, Bejan number (a dimensionless pressure drop number), spacing, and optimal cylinder diameters. The numerical solution involved a range of smaller cylinder diameters between 0.1 and 1, while the diameters of the large-scale cylinders remained constant, with a range of dimensionless pressure numbers between 10 and 104. The author concluded that enhancements in heat transfer were more significant with the configuration of rotating cylinders aligned on the same axis of rotation compared to the configuration with cylinders aligned along the plane of the leading edge.

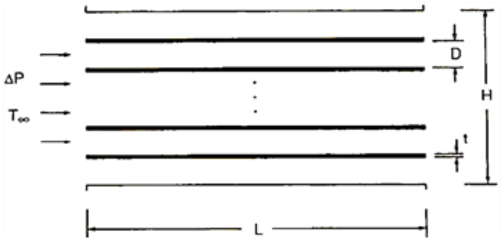
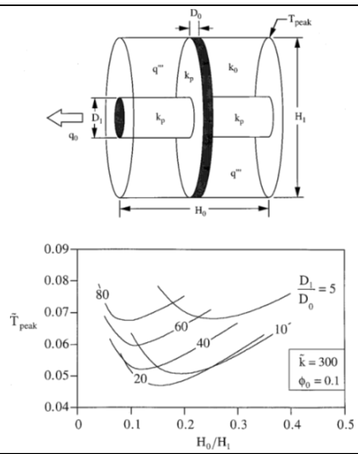
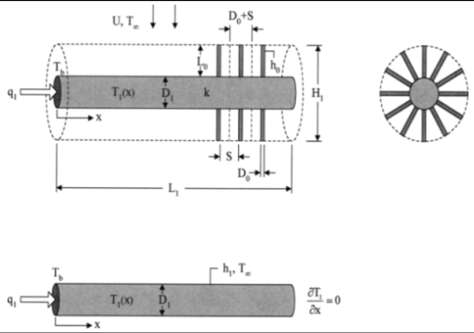
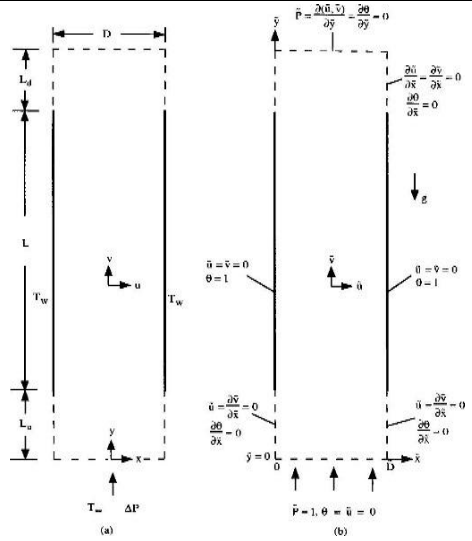
Page et. al [33] Studied the thermal characteristics of a staggered counter-rotating arrangement of multi-scale cylinders undergoing the objective of natural convection cooling was to optimize the rate of HTD (defined as the thermal conductance rate per unit volume). Through the utilization of a numerical model, the governing equations governing flow fields and temperature were resolved, together with a mathematical optimization algorithm was employed to determine the optimal configuration for flow patterns with two degrees of freedom. The multi-scale arrangement of the cylinder assembly underwent optimization for each Rayleigh number and cylinder rotation speed, taking into account the two degrees of freedom. Emphasis was placed on the strategic positioning of smaller cylinders at the inlet of the assembly, within wedge-shaped flow regions housing unused fluid for heat transfer, was implemented to introduce additional length scales to the flow configuration. The study revealed that, in comparison to stationary cylinders, there was almost negligible impact the maximum of heat transfer density rate of cylinder rotation at each Rayleigh number, except at high cylinder rotation speeds where suppression an increase in the rate of (HTD) was noted. However, the optimized spacing diminished as the cylinder rotation speed raised for each Rayleigh number. Findings additionally revealed that the peak (HTD) for a multi-scale layout (without cylinder rotation) exceeds that of a single-scale layout (with rotating cylinders), except under extremely low Rayleigh numbers.

Mustafa A.W. [34] A study explored the maximization of HTD (Heat Transfer Density) in a diamond- shaped pin fins arrangement with multi-scale exposed to mixed convection using constructal design methodology with a fixed pressure gradient. The study used the Rayleigh number to Bejan number ratio (Ra/Be), as well as included two sizes of fins within a designated vertical crossflow volume. Smaller fins were positioned between the larger fins in the unused entry region, both preserved at a constant temperature. Applying the finite volume method, the governing equations for buoyancy – pressure driven flow were solved for steady, two- dimensional, laminar, incompressible flow. The fin's leading- edge angles varied between 30° to 60°. With the Rayleigh number fixed at Ra = 105, the study examined Bejan numbers from 104 to 106, Ra/Be ratios from 0.1 to 10, and used air with a Prandtl number of 0.71. Two instances of maximizing thermal conduction density were identified: one determining optimal spacing between larger fins, and the other identifying optimal height of smaller fins at a fixed leading-edge angle of 30°. It was found that at a given Ra/Be, the optimum spacing between larger fins was consistent across all fin angles, indicating fewer fins with increasing angle. Results showed HTD could be maximized twice for all smaller fin angles (30°, 45°, and 60°) and Ra/Be ratios (0.1, 1, and 10), with the highest second HTD peak at a 30° leading-edge angle for all Ra/Be ratios..

