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Vibration Analysis of Sudden Enlargement Pipe Conveying Fluid with Presence of Heat Flux

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Abstract:
The vibrated pipe conveying fluid with sudden enlargement and exposed to heat flux is studying in this paper. The governing equations of motion for this system are derived by using beam theory. The effect of external force that applied at the mid length (at the enlargement) of the pipe is studied. The transfer matrix method is a technique that used in this study. From this technique it can be compute the natural frequencies, mode shapes, deflection, slope, bending moment, shear force, velocity, and pressure for different cases of pipe conveying fluid (with and without heat flux), also the effect of forced vibration on these parameter are presented. Different types of supports are used to show the effect of changing the support's type on the behavior of this system at different fluid velocities and heat flux. Also the effect of change the values of fluid velocities and heat flux on the Coriolis and compressive force are studied. The results of this study are compared with the results that found from ANSYS program, also another comparison is made with the last investigation. Those comparisons show good agreement.

Keyword: Sudden Enlargement Pipe, Vibration in Pipe, Pipe Conveying Fluid with Heat Flux

تحليل الاهتزازات في انبوب ذو توسع مفاجئ بنقل مائع بوجود فيض حراري

الخلاصة
تم دراسة الاهتزاز لانبوب ذو توسع مفاجيء ويجري فيه مائع ومعرض الى فيض حراري. معادلة الحركة لهذا النظام اشتقت من نظرية (beam). تم دراسة تاثير القوة الخارجية المسلطة في منتصف الطول (عند التوسع) للانبوب. طريقة المصفوفة الانتقالية هي التقنية المستخدمة في هذا البحث. من خلال هذه التقنية يمكن حساب الترددات الطبيعية, نسق الاهتزاز, الانحراف, الميل, عزم الانحناء, قوة القص, السرعة والضغط لمختلف حالات الانبوب الذي يجري فيه مائع (بوجود وعدم وجود فيض حراري), كذلك تم عرض تاثير الاهتزاز القسري على هذه الثوابت. انواع مختلفة من التثبيتات استخدمت لتبين تاثير تغيير نوع التثبيت على سلوك النظام عند قيم مختلفة من سرع المائع والفيض الحراري. كذلك درس تاثير تغيير قيم سرع المائع والفيض الحراري على قوى (Coriolis and compressive). تمت مقارنة النتائج التي تم الحصول عليها من هذه الدراسة مع النتائج التي تم الحصول عليها من برنامج (ANSYS), كذلك تم عمل مقارنة اخرى مع بحث سابق. هذه المقارنات بينت تطابق جيد.

A_p	Cross-sectional area of the pipe	m^2
D_h	Hydrualic diameter	m
D_1	Inner diameter for the first part of the pipe (before the enlargement)	m
D_2	Inner diameter for the second part of the pipe (after the enlargement)	m
E	Modulus of elasticity for pipe	N/m^2
E_m	Mean modulus of elasticity for pipe	N/m^2
F_o	External force	N
f	Friction factor	-
G	Modulus of rigidity	N/m^2
I	Second moment of area for pipe	m^4
I_m	Mean second moment of area for pipe	m^4
L	Length of the pipe	m
L_i	Element length	m
L_m	Mean element length	m
M	Bending moment	N.m
m_f	Mass of fluid per unit length	kg/m

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m_p	Mass of pipe per unit length	kg/m
N	Thermal forces in the pipe	N
P	Fluid pressure	N/m^2
P_1	Inlet pressure to the pipe	N/m^2
P_2	Outlet pressure from the pipe	N/m^2
P_{L1}	losses for first part of pipe(before enlargement)	N/m^2
P_{L2}	Losses for second part of pipe(after enlargement)	N/m^2
P_{Le}	losses at enlargement	N/m^2
U	Velocity component in axial direction	m/s
u_1	Inlet fluid velocity to the pipe	m/s
u_2	Fluid velocity after enlargement	m/s
V	transverse shear force in the pipe	N
W	Coriolis &Compressive forces	N
Y	Transverse displacement of pipe	m
	Dimensionless Groups	
Re	Reynolds number based on hydrualic diameter and velocity	$U.D_h/u$
C_e	Constant related to losses in enlargement	$\left[1-\left(A_1/A_2\right)\right]^2$
	Greek Letters	
β	Dimensionless parameters	$b=\left(\dfrac{m_f}{m_f+m_p}\right)^{1/2}$
γ	Dimensionless parameters	$g=\left(\dfrac{L_m^{\ 2}}{E\cdot I}\right)\cdot P\cdot A_p$
h	area ratio for pipe	$h=u_1/u_2$
θ	Slope of the pipe	rad
ρ	Density of the fluid	kg/m^3
τ	Shear stress on the internal surface of the pipe	N
υ	Kinematic viscosity of the fluid	m^2/s
f	Dimensionless parameters	$f=\dfrac{L_m}{(EI)_m}$
j	Dimensionless parameters	$j=\dfrac{L_m^2}{(EI)_m}$
c	Numerical factor	-
	Subscripts	
m	Mean diameter or mean radius of the pipe	-
	Superscripts	
$\bar{}$	Dimensionless notation	-
L,R	Left and right of the state vector	-

Introduction:

Heat flux constitutes an important limitation on the operation of boiling heat transfer systems. In heat flux controlled systems such as nuclear reactors, the consequence is a substantial increase in wall temperature, which may result in a physical failure of heat transfer systems. Many component as (steam generators, condenser, piping system, and nuclear fuels) are subjected to high axial or cross flow which could often cause vibration problems, resulting in wear and fretting damage to those systems. Furthermore, the development of advanced nuclear reactors and the use of high strength materials cause structures to become more slender and more susceptible to it. Recently, Wu and Shin [1] studied the dynamic analysis of multispan pipe conveying fluid subjected to external force and using the transfer matrix method, they concluded that this method independent of the number of the spans for the system. Reddy and Wang [2] derived the pipe conveying fluid by using Timoshenko beam theories and extended Hamilton's principle, they found the forces due to the flowing fluid in the beam and the kinetic energy of the flowing fluid. Yoon and Son [3] studied the vibration system consist of a rotating cantilever pipe conveying fluid and a tip mass, they concluded that the natural frequencies of the cantilever pipe conveying fluid are proportional to the angular velocity of the pipe and the tip mass. The non linear dynamics of a vertical cantilevered pipe conveying fluid was studied by Paidoussis et al. [4], they found that the system losses stability by flutter. An experimental study by Ismael et al. [5] for the effect of the forced and free vibration on simply supported annulus pipe conveying fluid and it's effect on the forced convection heat transfer coefficient, they concluded that the heat transfer rate increases as frequency increases and decreases as the Reynolds number increases, the thermal force have predominance effect on the natural frequency of the vibrated system, and the effect of heat flux is greater than the flow velocity effect on the natural frequency of the system. In this research the vibration in a pipe conveying fluid with a sudden enlargement combined with the effect of heat flux are studied.

Governing Equation of Motion:

The following assumptions are adopted in deriving the equation of motion for free vibration of undamped single span pipe conveying fluid , based on the beam theory :

1. The cylinder is modeled as a beam with it’s actual second moment of area.
2. Uniform axial flow.
3. The effect of gravity and material damping are negligible.
4. The rotary inertia and shear deformations are negligible.
5. The material is assumed to be isotropic and the (E, k, α,…etc) are dependent of temperature.
6. The velocity distribution through the cross-section of the pipe is negligible.
7. The pipe is considered to be horizontal.

The equation of motion for free vibration of pipe conveying fluid may be written as:

$$E\cdot I\frac{\partial^4 y}{\partial x^4}+\left(P\cdot A_p+m_f\cdot u_1\cdot u_2+N\right)\frac{\partial^2 y}{\partial x^2}+2\cdot m_f\cdot u_1\frac{\partial^2 y}{\partial x\,\partial t}+\left(m_f+m_p\right)\frac{\partial^2 y}{\partial t^2}=0$$

Where:

$E\cdot I\frac{\partial^4 y}{\partial x^4}$ = stiffness term.

$\left(P\cdot A_p+m_f\cdot u_1\cdot u_2+N\right)\frac{\partial^2 y}{\partial x^2}$ =curvature term.

$2\cdot m_f\frac{\partial^2 y}{\partial x\,\partial t}$ = Coriolis term which results from the rotation of fluid element due to the system lateral motion.

$\left(m_f+m_p\right)\frac{\partial^2 y}{\partial t^2}$ = inertia term.

N = thermal force = $A_P\cdot E_P\cdot a\cdot\Delta T$

The equation of motion for forced vibration of pipe conveying fluid may be written as:

$$E\cdot I\cdot y^{iv}+\left(P\cdot A_p+m_f\cdot u_1\cdot u_2+N\right)y''+2\cdot m_f\cdot u_1\cdot\dot{y}'+\left(m_f+m_p\right)\ddot{y}=F(x,t)$$

Where: F(x,t) is the external harmonic force being applied normally on the pipe axis in the (y – direction).

The dimensionless variables are:

$$\overline{X} = \frac{x}{L_m}, \quad \overline{Y} = \frac{y}{L_m},$$

$$b = \left(\frac{m_f}{m_f + m_p}\right)^{1/2}$$

$$\overline{U} = \left(\frac{m_f}{E \cdot I}\right)^{1/2} \cdot u_f \cdot L_m$$

$$g = \left(\frac{L_m^2}{E \cdot I}\right) \cdot P \cdot A_p$$

$$t = \left(\frac{E \cdot I}{m_f + m_p}\right)^{1/2} \left(\frac{t}{L_m^2}\right)$$

Then the equations of motion for free and forced vibration becomes:

$$\overset{iv}{\overline{Y}} + \left(g + \overline{U}_1 \cdot \overline{U}_2\right)\overline{Y}'' + \overline{N} \cdot L_m \cdot \overline{Y}'' + 2 \cdot \overline{U} \cdot b \cdot \dot{\overline{Y}}' + \frac{1}{L_m}\ddot{\overline{Y}} = 0$$

$$\overset{iv}{\overline{Y}} + \left(g + \overline{U}_1 \cdot \overline{U}_2\right)\overline{Y}'' + \overline{N} \cdot L_m \overline{Y}'' + 2 \cdot \overline{U} \cdot b \cdot \dot{\overline{Y}}' + \frac{1}{L_m}\ddot{\overline{Y}} = F\left(\overline{X}, t\right)$$

The values of young modulus of elasticity (E) at any temperature can be found from the equation:

$$E = 1.09939E+02 - 20917459E-02*T - 5.7084838E-05*T^2 + 3.7569199E-09*T^3$$

Investigation of the Flow Stream:

The value of inlet velocity (u1) can be found from inlet Reynolds number where:

$$u_1 = \frac{m \cdot Re}{r \cdot D_1}$$

while the velocity through the enlargement (u2) of the pipe can be determined by using the following formula:

$$u_2 = h\, u_1$$

Where: h =area ratio for pipe.

The pressure drop (ΔP) due to friction for flow in pipe for any uniform cross section given as follows:

$$\Delta p = f \frac{L}{D_h} \cdot \frac{r \cdot u^2}{2}$$

Where: (f) is the friction factor for laminar flow in pipe given by:

f =64/Re

Re = Reynolds number $= \dfrac{u \cdot D_h}{u}$

Dh=inner diameter of pipe (flow diameter).

Since the flow get out to atmosphere; therefore, the out let pressure of the pipe (P2) = 1 atm, and the inlet pressure to the pipe (P1)can be found from Bernolli's equation as follows:

$$\frac{P_1}{r \cdot g} + \frac{u_1^2}{2 \cdot g} + Z_1 = \frac{P_2}{r \cdot g} + \frac{u_2^2}{2 \cdot g} + Z_2 + \text{losses}$$

For horizontal pipe (z1=z2=0) substitute in above equation gives:

$$P_1 = P_2 + \left(\frac{u_2^2 - u_1^2}{2} + \text{losses}\right) * r$$

Where: losses = PL1+PL2+PLe

PL1= losses for the first part of pipe (before enlargement).

PL2= losses for the second part of pipe (after enlargement).

