The Approximate Solution To The Dynamics of The Observed-State Feed Back Control System by Walsh Functions

الحل التقريبي لديناميكية مراقبة نظام السيطرة لاستجابة الحالة بواسطة دوال والش

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المستخلص

يتعامل البحث الحالي بتطبيق دوال والش بالأعتماد على تكامل مصفوفة العمليات لديناميكية مراقبة نظام السيطرة لأستجابة الحالة. تستخدم طريقة والش نظرا لبساطة تحليلها ودراستها وتنفيذها بسهولة استنادا الى بعض الخطوات للوصول الى حل ديناميكية مراقبة نظام السيطرة لأستجابة الحالة . تم اخذ مثال لشرح الطريقةالتي تعتمد على برنامج الماتلاب. أظهرت النتائج أن الطريقة المقترحة عالية الدقة والكفاءة عند مقارنتها بالحل الدقيق الموجود.

الكلمات المفتاحية:دوال والش ,متسلسلة والش, ديناميكية مراقبة نظام السيطرة لأستجابة الحالة,دوال راديميجر , ضرب كرونكر

Abstract:

The present paper deals with the application of Walsh functions based on integration operational matrix to the dynamics of the observed –state feedback control system. Walsh method is used because its simple analysis, studying and easier implementation based on some steps to reach the solution of the observed-state feedback control system . An example is taken to explain the method depending on Matlab programming. The results show that the proposed method is very accurate and efficient when compared to existing exact solution.

Keywords: Walsh functions, Walsh series, The observed-state feedback control system, Rademacher functions. Kroncker product

1. Introduction

Walsh functions have been widely used in the analysis of communication theory (Multiplexing system, coding system and non-sinusoidal electromagnetic radiation), signals processing (spectroscopy, speech processing, Medical applications, and seismology) and in transform spectroscopy rather than sinusoidal functions,[2-6,8-11].

In control system, Walsh functions been used since 1975 where many authors applied them in many different problems [3-5], such as the time-domain-synthesis problem, the time-varying feed gains of linear systems problem, and optimal problem. Also in recent years Walsh functions were defined in hybrid orthogonal functions which called Hybrid Walsh functions and been applied in many real life problems, [1,14-15,18].

The Laplace transform method is an approach used to solve homogeneous and non homogeneous state equations. In [13] this method is described to solve the control system problems associated with the repeated integration and operational matrix which relates : (a)piecewise constant orthogonal functions(block-pulse functions, Haar functions , Walsh functions), (b) orthogonal polynomials(Legendre polynomials, Laguerre polynomials , Hermite polynomials, Tchebycheff polynomials of the first kind , Tchebycheff polynomials of the second kind, Jacobi polynomials, Gegenbauer polynomials),and (c) sine-cosine(Fourier) functions.

In this paper, Walsh series is used for the determination of suboptimal feedback laws for the linear systems with quadratic performance criteria, then applied them for solving linear dynamic systems.

2- Study problem:

The study of the problem can be formulated by the following questions:

a) Can the concept of Walsh functions be used , studied and analysed to obtain the numerical to general approach solution solve the observed-state feedback control system?
b) Is it possible to find the integral operational matrix of Walsh functions in terms of integral operational matrix of Block pulse functions?
c) How to characterize the approximate solution by Walsh functions?

3- Study supposal:

In this paper will relate and define the Walsh functions based on the product of Rademacher functions and their integral operational matrix and represent any square function f(t) in terms of Walsh series, as well as apply Walsh series compute to the approximate numerical solution of the observed-state feedback control system. Finally an example is considered to explain the method.

4- Importance of the study:

In the present work a new approach for estimating the error vector in the dynamics of the observed-state feedback control system applying Walsh functions. Also can be partly classified into three importance, first it can use another representation method for computing the integrational operational Walsh matrix based on Block pulses functions. Second it present the numerical solution of the observed-state feedback control system. Finally, The results show that the proposed method is very accurate and efficient when compared to with existing exact solution.