Mustafa and Abdul Elqadir [35] Conducted a study of two-scale elliptical tubes with constructal design in cross- flow principles. Within a predefined domain characterized by a specified length and height, larger tubes were positioned. Strategically, smaller tubes were introduced among the largest tubes, particularly at the area designated for entrance at a mid-leading-edge to leading edge distance among the largest tubes. Variations in the spacing between the larger tubes, the semiminor axis of the larger tubes, the major axis of the smaller tubes, and the semiminor axis of the smaller tubes were systematically explored within the domain to determine the optimal configuration. Two distinct optimal layouts were identified: one without the presence of the smaller tubes and another incorporating the smaller tubes. Both the larger and smaller tubes maintained at uniform surface temperature, with the flow induced by a fixed pressure gradient. The governing equations for steady, laminar, incompressible and two- dimensional flow were resolved using the finite volume technique. In scenarios devoid of the smaller tubes, the Bejan number (dimensionless pressure drop) ranges from 103 to 105, while in the presence of the smaller tubes, the Bejan number was fixed at Be = 105. The dimensionless semiminor axis of the larger tubes spans from 0.1 to 0.4. The cooling medium for the tube row was air, characterized by a Prandtl number of 0.7. The findings indicated that, for various semiminor axes of the larger tubes, the thermal conduction rate undergoes enhancement when the smaller tubes were positioned among the larger tubes.

Mustafa et. al [36] Investigated of the constructal design displaying multi-scale, vertical triangular fins in natural convection was undertaken in this study. The design was comprised consisting of two parts, the first part focused on single-scale triangular fins, aiming to achieve fins with the maximum heat transfer density across three angles of fins (15°, 30°, and 45°). These fins are single- scale were arranged horizontally and treated as fins with an isothermal. The degree of freedom through the first design included a fin angle and fin height as the constraint the fin-to- fin spacing as the desired amount. The second part involved multi-scale fins, where a small fin was positioned within the large, optimally shaped fins in the initial phase. In this subsequent phase, both the large and small scale fins were maintained at a constant angle of 15°. Based on the first part, the optimal fin-to- fin spacing served as a constraint at the second phase. The design considered the Rayleigh numbers for Ra are 103, 104 and 105. The finite volume approach was used to solve the two- dimensional equations of mass, momentum and energy for natural convection. The outcomes revealed a notable benefit in the small scale fins, the presence of the small fins between the large fins results in a percentage increase in heat transfer density of 10.22% at Ra=103 and 50.6% at Ra=105.

**Table (1):** Summery of Constructal Designs from Different Shapes of Tubes.

| Study | Configuration | Range of (Ra) or (Be) | Conclusions |
|---|---|---|---|
| Bejan and Sciubba[4] |  | // | • The optimal board-to-board spacing in a stack of parallel boards cooled by laminar forced convection is proportional to the board length raised to the power of 1/2, the property group, and the pressure head maintained across the stack.<br>• The total heat transfer rate is proportional to the pressure head, the total thickness of the stack, and the maximum allowable temperature difference between the board and the coolant inlet<br>• The ratio between the total heat transfer rate and pumping power decreases monotonically as the spacing between the boards increases |
| Alebrahim and Bejan [ 5] |  | // | • The optimization by adopting the internal geometric aspect ratios that were optimized independently at the elemental-volume level. |
| Almogbel and Bejan [6] |  | // | • The optimized tree construct of cylindrical assemblies of pin fins maximizes global conductance with fixed total volume and fin material, showing robustness. |
| Bello and Bejan [7] |  | $10^5 \leq Ra \leq 10^7$<br>Be = 1 and $10^5$ | • correlated optimal spacings for mixed convection with those for natural and forced convection, showing the existence of optimal spacings in all cases<br>• The transition between forced and natural convection in the study was governed by the ratio Ra / Be, highlighting the roles of Ra and Be in driving convection. |