PLe= losses at enlargement = $\frac{1}{2} \cdot C_e \cdot r \cdot u_1^2$

Ce= constant $= \left[1 - \frac{A_1}{A_2}\right]^2$

The Transfer Matrix Method:

In this method it can be convert the system to the mathematical model consist of number of stations represented by point matrix where the mass concentrated at each station, each station joint with massless element which represented by field matrix, then it can be found the equations of {deflection(Y), slope(q), bending moment(M), shear force(V), velocity(U), pressure(P)} for the vibrated pipe conveying fluid, these equations are:

The Equations for Field Matrix:

$$\overline{Y}_i^L = \overline{Y}_{i-1}^R - \frac{\overline{q}_{i-1}^R}{L_m}\left[L_i + \frac{\overline{N}_{i-1}^R}{j}\left(\frac{L_i^3}{6\,EI} - \frac{x \cdot L_i}{GA_{Pi}}\right)\right] - \overline{M}_{i-1}^R\left(\frac{L_i^2}{2 \cdot f \cdot L_m \cdot EI}\right) - \overline{V}_{i-1}^R \cdot \frac{1}{j \cdot L_m}\left(\frac{L_i^3}{6\,EI} - \frac{x \cdot L_i}{GA_{Pi}}\right) + \frac{\overline{W}_i}{j \cdot L_m}\left(\frac{L_i^3}{48\;EI} - \frac{x \cdot L_i}{GA_{Pi}}\right)$$

$$\overline{q}_i^L = \overline{q}_{i-1}^R\left(1 + \overline{N}_{i-1}^R \cdot \frac{L_i^2}{2 \cdot j \cdot EI}\right) + \overline{M}_{i-1}^R \cdot \frac{L_i}{f \cdot EI} + \overline{V}_{i-1}^R \cdot \frac{L_i^2}{2 \cdot j \cdot EI} - \overline{W}_i \cdot \frac{L_i^2}{8 \cdot j \cdot EI}$$

$$\overline{M}_i^L = \overline{q}_{i-1}^R \cdot \overline{N}_{i-1}^R \frac{L_i \cdot f}{j} + \overline{M}_{i-1}^R + \overline{V}_{i-1}^R \cdot \frac{L_i \cdot f}{j} - \frac{\overline{W}_i}{j} \cdot \frac{L_i \cdot f}{2}$$

$$\overline{V}_i^L = \overline{q}_{i-1}^R \left[\overline{N}_{i-1}^R \left(1 - \overline{N}_i^L \frac{L_i^2}{2j \cdot EI} \right) - \overline{N}_i^L \right] - \overline{M}_{i-1}^R \left(\frac{L_i}{f \cdot EI} \right) + \overline{V}_{i-1}^R \left(1 - \overline{N}_i^L \frac{L_i^2}{2j \cdot EI} \right) - \overline{W}_i \left(1 - \overline{N}_i^L \frac{L_i^2}{8 \cdot j \cdot EI} \right)$$

$$\overline{U}_i = \left[\frac{m_f}{EI} \right]^{1/2} . U_i . L_m \quad \& \quad \overline{P}_i = \frac{P_i}{P_{inlet}}$$

Where: $j = \dfrac{L_m^2}{(EI)_m}$,

$$f = \frac{L_m}{(EI)_m}$$

$$L_m = \frac{\sum_{n=1}^{n} L_i}{n-1}, \quad E_m = \frac{\sum_{n=1}^{n} E_i}{n-1},$$

$$I_m = \frac{\sum_{n=1}^{n} I_i}{n-1}$$

The Equations for the Particular Node:

$$\overline{Y}_i^L = \overline{Y}_i^R \,\&\, \overline{q}_i^L = \overline{q}_i^R \,\&\, \overline{M}_i^L = \overline{M}_i^R$$

$$\overline{V}_i^R = -(m_P + m_f)_i \cdot \Omega^2 \cdot \frac{L_m^3}{(EI)_m} \cdot \overline{Y}_i - F_o \cdot \frac{L_m^2}{(EI)_m} + \overline{V}_i^L$$

$$\overline{U}_i^L = \overline{U}_i^R \,\&\, \overline{P}_i^L = \overline{P}_i^R$$

The Equations for the Supported Node:

$$\overline{Y}_i^L = \overline{Y}_i^R \,\&\, \overline{q}_i^L = \overline{q}_i^R \,\&\, \overline{M}_i^L = \overline{M}_i^R$$

$$\overline{V}_i^R = -\left[(m_P + m_f)_i \cdot \Omega^2 - K \right] \cdot \frac{L_m^3}{(EI)_m} \cdot \overline{Y}_i + \overline{V}_i^L$$

$$\overline{U}_i^L = \overline{U}_i^R \,\&\, \overline{P}_i^L = \overline{P}_i^R$$

The Equations for the Sudden Enlargement:

$$\overline{Y}_i^L = \overline{Y}_i^R \,\&\, \overline{q}_i^L = \overline{q}_i^R \,\&\, \overline{M}_i^L = \overline{M}_i^R$$

$$\overline{U}_i^L = h . \overline{U}_i^R$$

$$\overline{P}_i^L = \overline{P}_i^R + \frac{C_e . r . u^2}{2 . P_{inlet}}$$

Where: $h = \dfrac{A_1}{A_2}$, A1= area of the pipe before enlargement, A2= area of the pipe after enlargement, Pinlet= inlet pressure to the pipe, u= fluid velocity, Ce= constant $= \left[1 - \frac{A_1}{A_2} \right]^2$

Results and Discussion:

To embrace the theoretical work of this paper a programs have been developed by using FORTRAN 90 to determine (mode shapes, natural frequencies, deflection, slope, bending moment, shear force) for different values of heat flux and fluid velocity.

A- Results of the Effect of Induced Vibration on the Pipe Conveying Fluid:

*The deflection at mid length of pipe conveying fluid with various velocities and heat flux and for different kinds of supports (simply, flexible, rigid) are presented at Figs.(1) to (3), also it may be observed the values of the natural frequencies from the peaks of these Figurers which are given in tables (A) for different cases of pipe supports and heat flux. It can be notice from these Figurers and tables the following:

**Figs.(1a&b), (2a&b), (3a&b) and tables (A1&2) show that the values of natural frequencies for the case of vibrated pipe conveying fluid without heat are less than the values of the natural frequencies for the case of vibrated pipe without (fluid and heat), that's may be due to the effect of the fluid mass which is added to the mass of the system and proportional inversely with the natural frequencies.

**Figs.(1b), (2b), (3b) and table (A1) show that the values of natural frequencies for the case of vibrated pipe conveying fluid without heat are still constant with the increasing of Reynolds number because increasing Reynolds number lead's to increasing fluid velocity and this increasing (when there is no heat) doesn't affect the properties of pipe material (stiffness, mass). To prove this phenomenon a comparison are making between the results of the present work and last investigation [6] where this investigation take simply supported straight pipe conveying fluid with orifice, this comparison show in Fig.(4), it can be seen that the same phenomenon in both works also it can be seen that the values of the natural frequencies (1st, 2nd, 3rd) for the vibrated pipe conveying fluid for present work are close to the values of natural frequency for the last investigation, the small difference in the values occur due to the sudden enlargement in the present investigation and using orifice in the last investigation. Also Fig.(5) shows that the first, second, and third natural frequencies don't change with the flow rates change .

**When the pipe conveying fluid is heated and compared with the pipe without (fluid and heat) which represented at Figs.(1a,c,d,e,f),(2a,c,d,e,f), (1a,c,d,e,f) and tables (A1&3), it can be seen that the values of the natural frequencies for the case of heated pipe conveying fluid is less than the values of natural frequencies for the case of pipe without (fluid and heat) because of the existence of the effect of Coriolis force which result from flowing fluid combined with the effect of heat on the vibrating system which lead's to the existence of the thermal force.

**By comparing the Figs.(1b,c,d,e), (2b,c,d,e), (3b,c,d,e) and tables (A2&3) which represent the case of pipe conveying fluid without heat and the case of heated pipe conveying fluid, it may be observed that the values of natural frequencies for the case of existence (fluid and heat) are less than the values of natural frequencies for the case of existence fluid and without heat, that's because the effect of heat flux which increases the pipe temperature and in turn increases the thermal force in addition to that the young modulus of elasticity has an inverse proportion with the temperature; therefore, the young modulus of elasticity will decrease with temperature increasing, and this will cause a decrease in pipe stiffness, and hence the natural frequencies will be reduce.

**From Figs.(1c,d,e), (2c,d,e), (3c,d,e) and tables (A3) it can be noticed that for small values of heat flux applied on the vibrated pipe conveying fluid there are no effect on the system because the thickness of the pipe wall is very small comparing with the cross-sectional area of the fluid; therefore, the fluid will absorb most of the heat and it's effect on the properties of the pipe doesn't appear hence the values of the natural frequencies don't alter, while increasing of the heat flux lead's to increase the thermal force generated in the pipe which affect the pipe properties (stiffness) and hence reduces the natural frequencies (as explained before). Also it can be noticed that increasing the natural frequencies with Reynolds number increasing at the same heat flux because increasing Reynolds number means increasing fluid velocity and that's make the fluid absorb more quantities of heat flux, that's mean increasing cooling rate of the system and reducing the pipe temperature. Also the little increasing in temperature lead's to little decreasing in young modulus of elasticity $(E\, a\, 1/T)$, hence the pipe stiffness increases with Reynolds number increasing and that's lead to increase the values of the natural frequencies.

**Figs.(1f), (2f), (3f) and tables (A3) show that the increasing of heat flux at the same values of fluid velocity and excitation frequencies lead's to decrease the values of natural frequencies. Also it may be noticed that the pipe deflection increases as the thermal forces increases.

* Figs.(6) to (18) represent first, second, and third mode shapes for different kinds of supports and for the case of vibrated pipe without (fluid and heat) and the case of vibrated pipe conveying fluid also the case of heated pipe conveying fluid, these cases at range of Reynolds number (250) to (1500) and range of heat flux (10-15-20) kw/m2. It can be seen that the values of deflection increase as Reynolds number increases at the same natural frequencies, since increasing Reynolds number lead to increase fluid velocity, and hence increasing the values of the Coriolis force which in turn lead to increase the deflection; therefore, the deflection increases with the increasing of Reynolds number (for no heat).

*Figs.(10) to (18) represent the mode shapes for the case of heated pipe conveying fluid, it can be seen that increasing the values of deflection with the increasing of Reynolds number at the same heat flux and same natural frequency because increasing Reynolds number (increasing velocity) lead's to increase Coriolis force and then increase the deflection.

* Figs.(19) represent a comparison between the cases of pipe without (fluid and heat), and pipe conveying fluid without heat , and heated pipe conveying fluid with various (heat flux and fluid velocity). From these figures it can be noticed that increasing the deflection as Reynolds number increases for the case of pipe conveying fluid without heat (as explained before). Also it can be seen that the deflection at the case of pipe without (fluid and heat) is greater than the deflection for the case of pipe conveying fluid without heat flux and at Reynolds number equal (250) because Reynolds number (or velocity) is small that's made the values of Coriolis force small. But as Reynolds number increases (Re=1500) it can be seen that the deflection for the case of pipe conveying fluid without heat flux is greater than the deflection for the case of pipe without (fluid and heat) because increasing Reynolds number lead's to increase the Coriolis force and then the deflection. From these figures it can also be seen increasing the deflection for the case of heated pipe conveying fluid with increasing Reynolds number (as explained before). These Figures also show that the maximum deflection lies before the mid pipe since the diameter of the pipe after the enlargement (after the mid point) is greater than that before the enlargement and hence this portion of the pipe is stiffer than that before the enlargement.

*Figs.(20) show (bending moment, slope, shear force) for pipe with different excitation frequencies for different cases, these figures show increasing the (bending moment, slope, shear force) with increasing excitation frequencies.

* Figs.(21) show (bending moment, slope, shear force) for pipe at it's natural frequencies with various (heat flux and fluid velocity), it can be noticed from these figures the following:

1-Increasing of (bending moment, slope, shear force) of the pipe conveying fluid as increasing Reynolds number (increasing velocity) because increasing the velocity lead's to increase Coriolis force and this force increase (bending moment, slope, shear force) of the pipe.

2-When comparing the case of pipe without (fluid and heat) with the case of pipe conveying fluid without heat it can be seen that the (bending moment, slope, shear force) at the case of no (fluid and heat) is greater than the case of pipe conveying fluid at Reynolds number equal (250) but at increasing Reynolds number (1500) the (bending moment, slope, shear force) at the case of pipe conveying fluid will be greater than that the case of the pipe without (fluid and heat), that's because as Reynolds number increases, velocity increases and then the Coriolis force increases which in turn lead to increase the cited values of (bending moment, slope, shear force).

3-In spite of increasing fluid velocity as a result of Reynolds number increasing which in turn decrease the thermal forces, the (bending moment, slope, shear force) increases since the effect of increasing the Coriolis force is predominant.

4-These figures show that the values of bending moment just after the enlargement are greater than those before entering the enlargement, this may be related to the change in fluid momentum just before and after the down stream of the enlargement. This is true till the pipe return to it's original behavior after this region.

5-The values of the slope just after the enlargement are less than those before entering the enlargement, Also the values of shear force just after the enlargement are greater than those before entering the enlargement. This is related to the increase in the pipe stiffness due to the enlargement and as known increasing the stiffness will increase the slope and increase the shear force.

