

## Design charts for channel shaped reinforced concrete short columns subjected to axial compressive load and uniaxial bending

Dr. Sarmad S. Abdul-Qader AL-Badri\* & Dr. Asma'a A. Ahmad\*

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### Abstract

Eight design charts are presented for reinforced concrete short C-columns subjected to axial compressive load plus uniaxial bending. For design these charts can be used for determining the required column dimensions and amount of steel, while for analysis these charts can be used for estimating the loaded column capacity.

Four examples are given to explain the use of design charts for both design and analysis, two of which are design examples while the other two are analysis. It has been shown by these examples that the new proposed charts are very simple to use in structural applications.

**Keywords:** Columns, computer program; interaction diagram; reinforced concrete; uniaxial bending.

### منحنيات التصميم للأعمدة الخرسانية القصيرة المسلحة ذات مقطع على شكل حرف C والمعرضة لحمل أنضغاط محوري و عزم انحناء أحادي المحور

#### الخلاصة

يقدم البحث الحالي ثمانية منحنيات لتصميم الأعمدة الخرسانية المسلحة القصيرة على شكل حرف C و المعرضة لحمل أنضغاط محوري و عزم انحناء أحادي المحور. في التصميم يمكن استخدام المنحنيات لإيجاد متطلبات العمود من حيث الأبعاد و حديد التسليح بينما في التحليل يمكن استخدام المنحنيات لإيجاد قابلية تحمل العمود للأحمال.

ولغرض توضيح و شرح عملية استخدام منحنيات التصميم المقترحة في التصميم و التحليل تم وضع أربعة أمثلة ، اثنان منها تصميمية و الاثنان الباقية تحليلية . لوحظ من هذه الأمثلة أن منحنيات التصميم المقترحة تمتاز ببساطتها في التطبيق الإنشائي.

### Introduction

Arbitrarily shaped reinforced concrete members subjected to uniaxial or biaxial bending with axial compression are frequently used in multistory tall buildings and bridge piers.

In the last decades some methods have been presented for the ultimate strength analysis of various concrete sections, such as L-, T- and channel-shaped, under symmetrical bending or combined biaxial bending and axial compression<sup>(1-5)</sup>. These methods compute the ultimate flexural capacity of section. For design

purposes they require trial and error procedures.

The present research also aims at obtaining direct relationships between the compression load and the uniaxial bending capacities which can be used as ready design charts for short C-columns.

#### Research significance

This search deals with reinforced concrete C-shaped cross sections commonly are used as columns and enclosures of the elevator shafts.

The principal aims of this work is to present a method for analysing tied short columns under the combined action of

\* Building and Construction Engineering Department, University of Technology/ Baghdad

axial compressive load and uniaxial bending that is simple in concept and can be beneficially used in providing easy way to deal with charts for the design of such columns.

#### Description of the procedure

For columns subjected to uniaxial bending, the neutral axis (N.A) always remains parallel to the axis about which the moment is being applied. Since the position of the neutral axis depends on the value of the eccentricity ( $e$ ), therefore the variation of the neutral axis position may in general leads to the two possible cases of compression zone shown in figure (1).

#### Estimating concrete compression force

Depending on an equivalent rectangular compression block for concrete, defined by ACI-318M-05<sup>(6)</sup> the compressive force of the concrete is

$$C_c = \text{area of compression zone} * (a_1 f'_c) \quad \dots(1)$$

Where  $a_1$  = ratio of the stress in the equivalent rectangular stress block to the specified compressive strength of concrete = 0.85, and

$f'_c$  = specified cylinder compressive strength of concrete.

#### Estimating strain in steel reinforcement

Based on the chosen value of ultimate usable strain at extreme concrete compression fibre ( $\epsilon_{cu}$ ) which is equal to 0.003 based on the ACI-318M-05 Code<sup>(6)</sup> and the linear strain distribution across the depth of the cross section; (figure 2), a correlation between the strain ( $\epsilon_{si}$ ) in any arbitrary reinforcing bar and the depth of the compression zone ( $c$ ) can be obtained. Let ( $d_{si}$ ) denotes the distance from the extreme compression fibre to the centroid of any arbitrary reinforcing bar.

Referring to figure (2), the strain in any steel bar can therefore be obtained

$$\epsilon_{si} = \frac{\epsilon_{cu}}{c} \left| c - d_{si} \right| \quad \dots(2)$$

since steel can be idealized as elastic-perfectly plastic material with maximum value of stress ( $f_y$ ), therefore the stress in any steel bar is simply

$$f_{si} = \epsilon_{si} E_s \leq f_y \quad \dots(3)$$

where  $f_{si}$  = stress in the reinforcement of the layer  $i$ ,

$E_s$  = modulus of elasticity of the steel reinforcement, and

$f_y$  = specified yield strength of the reinforcement.

#### Equilibrium criteria

For a given eccentricity ( $e$ ), the value of the compressive load ( $P$ ) can be estimated from the following simple equilibrium equation

$$P = C_c + \sum C_{si} - \sum T_{si} \quad \dots(4)$$

The associated uniaxial bending moment ( $M$ ) can also be estimated by summing up the moment of the resulting forces on the cross section around the plastic centroid ( $PC$ ) of the section,

$$M = C_c * \text{its lever arm to the PC} + \sum C_{si} * \text{its lever arm to the PC} + \sum T_{si} * \text{its lever arm to the PC} \quad \dots(5)$$

where  $C_{si}$  and  $T_{si}$  represent the compressive and tensile force in the  $i^{th}$  reinforcing bar respectively, figure (2). The subscript ( $i$ ) refers to the reinforcing steel layer position.

#### Program description

The computer program is developed in Microsoft Quick-Basic Version 4.5. It is capable of producing points that describe the axial load versus moment interaction diagram for any short C-column under uniaxial bending.

