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ORIGINAL STUDY

Finding General Mathematical Formulas for Extraction the Minimal Path Sets of Complex Parallel-Series Networks

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ABSTRACT

Most real-world technological systems are highly complex, making it challenging to examine their reliability. Many systems can be represented as Complex Parallel-Series Networks (CPSN). The large number of components and subnetworks, along with their intricate connection, complicates the identification, evaluation, and potential failure of the CPSN. The concept of minimal path sets for a CPSN refers to a specific set of components whose failure leads to the failure of the whole CPSN. The primary research problem is to identify these minimal path sets, both for the overall CPSN and for its complex subnetworks. This paper presents a mathematical technique for analyzing subnetworks instead of the whole CPSN to reduce computational effort and simplifies intricate calculations. Specialized algorithms and techniques are presented and used to identify minimal path sets, with numerical results funded by general formulas. Numerical cases show the effectiveness and applicability of this technique for CPSN.

Keywords: Reliability, Complex system, Parallel-series system, Minimal path

1. Introduction

Reliability has its roots in the early 1930s when probability theories were applied to solve issues related to energy production [1–10]. During World War II, the Germans made a significant revolutionary move by adopting the concept of Reliability. This adoption was a major turning point in the field and demonstrated how historical events substantially impacted the development of reliability engineering. The adoption was meant to strengthen the dependability of their V1 and V2 rockets. The US Department of Defense formed an ad hoc dependability committee in 1950. Then, this committee was established [3, 11, 12].

From the beginning, the central problem of reliability theory has been ascertaining a system's reliability criteria from the reliability criteria of its constituent parts. This is a complex and crucial task, as the most crucial instrument for measuring any system's component efficiency is Reliability. Dependability, or signature, is predicated on the likelihood that one or more system components would malfunction. Despite the advancements in the field, a dynamic solution that can precisely tackle this issue and handle systems with more than 500 components is yet to be developed. The complexity of this task is evident when we consider that nondeterministic polynomial-time (NP)-hard issues rank among the most difficult problem types when viewed through the lens of computers. As a result, the time needed to assess a system's Reliability using the dependability of its constituent parts usually increases exponentially with the system's n-size, making this an ongoing challenge in the field [13–20]. One practical application of reliability theory is translating a system's pass/fail function into a reliability expression. This involves identifying each minimal path sets using probability,

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and graph theories. While complex, this process is crucial in understanding and improving a system's Reliability [4, 21]. The multipath problem in the WAN can be resolved by breaking the system into smaller units.

This reduces the time and effort needed to perform the computations necessary to determine the original system's path sets. Some have used the contact force matrix to calculate the system's MPSN. In this context, this system consists of two partial branches, each consisting of a network system connected in parallel so that the entire system is connected in series. This configuration is often used in reliability engineering to model and analyze complex systems [22–27].

The complexity of CPSN arises from the large number of components and subnet- works, as well as their intricate connections. This complexity makes it difficult to identify, evaluate, and understand potential failures within the CPSN. A minimal path sets in a CPSN is defined as a specific group of components whose failure results in the failure of the entire network. The main research problem is to identify these minimal path sets, both for the overall CPSN and for its various subnetworks. This paper introduces a mathematical technique to find all minimal path sets of CPSN by analyze subnetworks rather than the whole CPSN, which helps reduce computational effort and simplifies intricate calculations. By establishing generalization relations for CPSNs that align with the hierarchy of their subnetworks, this method could greatly enhance our comprehension and management of CPSNs.

The rest of this paper is organized as follows: Section 2 introduces research methodology to find all minimal path sets and provides an algorithm for constructing the minimal path sets of CPSN. Section 3 expanded CPSN system model. Section 4 generalization of complex parallel–series network Finally, Section 5 offers concluding remarks.