5- Study methodology:

Our work composed of many information such as, results relate the goal, for instance discussion, study and analysis of Walsh functions. The Matlab programming will be used to compute the approximate solution and compared it with the exact solution, as will be shown in example employed.

6- Theoretical consideration:

6-1 Walsh functions with their integration operational matrix

The American mathematician Walsh J.L. studied and analysed a set of orthogonal functions in 1923, which called Walsh functions, [16]. Each function takes only the values +1 or -1, except at jumps, where they takes the value 0.These functions can be defined by the following:-

a- A set of $m = 2^k$, for some $k \in N = \{1,2,3,\dots\}$ of Walsh functions can be defined in terms of finite products of Rademacher functions,[16] which defined as a set of orthogonal functions and are belongs to square waves of unit height with periods equal to 1, 1/2, 1/4 ,1/8, 1/16, 1/32,. . . , $2^{(1-i)}$:

$$W_0(t) \equiv R_0(t) \equiv 1, \ \forall t \in [0,1] \qquad . \quad . \quad . (1)$$

For $i > 0$, writing the binary expansion of $i : i = \sum_{r=0}^{[log_2 i]} i_r 2^r$, where, $i_r \in \{0,1\}$, so now

$$W_i(t) = R_{[log_2 i]+1}(t) \prod_{r=0}^{[log_2 i]-1} (R_{r+1}(t))^{i_r} \quad . \quad . \quad . \ (2)$$

Where, $R_i(t)$ is called the i^{th} Rademacher functions and written as:

$$R_i(t) = sgn\left(\sin\left(2^i \pi t\right)\right) \quad \forall t \in [0,1) \quad . \quad . \quad . \ (3)$$

$$sgn(t) = \begin{cases} +1 \ if \ t > 0 \\ -1 \ \ if \ \ t < 0 \\ \ 0 \ \ if \ t = 0 \end{cases} \quad . \quad . \quad . \ (4)$$

So the first eight of Walsh functions can be represented in terms of Rademacher functions as:

$$W_0(t) \equiv R_0(t)\ , W_1(t) \equiv R_1(t), W_2(t) \equiv R_2(t), W_3(t) \equiv R_2(t)R_1(t), W_4(t) \equiv R_3(t), W_5(t) \equiv R_3(t)R_1(t), W_6(t) \equiv R_3(t)R_2(t), W_7(t) \equiv R_3(t)R_2(t)R_1(t) \quad . \quad . \quad .(5)$$

Since the Walsh functions are complete orthonormal functions in Hilbert space $L^2[0,1)$,[2,7,16].

So it can be assembled as a square matrix of order m by dividing the closed interval [0,1] into m subintervals with length $1/m$, and denoting the collection points by:$t_s = \frac{2s-1}{2m}$, where $s = 1,2,3, \dots, m$ and

$$\boldsymbol{W}_c(t) = [\boldsymbol{W}(c, \tfrac{1}{2m}) \quad \boldsymbol{W}\left(c, \tfrac{3}{2m}\right) \quad \boldsymbol{W}\left(c, \tfrac{2s-1}{2m}\right) \ \ . \ . \ . \boldsymbol{W}(c, \tfrac{2m-1}{2m})\ ] \quad . \ . \ . \ (6)$$

Where, $c = 0,1,2, \dots, m-1$:

$$\boldsymbol{W}_m(t) = \begin{bmatrix} \boldsymbol{W}_0(t) \\ \boldsymbol{W}_1(t) \\ \boldsymbol{W}_2(t) \\ \vdots \\ W_{m-1}(t) \end{bmatrix} =$$