B- Results of the Effect of Supports Type on the Vibrated System:

*The values of natural frequencies for flexible support are less than that for simply and rigid support, also the values of natural frequencies for simply support are less than that for rigid support. This because the flexible support have the ability to move in Y-direction; therefore, it's flexibility is very high comparing with simply and rigid supports, that leads to decrease the pipe stiffness and hence it's natural frequency. While rigid support is tightly supported more than the other two kinds of supports [Y(0,t)=0&Y(L,t)=0] also there is no slope at the support's position of the pipe $\left[\frac{\partial Y}{\partial X}(0,t)=0 \,\&\, \frac{\partial Y}{\partial X}(L,t)=0\right]$ which leads to increase the stiffness of the pipe at the support's position and thus decreases the natural frequencies more than the other two kinds of supports.

*Figs.(2c,d,e) and tables (A3) represent the case of heated pipe conveying fluid for flexible support, it can be notice that the second and third values of natural frequency decreases with increasing Reynolds number. However it is expected that these values of the natural frequencies increases as Reynolds number increases. But as a matter of fact, the decreasing in these values is very small and may be due to the rounding of errors in the program's computation.

*From Figs.(1) to (3) and tables (A2&3) it can be seen that the effect of heat flux on the flexible support more than the other two supports because the flexibility of flexible support is higher and hence a compound effect will be predominant due to the decreasing of the stiffness system and heat flux. Also from Figs.(1f), (2f), (3f) and tables (A3) it can be noticed that increasing heat flux will lead to decrease the values of natural frequencies for flexible support an this decreasing is more than the decreasing of the values of natural frequencies for simply and rigid supports.

*Figs.(22) represent (bending moment, slope, shear force) along the pipe with different kinds of supports at the first natural frequency, it can be noticed that:

1-At the support's points the bending moment for rigid support is very high comparing with the other two types of supports because at flexible and simply supports $\left(\frac{\partial^2 Y}{\partial X^2}=0\right)$ at support's points, while at rigid support it's not equal to zero.

2-The slope at support's points for rigid support is equal to zero, whereas it has values at flexible and simply supports that's because the rigid support is tightly supported comparing with the other two kinds of supports; therefore, there is no inclined angle at support's regions, while simply and flexible supports are allowed the pipe to be inclined.

3-The shear force at support's points for flexible support is equal to zero, while it has values at simply and rigid supports because at flexible support $\left(\frac{\partial^3 Y}{\partial X^3}=0\right)$ at support's point but it's not equal to zero at the other kinds of supports, also because rigid support is tightly supported comparing with simply support; therefore, shear force will be greater than simply support as shown in Figures.

C- Results of the Fluid Flowing in Vibrated Pipe:

*Fig.(23) show Coriolis and compressive force for simply supported pipe conveying fluid due to forced vibration at mid length with various velocities and heat flux. It may be notice that:

1-These forces increase as the excitation frequencies increase for different values of heat flux and fluid velocity because the fluid elements are highly affected by the vibration of the pipe wall that's give an additional velocity component to the fluid elements resulting from transverse motion of pipe this component cause rotation to the fluid elements. This rotation motion increase the centrifugal force and since Coriolis has direct proportion to centrifugal force; therefore, Coriolis will increase with the increasing of the excitation frequencies.

2-Figs.(23a) to (23d) show increasing Coriolis and compressive force with increasing Reynolds number for different values of heat flux due to the effect of increasing fluid velocity which increase the centrifugal force where Coriolis force has direct proportion with centrifugal force which lead to increase the Coriolis force with the increasing of Reynolds number.

3-Fig.(23e) and Fig.(23f) show increasing of Coriolis and compressive force when applying heat flux because heat flux add energy to the system which increase the kinetic energy of the fluid elements which in turn increase Coriolis force in addition to the increase in pipe flexibility due to the applied heat. Increasing the flexibility of the system and Coriolis force lead to increase deformation in pipe.

D- Results of the Comparison Between the Ansys and Fortran 90 Programs:

The comparison make for different cases of pipe (without fluid and without heat flux, conveying fluid and without heat flux and different Reynolds number are used for the range (250-500-750-1000-1500), conveying fluid and exposed to heat flux at the range (10-15-20) kw/m2 and at the same range of Reynolds number).

The results show good agreement between the two programs (ANSYS and FORTRAN 90). The comparison presented in the following table (B)

Conclusions:

1- The values of natural frequencies for flexible support is less than that obtained for rigid support and simply support, also natural frequencies for simply support is less than that obtained for rigid support.

2- For all kinds of supports it's found that:

The natural frequencies for pipe conveying fluid without heat is less than the natural frequencies for pipe without fluid and heat.

The natural frequencies for pipe conveying fluid without heat steady constant with the increasing of Reynolds number (increasing velocity).

The natural frequencies for heated pipe conveying fluid is less than the natural frequencies for pipe without fluid and heat.

The natural frequencies for heated pipe conveying fluid is less than the natural frequencies for pipe conveying fluid without heat.

The little values of heat flux don't affect the vibrated system whereas the higher values of heat flux leads to decrease the natural frequencies of the vibrated system.

The increasing of Reynolds number at the same heat flux leads to increasing the natural frequencies for all kinds of supports except flexible support where the second and third natural frequencies will decrease with increasing Reynolds number at the same heat flux.

The effect of heat flux is greater than the effect of fluid velocity on the natural frequencies of the system.

The heat transfer rate increases as the Reynolds number increases.

The local wall temperature decreases as Reynolds number increases or the heat flux decreases.

The heat flux has more effect on the flexible support than the other two types (simply and rigid) supports.

3- The deflection, bending moment, slope, and shear force are:

Increased with the increasing of excitation frequencies.

Increasing with the increasing of Reynolds number for the case of pipe conveying fluid with and without heat flux.

Reducing when the fluid flows in the pipe is with small Reynolds number comparing with the values of pipe without fluid and heat but when increasing Reynolds number, the values of these parameters will be greater than those for pipe without fluid and heat.

4- The fluid forces (Coriolis and Compressive) greatly affect the response of the undamped pipe under vibration. The Coriolis force increases with Reynolds number, excitation frequencies, and heat flux.

5- The results of the transfer matrix method by using FORTRAN 90 program and finite element method by using ANSYS program show a good agreement.

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Tables (A)

1- No Fluid&No Heat

Natural frequency	Flexible support	Simply support	Fixed support
Ω_1	163	168	445
Ω_2	475.5	888.5	1352.5
Ω_3	731.5	1768	2401

2- Fluid&No Heat

Re	natural frequency	Flexible support	Simply support	Fixed support
50	Ω_1	138.5	142.5	380
	Ω_2	390.5	768	1167.5
	Ω_3	649	1520	2064
500	Ω_1	138.5	142.5	380
	Ω_2	390.5	768	1167.5
	Ω_3	649	1520	2064
750	Ω_1	138.5	142.5	380
	Ω_2	390.5	768	1167.5
	Ω_3	649	1520	2064
1000	Ω_1	138.5	142.5	380
	Ω_2	390.5	768	1167.5
	Ω_3	649	1520	2064
1500	Ω_1	138.5	142.5	380
	Ω_2	390.5	768	1167.5
	Ω_3	649	1520	2064

3- Fluid&Heat (A)Q=10 (kw/m^2)

Re	natural frequency	Flexible support	Simply support	Fixed support
250	Ω_1	120.5	141.5	376
	Ω_2	422	760.5	1157
	Ω_3	657.5	1505.5	2045
500	Ω_1	121.5	141.5	376.5
	Ω_2	418	761.5	1158
	Ω_3	655.5	1507.5	2047.5
750	Ω_1	122	141.5	376.5
	Ω_2	417	761.5	1158.5
	Ω_3	654.5	1508	2048
1000	Ω_1	122	141.5	377
	Ω_2	416	762	1158.5
	Ω_3	654	1508	2048.5
1500	Ω_1	122.5	141.5	377
	Ω_2	415.5	762	1159
	Ω_3	654	1508.5	2049

B) Q=15 (kw/m^2)

Re	natural frequency	Flexible support	Simply support	Fixed support
250	Ω_1	122.5	141.5	377
	Ω_2	415.5	762	1159
	Ω_3	654	1508.5	2049
500	Ω_1	115.5	140.5	374
	Ω_2	425.5	756.5	1150.5
	Ω_3	661	1497.5	2034
750	Ω_1	116	140.5	374.5
	Ω_2	424	757	1151
	Ω_3	660	1498	2035
1000	Ω_1	116	140.5	374.5
	Ω_2	423.5	757	1151.5
	Ω_3	659.5	1498.5	2035.5
1500	Ω_1	116.5	140.5	374.5
	Ω_2	422.5	757	1151.5
	Ω_3	658.5	1499	2036

(C) Q=20 (kw/m^2)

Re	natural frequency	Flexible support	Simply support	Fixed support
250	Ω_1	116.5	140.5	374.5
	Ω_2	422.5	757	1151.5
	Ω_3	658.5	1499	2036
500	Ω_1	110	139.5	371.5
	Ω_2	430.5	751	1142.5
	Ω_3	667	1486.5	2019
750	Ω_1	110.5	139.5	371.5
	Ω_2	429	751.5	1143
	Ω_3	665.5	1487.5	2020.5
1000	Ω_1	111	139.5	372
	Ω_2	428.5	752	1143.5
	Ω_3	665	1488	2021.5
1500	Ω_1	111	139.5	372
	Ω_2	427.5	752	1144
	Ω_3	664	1488.5	2022

Tables (B)

Q (w/m^2)	Re	Ω_1 (Hz)		Ω_2 (Hz)		Ω_3 (Hz)	
		Transfer Matrix Method	Finite Element Method	Transfer Matrix Method	Finite Element Method	Transfer Matrix Method	Finite Element Method
0	0	26.7	26.02	141.4	141.4	281.38	284.8
0	250	22.67	22.06	122.23	122.8	241.92	243.6
0	1500	22.67	22.06	122.23	122.8	241.92	243.6
10	250	22.52	21.88	121.037	121.1	239.61	240.4
10	500	22.52	21.7	121.196	121.2	239.93	240.6
10	750	22.52	21.7	121.196	121.2	240.01	240.8
10	1000	22.52	21.7	121.276	121.4	240.01	240.8
10	1500	22.52	21.7	121.276	121.4	240.09	240.9
15	250	22.36	21.7	120.162	120.4	237.86	238.8
15	500	22.36	21.7	120.4	120.6	238.34	239.2
15	750	22.36	21.7	120.48	120.6	238.414	239.4
15	1000	22.36	21.7	120.48	120.6	238.49	239.4
15	1500	22.36	21.7	120.48	120.6	238.57	239.4
20	250	22.202	21.52	119.21	119.6	236.03	237.0
20	500	22.202	21.52	119.53	119.8	236.58	237.6
20	750	22.202	21.52	119.61	119.8	236.75	237.8
20	1000	22.202	21.52	119.69	119.9	236.82	237.8
20	1500	22.202	21.52	119.69	120.0	236.902	238.0