Input data for program include: the material and section properties, and the area and coordinates of each longitudinal bar. The output of the program consists of a series of data points ( $P$  and  $M$  values) that could be used in drawing the interaction diagram for the column.

The program assumed a linear variation of strain over the depth of the section. Strain hardening of steel, tensile strength of concrete, and slenderness effects are ignored. In addition, the

output does not include the capacity reduction factor ( $f$ ).

#### Flowchart

In order to simplify the analysis procedure of columns, flowchart is presented which demonstrate the steps that are followed for the analysis of channel short columns subjected to axial compression load plus uniaxial bending. The flowchart is used as step by step guides for manual computations.

#### Design charts

In order to make direct use of the present method in the design of reinforced concrete C-columns subjected to axial load and uniaxial bending moment, charts are constructed in a manner analogous to those given in text books for the case of rectangular or circular columns with uniaxial bending. Charts 1 through 8 have been prepared for the case of uniaxial bending of C-columns. These charts are designed to cover a wide range of the cross sectional parameters.

Figure (3) shows a cross section of a typical reinforced concrete C-columns. For convenience, these design charts are presented in this study which cover the following cases

$$\frac{b}{2t} = 2 \text{ and } 4$$

$$g = 0.6, 0.7, 0.8 \text{ and } 0.9$$

$$\text{No. of bars} = 16$$

$$f'_c \leq 30 \text{ MPa and } f_y = 414 \text{ MPa}$$

For values of ( $\frac{b}{2t}$ ) and ( $g$ ) other than

those listed above, linear interpolation between neighbouring values can be used.

#### Examples

##### Example (1) – Design problem

A short reinforced concrete C-column subjected to nominal compressive load of 3000 kN acting at a position with eccentricity  $e = 220 \text{ mm}$ . Use  $f'_c = 28 \text{ MPa}$ ,  $f_y = 414 \text{ MPa}$ , No. of bars = 16,  $b = 800 \text{ mm}$ ,  $t = 200 \text{ mm}$ ,  $h = 550 \text{ mm}$  and cover = 60 mm. It is required to

determine the steel ratio by using the proposed charts.

Solution:-

$$m = \frac{f_y}{0.85 * f'_c} = \frac{414}{0.85 * 28} = 17.39;$$

$$\frac{b}{2t} = \frac{800}{2 * 200} = 2,$$

$$g = \frac{550 - 2 * 60}{550} = 0.78;$$

$$\frac{e}{h} = \frac{220}{550} = 0.4;$$

$$\frac{Pn}{f'_c (2t) h} = \frac{3000 * 10^3}{28 * 2 * 200 * 550} = 0.487$$

From figure (6), ( $g = 0.7$  and  $\frac{b}{2t} = 2$ )

$$\text{with } \frac{e}{h} = 0.4 \text{ and } \frac{Pn}{f'_c (2t) h} = 0.487,$$

$$\text{read } r_t m = 0.285$$

From figure (7), ( $g = 0.8$  and  $\frac{b}{2t} = 2$ )

$$\text{with } \frac{e}{h} = 0.4 \text{ and } \frac{Pn}{f'_c (2t) h} = 0.487,$$

$$\text{read } r_t m = 0.23$$

Interpolating ( $g = 0.78$  and  $\frac{b}{2t} = 2$ ):

$$r_t m = 0.241; \therefore r_t = \frac{0.241}{17.39} = 0.0139$$

##### Example (2) – Design problem

A short reinforced concrete C-column subjected to nominal compressive load of 3500 kN acting at a position with eccentricity  $e = 290 \text{ mm}$ . Use  $f'_c = 25 \text{ MPa}$ ,  $f_y = 414 \text{ MPa}$ , No. of bars = 16,  $h = 580 \text{ mm}$ , cover = 58 mm and steel ratio ( $r_t = 0.045$ ). It is required to determine values of dimensions ( $b$  and  $t$ ) by using the proposed charts.

Solution :-

$$m = \frac{414}{0.85 * 25} = 19.48;$$

$$r_t m = 0.045 * 19.48 = 0.877;$$

$$g = \frac{580 - 2 * 58}{580} = 0.8;$$

$$\frac{e}{h} = \frac{290}{580} = 0.5$$

$$\text{assume } \frac{b}{2t} = 2 \text{ and } b = 800 \text{ mm}$$

$$\text{From figure (7), (g = 0.8 and } \frac{b}{2t} = 2)$$

$$\text{with } \frac{e}{h} = 0.5 \text{ and } r_t m = 0.877; \text{ read}$$

$$\frac{P_n}{f'_c (2t) h} = 0.62 \text{ therefore}$$

$$P_n = 0.62 * 25 * 2 * 200 * 580 * 10^{-3} \\ = 3596 \text{ kN}$$

Since this value agrees closely with actual nominal load, therefore  $b = 800 \text{ mm}$  and  $t = 200 \text{ mm}$  are acceptable

#### Example (3) – Analysis problem

A short reinforced concrete C-column subjected to nominal compressive load of ( $P_n$ ) acting at a position with eccentricity  $e = 240 \text{ mm}$ . Use  $f'_c = 30 \text{ MPa}$ ,  $f_y = 414 \text{ MPa}$ , No. of bars = 16,  $b = 1600 \text{ mm}$ ,  $t = 200 \text{ mm}$ ,  $h = 800 \text{ mm}$ , cover = 60 mm and steel ratio ( $r_t = 0.049$ ). It is required to determine the allowable nominal compressive load ( $P_n$ ) by using the proposed charts.

