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Odd Generalized Rayleigh- Exponential Distribution Statistical Properties with Real Data Application

Ali Mahmood Monef *

Ministry of Education, Kirkuk Education Directorate, Kirkuk, Iraq.
*Corresponding author : 909alimahmood909@gmail.com

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Abstract

In this paper we offer a new distribution due to the most common problem for researchers is to determine a statistical model for analyzing lifetime data. There are many ways to add a scaling parameter or a shape parameter by generalizing the distributions. So statistical model called the Generalized Rayleigh Exponential Distribution (OGRE) is presented. Some different properties of this distribution including moments, moment generating function, hazard function, survival function, ordered statistics and piecewise function are studied. Maximum likelihood estimators are used to estimate the parameters. In addition, the practical significance of OGRE is demonstrated by two real data sets and the results are compared with a set of other related distributions. simulation is also performed.

1. Introduction:

The generalized Rayleigh distribution is an extension of the Rayleigh distribution, characterized by its ability to model a wider range of data types. It is particularly useful in scenarios were data exhibit non-linear relationships, allowing for flexibility in capturing complex patterns. This distribution is defined by its probability density function, which incorporates parameters that adjust the shape and scale, making it suitable for various applications, including reliability analysis and signal processing. Understanding its properties enhances statistical modeling, particularly in fields requiring nuanced data interpretation.

The exponential distribution is one of the most common classical distributions with one parameter and has been widely used to analyze lifetime data. However, the fixed-shaped exponential distribution of the hazard function is not suitable for data with different shapes of hazard rates as decreasing, increasing, bathtub, or unimodal shaped failure rates, often encountered in engineering and reliability, among others.

Several generalized exponential distributions with additional parameters have been developed to enhance modeling flexibilityGeneralized Rayleigh Distribution: Introduced as a two-parameter distribution, it includes shape and scale parameters, allowing for skewness and flexibility in lifetime data analysis [1]. Generalized Rayleigh-Truncated Negative Binomial Distribution: This four-parameter model incorporates negative binomial characteristics, providing better fits for various datasets compared to traditional models [2]. Modified Slashed Generalized Rayleigh Distribution: This three-parameter family extends the generalized Rayleigh distribution, focusing on structural properties and reliability analysis [3]. These distributions are valuable in fields like reliability engineering and survival analysis due to their adaptability to different data characteristics. Kundu and Gupta was studied The Parameter estimation using the Bayes method for the generalized exponential distribution [4]. Raqab and Madi used Bayesian estimation and prediction of the generalized exponential distribution, using prior information, were considered [5]. Rodriguez et al. estimated the parameters using more than one method of the Poisson exponential distribution [6].

Ristić and Kundu relied on Marshall Olkin's method to introduce a shape parameter into two-parameter generalized exponential distribution [7]. Oguntunde and Adejumo present a new two-parameter model that represents a generalization of the inverse exponential distribution using the transformed quadratic map [8]. the Marshall Olkin exponential by Marshall and Olkin [9]. Harris extended exponential by Pinho et al. [10]. modified exponential by Rasekhi et al.[11]. Kumaraswamy transmuted exponential by Afify et al. [12]. odd log-logistic Lindley exponential by Alizadeh et al. [13]. The beta exponential distribution by Nadarajah and Kotz [14]. Extended exponential distribution by Kumar et al. [15]. The gamma-exponentiated exponential distribution by Ristić and Balakrishnan [16]. Polynomial-exponential distribution by Sah and Sahani [17]. Lindley–exponential distribution by Bhati et al. [18].

The aim of this research is to find a new distribution that gives the best results in data modeling and analysis and helps in understanding and interpreting the results. The results are also displayed visually, such as tables and graphs.

2. Odd Generalized Rayleigh- Exponential distribution (OGR-E):

In this section we will expand a family odd Generalized Rayleigh family [19] with an exponential distribution, where the cumulative distribution function CDF of the family is known as.

$$G(x,\zeta)_{OGR-G} = \left[1-e^{-\rho\left(\frac{[F(x,\zeta)]^2}{1-F(x,\zeta)}\right)}\right] \tag{1}$$

The probability density function PDF is known as:

$$g(x,\zeta)_{OGR-G} = 2\delta\rho[F(x,\zeta)]^3 f(x,\zeta)(2-F(x,\zeta))(1-F(x,\zeta))^{-3}$$

$$\times\left[1-e^{-\rho}\left(\frac{F(x,\zeta)]^2}{1-F(x,\zeta)}\right)^2\right]^{\delta-1} e^{-\rho}\left(\frac{F(x,\zeta)]^2}{1-F(x,\zeta)}\right)^2 \tag{2}$$

The cumulative distribution function and the probability density function of the exponential distribution are also known as

$$G(x,\beta) = 1-e^{-\beta x} \tag{3}$$

$$g(x,\beta) = \beta e^{-\beta x}, x>0, \delta>0 \tag{4}$$

By substituting equation 3 into equation 1 we obtain CDF for the distribution OGR-E as

$$G(x,\rho,\beta,\delta) = \left[1-e^{-\rho\left(\frac{[1-e^{-x,\beta}]^2}{e^{-\beta x}}\right)^2}\right]^{\delta} \tag{5}$$

To expand the function C D F we perform some mathematical operations, from which we obtain:

$$G(x,\rho,\beta,\delta) = \Pi\iota,q,t^{e^{-\beta x(t-2q)}} \tag{6}$$

Where

$$\Pi r,q,t = \sum_{\iota=q=t=0}^{\infty}(-1)^{i+q+t}\frac{(\delta\rho)^q}{q!}\binom{\delta}{i}\binom{4q}{t}$$

And By substituting equation 3 and 4 into equation 2 we obtain PDF for the distribution OGRE as

$$g(x,\rho,\beta,\delta) = 2\delta\rho\beta e^{\beta x}[1-e^{-\beta x}]^3(1+e^{-\beta x})$$
$$e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}\left[1-e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}\right]^{\delta-1} \tag{7}$$

In the same way to expand the function P D F, we get:

$$g(x,\rho,\beta,\delta) = ¥_{\iota,q,t}\beta e^{-\beta x(t-1-2q)} + ¥_{\iota,q,t}\beta e^{-\beta x(t-2q)} \tag{8}$$

Where

$$¥_{\iota,q,t} = 2\sum_{\iota=q=t=0}^{\infty}\frac{(-1)^{\iota+q+t}\rho^{q+1}(1+\iota)^q}{q!}\binom{\delta-1}{\iota}\binom{3+4q}{t}$$

The probability density function can be shown in Figure 1 and the cumulative distribution function in Figure 2.