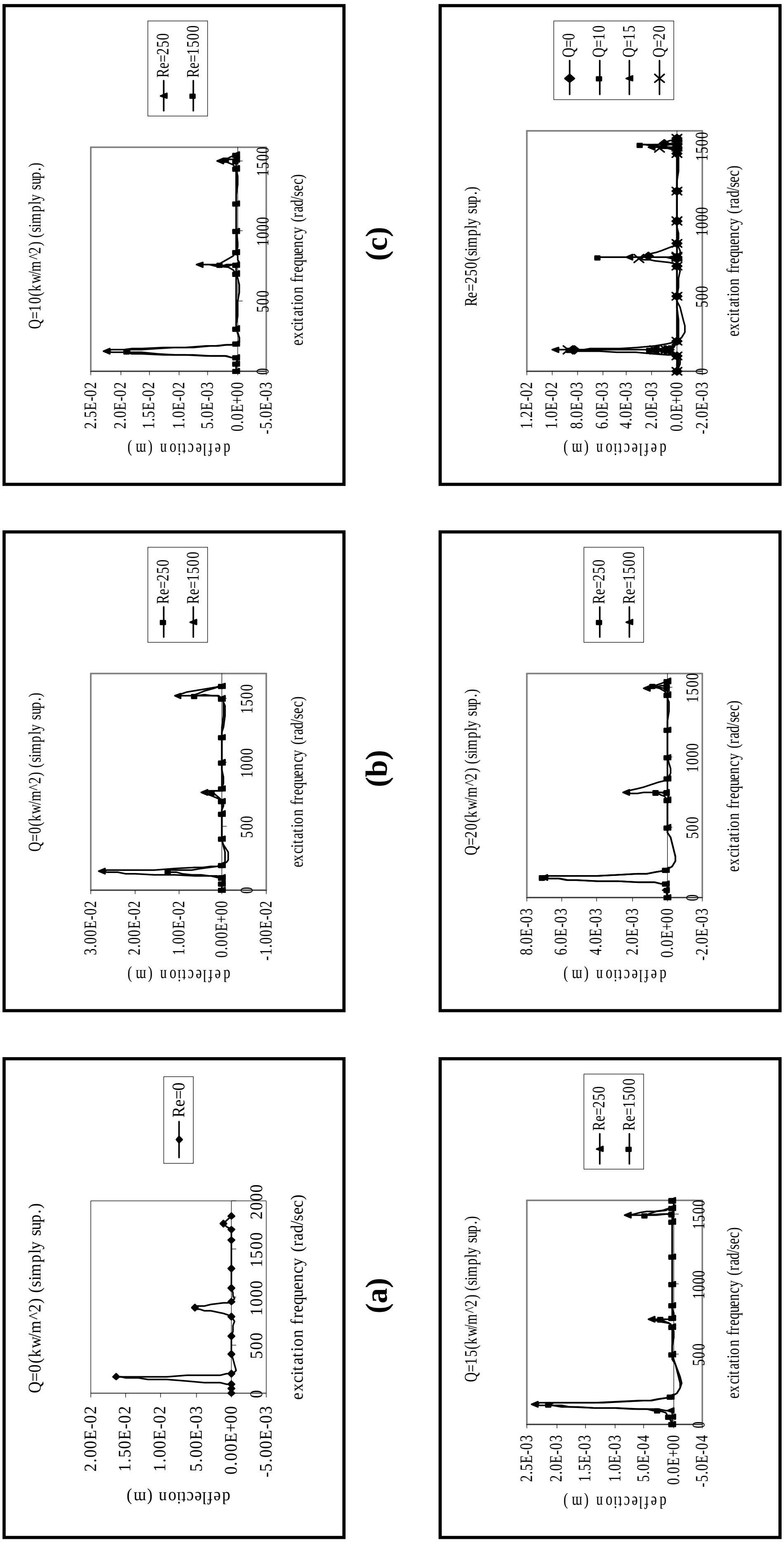


Fig.(1) Deflection for simply support pipe conveying fluid with various excitation frequencies at mid length of pipe for various velocities and heat flux

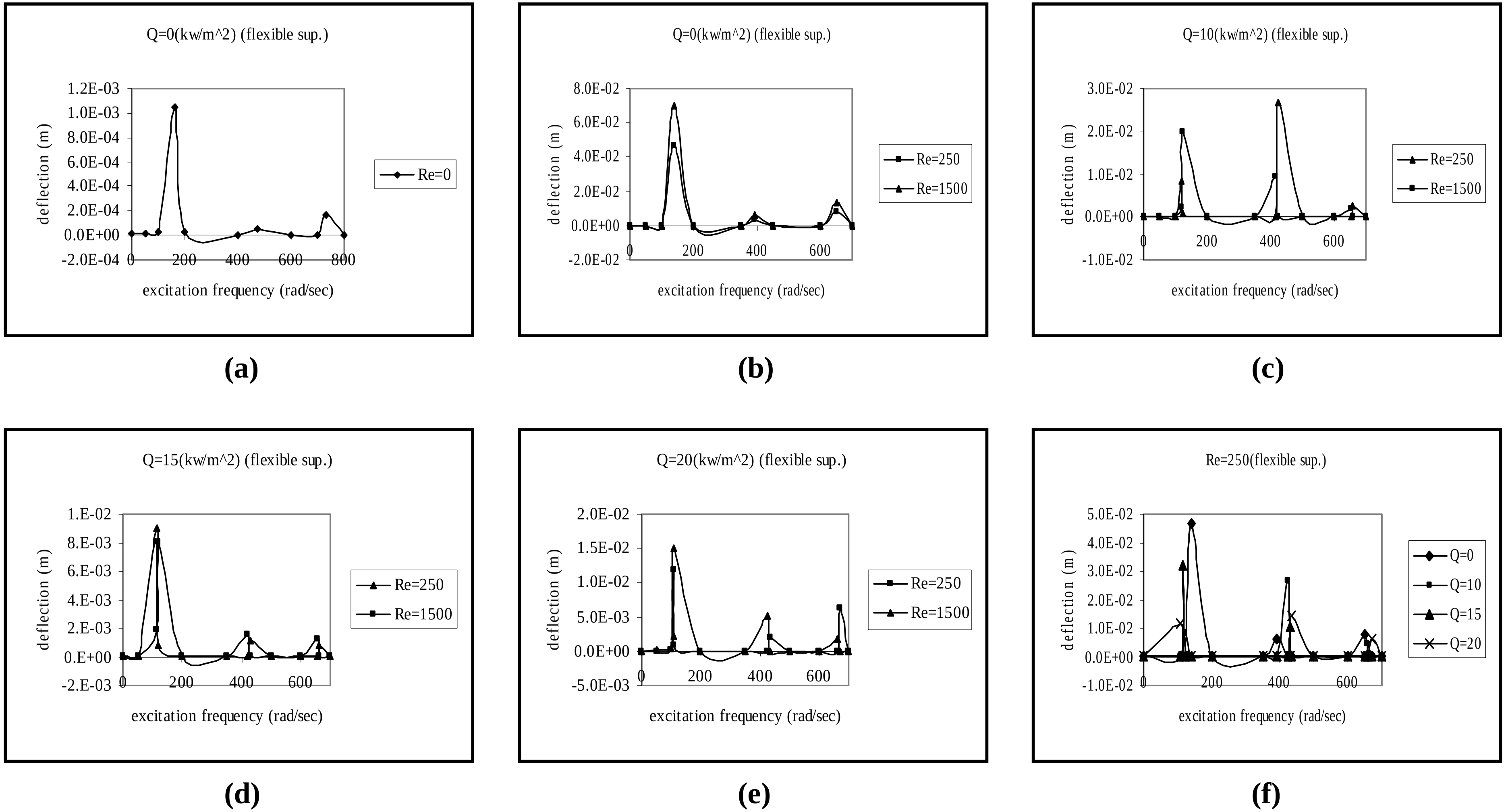


Fig.(2) Deflection for flexible support pipe conveying fluid with various excitation frequencies at mid length of pipe for various velocities and heat flux

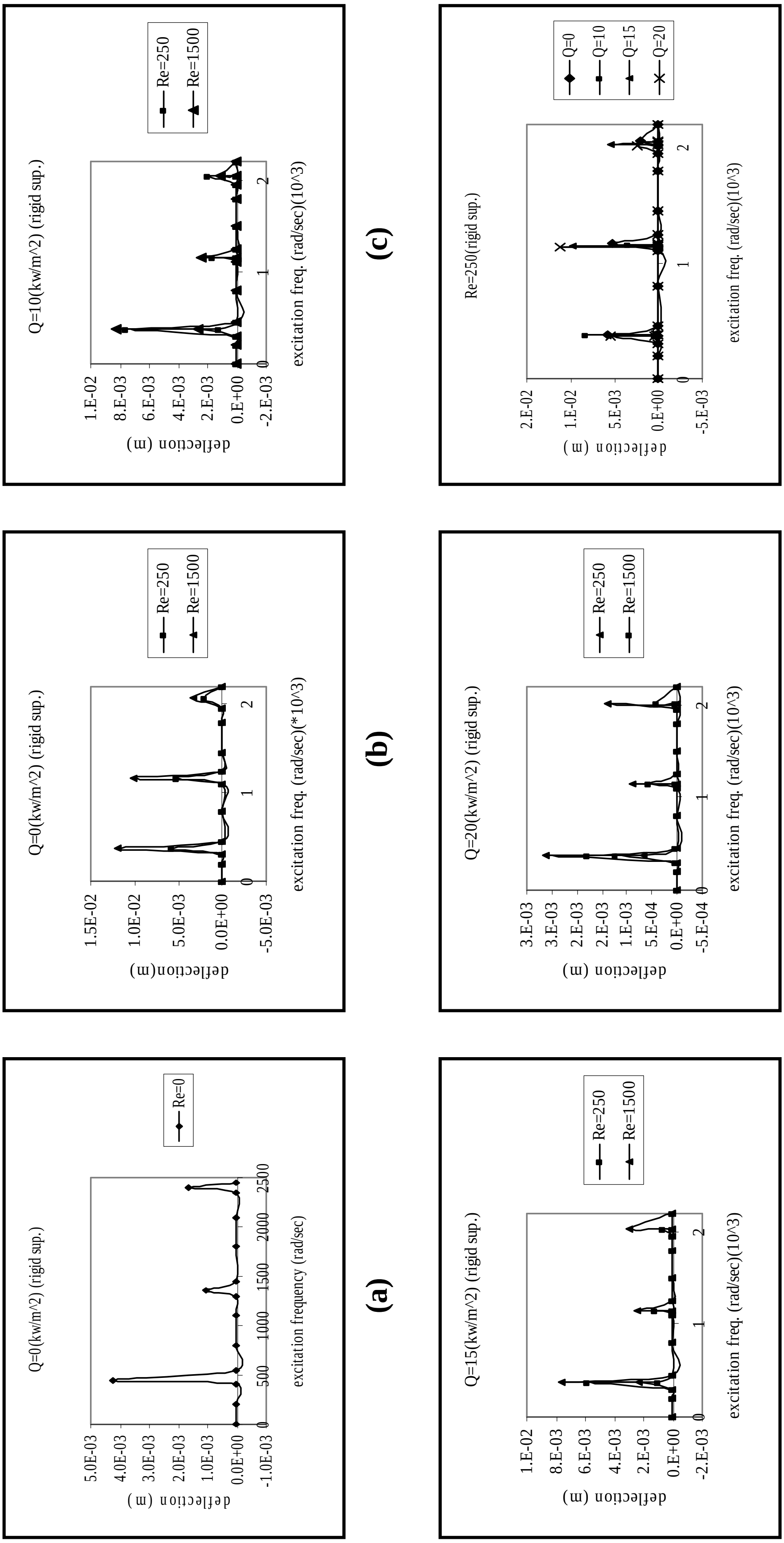


Fig.(3) Deflection for rigid support pipe conveying fluid with various excitation frequencies at mid length of pipe for various velocities and heat flux

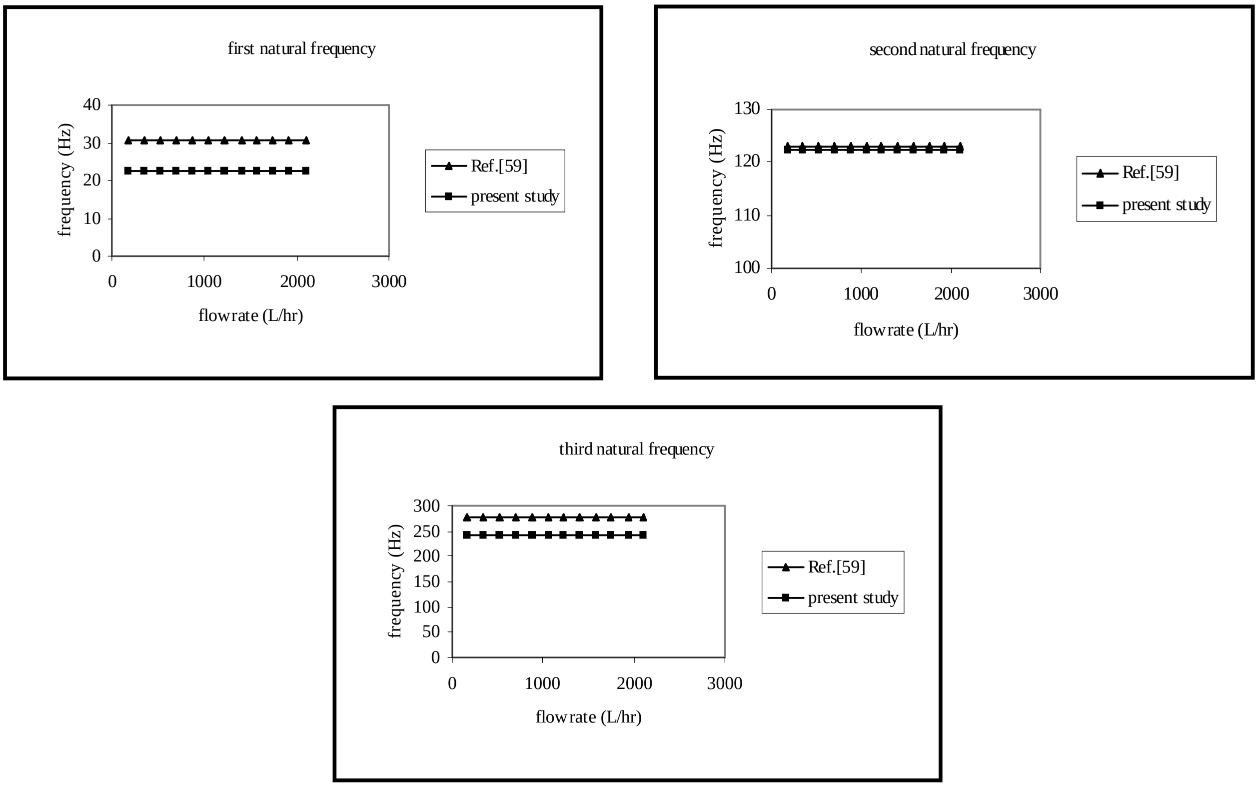


Fig.(4)Comparison between the present study and Ref.[6]