Solution:-

$$m = \frac{414}{0.85 * 30} = 16.23;$$

$$r_t m = 0.049 * 16.23 = 0.80;$$

$$g = \frac{800 - 2 * 60}{800} = 0.85;$$

$$\frac{b}{2t} = \frac{1600}{2 * 200} = 4; \frac{e}{h} = \frac{240}{800} = 0.3$$

$$\text{From figure (11), (g = 0.8 and } \frac{b}{2t} = 4)$$

$$\text{with } \frac{e}{h} = 0.3 \text{ and } r_t m = 0.8; \text{ read}$$

$$\frac{P_n}{f'_c (2t) h} = 1.19$$

$$\text{From figure (12), (g = 0.9 and } \frac{b}{2t} = 4)$$

$$\text{with } \frac{e}{h} = 0.3 \text{ and } r_t m = 0.8; \text{ read}$$

$$\frac{P_n}{f'_c (2t) h} = 1.26$$

$$\text{Interpolating (g = 0.85 and } \frac{b}{2t} = 4):$$

$$\therefore \frac{P_n}{f'_c (2t) h} = 1.225 \text{ therefore}$$

$$P_n = 1.225 * 30 * 2 * 200 * 800 * 10^{-3} \\ = 11760 \text{ kN}$$

#### Example (4) – Analysis problem

A short reinforced concrete C-column subjected to nominal compressive load of 3500 kN acting at a position with eccentricity  $e$ . Use  $f'_c = 20 \text{ MPa}$ ,  $f_y = 414 \text{ MPa}$ , No. of bars = 16,  $b = 1050 \text{ mm}$ ,  $t = 175 \text{ mm}$ ,  $h = 1000 \text{ mm}$ , cover = 50 mm and steel ratio ( $r_t = 0.0328$ ). It is required to determine the allowable nominal moment by using the proposed charts.

Solution :-

$$m = \frac{414}{0.85 * 20} = 24.35;$$

$$r_t m = 0.0328 * 24.35 = 0.80;$$

$$\frac{b}{2t} = \frac{1050}{2 * 175} = 3;$$

$$g = \frac{1000 - 2 * 50}{1000} = 0.9;$$

$$\frac{P_n}{f'_c (2t) h} = \frac{3500 * 10^3}{20 * 2 * 175 * 1000} = 0.5$$

$$\text{From figure (8), (g = 0.9 and } \frac{b}{2t} = 2)$$

$$\text{with } \frac{P_n}{f'_c (2t) h} = 0.5 \text{ and } r_t m = 0.8,$$

$$\text{read } \frac{e}{h} = 0.7$$

From figure (12), (  $g = 0.9$  and  $\frac{b}{2t} =$

$$4) \quad \text{with} \quad \frac{P_n}{f_c' (2t) h} = 0.5 \quad \text{and}$$

$$r_t m = 0.8, \text{ read } \frac{e}{h} = 0.95$$

Interpolating (  $g = 0.9$  and  $\frac{b}{2t} = 3$  ):

$$\text{therefore } e = 0.825 * 1000 = 825 \text{ mm}$$

$$\therefore M_{allowable} = 3500 * 825 * 10^{-3} \\ = 2887.5 \text{ kN.m}$$

### Conclusions

The analysis and design of reinforced concrete C-sections subjected to axial compression and uniaxial bending are tedious and time consuming because

1. In the analysis, a trial and error procedure is required to find the depth of the neutral axis satisfying the equilibrium conditions.
2. In the design process, a trial and error procedure is required to find

the steel ratio (  $r_t$  ) satisfying the strength requirements.

While the simplicity of the present approach enabled the construction of new design charts can be used directly in design.

### Notation

$b$	width of compression face of the cross section,
$c$	depth of compression zone,
$C_c$	calculated force for compression region,
$d_{si}$	distance from the extreme compression fibre to the centroid of any arbitrary reinforcing bar,
$f_c'$	specified compressive strength concrete of cylinder,
$f_y$	specified yield strength of reinforcement,
$g$	ratio of center-to-center distance between exterior layers of longitudinal reinforcement to overall depth of section,
$h$	overall thickness of the cross-section in plane of bending,
$m$	$= \frac{f_y}{0.85 f_c'}$
$M_n$	nominal flexural strength about the axis of bending,
$N.A$	neutral axis
$P_n$	nominal allowable load in uniaxial bending with eccentricity $e$ ,
$a_1$	ratio of the stress in the equivalent rectangular stress block to the specified

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compressive strength of concrete  $=0.85$ ,  
 $b_1$  ratio of the depth of the equivalent stress block to neutral axis depth,  
 $e_{cu}$  specified ultimate compressive strain of concrete  $=0.003$ , and  
 $r_t$  gross ratio of reinforcement.

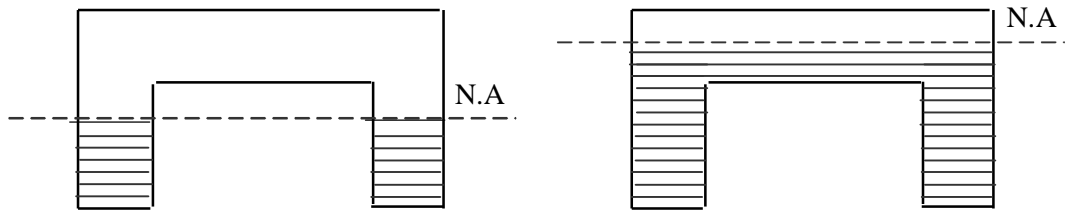


Figure (1) The two possible cases of compression zone.