$$\begin{bmatrix} \boldsymbol{W}\left(0, \frac{1}{2m}\right) & \boldsymbol{W}\left(0, \frac{3}{2m}\right) & \boldsymbol{W}\left(0, \frac{2s-1}{2m}\right) & . \ . \ . & \boldsymbol{W}(0, \frac{2m-1}{2m}) \\ \boldsymbol{W}\left(1, \frac{1}{2m}\right) & \boldsymbol{W}\left(1, \frac{3}{2m}\right) & \boldsymbol{W}\left(1, \frac{2s-1}{2m}\right) & . \ . \ . & \boldsymbol{W}(1, \frac{2m-1}{2m}) \\ \boldsymbol{W}\left(2, \frac{1}{2m}\right) & \boldsymbol{W}\left(2, \frac{3}{2m}\right) & \boldsymbol{W}\left(2, \frac{2s-1}{2m}\right) & . \ . \ . & \boldsymbol{W}(2, \frac{2m-1}{2m}) \\ \boldsymbol{W}\left(3, \frac{1}{2m}\right) & \boldsymbol{W}\left(3, \frac{3}{2m}\right) & \boldsymbol{W}\left(3, \frac{2s-1}{2m}\right) & . \ . \ . & \boldsymbol{W}(3, \frac{2m-1}{2m}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{W}\left(m-1, \frac{1}{2m}\right) & \boldsymbol{W}\left(m-1, \frac{3}{2m}\right) & \boldsymbol{W}\left(m-1, \frac{2s-1}{2m}\right) & . \ . \ . & \boldsymbol{W}(m-1, \frac{2m-1}{2m}) \end{bmatrix} \ . \ . \ .(7)$$

Where, $\boldsymbol{W}_m(t)$ is called the Walsh matrix of order $m = 2^k$, for some $k \in N$ and $\boldsymbol{W}_0(t)\ , \boldsymbol{W}_1(t)$, . . . , $\boldsymbol{W}_{m-1}(t)$ are the Walsh vectors. For example, consider the Walsh matrices of order 2,4, and 8 respectively:

$$\boldsymbol{W}_2(t) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad . \ . \ . \ (8)$$

$$\boldsymbol{W}_4(t) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & -1 \end{bmatrix} \quad . \ . \ . \ (9)$$

$$\boldsymbol{W}_8(t)=\begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix} \quad . \; . \; . \; (10)$$

b- Any square integrable function $f(t)$ can be represented as an infinite series in terms of Walsh functions [18]:

$$f(t)=\sum_{i=0}^{\infty}\alpha_i \boldsymbol{W}(i,t) \qquad . . . (11)$$

Where, $\boldsymbol{W}(i,t)$ are the Walsh functions and the Walsh coefficients $\alpha_i$can be written as:

$$\alpha_i=\frac{1}{\gamma_i}\int_0^1 f(t)\boldsymbol{W}(i,t)dt \qquad . \; . \; . (12)$$

And , $\gamma_i=\int_0^1 \boldsymbol{W}(i,t)\boldsymbol{W}(i,t)dt=m$ which called the normalized factors for Walsh functions .

Eq.(11) was also called the spectrum of $f(t)$ with respect to the system of orthogonal Walsh functions $\{\boldsymbol{W}(i,t)\}_{i=0}^{\infty}$. This compact (discrete) form of eq.(11) can be defined by:

$$f(t)=\sum_{i=0}^{m-1}\alpha_i \boldsymbol{W}(i,t)=\boldsymbol{\alpha^T}\,\boldsymbol{W}_m(t) \quad . \; . \; .(13)$$

Where, $\boldsymbol{\alpha^T}=[\,\alpha_0 \;\; \alpha_1 \;\; \alpha_2 \;\; . \; . \; . \alpha_{m-1}]$ is called the coefficients vector and $\boldsymbol{W}_m(t)=[\boldsymbol{W}_0(t) \;\; \boldsymbol{W}_1(t) \;\; \boldsymbol{W}_2(t) \; . \; . \; . \; \boldsymbol{W}_{m-1}(t)]$ is the Walsh vector.