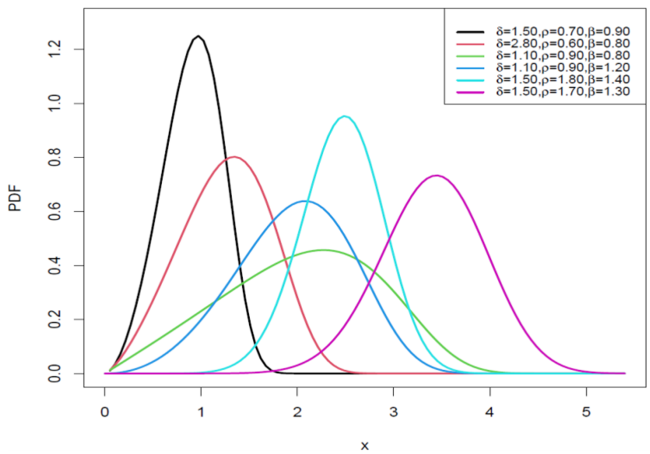

Figure 1. PDF plot for the OGRE distribution.

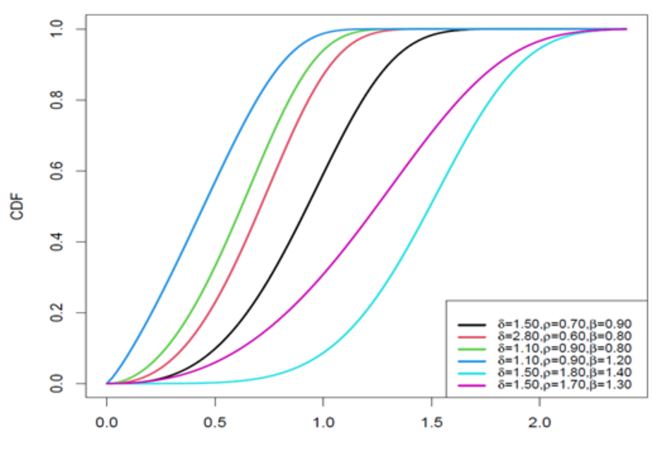

Figure 2. CDF plot for the OGRE distribution.

3. Survival function:

The survival function can be written in the form:

$$S(x) = 1 - G(x) \tag{9}$$

$$S(x,\rho,\beta,\delta)_{ORGE} =$$

$$1 - G(x,\rho,\beta,\delta)_{ORGE} = 1 - \left[1 - e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}\right]^{\delta} \tag{10}$$

4. Hazard function:

The formula [20] can be used to find the risk function in the form

$$h(x,\rho,\beta,\delta) = \frac{g(x,\rho,\beta,\delta)}{S(x,\rho,\beta,\delta)} \tag{11}$$

$$h(x,\rho,\beta,\delta) =$$

$$\frac{2\delta\rho\beta e^{\beta x}[1-e^{-\beta x}]^3(1+e^{-\beta x})e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}}{1-} \times \frac{\left[1-e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}\right]^{\delta-1}}{\left[1-e^{-\rho\left(\frac{[1-e^{-\beta x}]^2}{e^{-\beta x}}\right)^2}\right]^{\delta}} \tag{12}$$

5. Some properties:

In this section, we will find some properties of the proposed distribution (OGRE), such as moments, the moments generating function, the Quantile function, and ordered statistics.

5.1 Moments:

Mathematical expectation is one of the statistical properties that is used in moments. It is divided into two parts. The initial part is called the torque about the origin. The next part is called the central moments. We can obtain the moments about the origin through the following relationship based on the probability density function from equation 7.

$$M_r = E(x^r) = \int_{-\infty}^{\infty} x^r g(x,\rho,\beta,\delta)dx \tag{13}$$

$$M_r = \int_{-\infty}^{\infty} x^r (¥_{\iota,q,t}\beta e^{-\beta x(t-1-2q)} + ¥_{\iota,q,t}\beta e^{-\beta x(t-1-2q)})dx \tag{14}$$

$$M_r = \frac{¥_{\iota,q,t}}{\beta^r(t-1-2q)}\Gamma(r+1) + \frac{¥_{\iota,q,t}}{\beta^r(t-2q)}\Gamma(r+1)$$

Table 1. represents the values of the first, second, third and fourth moments.

δ	ρ	β	μ_1	μ_2	μ_3	μ_4
2	1.5	0.2	0.1714	0.6274	2.4250	9.8105
2	1.5	0.3	0.1143	0.2788	0.7185	1.9378
2	1.7	0.2	0.1621	0.6131	2.4311	10.038
2	1.7	0.3	0.1080	0.2724	0.7203	1.9828
2.5	1.5	0.2	0.1892	0.6621	2.4444	9.4434
2.5	1.5	0.3	0.1261	0.2942	0.7242	1.8653
2.5	1.7	0.2	0.1798	0.6495	2.4590	9.6911
2.5	1.7	0.3	0.1198	0.2886	0.7286	1.9143
3	1.5	0.2	0.2042	0.6882	2.4468	9.0991
3	1.5	0.3	0.1361	0.3058	0.7249	1.7973
3	1.7	0.2	0.1947	0.6773	2.4679	9.3589
3	1.7	0.3	0.1298	0.3010	0.7312	1.8486

5.2 Moments Generating Function:

The function that generates the moments is of great importance because it generates all the moments. It is symbolized by M_x (t) for the new distribution (OGRE) and its formula is:

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} g(x)dx = \int_{-\infty}^{\infty} e^{tx} g(x,\rho,\beta,\delta)_{OGRE}dx \tag{15}$$

$$M_x(t)_{OGRE} = \sum_{r=0}^{\infty}\frac{t^r}{r!}\int_0^{\infty} x^r g(x,\rho,\beta,\delta)_{OGRE}dx = \sum_{r=0}^{\infty}\frac{t^r}{r!}E(x^r)$$

$$M_x(t)_{OGRE} = \sum_{r=0}^{\infty}\frac{t^r}{r!}\left(\frac{¥_{\iota,q,t}}{\beta^r(t-1-2q)}\Gamma(r+1) + \frac{¥_{\iota,q,t}}{\beta^r(t-2q)}\Gamma(r+1)\right) \tag{16}$$

5.3 Quantile function:

The quantile function is the inverse of the cumulative distribution function and is of great importance because it is considered one of the ways to determine the probability function, as well as to find the kurtosis and skewness for distributions with large skewness or that do not have moments, and it is found from the relationship $Q(u) = F^{-1}$ (x) [21].

$$x = \frac{ln\left(\sqrt{\frac{ln\left(1-u^{\frac{1}{\delta}}\right)}{\rho}}+2\right)^{-1}}{\beta^2} \tag{17}$$

5.4 Order Statistic:

Order statistics are very important in many areas of practice and statistical theory. Let $x_1, x_2, ..., x_n$ be a random sample of size n from the distribution (OGRE) which has a cumulative function in equation 6 and a probability density function in equation 8 and its ordered value is denoted by $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ where we can obtain statistics ranked according to [22].

$$f_{\tau,n}(x) = \frac{n!}{(\tau-1)!(n-\tau)!}f(x)[F(x)]^{\tau-1}[1-F(x)]^{n-\tau} \tag{18}$$

$$f_{\tau,n}(x,\rho,\beta,\delta)_{OGRE} = \frac{n!}{(\tau-1)!(n-\tau)!}f(x,\rho,\beta,\delta)_{OGRE}[F(x,\rho,\beta,\delta)_{OGRE}]^{\tau-1}[1-F(x,\rho,\beta,\delta)_{OGRE}]^{n-\tau} \tag{19}$$

$$f_{\tau,n}(x,\rho,\beta,\delta)_{OGRE} = \frac{n!}{(\tau-1)!(n-\tau)!}(¥_{\iota,q,t}e^{-\beta x(t-1-2q)} + ¥_{\iota,q,t}e^{-\beta x(t-2q)}) \times [\Pi_{\iota,q,t}e^{-\beta x(t-2q)}]^{\tau-1}[\Pi_{\iota,q,t}e^{-\beta x(t-2q)}]^{n-\tau} \tag{20}$$

When τ=1 we get the smallest ranked statistic such as:

$$f_{\tau,n}(x,\rho,\beta,\delta)_{OGRE} = \frac{n!}{(n-1)!}(¥_{\iota,q,t}e^{-\beta x(t-1-2q)} + ¥_{\iota,q,t}e^{-\beta x(t-2q)}) \times [\Pi_{\iota,q,t}e^{-\beta x(t-2q)}]^{n-1} \tag{21}$$

When τ=n we get the greater ranked statistic such as:

$$f_{n,n}(x,a,b)_{OGRE} = \frac{n!}{(n-1)!}(¥_{\iota,q,t}e^{-\beta x(t-1-2q)} + ¥_{\iota,q,t}e^{-\beta x(t-2q)}) \times [\Pi_{\iota,q,t}e^{-\beta x(t-2q)}]^{n-1} \tag{22}$$

6. Maximum Likelihood Estimation:

To estimate the parameters of our new distribution, we will use the maximum likelihood method, as it is one of the best methods for estimating the parameters, through the partial derivation of the maximum likelihood function. First, we will take a random sample [23]. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from (OGRE) distribution.

$$L = L(\rho,\beta,\delta|x_i) = \Pi_{i=1}^{n} g(x,a,b)_{OGRE}$$
$$L = L(\rho,\beta,\delta|x_i)$$
$$= \Pi_{i=1}^{n}\Big(2\delta\rho\beta e^{\beta x}[1-e^{-\beta x}]^3(1+e^{-\beta x})e^{-\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2}$$
$$\times\left[1-e^{-\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2}\right]^{\delta-1}\Big) \tag{23}$$

$$lnL = nln2+nln\delta+nln\rho+nln\beta+\sum_{i=o}^{n}\beta x_i+\sum_{i=o}^{n}ln(1+e^{-\beta x_i})$$

$$+3\sum_{i=o}^{n}ln(1+e^{-\beta x_i})-\sum_{i=o}^{n}\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2$$
$$+(\delta+1)\sum_{i=o}^{n}ln\big(1-e^{\beta x_i}+e^{-\beta x_i}-2)^2\big)$$
$$\frac{\partial lnL}{\partial\delta}=\frac{n}{\delta}+\sum_{i=o}^{n}ln\big(1-e^{\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2}\big)$$
$$\frac{\partial lnL}{\partial\rho}=\frac{n}{\rho}-\sum_{i=o}^{n}\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2+$$
$$\sum_{i=o}^{n}\frac{(\delta-1)(e^{\beta x_i}+e^{-\beta x_i}-2)^2(e^{\beta x_i}+e^{-\beta x_i}-2)^2}{1-e^{-\rho(\beta x_i+e^{-\beta x_i}-2)^2}}$$

$$\frac{\partial lnL}{\partial\beta}=\frac{n}{\beta}\sum_{i=o}^{n}chi_i+\sum_{i=o}^{n}\frac{3e^{-\beta x}x_i}{(1-e^{-\beta x^i})}$$
$$-2\rho\sum_{i=o}^{n}(e^{\beta x_i}+e^{-\beta x_i}-2)^2(e^{\beta x_i}x_i-e^{-\beta x_i}x_i)$$

$$+\sum_{i=o}^{n}\left[\frac{(\delta-1)(e^{\beta x_i}+e^{-\beta x_i}-2)^2 2\rho(e^{\beta x_i}+e^{-\beta x_i}-2)^2}{1-}\right.$$
$$\left.\times\frac{(e^{\beta x_i}x_i-e^{-\beta x_i}x_i)}{e^{-\rho(\beta x_i+e^{-\beta x_i}-2)^2}}\right]$$

By equating the derivatives to zero, we get:

$$\frac{\partial lnL}{\partial\delta}=\frac{\partial lnL}{\partial\rho}=\frac{\partial lnL}{\partial\beta}=0$$

It becomes clear to us after equating them to zero that it is difficult to find solutions to these equations manually, so we will resort to numerical methods to find approximate solutions. This is done through computer programs in order to find the maximum potential capabilities, such as the R program.