2. Basic concepts about minimal path sets system

Connection matrix of graph G with the vertex set $x = \{x_1,\ x_2, \ldots, x_n\}$, to create a connection matrix with k minimal path sets (MPS) $(p_1, p_2, \ldots, p_n)$ arrange the components in rows and the minimal paths in columns. Compute each minimal path using a connection matrix (P) [4, 28].

$$|P| = \begin{array}{c} p_1 \\ p_2 \\ p_k \end{array} \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{k1} & \cdots & x_{kn} \end{bmatrix}, x_{ij} = 1 \text{ iff } x_{ij} \in p_i \text{ otherwise} \qquad (1)$$

$x_{ij} = 0$ and $i = 1, 2, \ldots, m,\ \ j = 1, 2, \ldots, n$

2.1. Propose algorithm to extract minimal path sets

This section will review the phases of the suggested algorithm, which will assist in determining the minimal path sets for the given Complex Parallel-Series Networks (CPSN) system [8, 28, 29].

Step 1: The algorithm's initial step is the connection matrix (P), which serves as the logic graph.

Step 2: The connection matrix [P] is enhanced by the addition of a p × p diagonal unity matrix [U], a crucial step in the algorithm.

Step 3: Remove the column that belongs to a row that corresponds to n and any other column or row devoid of variables.

Step 4: Using the remaining rows and columns, determine the system success determinant S. Each algebraic variable is converted into the matching Boolean variable at this stage.

Step 5: Extend the determinant S using Boolean sum and product operations.

Step 6: Define the system success determinant |P| of size (p^{n-1}).

2.2. Complex parallel-series networks

A system comprising several complex subsystems connected in parallel, with these parallel configurations then linked in series, is known as a Complex Parallel-Series Networks CPSN.

Consider a system comprising four complex subsystems, interconnected in a parallel-series configuration. This means each subsystem is connected in parallel, and these parallel groups are then linked in series with one another, as shown in Fig. 1.

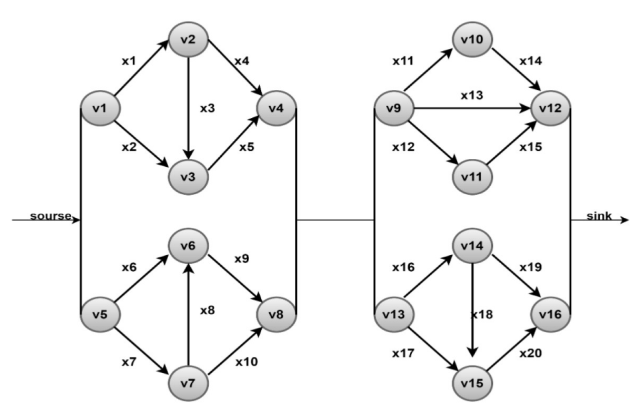

Fig. 1. (CPSN) system.

2.3. Finding the total number of all minimal paths of Fig. 1

According to the proposed algorithm, the following steps Eq. (2) has been obtained from step 2

$$|P| = \begin{bmatrix} 1 & x_1 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & x_5 & x_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_3 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & x_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_8 & 1 & x_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{11} & x_{12} & x_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & x_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{16} & x_{17} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{18} & x_{19} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{20} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Delete the columns and rows that do not have any variables to form the matrix in Eq. (3)

$$|P|^2 = \begin{bmatrix} x_1 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & x_5 & x_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_8 & 1 & x_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_{11} & x_{12} & x_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & x_{14} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{16} & x_{17} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{18} & x_{19} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{20} \end{bmatrix} \quad (3)$$

Where $|p|^2$ represents step (2) after elimination, our results are the determinant of the matrix, as shown. This process is carried out till the success matrix is reached of size (p^{n-1}) [7–13]. Thus, for this system the process finished when the size of matrix reach to (p^{15}). Then the number of minimal paths for the total system is (36) shown in the Table 1.

Table 1. Minimal paths of system (CPSN) of Fig. 1.