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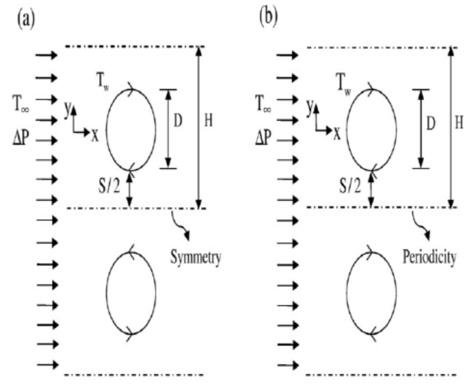
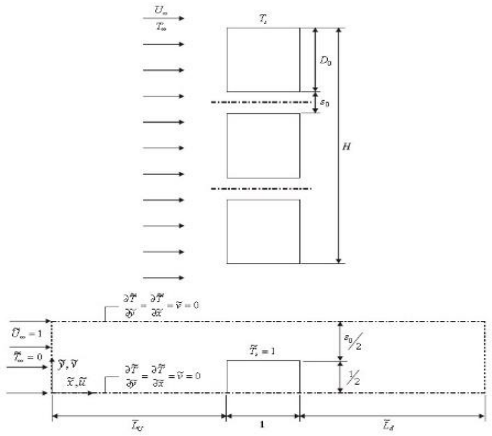
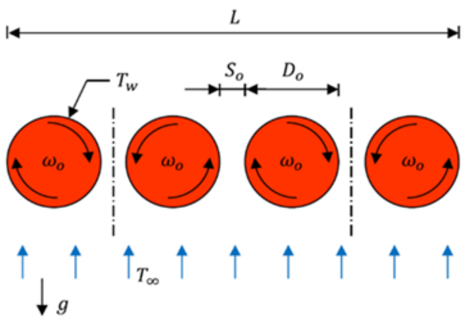
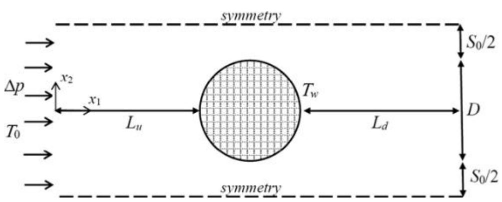
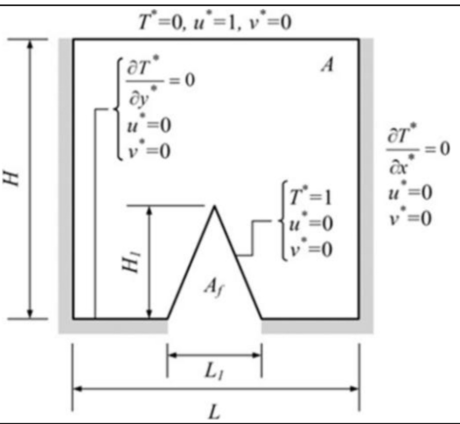
| Rotating Cylinders in Forced Convection Joucaviel et al. [8] |  | 10 ≤ Be ≤ $10^4$ | • The optimization of spacing between sequential cylinders was conducted in each case.<br>• The counter-rotation minimized heat exchangers dimensions and maximize the heat transfer efficiency, while the co-rotating configuration was useless. |
| Dos Santos et al. [9] |  | 60 ≤ Re ≤ 160 | • The maximum heat transfer density that occurs via a reduction in an ideal distance caused by an increase in the Rayleigh number. |
| Rotating Cylinders in Natural Convection Page et al. [10] |  | 10 ≤ Ra ≤ $10^4$ | • The optimal spacing deceases with Rayleigh number.<br>The heat transfer density increases with Rayleigh number |
| Klein et al. [11] |  | // | • The Bejan number (Be) is concluded to be a suitable dimensionless group for pressure drop in non-Newtonian fluids. It can be applied in experiments to assess and forecast the transfer of heat density in relation to heat exchanger tube bundle structural design. |
| Razera et al. [12] |  | $Ra_H = 10^5$ and $Re_H < 200$ | • The thermal efficiency escalates with elevated Reynolds and Rayleigh numbers, and diminishes with a reduction in the fin area ratio.<br>• Under a Rayleigh number of 105, the triangular fin exhibits superior performance compared to rectangular fins at lower Reynolds numbers, whereas rectangular fins demonstrate superior performance at higher Reynolds numbers. |