Now the approximation of the integral of a Walsh vector $\boldsymbol{W}_m(t)$ can be represented mathematically [18] as:

$$\int_0^t \boldsymbol{W}_m(x)dx \approx \boldsymbol{E}_m\boldsymbol{W}_m(t) \qquad . \; . \; . \; (14)$$

Where, $\boldsymbol{E}_m$ is a $m\times m$ square operational matrix of integration uniquely determined by $\boldsymbol{W}_c(t)$, Where, $c=0,1,2,\ldots,m-1$, which is given by:

$$\boldsymbol{E}_m=\boldsymbol{W}_m(t)\boldsymbol{E}_m^{*}(\boldsymbol{W}_m(t))^{-1} \quad . \; . \; . \; (15)$$

Where,

$$\boldsymbol{E}_m^{*}=\frac{1}{m}\begin{bmatrix} \frac{1}{2} & 1 & 1 & . \; . \; . & 1 \\ 0 & \frac{1}{2} & 1 & . \; . \; . & 1 \\ 0 & 0 & \frac{1}{2} & . \; . \; . & 1 \\ \vdots & \vdots & \vdots & . \; . \; . & \vdots \\ 0 & 0 & 0 & . \; . \; . & 1/2 \end{bmatrix} \quad . \; . \; . \; (16)$$

$\boldsymbol{E}_m^{*}$ is called the operational square matrix for Block pulse functions,[17].

It can be noted that eq.(14) was computed as Wu, [17] in integration operational matrix for orthogonal bases vector based on $\boldsymbol{E}_m^{*}$.

Therefore the operational matrices for Walsh functions of order 4 and 8 can be represented respectively as:

$$\boldsymbol{E}_4 = \begin{bmatrix} 1/8 & -1/4 & -1/8 & 0.00 \\ 1/4 & 0.00 & 0.00 & -1/8 \\ 1/8 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1/8 & 0.00 & 0.00 \end{bmatrix} \quad ...(17)$$

$$\boldsymbol{E}_8 = \begin{bmatrix} +1/2 & -1/4 & -1/8 & 0.00 & -1/16 & 0.00 & 0.00 & 0.00 \\ +1/4 & 0.00 & 0.00 & -1/8 & 0.00 & -1/16 & 0.00 & 0.00 \\ +1/8 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -1/16 & 0.00 \\ 0.00 & +1/8 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -1/16 \\ +1/16 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & +1/16 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & +1/16 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & +1/16 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \quad ...(18)$$

7- **Application and Results:**

Consider the completely state controllable and completely observable system which defined by the following equations,[12]:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + Bu(t)\,,\ y = C\boldsymbol{x}(\boldsymbol{t}) \quad ...(19)$$

Where, $\dot{\boldsymbol{x}}(t)$: is the vector function of the state vector for the plant with n components $\boldsymbol{x_i}(t)$; $\boldsymbol{x_1}(t), \boldsymbol{x_2}(t), \boldsymbol{x_3}(t), \dots, \boldsymbol{x_n}(t)$:; $u(t)$: control signal (scalar); $\boldsymbol{A}$ is an $n \times n$ constant square matrix ; $\boldsymbol{B}$ is an $n \times 1$ constant matrix ; $y(t)$: output signal (scalar) and C is an $1 \times n$ constant matrix. For the state – feed back control based on the observed state $\widehat{\boldsymbol{x}}(t)$,

$$u(t) = -K\widehat{\boldsymbol{x}}(t) \quad ...(20)$$

Where, K is $(1 \times n)$ the state feedback gain matrix which can be determined. With this control, the state equation becomes:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) - BK\widehat{\boldsymbol{x}}(t) \quad ...(21)$$

Arranging eq.(21) by adding subtracting $-BK\boldsymbol{x}(t)$, will obtain

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) - BK\boldsymbol{x}(t) + BK\boldsymbol{x}(t) - BK\widehat{\boldsymbol{x}}(t)$$

$$= (A - BK)\boldsymbol{x}(t) + BK\left(\boldsymbol{x}(t) - \widehat{\boldsymbol{x}}(t)\right) \quad ...(22)$$

where the difference between $\boldsymbol{x}(t)$ and $\widehat{\boldsymbol{x}}(t)$ is the error vector $\boldsymbol{e}(t)$:

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \widehat{\boldsymbol{x}}(t) \quad ...(23)$$

Substituting the error vector $\boldsymbol{e}(t)$ and $\boldsymbol{A}^* = (A - BK), \boldsymbol{B}^* = BK$ into eq.(21) gives:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}^*\boldsymbol{x}(t) + \boldsymbol{B}^*\boldsymbol{e}(t) \quad ...(24)$$

The observer error equation is given by, [12]:

$$\dot{\boldsymbol{e}}(t) = (A - K_e C)\boldsymbol{e}(t) = M\boldsymbol{e}(t) \quad ...(25)$$

Where, K_e is called the observer gain matrix, can be determined and $M = (A - K_e C)$.