7. Simulation:

This section utilizes a Monte Carlo experiment to examine the asymptotic behavior of MLEs for parameters of the OGRE distribution. The study investigates four different sets of parameter value: (δ = 0.7, ρ = 0.25, β = 0.015), (δ = 0.6, ρ =0.35, β = 0.025), (δ = 0.4, ρ = 0.4, β = 0.03), (δ = 0.8,ρ =0.5, β =0.035).

Four sample sizes are considered n=100, 200, 300, and 400, and which displays the mean estimates, root mean squared errors (RMSEs), and average bias (Bias) of the MLEs. As anticipated, the MLEs converge to the correct parameters, and the RMSEs decrease as the sample size (n) increases.

Table 2 shows the mean estimates, root mean square error values, and average bias.

8. Application:

In this part, the suggested distribution is tested against actual data to validate its superiority and applicability over the following distribution: The following probability distributions are considered:

1. Truncated Inverse Weibull Exponential (TIWE) [24].
2. Odd Burr XII Exponential (OBXIIE) (New).
3. Beta Exponential (BeE) [14].
4. Kumaraswamy Exponential (KuE) (New).
5. Exponential Generalized Exponential (EGE) (NEW).
6. Weibull Exponential (WeE) [25].
7. Gompertz Exponential (GoE) (New).
8. Burr type X (Bx) [26].

We take into account, any selection factors to compare the OGRE with the other models mentioned earlier. Finding the best model to fit the Data I and Data II. Specifically, we use the following criteria: criterion, log-likelihood, AIC (Akaike Information Criterion), AIC (corrected Akaike Information Criterion), BIC (Bayesian Information Criterion), HQIC (Hannan-Quinn Information Criterion), A (Anderson Darling), W (Cramer-Von-Messes), and KS (Kolmogorov-Simirnov).

The Data I: the failure stresses of 20 mm-long single carbon fibers are demonstrated by 69 observations in Data I, according to [27], the data are:

Table 2. Monte Carlo Simulation Results for the OGRE distribution.

		(δ = 0.7 , ρ = 0.25, β = 0.015)			(δ = 0.6, ρ = 0.35, β = 0.025)		
Para.	n	Mean	RMSE	Bias	Mean	RMSE	Bias
	100	0.88463	0.45297	0.18463	0.88215	1.28222	0.28215
	200	0.76366	0.36032	0.06366	0.78460	1.06829	0.18460
δ	300	0.74371	0.32477	0.04371	0.78330	1.00768	0.18330
	400	0.72369	0.15961	0.02369	0.77016	0.69302	0.17016
	100	0.26728	0.02733	0.01728	0.37651	0.04265	0.02651
	200	0.261517	0.01852	70.011517	0.36674	70.02555	0.01474
ρ	300	0.26072	0.01630	0.01072	0.36525	0.02352	0.01525
	400	0.25707	0.01383	0.00707	0.36090	0.01817	0.01090
	100	0.01451	0.01134	0.00489	0.02445	0.00397	0.00463
	200	0.01438	0.00852	0.00417	0.02449	0.00349	0.00360
β	300	0.01427	0.00668	0.00296	0.02444	0.00257	0.00355
	400	0.01391	0.00496	0.00286	0.02431	0.00213	0.00284
		(δ = 0.4 , ρ = 0.4, β = 0.03)			(δ = 0.8, ρ = 0.5, β = 0.35)		
Para.	n	Mean	RMSE	Bias	Mean	RMSE	Bias
	100	1.12151	2.08817	0.72151	1.35420	1.69714	0.55420
	200	0.74026	1.46110	0.34026	1.28380	1.69015	0.48380
δ	300	0.72455	1.43767	0.32452	0.95581	1.03346	0.15581
	400	0.57576	0.90659	0.17452	0.91706	0.58053	0.11706
	100	0.42976	0.04806	0.02976	0.53462	0.05933	0.03462
	200	0.41839	0.03187	0.01839	0.52267	0.04041	0.02267
ρ	300	0.41730	0.02903	0.01730	0.52097	0.03650	0.02097
	400	0.41181	0.02324	0.01181	0.51688	0.02961	0.01688
	100	0.02956	0.00614	0.00930	0.03468	0.00599	0.00434
	200	0.02933	0.00505	0.00861	0.03456	0.00542	0.00319
β	300	0.02928	0.00490	0.00371	0.03448	0.00396	0.00101
	400	0.02915	0.00341	0.00248	0.03439	0.00291	0.00060

(1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585) The results in Table 3 demonstrate the OGRE model superiority; the suggested and extended model provides a true picture due to its low scores on the statistical and informative measures. The model scores include –LL, AIC, CAIC, BIC, HQIC, W, A, and KS.

Figure 3 shows the histogram and curves of the new distribution with other distributions. of the Data I.

Figure 4 shows the experimental function of the new distribution and other distributions.

The Data II: According to [28], the Data II contains 65 observations that demonstrate the failure stresses (in GPa) of 50 mm-long single carbon fibers. The data are:

(1.339, 1.549, 1.574 ,1.589, 1.613, 1.746, 1.753, 1.764, 1.807, 1.812, 1.840, 1.852, 1.852, 1.862, 1.864, 1.952, 1.974, 2.051, 2.058, 2.088, 2.125, 2.171, 2.172, 2.211, 2.270, 2.272, 2.299, 2.308, 2.356, 2.390, 2.410, 2.430, 2.431, 2.458, 2.497, 2.558, 2.577, 2.593, 2.601, 2.620, 2.633, 2.670, 2.682, 2.705, 2.735, 2.785, 3.020, 3.042, 3.116, 3.174). The results in Table 4 demonstrate the OGRE model superiority; the suggested and extended model provides a true picture due to its low scores on the statistical and informative measures. The model scores include –LL, AIC, CAIC, BIC, HQIC, W, A, and KS.