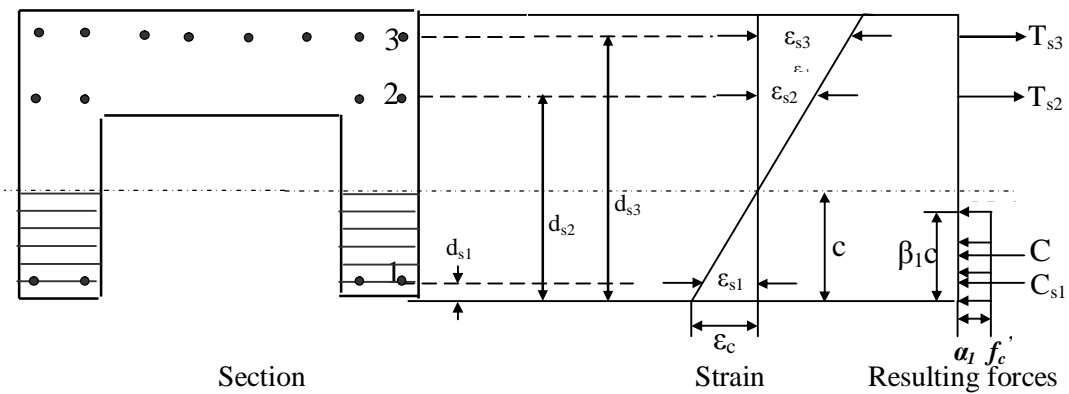


Figure (2) Eccentrically loaded C-columns.

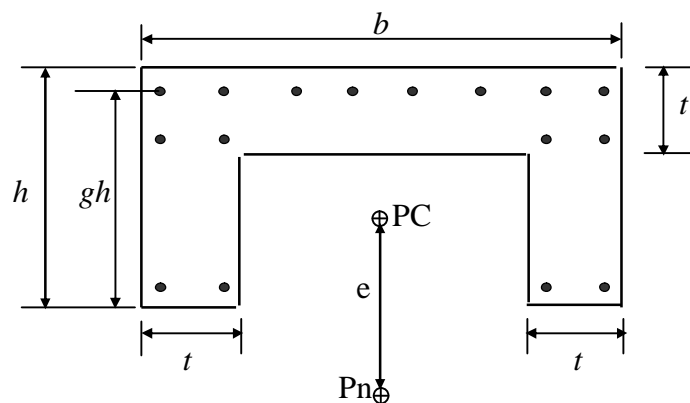
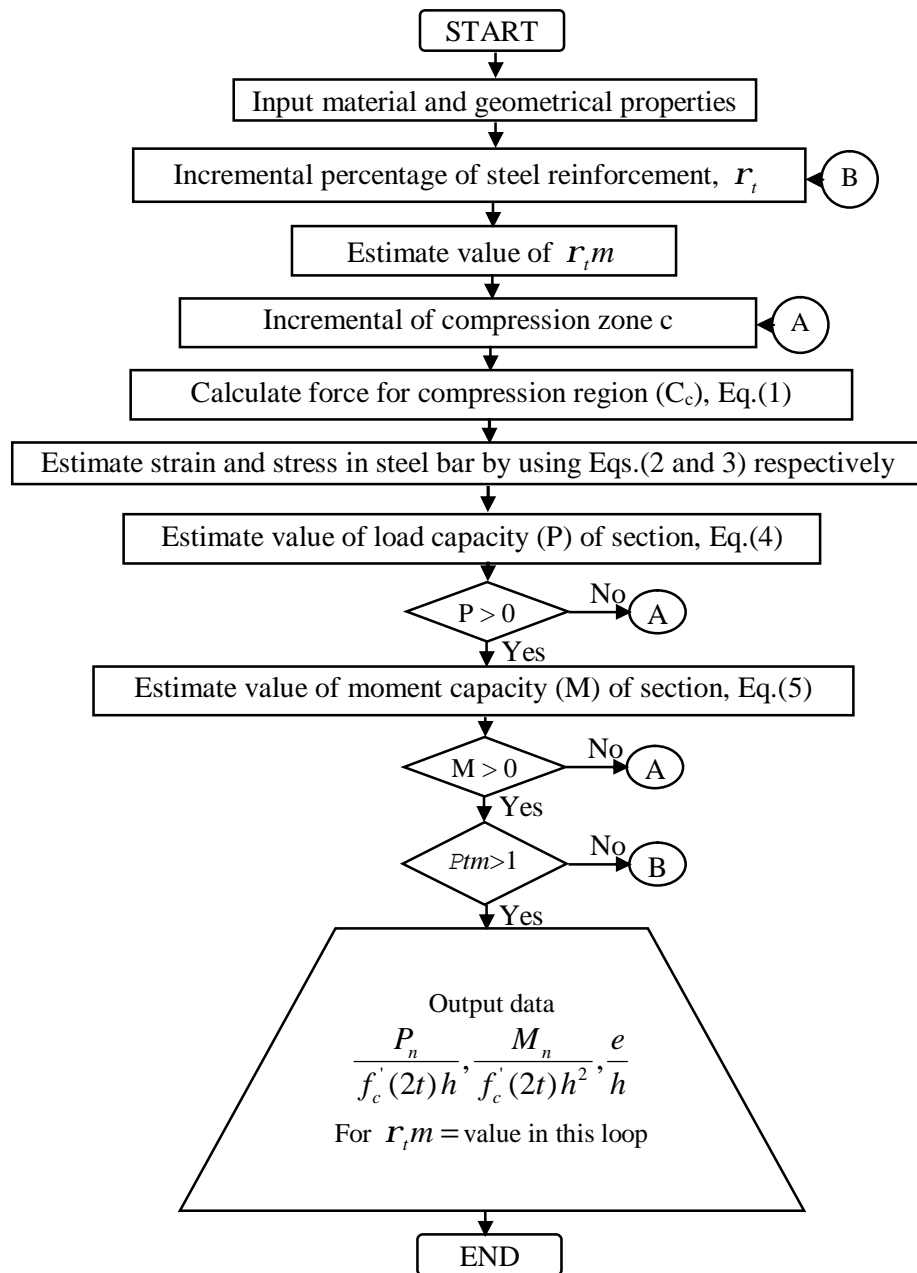


Figure (3) Symbol details of reinforced concrete C-column  
subjected to uniaxial bending.