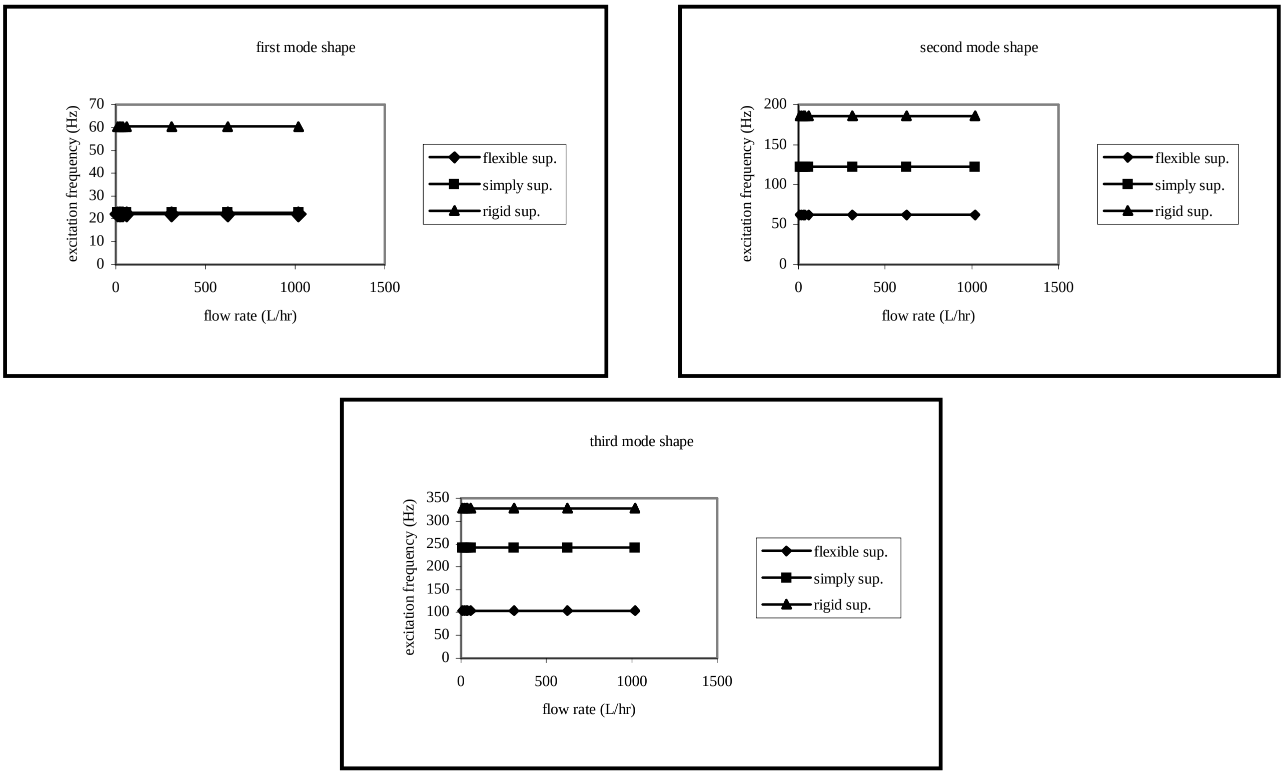


Fig.(5) Comparison for different kinds of supports for pipe conveying fluid without heat flux

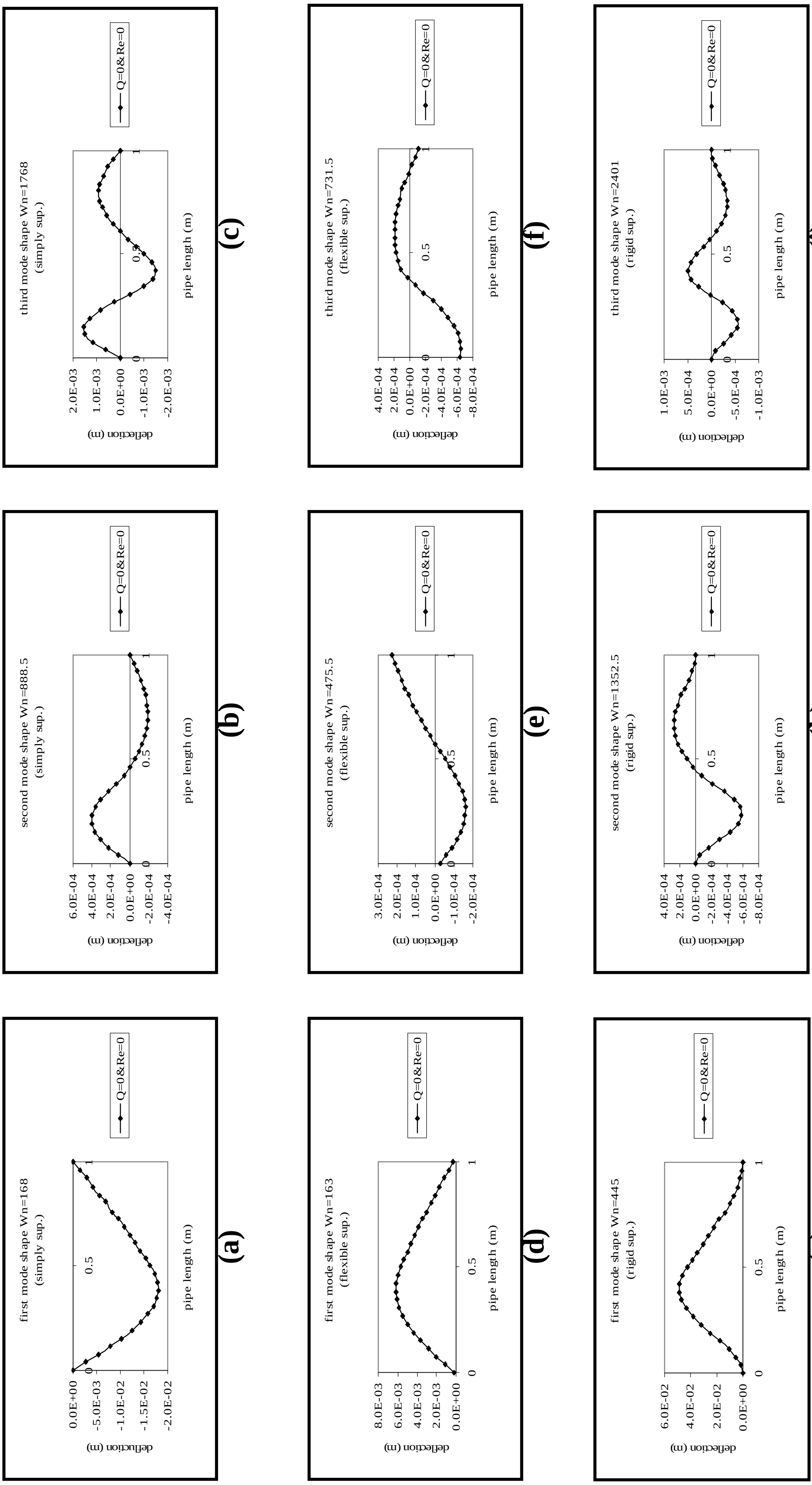
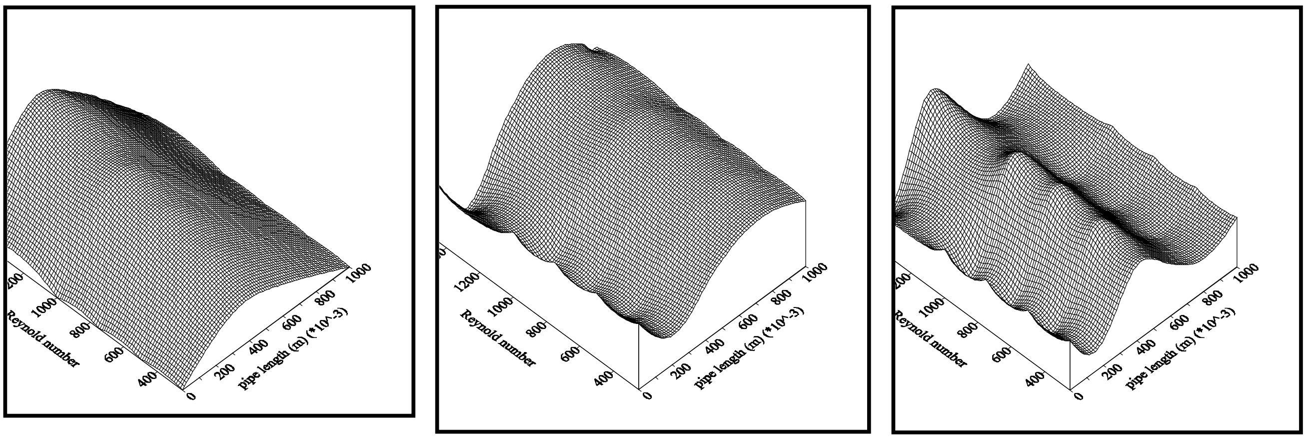
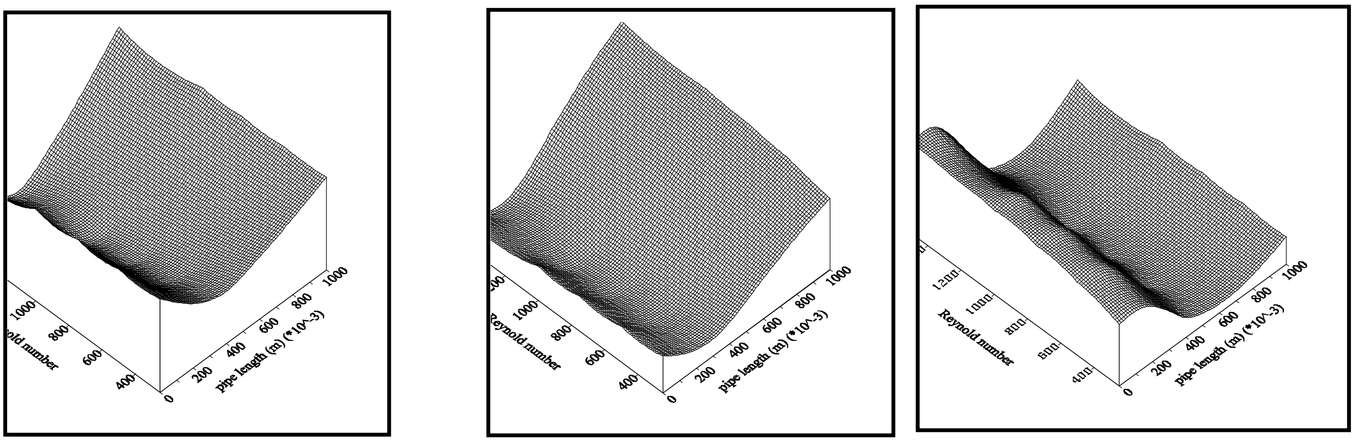


Fig.(6) The mode shapes for the pipe without (fluid&heat) and for different kinds of supports



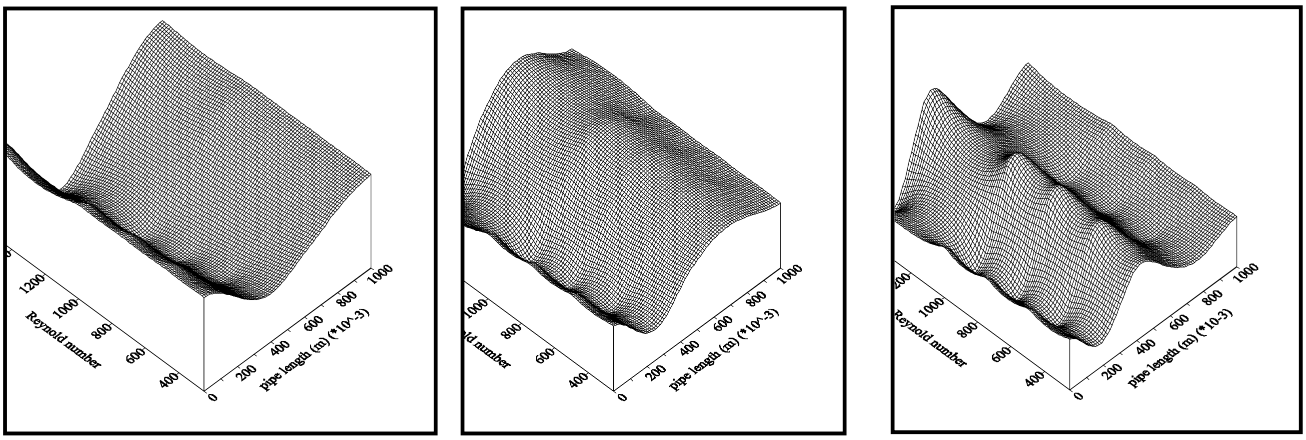
(a) First mode shape **(b) Second mode shape** **(c) Third mode shape**