Combining equations (21) and (25), and arrange them in matrix form:

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - K_e C \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{e}(t) \end{bmatrix} \quad ...(26)$$

In order to describes the dynamics of the observed-state feedback control system matrix.

To estimate the state feedback control based on the observed state $\widehat{\boldsymbol{x}}(t)$, first approximate the error vector $e(t)$ by a set of m orthogonal Walsh functions $\{W_i(t)\}_{i=0}^{m-1}$:

$$\boldsymbol{e}(t)=\begin{bmatrix}\boldsymbol{e}_1(t)\\ \boldsymbol{e}_2(t)\end{bmatrix}\approx \boldsymbol{e}^*(t)=\begin{bmatrix}\boldsymbol{e}^*{}_1(t)\\ \boldsymbol{e}^*{}_2(t)\end{bmatrix}=\sum_{i=0}^{m-1}\sigma_i\,W_i(t)=\sigma\boldsymbol{W}(t)\quad ...(27)$$

Integrating both sides of eq.(25) over the interval [0,t] and using eq.(27), will obtain

$$e(t)-e_0=M\sigma\boldsymbol{E}_m\boldsymbol{W}(t)\quad ...(28)$$

$$\sigma\boldsymbol{W}(t)-e_0\boldsymbol{W}(t)=M\sigma\boldsymbol{E}_m\boldsymbol{W}(t)\quad ...(29)$$

Where, $e_0=[e(0),0,0,\dots,0]$, $e(0)$ is the initial value of eq.(13), gives

$$\sigma-M\sigma\boldsymbol{E}_m=e_0\quad ...(30)$$

By solving σ based on Kroncker product, will have

$$\begin{bmatrix}\sigma_0\\ \sigma_1\\ \sigma_2\\ \vdots\\ \sigma_{m-1}\end{bmatrix}=[I-\boldsymbol{E}_m^T\otimes M]^{-1}\begin{bmatrix}e_0\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}\quad ...(31)$$

Where, I is the identity matrix, and $\boldsymbol{E}_m^T\otimes M$ is the Kroncker product defined by:

$$\boldsymbol{E}_m^T\otimes M=\begin{bmatrix}M_{11}\boldsymbol{E}_m^T & M_{21}\boldsymbol{E}_m^T & ... & M_{m1}\boldsymbol{E}_m^T\\ M_{12}\boldsymbol{E}_m^T & M_{22}\boldsymbol{E}_m^T & ... & M_{m2}\boldsymbol{E}_m^T\\ \vdots & \vdots & ... & \vdots\\ M_{1m}\boldsymbol{E}_m^T & M_{2m}\boldsymbol{E}_m^T & ... & M_{mm}\boldsymbol{E}_m^T\end{bmatrix}\quad ...(32)$$

After σ is determined, the solution $\dot{\boldsymbol{e}}(t)$ is obtained. The solution $\boldsymbol{e}$ is easily found by substituting σ into eq.(29).