No	Order	Count	Example
1	4	1	$x_1,\ x_4, x_{11},\ x_{14}$
2	5	17	$x_1,\ x_3, x_5,\ x_{11}, x_{14}$
3	6	36	$x_7,\ x_8, x_9,\ x_{16}, x_{18},\ x_{20}$

2.4. Partitioning of (CPSN) system

To study the relationship between the total system and the partial systems that make up that system, we resort to the partition method [30–37], which is an effective method for found the relationship between the system and its components (subsystems). The partition method for this type of system is as follows:

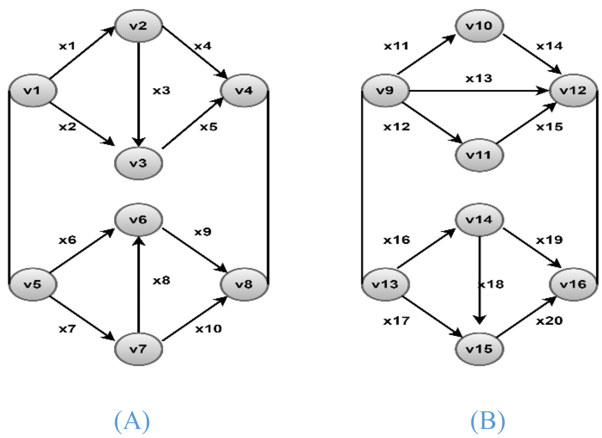

Fig. 2. Partitioning of (CPSN) system into block diagrams A and B.

Step 1: Divide the overall system into parallel Block Diagram (A, B) as shown in Fig. 2.

Step 2: Divide the subsystems in Fig. 2 (A and B) from Step 1 into Subsystem A and subsystem B, as in the following Fig. 3.

Concerning Fig. 3, the first parenthesis represents the MPS of subsystem S_1, and the second parenthesis represents the minimal paths of subsystem S2 of Block Diagram A. Hence, can conclude all MPS of Block Diagram A as follows:

$$P_1^{S_1} = \{x_1x_4\},\ P_2^{S_1} = \{x_2x_5\},\ P_3^{S_1} = \{x_1x_3x_5\}$$

$$P_1^{S_2} = \{x_6x_9\},\ P_2^{S_2} = \{x_7x_8x_9\},\ P_3^{S_2} = \{x_7x_{10}\}$$

Similarly, the sets of all minimal paths of Block Diagram B, which consists of subsystems S_3 and S_4, are

$$P_1^{S_1} = \{x_{13}\},\ P_2^{S_1} = \{x_{11}x_{14}\},\ P_3^{S_1} = \{x_{12}x_{15}\}$$

$$P_1^{S_2} = \{x_{16}x_{19}\},\ P_2^{S_2} = \{x_{17}x_{20}\},\ P_3^{S_2} = \{x_{16}x_{18}x_{20}\}$$

2.5. The total numbers of minimal paths of the Blok Diagrams A and B

The minimal number of paths for block diagrams A and B shown in the Fig. 2 can be calculated from the subsystems that make up both of them connected in parallel. That is, block diagram A consists of subsystems S1 and S2, as shown in the Fig. 3, connected in parallel. Block diagram B consists of subsystems S_3 and S_4, as shown in the figure, connected in parallel. So, from our previous studies [38–42], the total number of MPS for both block diagrams A and B was six MPS. This is in accordance with the mathematical relationship that governs complex