Fig.(7)The mode shapes for pipe conveying fluid with different velocities and without heat flux (simply support)



(a) First mode shape **(b) Second mode shape** **(c) Third mode shape**

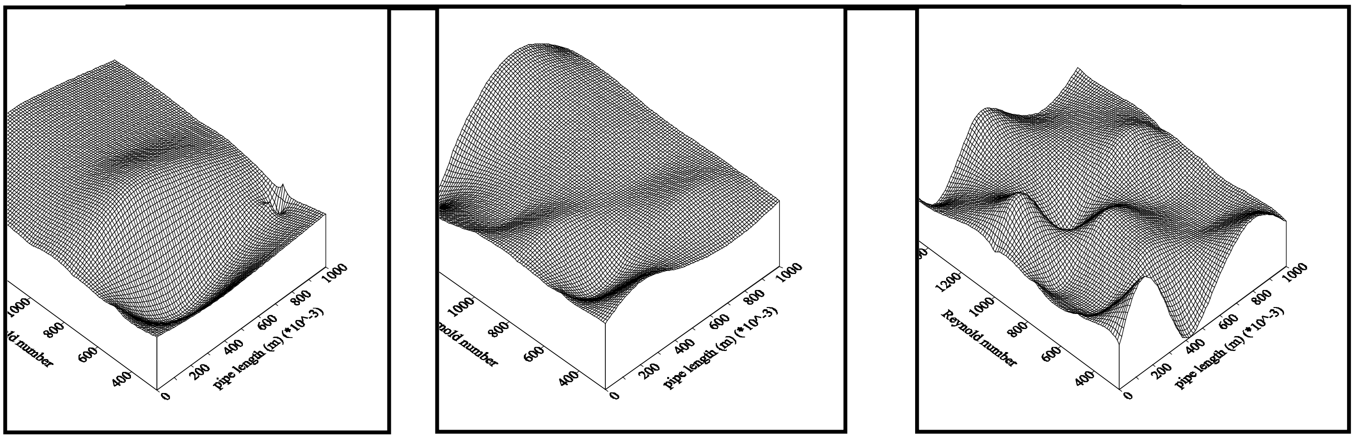
Fig.(8)The mode shapes for pipe conveying fluid with different velocities and without heat flux (flexible support)



(a) First mode shape **(b) Second mode shape** **(c) Third mode shape**

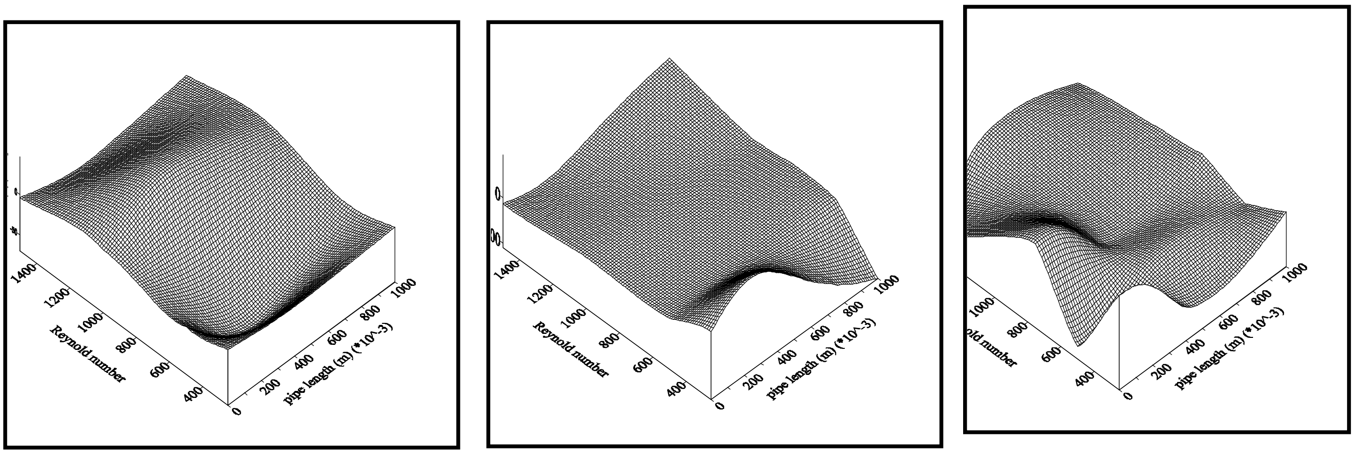
Fig.(9)The mode shapes for pipe conveying fluid with different

velocities and without heat flux (rigid support)



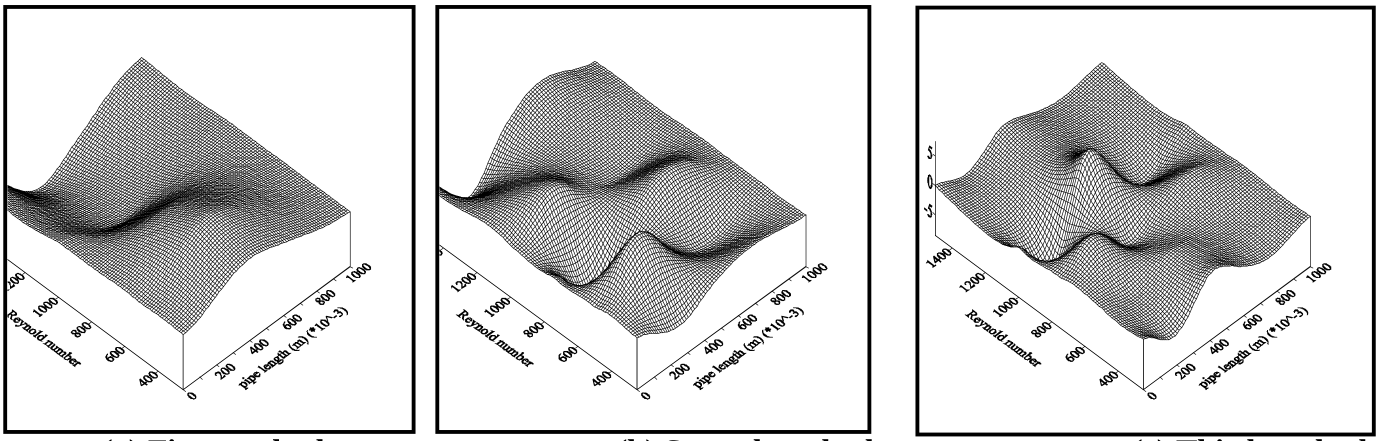
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(10)The mode shapes for pipe conveying fluid with different velocities and with heat flux=10kw/m^2 (simply support)



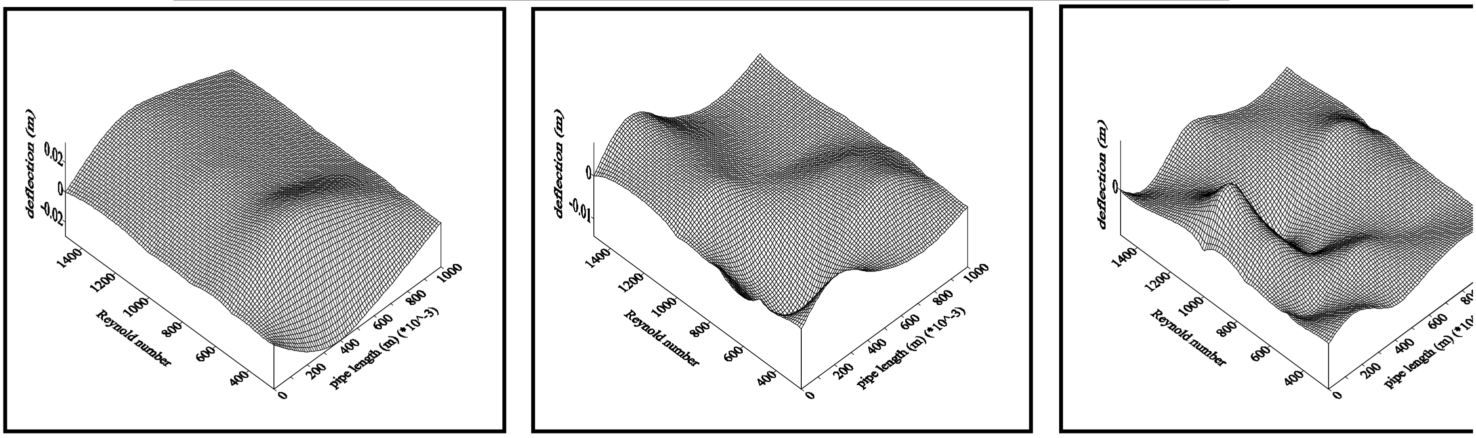
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(11)The mode shapes for pipe conveying fluid with different velocities and with heat flux=10kw/m^2 (flexible support)



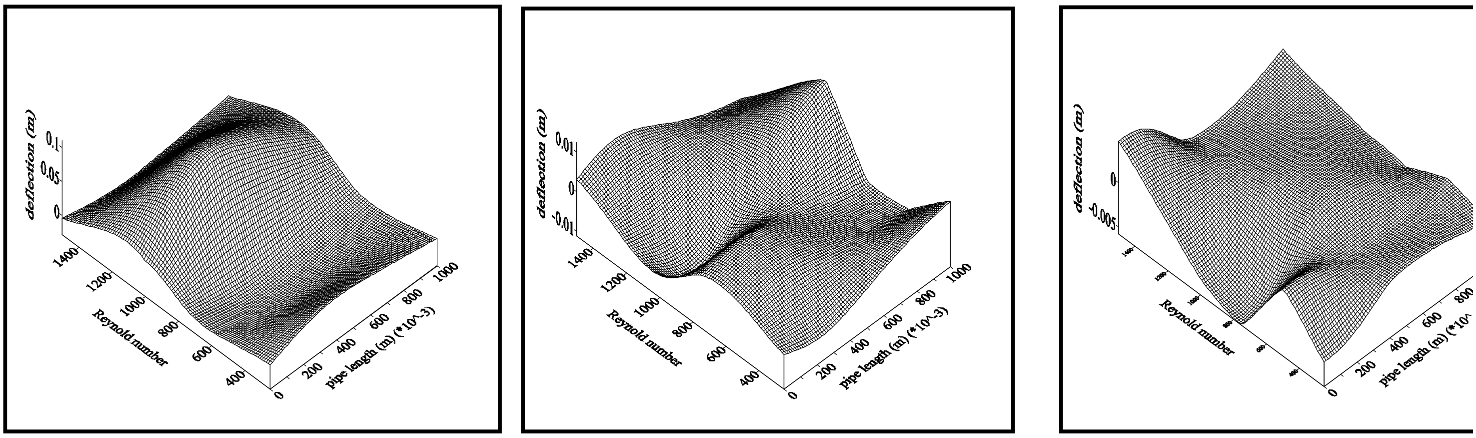
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(12)The mode shapes for pipe conveying fluid with different velocities and with heat flux=10kw/m^2 (rigid support)