**Figure (4) Flowchart analysis of channel shaped short columns subjected to axial compressive load plus uniaxial bending.**



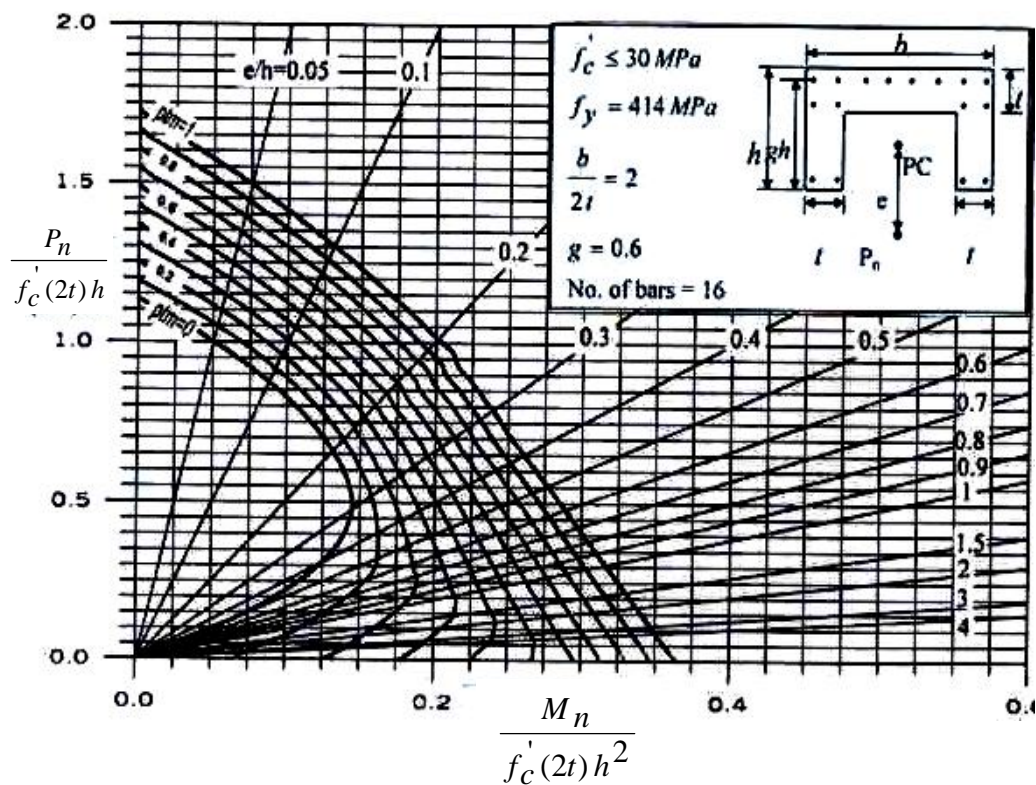


Figure (5) Column strength interaction diagram for C-section with  $g=0.6$

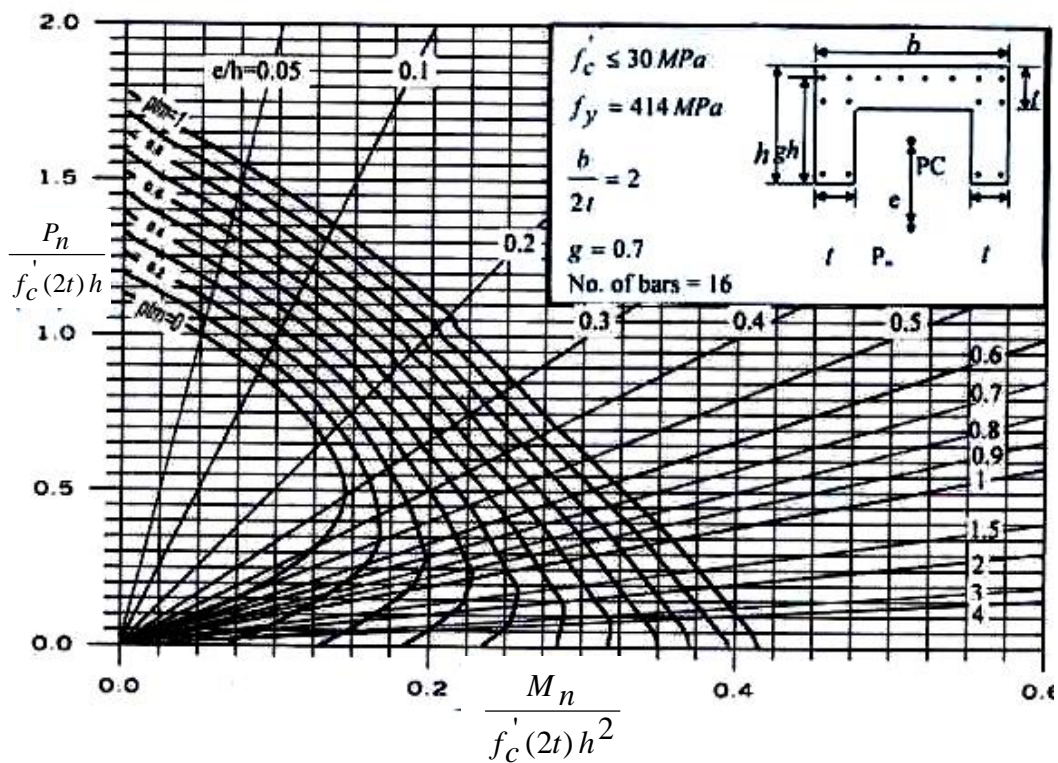