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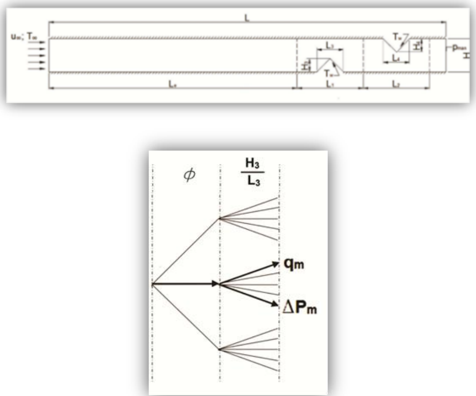
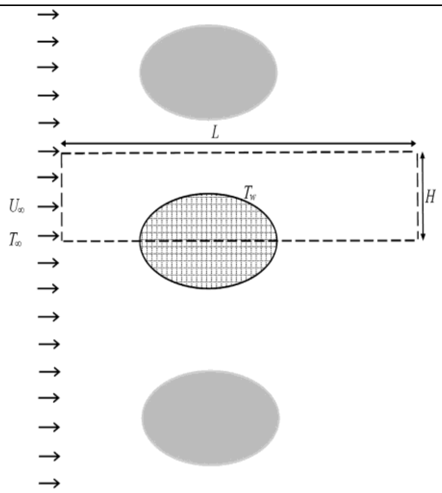
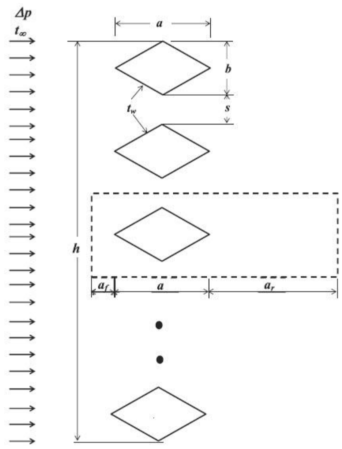
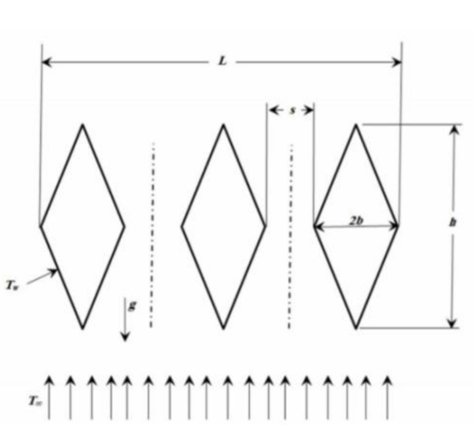
| Feijó et al. [13] |  | ($Re_H$ = 100) | •The optimum geometry of the fins depends on the specific objective of the problem, with different geometries for fluid dynamics and thermal optimization. |
| Hermany et al. [14] |  | Re = 1 | • Elliptic tubes may be a good alternative for heat transfer with mild viscoplastic fluids when there is a need to reduce pressure drop or pumping power.<br>• Elliptic tubes can be used as an alternative in heat transfer applications with mild viscoplastic fluids to reduce pressure drop or pumping power. However, for maximizing heat transfer, circular cross-section tubes are more effective. |
| Rhombic tubes in forced convection Mustafa A.W. [15] |  | $10^3 \leq Be \leq 10^5$ | • The tube axis ratio (minor axis/major axis) for a certain Bejan number maintains the optimal spacing.<br>• The optimal spacing decreases with Bejan number.<br>• The heat transfer density increases with Bejan number. |
| Rhombic tubes during convection in nature Mustafa et al. [16] |  | $10^3 \leq Ra \leq 10^5$ | • As the Rayleigh number rises, the ideal distance between diamond-shaped tubes cooled by free convection decreases.<br>• The heat transmission density reaches its maximum at e=0 and its lowest at e=0.5 as the Rayleigh number grows. When the axis ratio is between 0.1 and 0.5, the ideal spacing remains almost constant, indicating that as the axis ratio rises, fewer tubes are needed. |

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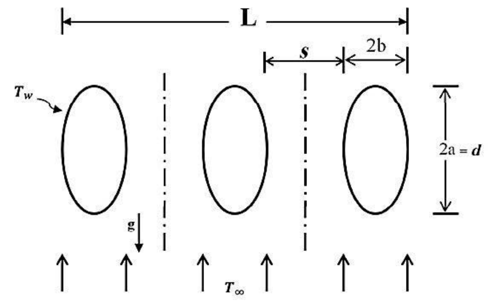
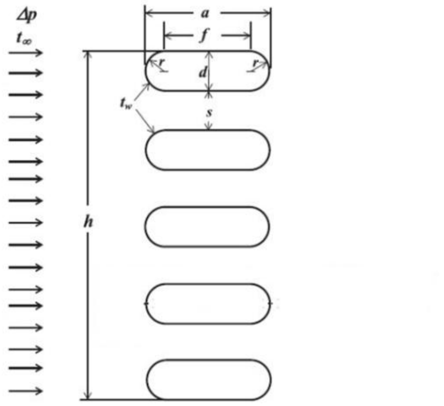
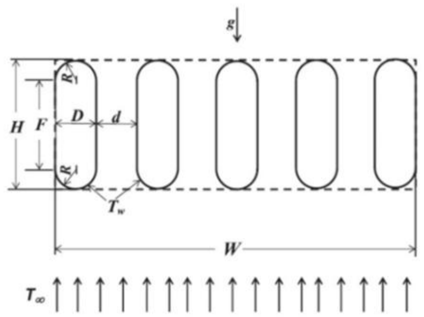
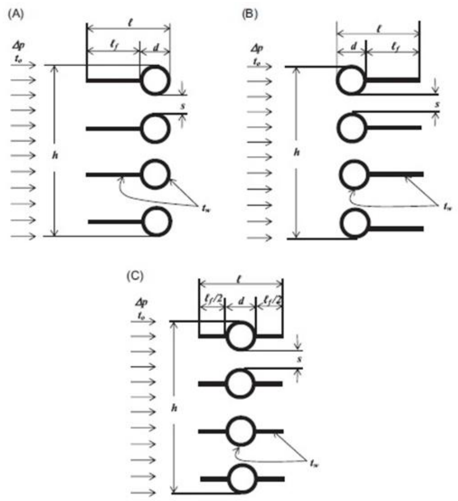
| Elliptic tubes in natural convection Mustafa and Zahi [17] |   | $10^3 \le Ra \le 10^5$ | • For all axis ratios, the ideal spacing is smaller as the Rayleigh number rises.<br>• For all Rayleigh numbers, the maximum heat transfer density is found at ε = 0 (flat plate) and the lowest at ε = 0.5 (circular tube).<br>• At a constant Rayleigh number, the axis ratio has no effect on the ideal spacing. |
| Flat tubes in forced convection Mustafa and Ghani [18] |   | $10^3 \le Be \le 10^5$ | • The Bejan number causes the ideal spacing to decrease.<br>• At a certain Bejan number, the optimal spacing remains constant regardless of the tube flatness (flat part (f) / tube length (a)). |
| In natural convection, flat tubes Mustafa et al. [19] |   | $10^3 \le Ra \le 10^5$ | • The number of small diameter tubes must be more than the number of large diameter tubes for the same Rayleigh number and same domain to make the heat flow from the tubes to the coolant easier. |
| Finned tubes in forced convection Abbas et al. [20] |   | $10^3 \le Be \le 10^5$ | • The optimal separation distance diminishes with an increase in the Bejan number.<br>• The rate of thermal conduction density rises with an increase in the Bejan number.<br>• The front fin configuration yields the highest thermal conduction density. |