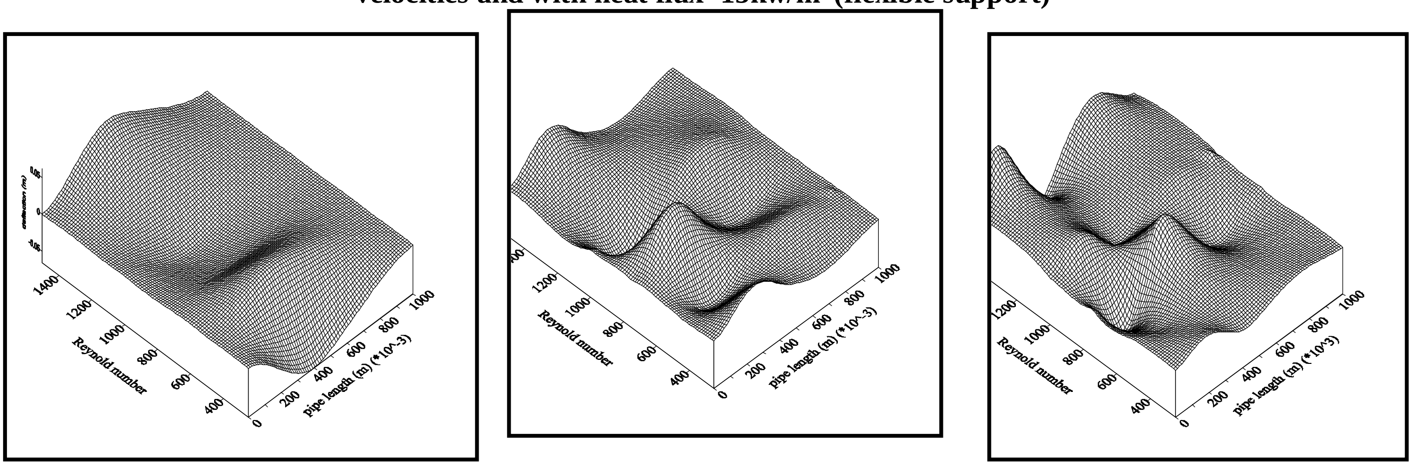
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(13)The mode shapes for pipe conveying fluid with different velocities and with heat flux=15kw/m^2 (simply support)



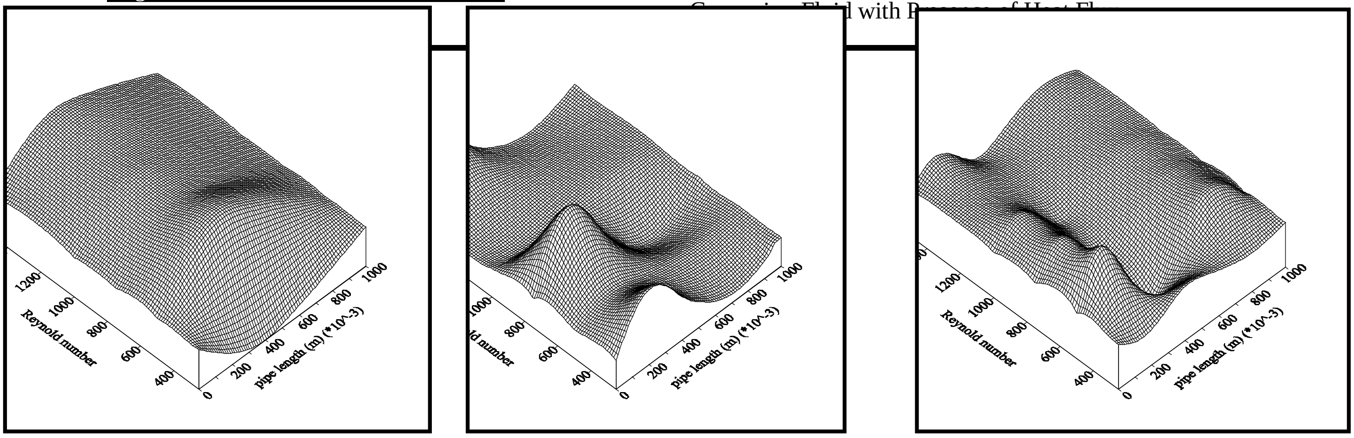
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(14)The mode shapes for pipe conveying fluid with different velocities and with heat flux=15kw/m^2 (flexible support)



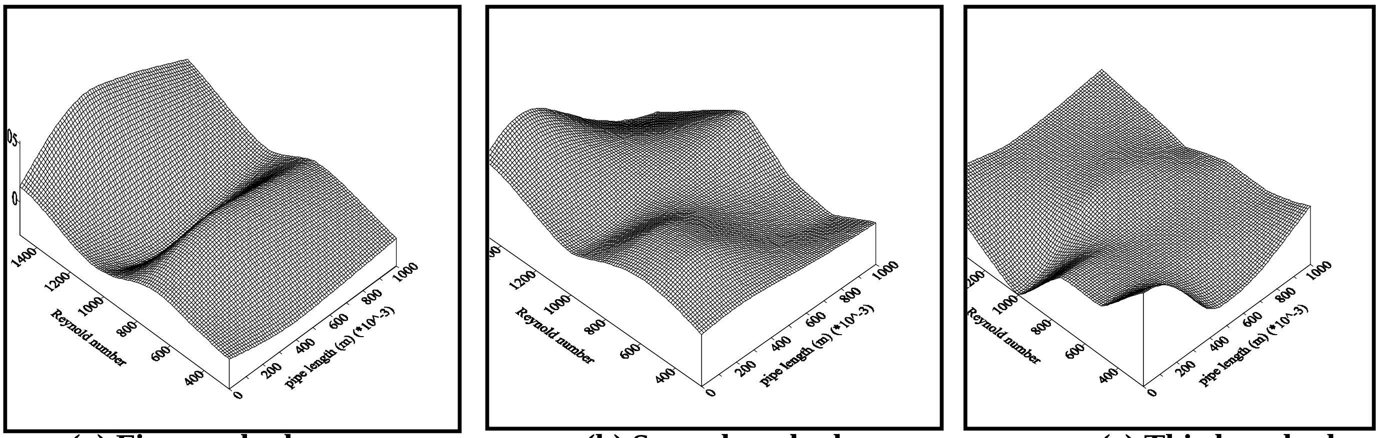
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(15)The mode shapes for pipe conveying fluid with different velocities and with heat flux=15kw/m^2 (rigid support)



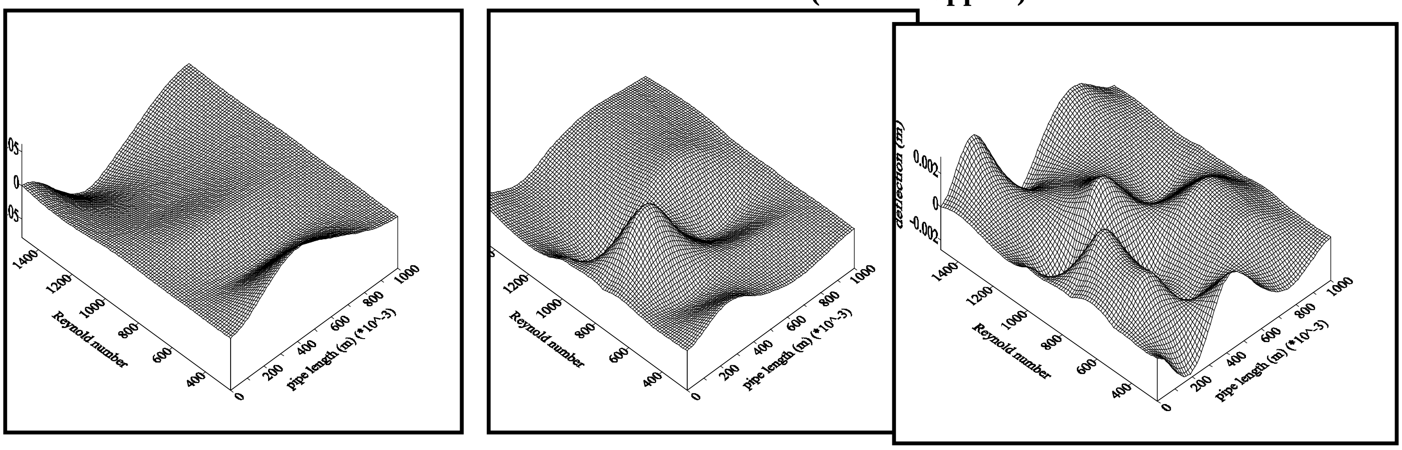
(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(16)The mode shapes for pipe conveying fluid with different velocities and with heat flux=20kw/m^2 (simply support)



(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(17)The mode shapes for pipe conveying fluid with different velocities and with heat flux=20kw/m^2 (flexible support)



(a) First mode shape (b) Second mode shape (c) Third mode shape

Fig.(18)The mode shapes for pipe conveying fluid with different velocities and with heat flux=20kw/m^2 (rigid support)

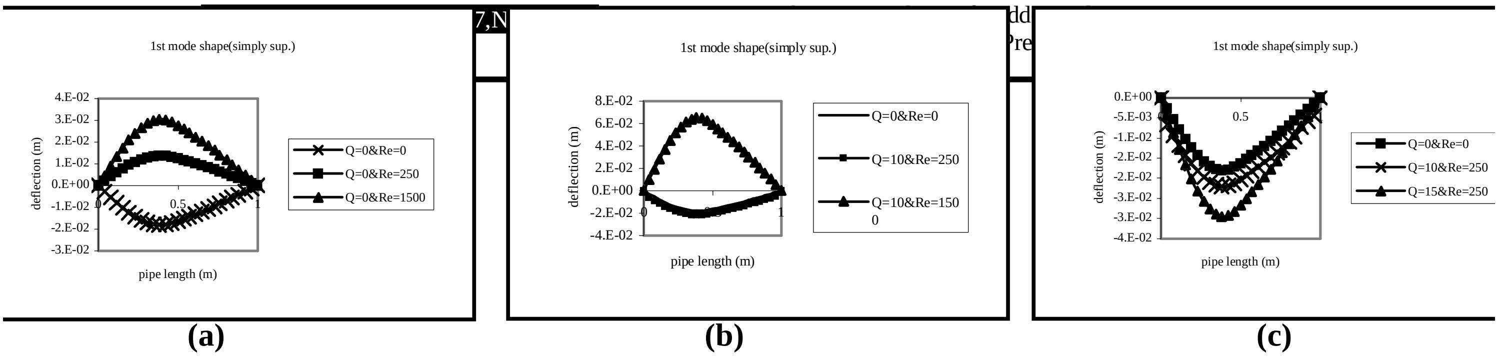


Fig.(19)Comparison between different cases of pipe at first natural frequency(simply support)

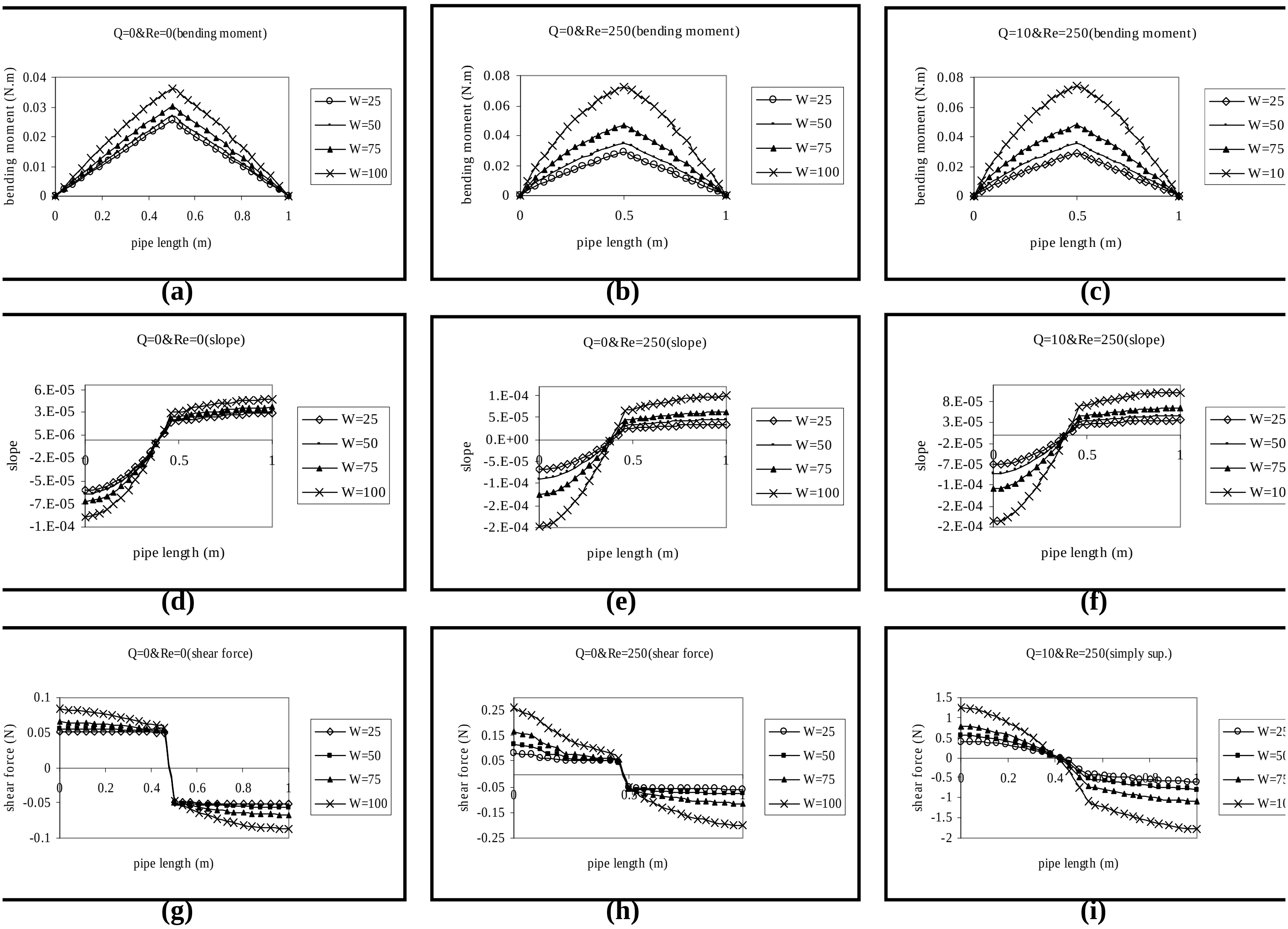


Fig.(20) The(bending moment, slope, shear force) for simply support pipe conveying fluid with various excitation frequencies and different velocities and heat flux