Figure (6) Column strength interaction diagram for C-section with  $g=0.7$



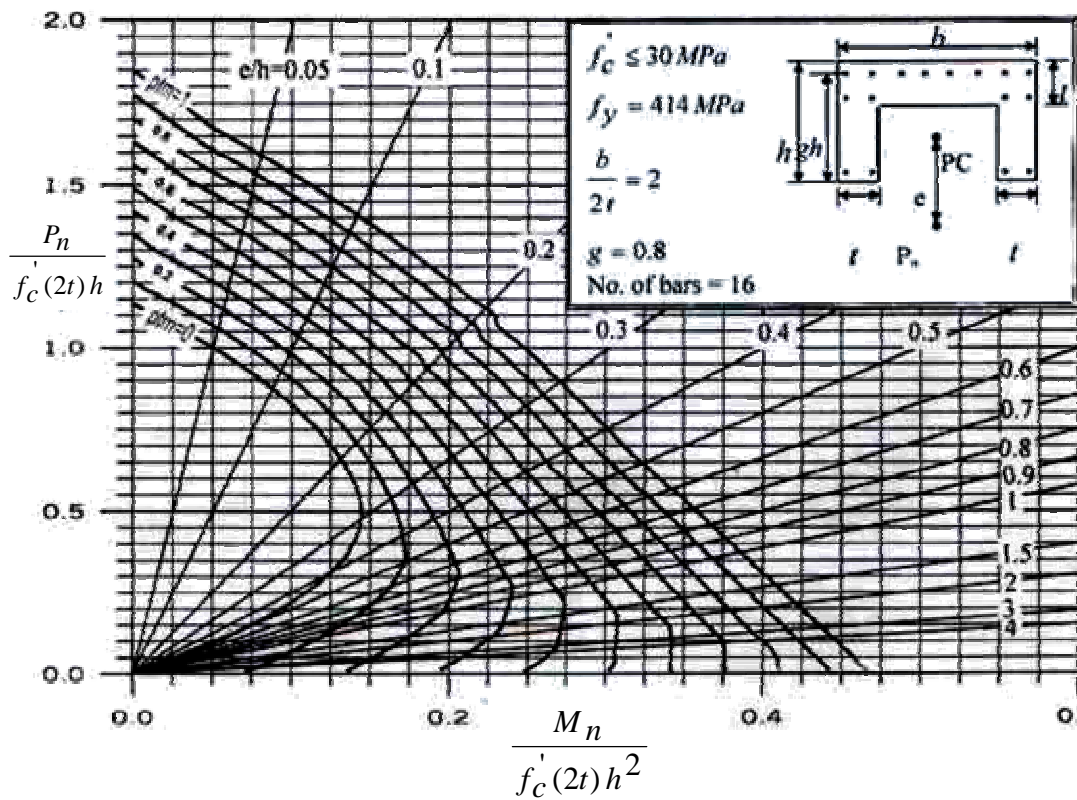


Figure (7) Column strength interaction diagram for C-section with  $g=0.8$

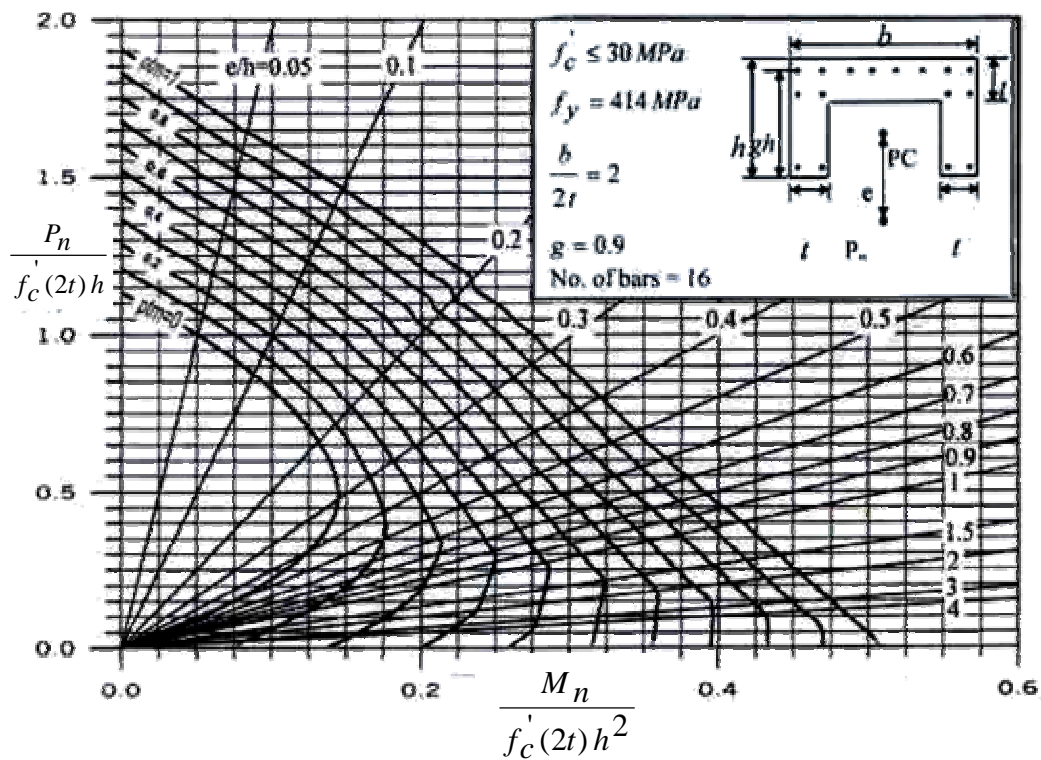