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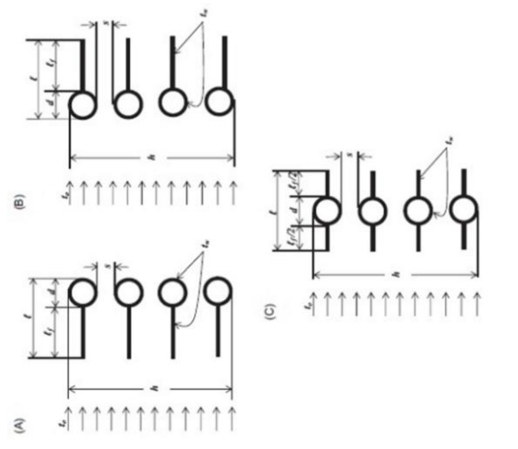
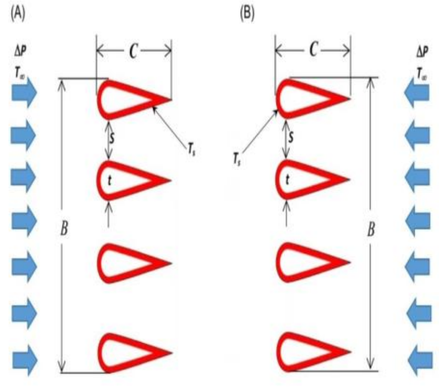
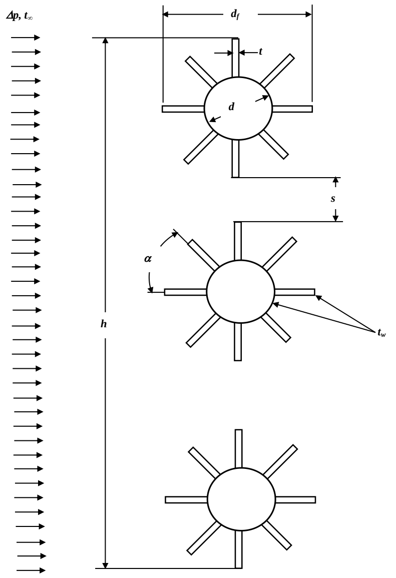
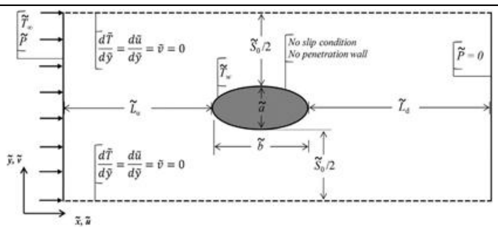
| Finned tubes in natural convection Mustafa and Ismael [21] |  | $10^3 \leq Ra \leq 10^5$ | • The optimal separation distance among finned tubes undergoing free convection cooling diminishes as the Rayleigh number escalates.<br>• The maximal thermal conduction rate density amplifies with an increase in the Rayleigh number.<br>• At a constant Rayleigh number, the ideal separation distance remains unchanged despite variations in tube position.<br>• The quantity of finned tubes installed within a fixed volume remains constant across all tube positions |
| Anas and Mussa [22] |  | $10^3 \leq Be \leq 10^5$ | • In the scenario of trailing edge flow direction, the peak thermal conduction rate density surpasses that observed in instances of leading edge flow direction, across all wing thicknesses and Bejan numbers. |
| Mustafa et al. [23] |  | $Be = 10^3$ and $Be = 10^5$ | • For both Bejan numbers of $10^3$ and $10^5$:<br>1. The greatest maximum volumetric thermal conduction density is observed at N = 2, decreasing with an increase in the number of fins.<br>2. The lowest values of the maximum volumetric thermal conduction density occur when vertical fins are present at N = 4, 8, and 12. |
| Razera A.L. et. al [24] |  | *Be = 10, and 5 ×10* | • The optimal configurations notably enhance the thermal conduction density, elevating it by 50% to 97% in comparison to lower-tier cases<br>• The optimal configuration exhibits variability with the Bejan number and aspect ratio. |