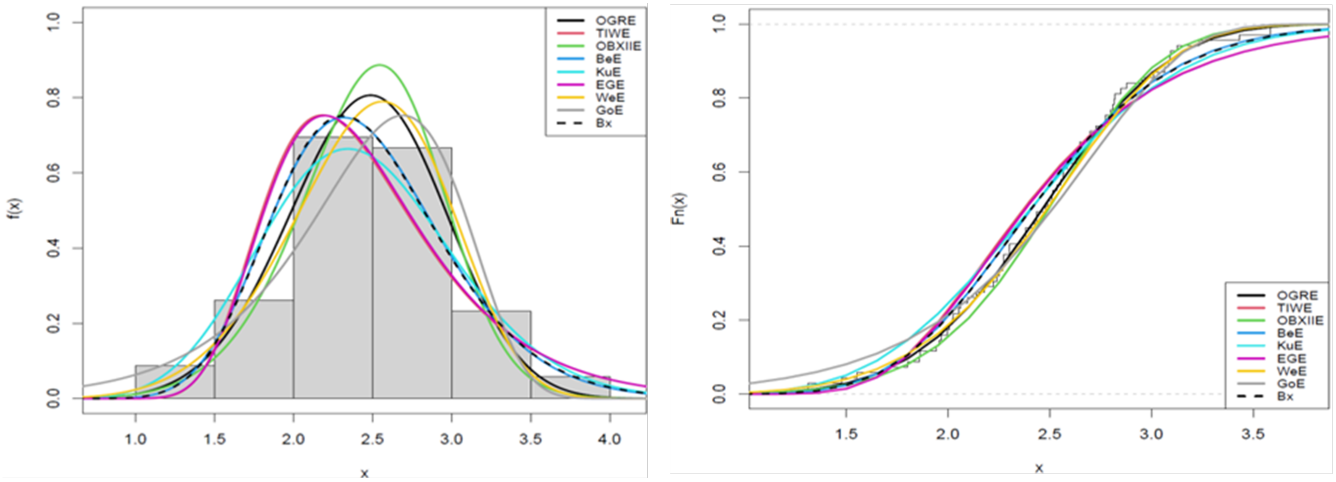

Figure 3. Estimated PDF, and CDF for Data I.

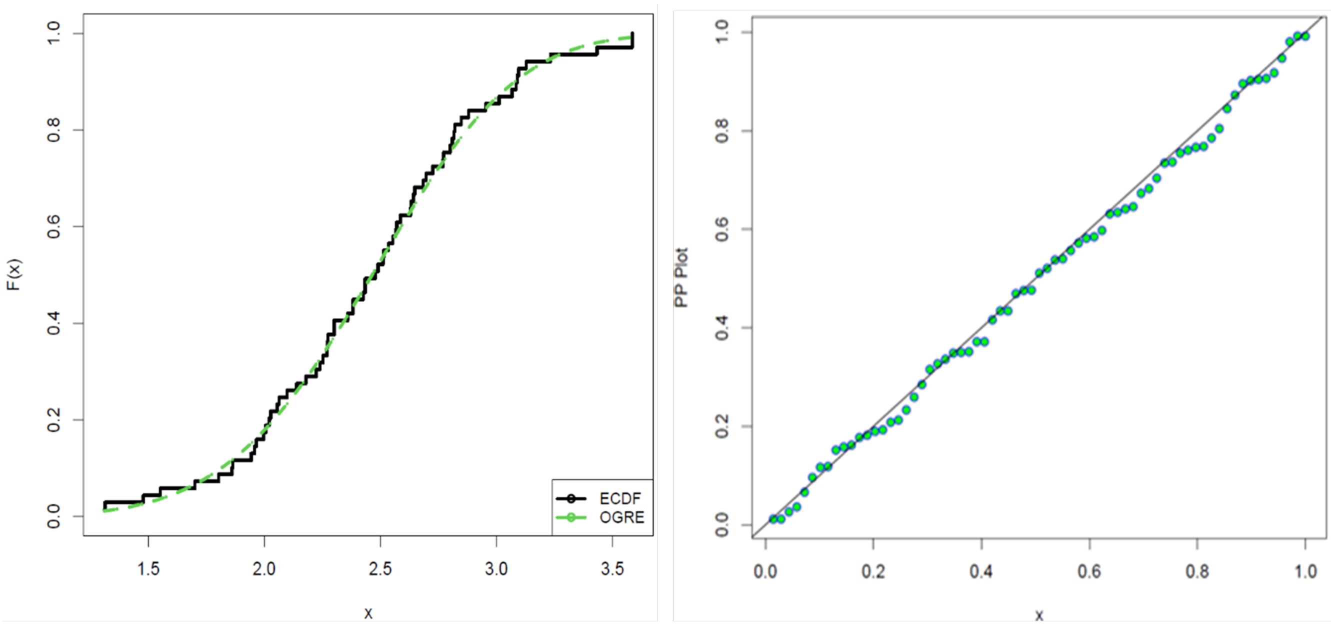

Figure 4. CDF plot for the OGRE distribution.

Table 3. Data I goodness-of-fit, and information criteria.