Figure (8) Column strength interaction diagram for C-section with  $g=0.9$

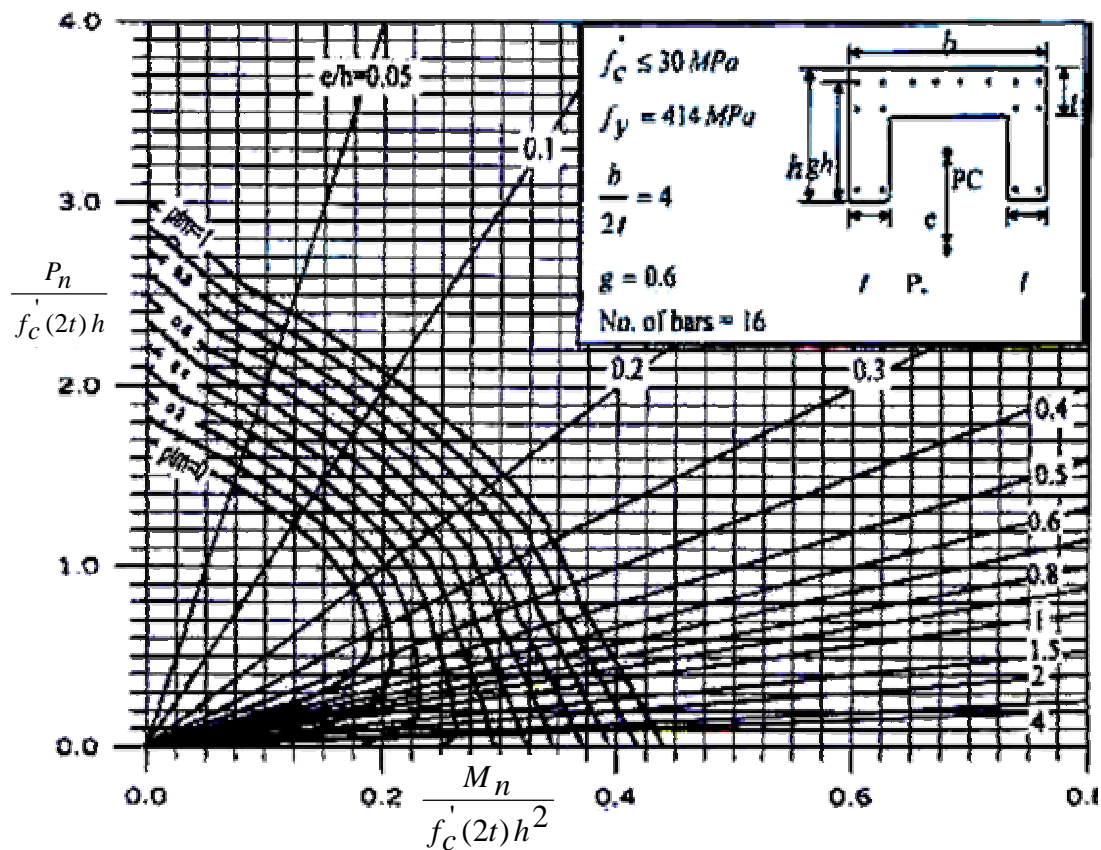


Figure (9) Column strength interaction diagram for C-section with  $g=0.6$

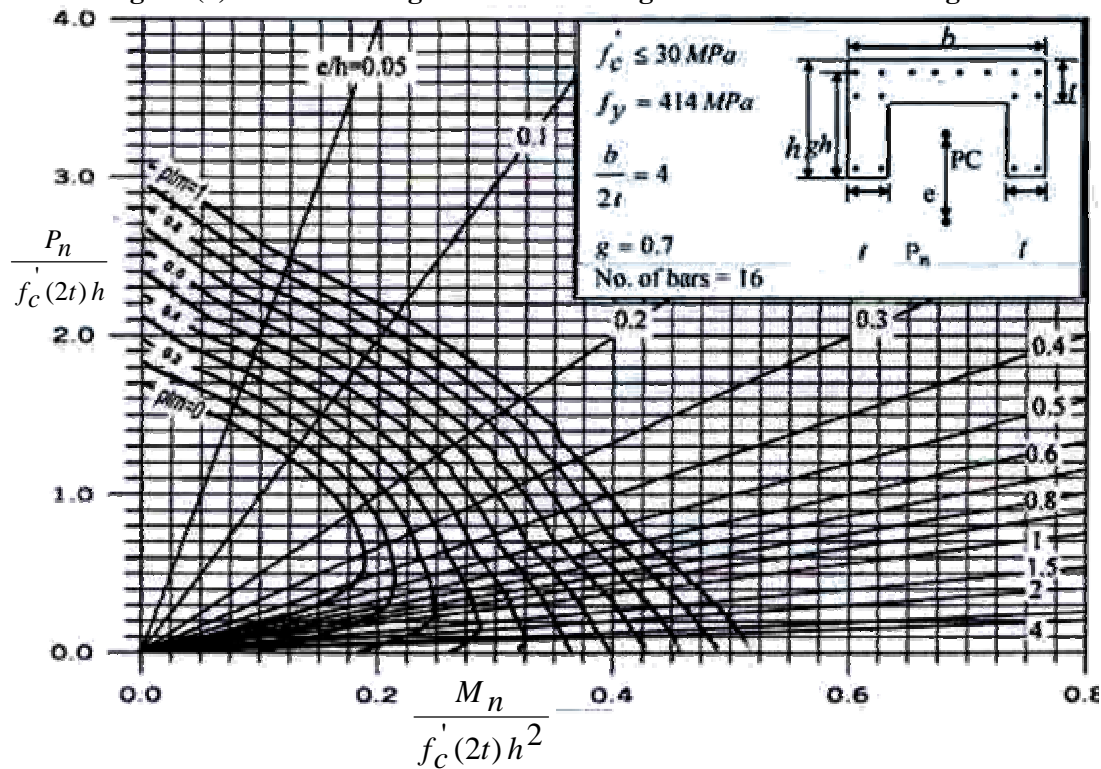


Figure (10) Column strength interaction diagram for C-section with  $g=0.7$