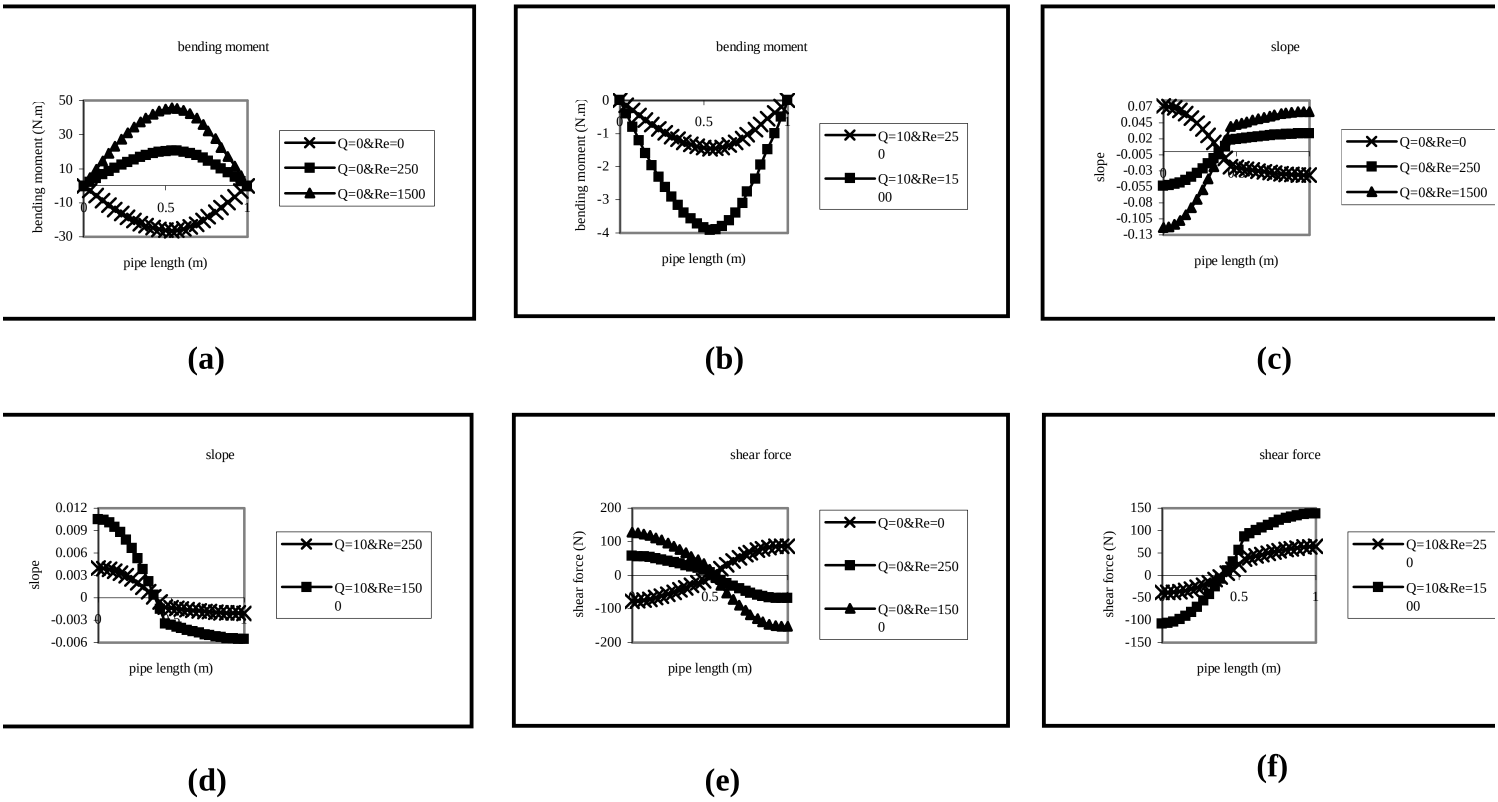


Fig.(21)Comparison the (bending moment, slope, shear force) for simply support pipe conveying fluid at the first natural frequency and different velocities and heat flux

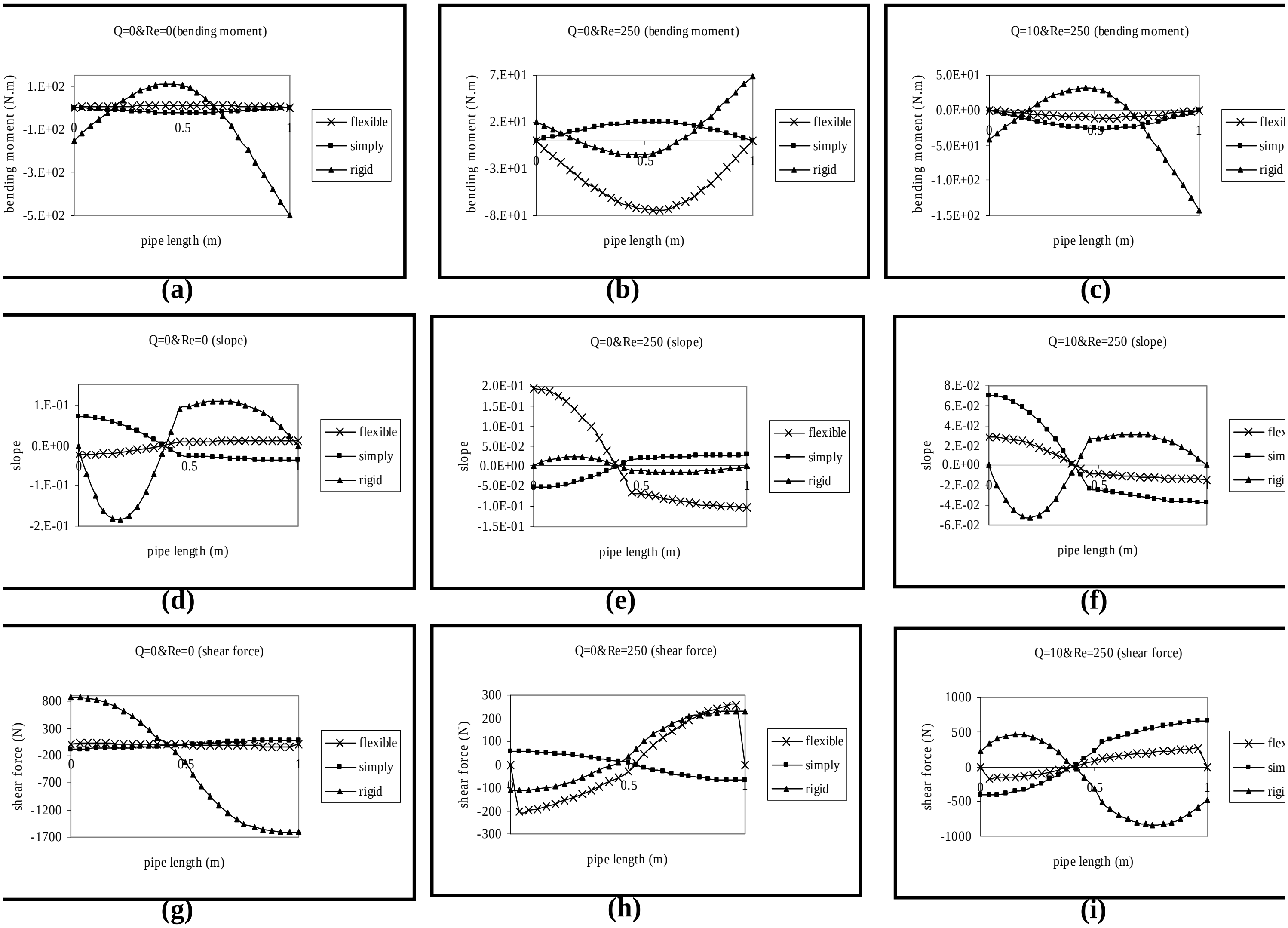


Fig.(22) Comparison the (bending moment, slope, shear force) for different kinds of supports with various velocities and heat flux at first natural frequency

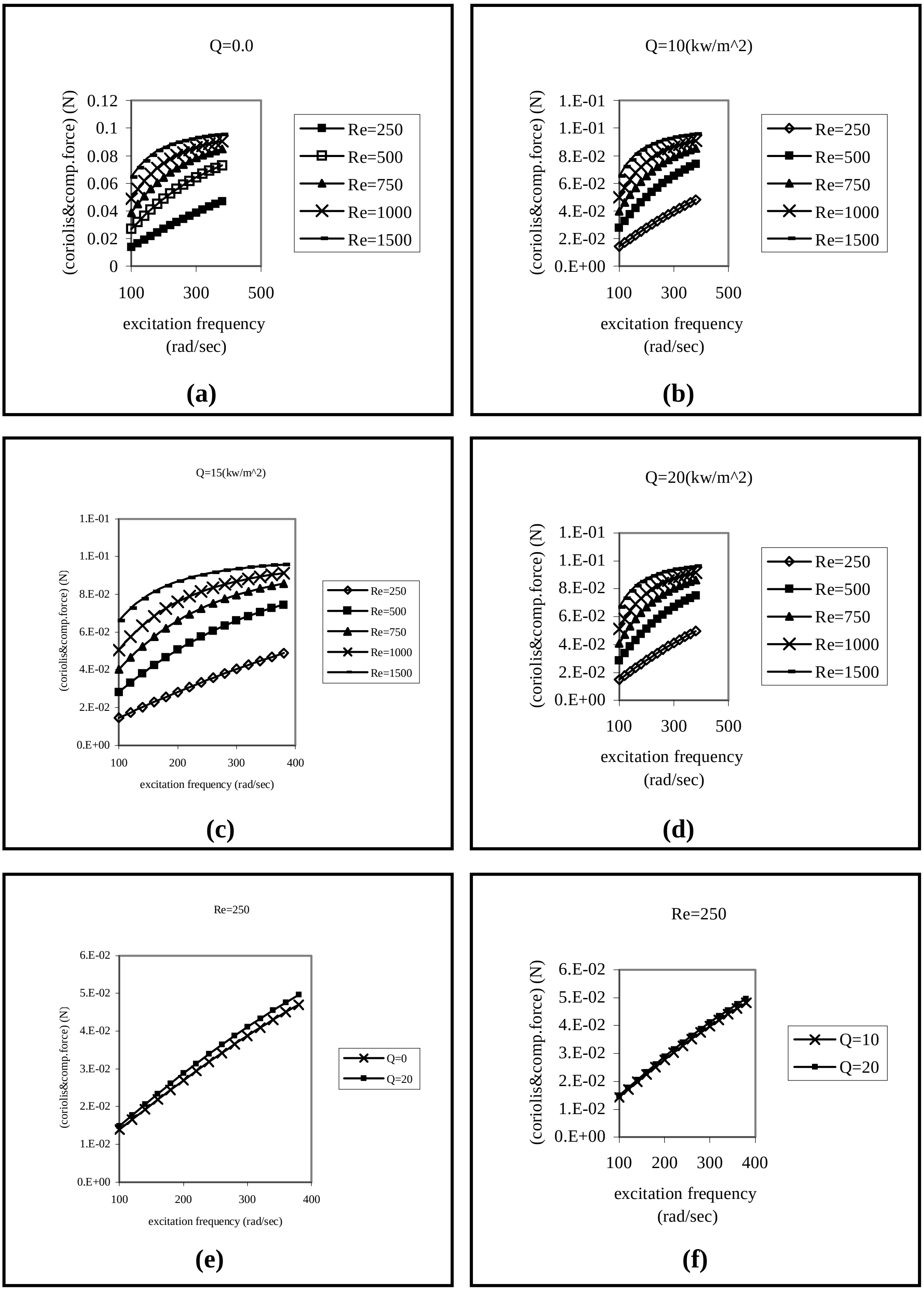


Fig.(23) Coriolis and compressive force for simply support pipe conveying fluid due to forced vibration at mid length with various velocities and heat flux