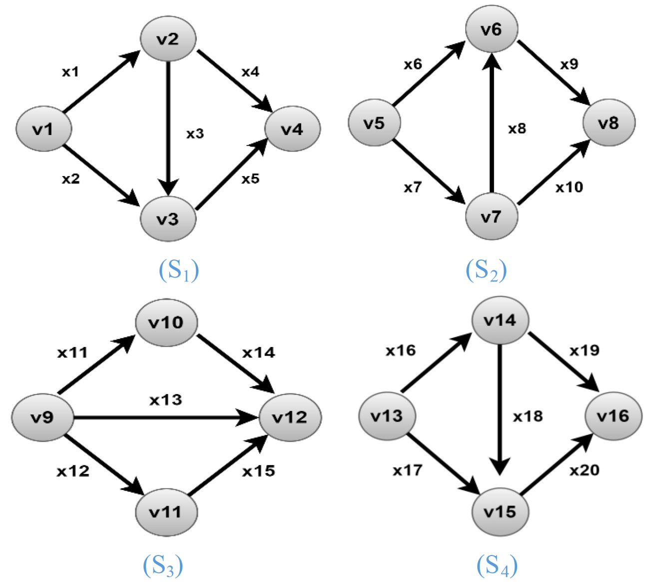

Fig. 3. Partitioning of block diagrams A and B.

parallel systems.

$$P_K^{S_{AB}} = \sum_{i=1}^{6} P_i^{S_A} + \sum_{j=1}^{6} P_j^{S_B}, \tag{4}$$

where $k = i + j$.

Fig. 1 shows that Block Diagrams A and B are connected in series, prompting us to return to our previous studies [12] according to the mathematical relationship that governs complex-series systems.

$$P_K^{S_{AB}} = \left(\sum_{i=1}^{6} P_i^{S_A}\right) \times \left(\sum_{j=1}^{6} P_j^{S_B}\right), \tag{5}$$

where $k = i + j$.

From what has been presented, obtain a mathematical relationship combining Eqs. (4) and (5) through the following mathematical formula.

$$P_K^{S_{AB}} = \left(\sum_{i=1}^{3} P_i^{S_1} + \sum_{j=1}^{3} P_j^{S_2}\right) \times \left(\sum_{r=1}^{3} P_r^{S_3} + \sum_{t=1}^{3} P_t^{S_4}\right), \tag{6}$$

where $k = [i + j] \times [r + t]$.

$P_K^{S_{AB}}$ are minimal paths of Fig. 1, $P_i^{S_1}$ minimal paths of Fig. 3(S_1), $P_j^{S_2}$ minimal paths of Fig. 3(S_2), $P_r^{S_3}$ minimal paths of Fig. 3(S_3), $P_t^{S_4}$ minimal paths of Fig. 3(S_4). From above, we notice that the number of minimal paths for the total system is (36), which is the same number extracted in (2.3).

3. Expanded (CPSN) system model

In this section, we will expand the system into a larger system by adding complex subsystems as follows:

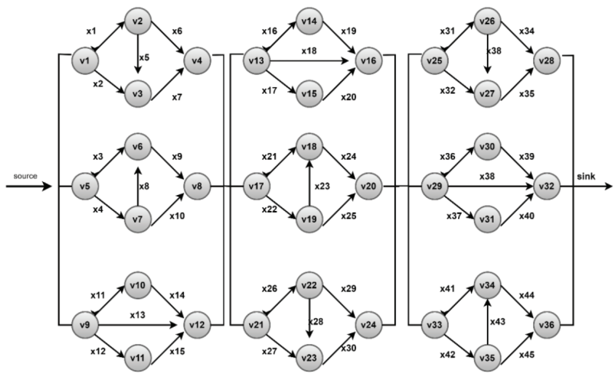

Fig. 4. (CPSN) systems.

As explained in the previous section, first, the total set of minimal paths for the extended system shown in the (CPSN) network shown in Fig. 4 was extracted and calculated with reference to the proposed algorithm. We constructed the connectivity matrix [P] (P^{35}); Thus, from step 4 of the proposed algorithm, we obtained 729 minimal path sets, and we will give an example of a few of them [43–47].

Note: Due to the matrix's large size and the large number of minimal paths for the total system, it has been hidden. The total number of minimal paths is sorted by the order of each path shown in the Table 2.