Model	MLEs	-LL	AIC	CAIC	BIC	HQIC	W	A	KS
OGRE	$\hat{\delta}$:1.0890 $\hat{\rho}$:1.8071 $\hat{\beta}$:0.2322	48.90	103.81	104.18	110.51	106.47	0.0174	0.1567	0.0431
TIWE	$\hat{\delta}$:1.7700 $\hat{\rho}$:4.9150 $\hat{\beta}$:2.0141	54.99	115.98	116.35	122.68	118.64	0.1606	1.0953	0.1022
OBXIIE	$\hat{\delta}$:3.2340 $\hat{\rho}$:5.6141 $\hat{\beta}$:0.2867	49.55	105.11	105.48	111.81	107.77	0.0216	0.1820	0.0672
BeE	$\hat{\delta}$:2.5542 $\hat{\rho}$:3.6248 $\hat{\beta}$:0.5069	50.65	107.31	107.68	114.01	109.97	0.0521	0.3816	0.0726
KuE	$\hat{\delta}$:1.0364 $\hat{\rho}$:7.2220 $\hat{\beta}$:0.6801	51.86	109.73	110.10	116.43	112.39	0.0288	0.2260	0.0973
EGE	$\hat{\delta}$:1.4306 $\hat{\rho}$:7.4412 $\hat{\beta}$:1.4213	54.62	115.24	115.61	121.94	117.89	0.1504	1.0268	0.0949
WeE	$\hat{\delta}$:5.6041 $\hat{\rho}$:0.1485 $\hat{\beta}$:0.0559	49.61	105.23	105.60	111.93	107.89	0.0339	0.2724	0.0568
GoE	$\hat{\delta}$:0.0171 $\hat{\rho}$:4.1384 $\hat{\beta}$:0.4924	53.62	113.25	113.61	119.95	115.90	0.1177	0.8259	0.0845
Bx	$\hat{\delta}$:0.6505 $\hat{\rho}$:7.7171	50.77	105.54	105.72	110.01	107.32	0.0548	0.3965	0.0723

Figure 5 shows the histogram and curves of the new distribution with other distributions. of the Data II.

Figure 6 shows the experimental function of the new distribution and other distributions

9. Conclusions:

In this research, a new distribution is presented, which is an extension of the odd generalized Rayleigh family, which can model and analyze data well. Many of its statistical properties have been studied, such as the survival function, the risk function, the quantile function, the moments, the moment-generating function, and the ordered statistics. The parameters have also been estimated by the maximum likelihood method, and applied Monte Carlo simulation to examine the asymptotic behavior of MLEs for parameters of the OGRE distribution, and application of the distribution to two sets of real data have been studied and compared with several distributions to clarify and prove its flexibility in modeling data. We have noticed that it gives better results than other distributions.

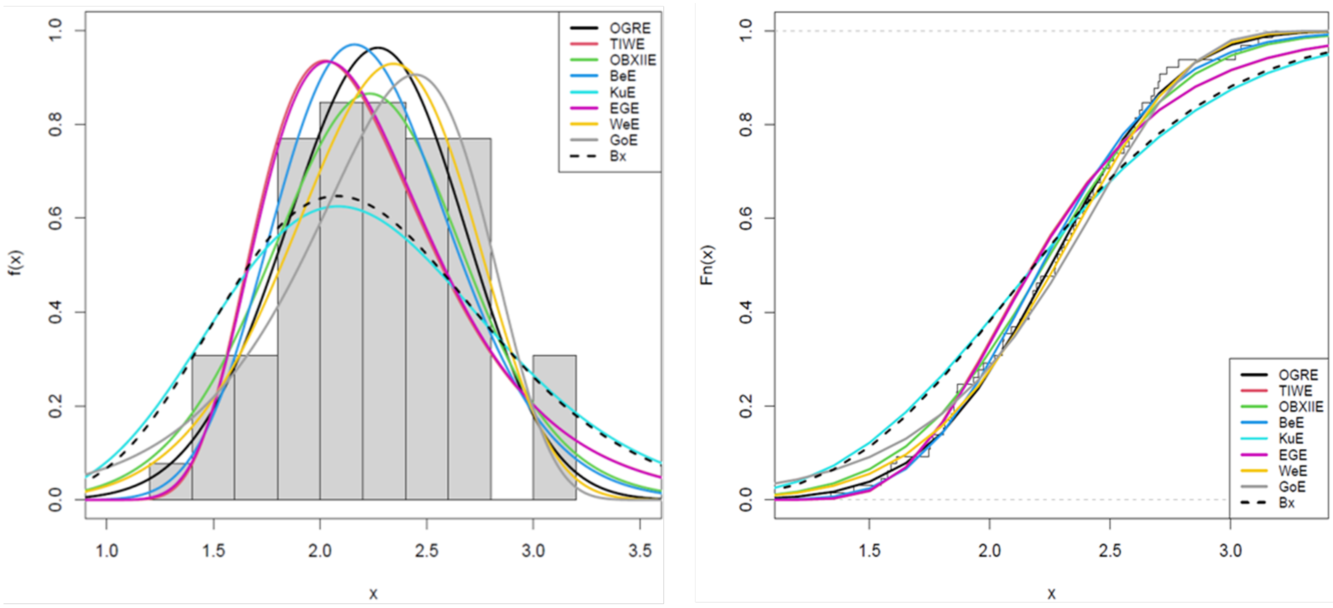

Figure 5. Estimated PDF, and CDF for Data II.