Secondly, the state vector $\boldsymbol{x}(t)$ can be approximated by a set of m number of orthogonal Walsh functions $\{W_i(t)\}_{i=0}^{m-1}$ as follows:

$$\boldsymbol{x}(t)=\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\end{bmatrix}\approx \boldsymbol{x}^*(t)=\begin{bmatrix}\boldsymbol{x}^*{}_1(t)\\ \boldsymbol{x}^*{}_2(t)\end{bmatrix}=\sum_{i=0}^{m-1}\theta_i\,W_i(t)=\boldsymbol{\theta}\boldsymbol{W}(t)\quad ...(33)$$

Integrate both sides of eq.(24) over the interval [0,t] and using eq.(33), will have

$$\boldsymbol{x}(t)-\boldsymbol{x}_0=\boldsymbol{A}^*\boldsymbol{\theta}\int_0^t\boldsymbol{W}(t)dt+\boldsymbol{B}^*\boldsymbol{\sigma}\int_0^t\boldsymbol{W}(t)dt$$

$$\boldsymbol{x}(t)-\boldsymbol{x}_0=\boldsymbol{A}^*\boldsymbol{\theta}\boldsymbol{E}_m\boldsymbol{W}_m(t)+\boldsymbol{B}^*\boldsymbol{\sigma}\boldsymbol{E}_m\boldsymbol{W}_m(t)\quad ...(34)$$

$$\boldsymbol{\theta}\boldsymbol{W}_m(t)-\boldsymbol{x}_0\boldsymbol{W}_m(t)=\boldsymbol{A}^*\boldsymbol{\theta}\boldsymbol{E}_m\boldsymbol{W}_m(t)+\boldsymbol{B}^*\boldsymbol{\sigma}\boldsymbol{E}_m\boldsymbol{W}_m(t)\quad ...(35)$$

Where, $\boldsymbol{x}_0=[\boldsymbol{x}_{00},\boldsymbol{x}_{01},\dots,\boldsymbol{x}_{0m-1}]$, therefore,

$$\boldsymbol{\theta}-\boldsymbol{A}^*\boldsymbol{\theta}\boldsymbol{E}_m=\boldsymbol{x}_0+\boldsymbol{B}^*\boldsymbol{\sigma}\boldsymbol{E}_m=\boldsymbol{Q}\quad ...(36)$$

Finally, will obtain:

$$\begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \vdots\\ \theta_{m-1}\end{bmatrix}=[I-\boldsymbol{E}_m^T\otimes M]^{-1}\begin{bmatrix}q_0\\ q_1\\ q_2\\ \vdots\\ q_{m-1}\end{bmatrix}\quad ...(37)$$

Where, q_1 is the first column of ; q_2 is the second column of $\boldsymbol{Q}$, . . . ,etc. After $\boldsymbol{\theta}$ is determined from eq.(37) the solution of $\dot{\boldsymbol{x}}(t)$ in eq.(24) is obtained. The solution $\boldsymbol{x}(t)$ is easily found by substituting $\boldsymbol{\theta}$ into eq.(34).

7-1 Study Problem: Consider the completely state controllable and completely observable system defined by the equations[12]:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + Bu(t)\,,\ y = C\boldsymbol{x}(\boldsymbol{t}) \quad . . .(38)$$

Where, $A = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0], K_e = \begin{bmatrix} 16 \\ 84.6 \end{bmatrix}$.

The state-feedback gain matrix K for this case can be obtained as follows:

$$K = [29.6 \quad 3.6] \quad . . .(39)$$

Using this state-feedback gain matrix K, the control signal u is given by:

$$u = -K\boldsymbol{x}(t) = -[29.6 \quad 3.6]\begin{bmatrix} \boldsymbol{x}_1(\boldsymbol{t}) \\ \boldsymbol{x}_2(t) \end{bmatrix} \quad . . .(40)$$

Suppose that we use the observed-state feedback control instead of the actual-state feedback control:

$$u = -K\widehat{\boldsymbol{x}}(t) = -[29.6 \quad 3.6]\begin{bmatrix} \widehat{\boldsymbol{x}}_1(\boldsymbol{t}) \\ \widehat{\boldsymbol{x}}_2(t) \end{bmatrix} \quad . . .(41)$$

Table(1) : The general approximation solution of the completely state controllable and completely observable system.