3.1. Partitioning of expanded CPSN system

To reach the mathematical relationships that govern the complex subsystems and their relationship to the overall expanded CPSN system, which enables us to know the number of minimal paths for each subsystem, as was done in the previous sections, we divide the expanded system into subsystems and calculate the minimal paths for each subsystem. Here, we will have three Block Diagrams, A, B, and C, as follows:

Step 1: Divide the overall system into parallel Block Diagram as shown in Fig. 2.

For Fig. 5, the first arc represents the MPS of subsystem S_1, the second arc represents the minimal paths of subsystem S_2, and the third arc represents the minimal paths of subsystem S_3 for diagram A. Thus, all MPS for diagram A are derived as follows:

$$P_1^{S_1} = \{x_1x_4\},\ P_2^{S_1} = \{x_2x_5\},\ P_3^{S_1} = \{x_1x_3x_5\}$$
$$P_1^{S_2} = \{x_6x_9\},\ P_2^{S_2} = \{x_7x_8x_9\},\ P_3^{S_2} = \{x_7x_{10}\}$$
$$P_1^{S_3} = \{x_{13}\},\ P_2^{S_3} = \{x_{11}x_{14}\},\ P_3^{S_3} = \{x_{12}x_{15}\}$$

Table 2. Minimal paths of system CPSN of Fig. 1.

No	Order	Count	Example
1	3	1	$x_{13}\ x_{18}x_{38}$
2	4	18	$x_{13}\ x_{18}x_{31}\ x_{34}$
3	5	114	$x_1x_6x_{18}\ x_{31}x_{34}$
4	6	288	$x_1x_5x_7\ x_{18}x_{31}x_{34}$
5	7	228	$x_1\ x_5x_7\ x_{16}x_{19}\ x_{31}x_{34}$
6	8	72	$x_1\ x_5x_7\ x_{22}x_{23}\ x_{24}x_{31}x_{34}$
7	9	8	$x_1\ x_5x_7\ x_{22}x_{23}\ x_{24}x_{31}x_{33}x_{34}$

Similarly, the sets of all minimal paths of Block Diagram B, which consists of subsystems S_4, S_5, and S_6, are

$$P_1^{S_4} = \{x_{16}x_{19}\},\ P_2^{S_4} = \{x_{17}x_{20}\},\ P_3^{S_4} = \{x_{18}\}$$
$$P_1^{S_5} = \{x_{21}x_{24}\},\ P_2^{S_2} = \{x_{22}x_{25}\},\ P_3^{S_2} = \{x_{22}x_{23}x_{24}\}$$
$$P_1^{S_6} = \{x_{26}x_{29}\},\ P_2^{S_3} = \{x_{27}x_{30}\},\ P_3^{S_3} = \{x_{26}x_{28}x_{30}\}$$

Similarly, the sets of all minimal paths of Block Diagram C, which consists of subsystems S_7, S_8, and S_9, are

$$P_1^{S_7} = \{x_{31}x_{34}\},\ P_2^{S_7} = \{x_{32}x_{35}\},\ P_3^{S_7} = \{x_{31}x_{33}x_{35}\}$$
$$P_1^{S_8} = \{x_{36}x_{39}\},\ P_2^{S_8} = \{x_{38}\},\ P_3^{S_8} = \{x_{37}x_{40}\}$$
$$P_1^{S_9} = \{x_{42}x_{43}x_{44}\},\ P_2^{S_9} = \{x_{41}x_{44}\},\ P_3^{S_9} = \{x_{42}x_{45}\}$$