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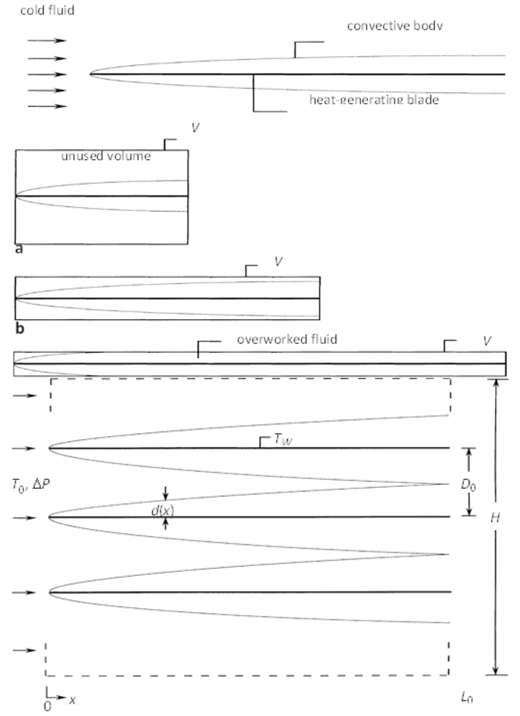
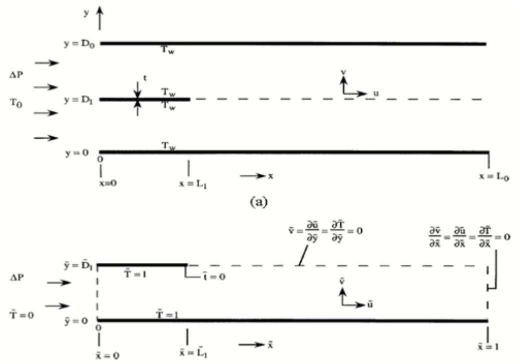
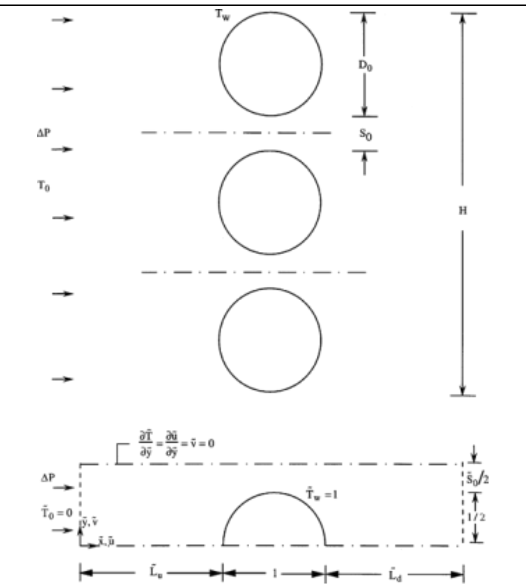
| Bejan and Fautrelle[25] |   | $10^3 \leq Be \leq 100 * 10^9$ | • The constructal multiscale structure offers practical implications for improving heat transfer efficiency in forced convection systems and can be extended to laminar natural convection. |
| Bello-Ochende and Bejan [26] |  | $10^4 < Be < 10^{11}$ | • Multi-scale flow structures can significantly increase heat transfer density in forced convection.<br>• The technique of inserting smaller plates between larger ones optimizes heat transfer efficiency. |
| Cylinders Cooled by Forced Convection Bello-Ochende and Bejan [27] |  | $10^3 \leq Be \leq 10^6$ | • The study's optimal multi-scale flow architectures have characteristics resembling tree-shaped designs, with many length scales arranged in a hierarchical fashion and dispersed unevenly throughout the available area. |