Time →	0.0625	0.1875	0.3125	0.4375	0.5625	0.6875	0.8125	0.9375
$\boldsymbol{x}_1(t)$	0.9701	0.7914	0.5187	0.2420	0.0151	0.1425	-0.2321	-0.2654
$\widehat{\boldsymbol{x}}_1(t)$	0.9701	0.8591	0.5685	0.2695	0.0286	0.1487	-0.2294	-0.2642
$\boldsymbol{e}_1(t)$	0.0000	-0.0677	-0.0498	-0.0275	-0.0135	-0.0062	-0.0027	-0.0012
$\boldsymbol{x}_2(t)$	-0.8061	-1.9406	-2.2967	-2.0600	-1.5469	-0.9776	-0.4723	-0.0823
$\widehat{\boldsymbol{x}}_2(t)$	0.6654	-0.8579	-1.6993	2.3531	1.6817	-0.9181	-0.4468	-0.0716
$\boldsymbol{e}_2(t)$	-1.4715	-1.0827	-0.5974	-0.2931	-0.1348	-0.0595	-0.0255	-0.0107
$-K\boldsymbol{x}(t)$	-25.8130	-16.4393	-7.0854	0.2528	5.1219	7.7374	8.5704	8.1521
$-K\widehat{\boldsymbol{x}}(t)$	-31.1104	-22.3409	-10.7101	-16.4484	-6.9007	-1.0964	8.3987	8.0781
$\boldsymbol{x}^*{}_1(t)$	0.9687	0.8493	0.6433	0.4107	0.2001	0.0315	-0.0873	-0.1571
$\widehat{\boldsymbol{x}}^*{}_1(t)$	0.8367	0.8831	0.6611	0.4223	0.2093	0.0373	-0.0843	-0.1559
$\boldsymbol{e}^*{}_1(t)$	0.1320	-0.0338	-0.0178	-0.0116	-0.0092	-0.0058	-0.0030	-0.0012
$\boldsymbol{x}^*{}_2(t)$	-0.5000	-1.4106	-1.8878	-1.8308	-1.5394	-1.1576	-0.7424	-0.3738
$\widehat{\boldsymbol{x}}^*{}_2(t)$	0.8858	-0.5766	-1.4578	-1.6710	-1.3228	-1.0772	-0.7012	-0.3608
$\boldsymbol{e}^*{}_2(t)$	-1.3858	-0.8340	-0.4300	-0.1598	-0.2166	-0.0804	-0.0412	-0.0130
$-K\boldsymbol{x}^*$(t)	-26.8735	-20.0611	-12.2456	-5.5658	-0.3811	3.2350	5.2567	5.9958
$-K\widehat{\boldsymbol{x}}^*$(t)	-27.9552	-24.0640	-14.3205	-6.4845	-1.4332	2.7738	5.0196	5.9135

The results for numerical solution of present method is shown in table (1) for m = 8 , which confirms that with respect to Walsh functions operational matrix method approach produces the numerical solutions of errors $\boldsymbol{e}_1^*(t)$ and $\boldsymbol{e}_2^*(t)$ which are closer to the exact solutions $\boldsymbol{e}_1(t)$ and $\boldsymbol{e}_2(t)$. Better approximation is expected by choosing a larger value of m.

8- Conclusion

The Walsh functions method is found to be an effective tool for the approximate solution to the dynamics of the observed-state feedback control system. As shown from the analysis and is suitable for the completely state controllable and completely observable system, as well as, as appeared in solving the simple problem is stable in terms of error reducing versus step sizes, then it has fast computational time. The result is compared with the exact solutions. It is worth mentioning that Walsh solution provides excellent result even for small values of m=8. For large values of m=16,m=32, and m=64, we can also obtain the results closer to exact values.

So this approach can be extended to other orthogonal functions, such as Haar functions. They are much more complicated in terms of construction compared to the Walsh functions.

9- Recommendations

1- Using of another complete orthonormal systems instead of Walsh functions such as complete orthogonal polynomials approximation :Legendre polynomials , Laguerre polynomials, Tchebycheff polynomials of the first and second kind and others ….

2- Giving some practical examples in the completely state controllable and completely observable system and using orthogonal polynomials to get the suitable approximation for these examples, as well as, compared them with Walsh functions.

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