3.2. The total numbers of the minimal paths of expanded CPSN system model

The number of minimal paths of the CPSN block diagram system in the Fig. 4, the sum of the minimal paths of the Block Diagrams A, B and C, we notice that the subsystems that make up both are connected in parallel. i.e., Block Diagram A consists of subsystems S_1, S_2 and S_3, which are connected in parallel. Also, Block Diagram B consists of subsystems S_4, S_5 and S6connected in parallel. Also, Block Diagram C consists of subsystems S_7, S_8 and S_9 connected in parallel. So, from our previous studies [43–50], the total number of MPS of Block Diagram A is (9), and the total number of MPS of Block Diagram B is (9). and the total number of MPS of Block Diagram C is (9). This is according to the mathematical relationship that governs complex-parallel systems

$$P_K^{S_{AB}} = \sum_{i=1}^{9} P_i^{S_A} + \sum_{j=1}^{9} P_j^{S_B} + \sum_{l=1}^{9} P_l^{S_C}, \tag{7}$$

where $k = i + j + l$.

Fig. 4 clearly shows that block diagrams A, B, and C are connected in series, prompting us to return to

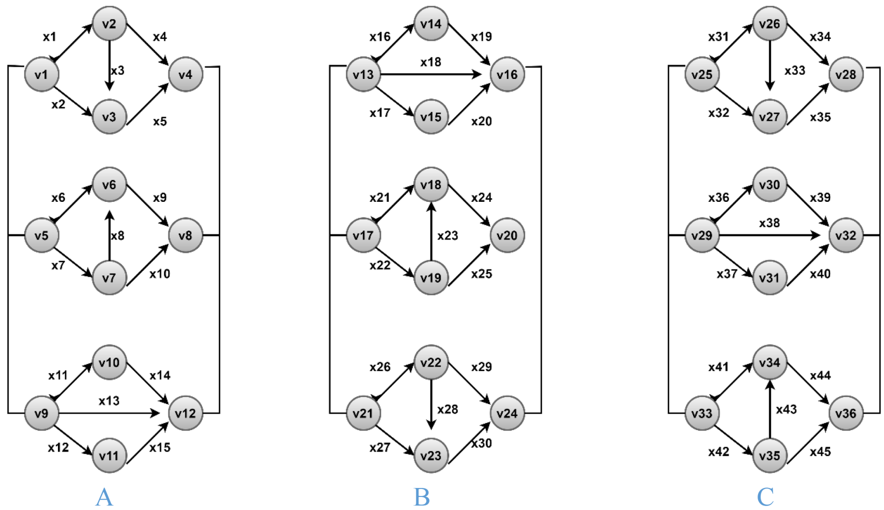

Fig. 5. Partitioning (CPSN) system of Fig. 4.

our previous studies [12] based on the mathematical relationship that governs complex series systems.

$$P_K^{S_{AB}} = \left(\sum_{i=1}^{9} P_i^{S_A}\right) + \left(\sum_{j=1}^{9} P_j^{S_B}\right) + \left(\sum_{l=1}^{9} P_l^{S_C}\right), \tag{8}$$

where $k = i + j + l$.

Through what was presented, a mathematical relationship was obtained that combines equations Eqs. (8) and (9) through the following mathematical formula.

From what has been presented, obtain a mathematical relationship combining Eqs. (4) and (5) through the following mathematical formula.

$$\begin{aligned} P_K^{S_{AB}} = & \left(\sum_{i=1}^{3} P_i^{S_1} + \sum_{j=1}^{3} P_j^{S_2} + \sum_{l=1}^{3} P_l^{S_3}\right) \\ & \times \left(\sum_{r=1}^{3} P_r^{S_4} + \sum_{t=1}^{3} P_t^{S_5} + \sum_{z=1}^{3} P_z^{S_6}\right) \\ & \times \left(\sum_{w=1}^{3} P_w^{S_7} + \sum_{y=1}^{3} P_y^{S_8} + \sum_{e=1}^{3} P_e^{S_9}\right), \end{aligned} \tag{9}$$

where $k = [i + j + l] \times [r + t + z] \times [w + y + e]$.