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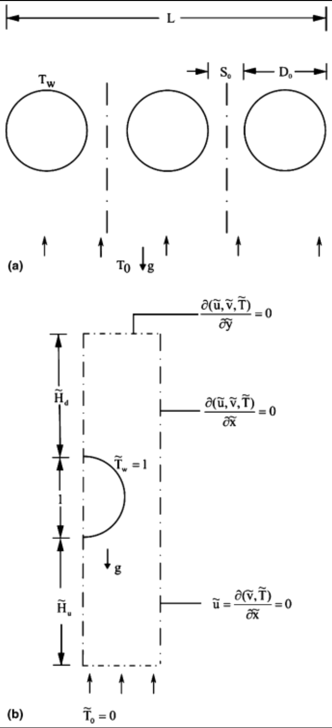
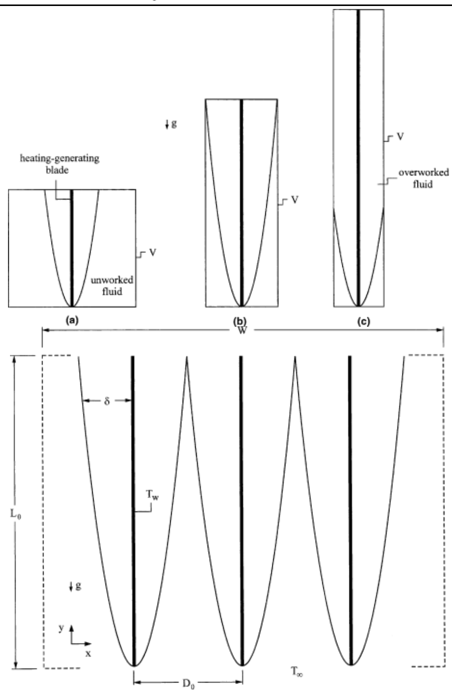
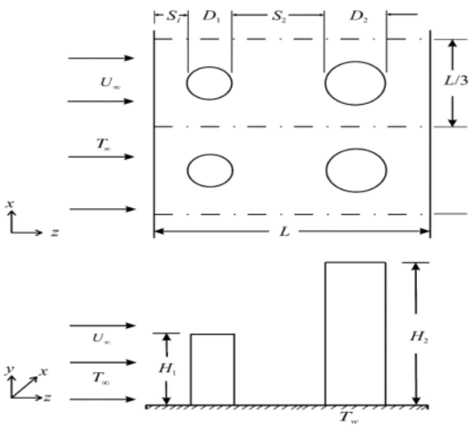
| Cylinders Cooled by Natural Convection Bello-Ochende et. al [28] |   | $10^3 \leq Ra \leq 10^5$ | • Heat transfer density in an area where groups of cylinders that are naturally cooled by convection can be increased by using cylinders of different sizes and optimizing their placement in the assembly. |
| Silva and Bejan [29] |   | $10^7 \leq Ra \leq 10^8$ | • It was found that when the flow structure's complexity was optimized, the rate of transfer of heat density was maximized. |
| Bello-Ochende et. al [30] |   | Re = 50 | • For short fins, the optimum pin fin design was found to be between (0.05) and (0.2), whereas the range of pin fin diameter ratios and the range The ratio of the large fin to tiny fin height ranged from 0.9 to 1.2. |

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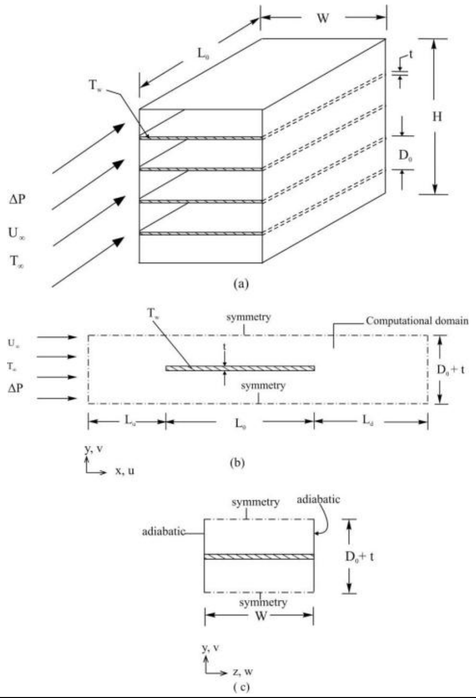
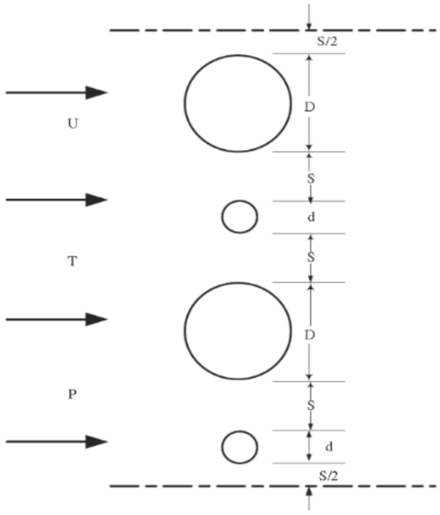
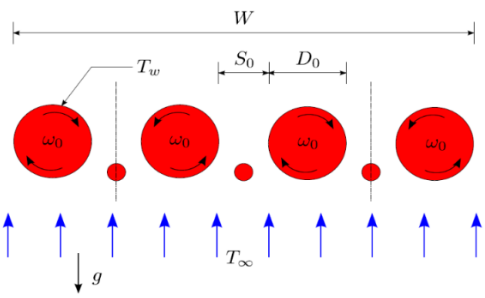
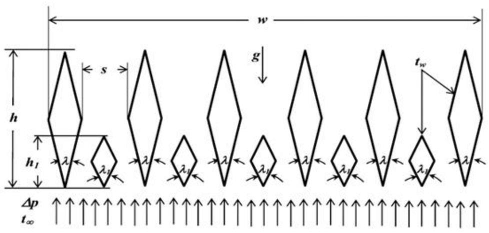
| Bello-Ochende et. al [31] |  | $10^5 \leq Be \leq 10^7$ | • The outcomes were somewhat consistent with the projected analytical results. |
| Bello-Ochende et. al [32] |  | $10 \leq Be \leq 10^4$ | • Improvements appeared to have a greater impact when the cylinders were oriented along the same rotational axis rather than the leading edge plane. |
| Page et. al [33] |  | $10^1 \leq Ra \leq 10^4$ | • At high values of Rayleigh numbers, the rotation of the cylinders showed no influence on the maximum heat transfer density when compared to a stationary for instance. It was found that when the cylinders' rotational speed increased, the optimum spacing decreased. |
| Mustafa A. W. [34] |  | $10^4 \leq Be \leq 10^6$<br>$0.1 \leq Ra/Be \leq 10$ | • The implementation of multi-scale layout caused twice as much heat transfer density as single-scale configuration. |