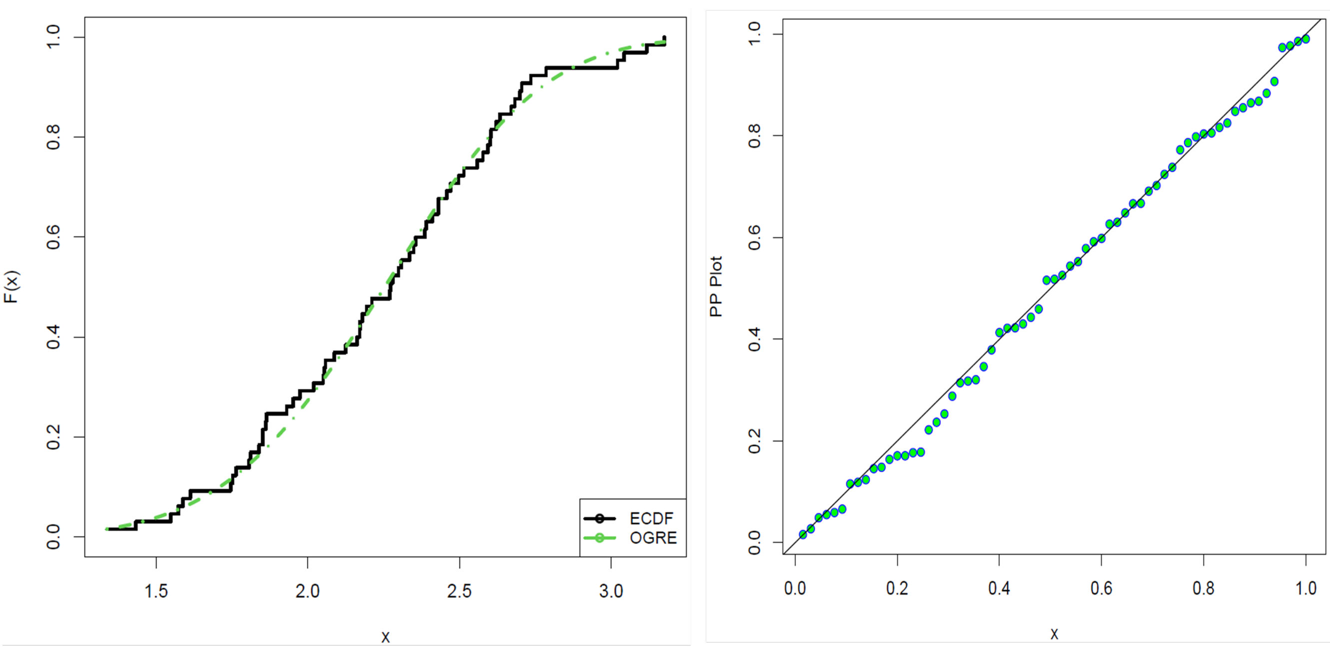

Figure 6. empirical CDF and PP Plot for Data II.

Table 4. Data I goodness-of-fit, and information criteria.

Model	MLEs	-LL	AIC	CAIC	BIC	HQIC	W	A	KS
OGRE	$\hat{\delta}$:4.6832 $\hat{\rho}$:2.0966 $\hat{\beta}$:0.3135	34.64	75.294	75.688	81.818	77.868	0.0234	0.1993	0.0680
TIWE	$\hat{\delta}$:6.1889 $\hat{\rho}$:2.5374 $\hat{\beta}$:2.4866	38.721	83.442	83.835	89.965	86.015	0.1539	0.9195	0.1033
OBXIIE	$\hat{\delta}$:3.0065 $\hat{\rho}$:2.5836 $\hat{\beta}$:0.3599	36.18	78.381	78.774	84.904	80.955	0.0319	0.2274	0.0692
BeE	$\hat{\delta}$:3.8757 $\hat{\rho}$:2.1892 $\hat{\beta}$:0.4202	35.15	76.310	76.704	82.834	78.884	0.0565	0.3440	0.0741
KuE	$\hat{\delta}$:3.1468 $\hat{\rho}$:4.6305 $\hat{\beta}$:0.7499	45.081	96.198	96.592	102.72	98.772	0.0475	0.2958	0.1428
EGE	$\hat{\delta}$:1.6828 $\hat{\rho}$:2.9422 $\hat{\beta}$:1.5038	38.363	82.726	83.119	89.249	85.299	0.1435	0.8546	0.0977
WeE	$\hat{\delta}$:6.0129 $\hat{\rho}$:0.4912 $\hat{\beta}$:0.2033	35.45	76.903	77.297	83.426	79.477	0.0273	0.2821	0.0763
GoE	$\hat{\delta}$:0.0112 $\hat{\rho}$:4.5570 $\hat{\beta}$:0.5391	38.91	83.826	84.219	90.349	86.400	0.0786	0.7055	0.0716
Bx	$\hat{\delta}$:0.6214 $\hat{\rho}$:4.0174	43.81	91.757	91.950	96.105	93.473	0.0496	0.3069	0.1354

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Data Availability Statement: All of the data supporting the findings of the presented study are available from corresponding author on request.

Declarations:
Conflict of interest: The authors declare that they have no conflict of interest.

Ethical approval: This research did not include any human subjects or animals, and as such, it was not necessary to obtain ethical approval.

References

[1] D. Kundu and M. Z. Raqab. Generalized rayleigh distribution: different methods of estimations. *Computational statistics data analysis*, 49(1): 187–200, 2005,doi:10.1016/j.csda.2004.05.008.

[2] D. R. Jensen. A generalization of the multivariate rayleigh distribution. *Sankhyā: The Indian Journal of Statistics*, Series A: 193–208, 1970.

[3] I. Barranco-Chamorro, Y. M. Iriarte, J. M. Gómez Astorga, and H. W. Gómez. A generalized rayleigh family of distributions based on the modified slash model. *Symmetry*, 13(7): 1226, 2021, doi:10.3390/sym13071226.

[4] D. Kundu and R. D. Gupta. Generalized exponential distribution: Bayesian estimations. *Computa-*

tional Statistics Data Analysis, 52(4): 1873–1883, 2008, doi:10.1016/j.csda.2007.06.004.

[5] M. Z. Raqab and M. T. Madi. Bayesian inference for the generalized exponential distribution. *Journal of Statistical computation and Simulation*, 75(10): 841–852, 2005, doi:10.1080/00949650412331299166.

[6] G. C. Rodrigues, F. Louzada, and P. L. Ramos. Poisson–exponential distribution: different methods of estimation. *Journal of applied statistics*, 45(1): 128–144, 2018, doi:10.1080/02664763.2016.1268571.

[7] M. M. Ristić and D. Kundu. Marshall-olkin generalized exponential distribution. *Metron*, 73: 317–333, 2015, doi:10.1007/s40300-014-0056-x.

[8] P. E. Oguntunde and A. O. Adejumo. The transmuted inverse exponential distribution. *International journal of advanced statistics and probability*, 3(1): 1–7, 2021, doi:10.14419/ijasp.v3i1.3684.

[9] A. W. Marshall and I. Olkin. A new method for adding a parameter to a family of distributions with application to the exponential and weibull families. *Biometrika*, 84(3): 641–652, 1997, doi:10.1093/biomet/84.3.641.

[10] L. G. B. Pinho, G. M. Cordeiro, and J. S. Nobre. The harris extended exponential distribution. *Communications in Statistics-Theory and methods*, 44(16): 3486–3502, 2015, doi:10.1080/03610926.2013.851221.