$P_K^{S_{AB}}$ are minimal paths of Fig. 4, $P_i^{S_1}$ minimal paths of Fig. 5 (S_1), $P_j^{S_2}$ minimal paths of Fig. 5 (S_2), $P_1^{S_3}$ minimal paths of Fig. 5 (S_3), $P_r^{S_4}$ minimal paths of Fig. 5 (S_4). $P_t^{S_5}$ minimal paths of Fig. 5 (S_5), $P_z^{S_6}$ minimal paths of Fig. 5 (S_6), $P_w^{S_7}$ minimal paths of Fig. 5 (S_7), $P_y^{S_8}$ minimal paths of Fig. 5 (S_8), $P_e^{S_9}$ minimal paths of Fig. 5 (S_9). From above, we notice that the number of minimal paths for the total system is (729), which is the same number extracted in (3).

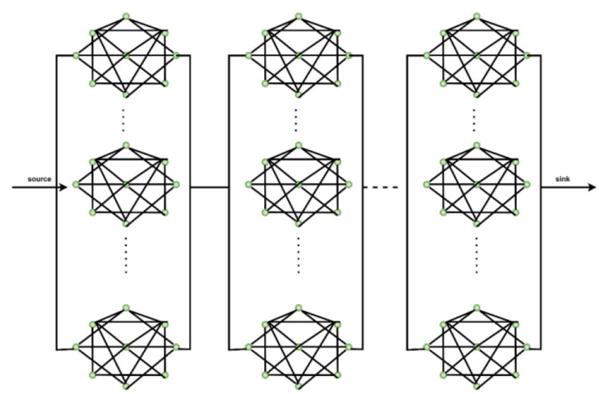

Fig. 6. Generalization (CPSN) systems.

4. Generalization of complex parallel-series

Suppose we have a system consisting of N complex subsystems linked together in parallel -Series configuration; i.e., CPSN configuration, as shown in Fig. 6.

Each subsystem has a certain number of minimal paths that were:

$$P_K^{S} = \left(\sum_{i=1}^{n} P_i^{S_A}\right) \times \left(\sum_{j=1}^{n} P_j^{S_B}\right) \times \cdots \times \left(\sum_{z=1}^{n} P_z^{S_x}\right), \tag{10}$$

where $k = i \times j \times \cdots \times z$.

Then the number of minimal paths for the total system is

$$N_{path} = (NP_{11} + NP_{21} + \cdots NP_{m1}) \times (NP_{12} + NP_{22} + \cdots NP_{m2}) \times \cdots \times (NP_{1n} + NP_{2n} + \cdots NP_{mn}) = \prod_{i=1}^{n} \left(\sum_{j=1}^{m} NP_j \right) \quad (11)$$

N_{path} represents the total number of minimal paths in the system, NP_j represents the number of minimal paths in the subsystem of (CPSN) system, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

If all units in all subsystems of (CPSN) are independent identical, then

$$N_{path} = NP_{ij}^{k} \quad (12)$$

where i $= 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

N it is the number of minimal paths in the subsystem, and m is the number of subsystems.

5. Conclusion

This study presents a new technique for identifying all minimal path sets in CPSNs by examining their respective subnetworks. This approach assists in evaluating the reliability of these networks. The findings indicate that concentrating on complex subnetworks, rather than analyzing the entire original CPSN, significantly reduces computational effort and simplifies intricate calculations. The proposed technique underscores the potential failure of the CPSN by construct all minimal path sets of CPSN and the potential for conducting reliability analyses of large CPSNs and their subnetworks. The proposed algorithms utilized to establish generalization relationships for CPSNs are essential in advancing our understanding of these complex systems. The relation- ships between subnetworks of CPSN can dramatically improve management capabilities. The potential for greater CPSN efficiency in vital sectors like telecommunications and logistics.

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Ethical approval

Not applicable.

Conflict of interest

The authors declare no conflict of interest.

Data availability

No datasets were generated or analyzed during the current study.

Author contribution

Mariem Hassan Lafta and Zahir Abdul Haddi Hassan contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.

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