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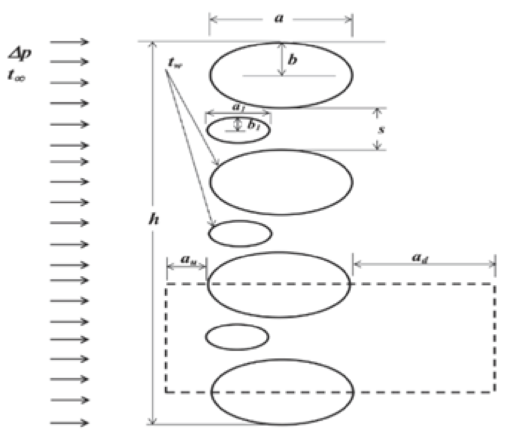
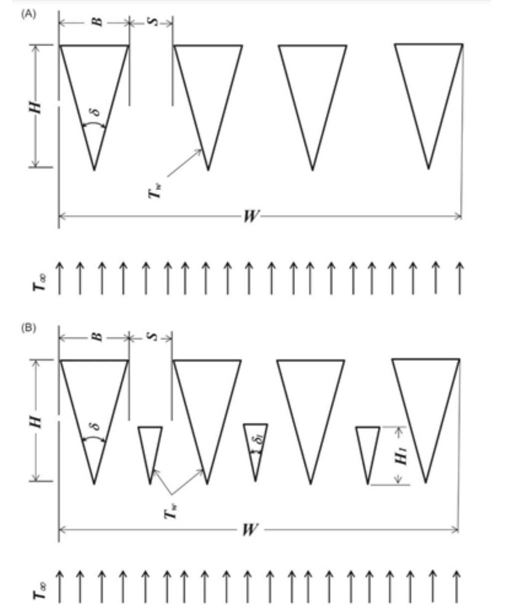
| Elliptic tubes in forced convection Mustafa and Abdul Elqadir [35] |  | $10^3 \leq Be \leq 10^5$ | • The optimum spacing maintains constant with the tube axis ratio (minor axis/major axis) at a certain Bejan number.<br>• The ideal spacing decreases with Bejan number.<br>• The heat transfer density increases with Bejan number. |
| Mustafa et. al [36] |  | *Ra* = $10^3$, $10^4$, and $10^5$ | • Placing small-scale fins between large fins led to a significant percentage increase in heat transfer density at different Rayleigh numbers. |

## 4. Conclusions

- The studies focused on improving the heat transfer intensity in different forms of pipe design, with the aim of enhancing the efficiency of heat exchangers in different applications.
- The most prominent research showed that the maximum heat transfer intensity increased with the rise of Rayleigh numbers for all axle ratios, peaking at zero axis ratio (flat plate) and decreasing at 0.5 axis ratio (circular tube).
- It was found that the optimal spacing remained constant at a given Rayleigh number regardless of the axis ratio, indicating a consistent relationship between these parameters in heat exchangers
- The study emphasized the importance of understanding the interplay between geometric parameters and performance optimization in forced convection scenarios for efficient heat transfer system design.
- Our previous study motivates us to design a row of triangular fins but in forced convection in order to reduce the flow separation behind the fins. The triangular fins have been selected in the current study because the triangular fins which have the same longitudinal length of the circular fins can be slimmer than the circular fins and so that the back separation region can be minimized by using constructal design.

## 5. References

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