[11] M. Rasekhi, M. Alizadeh, E. Altun, G. G. Hamedani, A. Z. Afify, and M. Ahmad. The modified exponential distribution with applications. *Pakistan Journal of Statistics*, 33(5): 17–21, 2023, doi:10.22034/APJCP.2017.18.1.17.

[12] A. Z. Afifya, G. M. Cordeiro, H. M. Yousof, Z. M. Nofal, and A. Alzaatreh. The kumaraswamy transmuted-g family of distributions: properties and applications. *Journal of Data Science*, 14(2): 245–270, 2016, doi:10.6339/JDS.201604$_1$4(2).0004.

[13] M. Alizadeh, A. Z. Afify, M. S. Eliwa, and S. Ali. The odd log-logistic lindley-g family of distributions: properties, bayesian and non-bayesian estimation with applications. computational. *statistics*, 35(1): 281–308, 2020, doi:10.1007/s00180-019-00932-9.

[14] S. Nadarajah and S. Kotz. The beta exponential distribution. *Reliability engineering system safety*, 91(6): 689–697, 2006, doi:10.1016/j.ress.2005.05.008.

[15] D. Kumar, S. Dey, and S. Nadarajah. Extended exponential distribution based on order statistics. *Communications in Statistics-Theory and Methods*, 46(18): 9166–9184, 2017, doi:10.1080/03610926.2016.1205625.

[16] M. M. Ristić and N. Balakrishnan. The gamma-exponentiated exponential distribution. *Journal of statistical computation and simulation*, 82(8): 1191–1206, 2012, doi:10.1080/00949655.2011.574633.

[17] B. K. Sah and S. K. Sahani. Polynomial-exponential distribution. *Mathematical Statistician and Engineering Applications*, 71(4): 2474–2486, 2022, doi:10.17762/msea.v71i4.807.

[18] D. Bhati, M. A. Malik, and H. J. Vaman. Lindley–exponential distribution: properties and applications. *Metron*, 73: 335–357, 2015, doi:10.1007/s40300-015-0060-9.

[19] A. Khalaf. A the new strange generalized rayleigh family: Characteristics and applications to covid-19 data. *Iraqi Journal For Computer Science and Mathematics*, 5(3): 92–107, 2024, doi:10.52866/ijcsm.2024.05.03.005.

[20] G. Feng. Time course and hazard function: A distributional analysis of fixation duration in reading. *Journal of eye movement research*, 3(2), 2010, doi:10.16910/jemr.3.2.3.

[21] R. J. Hyndman and Y. Fan. Sample quantiles in statistical packages. *The American Statistician*, 50(4): 361–365, 1996, doi:10.2307/2684934.

[22] S. S. Gupta. Order statistics from the gamma distribution. *Technometrics*, 2(2): 243–262, 1960.

[23] O. Barndorff-Nielsen. On a formula for the distribution of the maximum likelihood estimator. *Biometrika*, 70(2): 343–365, 1983, doi:10.1093/biomet/70.2.343.

[24] A. F. Khubbaz and M. A. Khaleel. Properties of truncated inverse weibull exponential distribution with application to lifetime data. *In AIP Conference Proceedings*, 2834(1), 2022, doi:10.1063/5.0161510.

[25] M. Bilal, M. Mohsin, and M. Aslam. Weibull-exponential distribution and its application in monitoring industrial process. *Mathematical Problems in Engineering*, 2021(1): 6650237, 2021, doi:10.1155/2021/6650237.

[26] I. W. Burr. Cumulative frequency functions. *The Annals of mathematical statistics*, 13(2): 215–232, 1942, doi:10.1214/aoms/1177731607.

[27] A. A. Khalaf, N. Noori, and M. Khaleel. A new expansion of the inverse weibull distribution: Properties with applications. *Iraqi Statisticians journal*, 52-62, 2024, doi:10.62933/sdkgcy19.

[28] M. E. Ghitany, D. K. Al-Mutairi, and S. M. Aboukhamseen. Estimation of the reliability of a stress-strength system from power lindley distributions. *Communications in Statistics-Simulation and Computation*, 44(1): 118–136, 2015, doi:10.1080/03610918.2013.767910.

الخصائص الإحصائية للتوزيع الأسي المعمم الفردي لرايلي مع تطبيق بيانات حقيقية

علي محمود منيف *

وزارة التربية، مديرية تربية كركوك، كركوك، العراق.

* الباحث المسؤول: 909alimahmood909@gmail.com

الخلاصة

في هذ الورقة نقدم توزيعًا جديدًا نظرًا لأن المشكلة الأكثر شيوعًا للباحثين هي تحديد نموذج إحصائي لتحليل بيانات العمر الافتراضي. هناك العديد من الطرق لإضافة معلمة مقياس أو معلمة شكل من خلال تعميم التوزيعات. لذلك يتم تقديم نموذج إحصائي يسمى التوزيع الاسي المعمم الفردي لرايلي (*OGRE*). تتم دراسة بعض الخصائص المختلفة لهذا التوزيع بما في ذلك العزوم ودالة توليد العزوم ودالة الخطر ودالة البقاء والإحصاءات المرتبة والدالة الكمية. يتم استخدام دالة الامكان الاعظم لتقدير المعلمات. بالإضافة إلى ذلك، يتم إثبات الأهمية العملية لـ (*OGRE*) من خلال مجموعتين من البيانات الحقيقية ويتم مقارنة النتائج بمجموعة من التوزيعات الأخرى ذات الصلة. يتم أيضًا إجراء المحاكاة.

الكلمات الدالة :دالة البقاء، دالة الخطر، الدالة الكمية، الخصائص الاحصائية، مقدرات الامكان الاعظم.

التمويل: لايوجد.

بيان توفر البيانات: جميع البيانات الداعمة لنتائج الدراسة المقدمة يمكن طلبها من المؤلف المسؤول.

اقرارات:

تضارب المصالح: يقر المؤلفون أنه ليس لديهم تضارب في المصالح.

الموافقة الأخلاقية: لم يتضمن هذا البحث أي تجارب على البشر أو الحيوانات، بالتالي لم يكن من الضروري الحصول على موافقة أخلاقية.