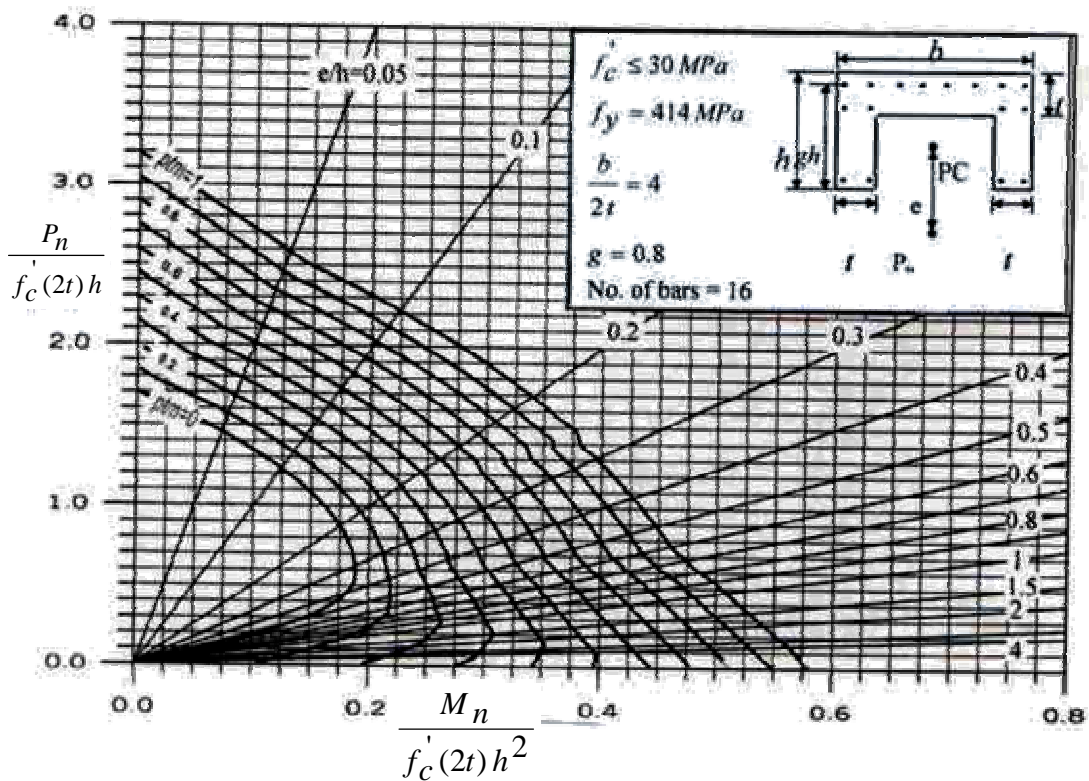


Figure (11) Column strength interaction diagram for C-section with  $g=0.8$

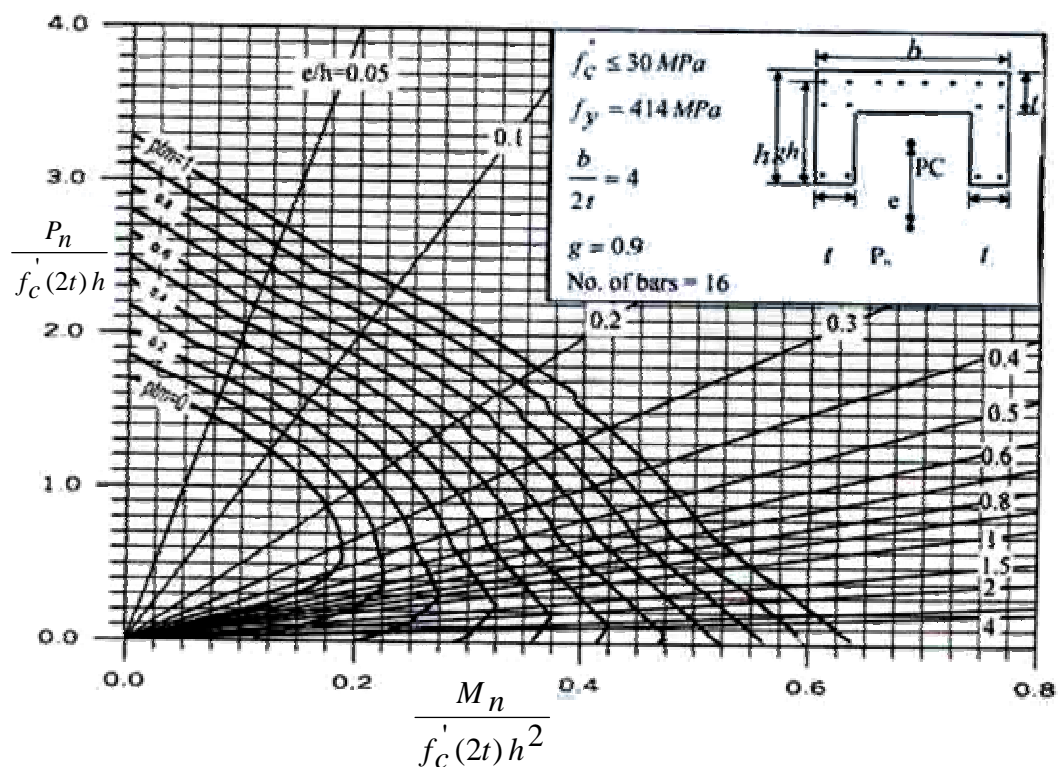


Figure (12) Column strength interaction diagram for C-section with  $g=0.9$